Exemplifying the specific properties of the inverse scattering series internal-multiple attenuation method that reside behind its capability for complex onshore and marine multiples

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The world of petroleum exploration constantly demands higher efficacy at every link in the data processing chain, from preconditioning to imaging and inversion. Within that chain, the removal of internal multiples constitutes a particularly resilient problem, whose resolution has only partially benefited from the advent of the data-driven technologies which have transformed the practice of free-surface multiple elimination.

Historically, internal multiples have received less attention than free-surface multiples. In offshore surveys, for example, their relative importance is often outweighed by dominant water-column multiples. However, as the demand for accuracy increases driven by improvements in imaging capability and removal of free-surface multiples, the interest in removing troublesome internal multiples rises in priority. Internal multiples can cause uncertainty in the interpretation process and can obscure both onshore and offshore exploration targets.

In all scenarios, the key to addressing the internal multiple problem consists of responding to a combination of several challenges, and exemplified in Weglein et al. in this special section of *TLE*. We seek a method that can accommodate an Earth with strong lateral variations and the wavefield phenomena it creates (such as multipathing, diffracted internal multiples). That capability is likely to be critical in offshore areas with a highly rugose (diffractive) water bottom and for internal multiples generated within salt bodies.

Other desirable characteristics are the ability to (1) operate independently of a priori information and (2) accommodate the broadest set of multiples without the user being required to identify the portion of the Earth responsible for the multiple's subevents. In a variety of situations, internal multiples are generated within alternating sequences of rocks and sediments with contrasting seismic properties. In certain geologic settings, those sequences can exist for several hundred or thousand meters and choosing one or more significant multiple generators represents a challenge by itself, which can only be confidently addressed using information from nearby well logs.

A further interest is in producing a simultaneous prediction of all internal multiples with equal accuracy at all offsets, because an accurate match between predicted and actual multiples (amplitude, phase, number of events) alleviates the burden on adaptive subtraction.

Currently available methods for internal multiple attenuation/removal can be divided into two groups. The first group of methods requires the user to identify the primaries as internal multiple subevents or the portion of the Earth responsible for the internal multiple's downward reflection. Typically, the interpretation consists in picking the traveltime of the event corresponding to a chosen downward reflector, often referred to as the internal multiple generator. The interpretation can be used directly to isolate the chosen generator from other events corresponding to deeper reflectors (pioneered by Key-

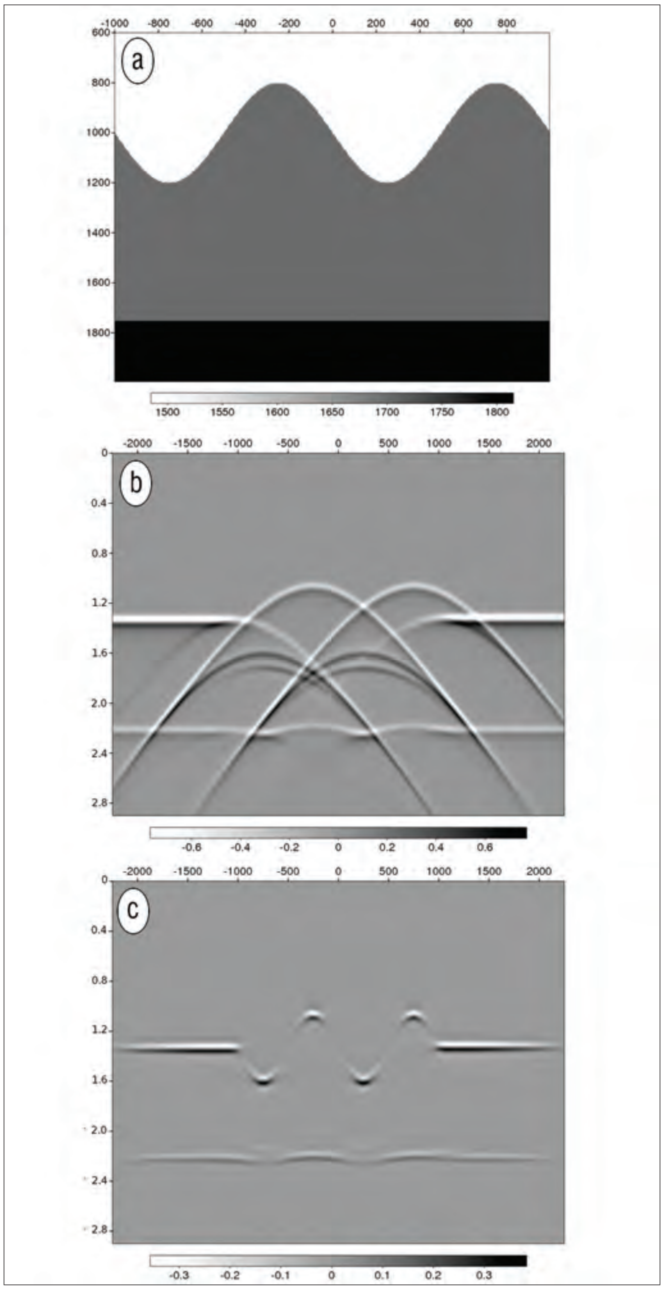

Figure 1. *(a) Velocity model; (b) zero-offset section of the input data; (c) zero-offset section of the water-speed* f-k *migration, first-order term in the ISS internal-multiple algorithm.*

dar et al., 1997, promulgated by Jakubowicz, 1998) or used to downward continue the wavefield (through common-focus-point operators) toward the generator (feedback methods, boundary approach) or toward a chosen reference level, i.e., layer approach (Berkhout and Verschuur, 2005; Verschuur and Berkhout, 2005). The reference level is chosen to separate the regions of the Earth that contain downward reflectors from those that contain upward reflectors in the construction of internal multiples. The second group of internal multiple attenuation/removal methods does not require generator identification and the internal multiples are constructed by combining three events that satisfy an automated constraint. In the method based on the inverse scattering series (ISS), the constraint is a deeper-shallower-deeper relationship in pseudodepth or vertical travel time (Araújo, 1994; Weglein et al., 1997; Weglein et al., 2003, Nita and Weglein, 2007). Ten Kroode (2002) proposed an asymptotic derivation of the results in Weglein et al. (1997), where the constraint is a longer-shorter-longer relationship between total traveltimes under the assumption of traveltime monotonicity (deeper events yield longer traveltime). The automated constraint enables the algorithms in the second group to predict internal multiples for all possible generators in one step and can be considered truly independent of subsurface information.

Through a set of examples, the analysis in this paper provides insights into the inner workings of the ISS algorithm, and an explanation for the success recently reported in a field data application on data from Saudi Arabia (Fu et al., 2010; and Luo et al. in this special section). The first example illustrates the case of internal multiples generated at a highly curved interface and demonstrates the advantage of the ISS internal multiple algorithm using vertical traveltime in contrast to total traveltime based algorithms. In the second numerical example, we illustrate the ability of the ISS internal multiple-attenuation algorithm to address all existing internal multiples generated by all downward reflectors in a single step. That ability is in contrast to the methods of the first group above, which are unable to match that removal efficacy.

Internal multiple attenuation using the inverse scattering series

The removal of internal multiples can be regarded as a particular task within the general inversion machinery of the ISS (Weglein et al., 2003). Within that framework, it is possible to identify a subset of ISS terms to suppress internal multiples starting from an input wavefield with all free-surface effects (source- and receiver-side ghosts and free-surface multiples) removed (Araújo et al., 1994; Weglein et al., 1997). ISS and all tasks within ISS (e.g., free-surface and the internal multiple-attenuation algorithms) are entirely data-driven tools which do not require information about the medium through which the multiples propagate, nor do they require moveout discrimination between primaries and multiples, nor interpretive intervention. The ISS internal multiple-attenuation algorithm predicts internal multiples for all horizons at once without needing or using information about the reflectors involved in generating them. Its leading-order term predicts the correct traveltimes and approximate amplitudes of all the internal multiples in the data. Ramírez and Weglein (2005) extended the theory from attenuation toward elimination by including more terms in the elimination subseries, thereby improving the amplitude prediction. Although the ISS free surface and internal-multiple algorithms were initially designed for a marine towed streamer experiment (had an acoustic reference medium of water), Coates and Weglein (1996) showed that all free surface and internal multiples with converted waves in their history were also predicted. The latter free-surface and internal multiple cases are using a reference medium for which S-waves don't even exist. This is not model matching, indirect inversion or modeling run backward! Matson (1997) extended ISS multiple removal to ocean-bottom and land data. Matson et al. (1999) and Weglein et al. (2003) were the first to apply the ISS free-surface and internal multiple algorithms to marine towed-streamer field data, while Fu et al. (2010) contains the first ISS internal multiple application on land data.

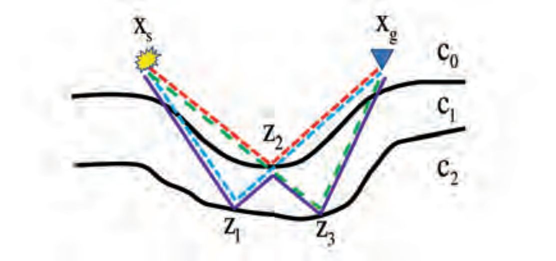

Figure 2. *An internal multiple (solid blue) satisfying monotonicity in vertical time but not in total traveltime. If wave speed* c_1 *is much greater than* c_0*, the (dashed blue) and (dashed green) primaries arrive at the surface earlier than the (dashed red) primary. The multiple is removed by the ISS method, but not by methods based on total traveltime monotonicity.*

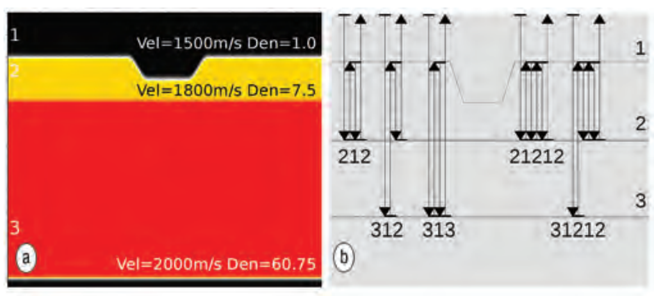

Figure 3. *(a) Earth model and (b) event labeling. Densities are chosen to yield a vertical-incidence reflection coefficient of 0.8 at all layer boundaries.*

Properties of the first-order term in the ISS internal multiple-attenuation algorithm—uncollapsed *f-k* migration

The algorithm starts with source- and receiver-side deghosted data absent of free-surface effects. Using only reference velocity, an uncollapsed migration (Stolt, 1978; Stolt and Weglein, 1985) maps the input data from time to pseudodepth (i.e., depth identified by imaging using a reference velocity). The concept of pseudodepth is similar to that of traveltime at vertical incidence. The pseudodepth is achieved in the frequency domain, where the temporal frequency (ω) observed in the surface recordings can be related to the vertical wavenumber ($k_z = q_g + q_s$)of a constant-velocity image, through the relationship

$$q_i = \mathrm{sgn}(\omega)\sqrt{(\omega / c_0)^2 - k_i^2}$$

for i=(g,s), where, c_0 is the chosen reference velocity k_i the horizontal wavenumber and the subscripts g and s characterize the Fourier domain variables on the receiver and source side, respectively (Clayton and Stolt, 1981).

Within constant velocity migration assumptions, the *f-k* Stolt migration correctly images the reflected wavefield generated by interfaces of any arbitrary shape, including diffractions and multipathing. One example of such phenomena is the bow-tie pattern generated by reflections over a sufficiently curved boundary. These effects are common in seismic exploration data and can occur in a variety of geologic features, including salt domes, faults, layer terminations, pinch-outs, fractured and/or irregular volcanic layers and for a rough sea bottom. As we mentioned, several internal multiple-removal algorithms require picking events and traveltimes. In some of those methods (Keydar et al., 1997), the picked traveltimes are directly used to mute the wavefield at earlier or later times with respect to the generator, and internal multiples are predicted using auto- and cross-correlation operations between traces from the resulting fields. In others (e.g., the feedback methods), the traveltimes are used to determine approximate redatuming operators. However, all these approaches are based on the implicit assumption that a one-to-one relationship exists between seismic events (their traveltime) and the Earth features that create them (such as layer boundaries). In the presence of diffractions and/or multipathing, a one-to-one relationship does not exist, as, e.g., a single curved interface can produce several seismic arrivals in a single seismic trace. Picking events, traveltimes, and generators may not be viable even in a normal-incidence experiment in a 1D Earth, since destructively interfering primary and multiple events are possible and often prevalent in land field data (Kelamis et al., 2006; Fu et al., 2010).

$$b_{3IM}(k_g, k_s, \omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk_1 e^{iq_2(z_g - z_s)} \int_{-\infty}^{\infty} dz_1 b_1(k_g, k_1, z_1) e^{i(q_g + q_1)z_1}$$
$$\times \int_{-\infty}^{\infty} dk_2 e^{iq_1(z_g - z_s)} \int_{-\infty}^{z_1 - \varepsilon} dz_2 b_1(k_1, k_2, z_2) e^{-i(q_1 + q_2)z_2} \int_{z_2 + \varepsilon}^{\infty} dz_3 b_1(k_2, k_s, z_3) e^{i(q_2 + q_s)z_3}$$

We present an example based on a simple three-layer Earth model where the shallowest interface is sine-shaped. The model in Figure 1a produces the data in Figure 1b where all seismic events except the second primary at 2.2 s originate at the shallow reflector. Clearly, it is an issue to pick a unique traveltime to represent the curved reflector, as many events are generated which interfere among themselves and even with the second primary. The ISS method provides a natural solution by using as input the uncollapsed prestack water-speed migration (Figure 1c) where the spatial (pseudodepth) relationship between seismic arrivals matches the spatial relationship of the reflectors in the actual Earth (Nita and Weglein, 2007). The sketch in Figure 2 describes another example of an internal multiple which would not be predicted if total traveltimes were the basis of the method. The multiple can be traced back to an Earth feature where the relationship between total traveltimes and vertical traveltimes (pseudodepth) is inverted due to the presence of a high-velocity layer at depth. Vertical travel times have a closer relationship to actual depth than total time, and hence represent a more effective way to approach the removal of actual internal multiples (see, e.g., Hsu, 2011). The latter vertical traveltime is the tool used in ISS internal multiple attenuation algorithms.

Properties of the leading (third) order term ISS internal multiple attenuation algorithm

Let z_1, z_2, and z_3 be the pseudodepths of three generic points in the data produced by the first-order term in the internal multiples series. The leading-order internal multiple prediction is composed of three events that satisfy a deeper-shallower-deeper condition in pseudodepth. As those points span the entire data volume, the leading-order attenuation algorithm (which is third-order in the imaged data) allows any combination such that z_1 is greater than z_2 and z_3 is greater than z_2 to contribute to the prediction (see equation on next page), where $b_1(k_g, k_s, z)$ corresponds to effective incident plane-wave data in the pseudodepth domain.

In contrast with the methods based on the convolution and correlation of wavefields, where the definition of the generator is static, the ISS algorithm's deeper-shallower-deeper constraint does not refer to any particular interface or event in the data. On the contrary, it applies to all of their water-speed images, allowing the simultaneous prediction of all first-order internal multiples from any depth without interpretation and traveltime picking of the data or knowledge of the medium.

In our second example, we demonstrate the properties of the ISS internal multiple-prediction algorithm using a set of acoustic finite-difference data. The model shown in Figure 3a consists of three interfaces, the first of which features a trench approximately 1.5 km long and 100 m deep. In Figure 3b, the travel paths of some internal multiples are drawn schematically using upgoing and downgoing arrows to represent wave propagation. In a zero-offset section of the data (Figure 4a), a first train of closely spaced internal multiples (characterized by the pattern 2[12]n) can be shown to originate from the energy reflected between the two shallow reflectors (1) and (2).

A deeper reflector (3) causes the entire train to begin again at around 1.4 s (3[12]n train) and once more at 2.1 s (313[12]n and 323[12]n trains). In general, even in a simple three-interface Earth model, the number of reverberations

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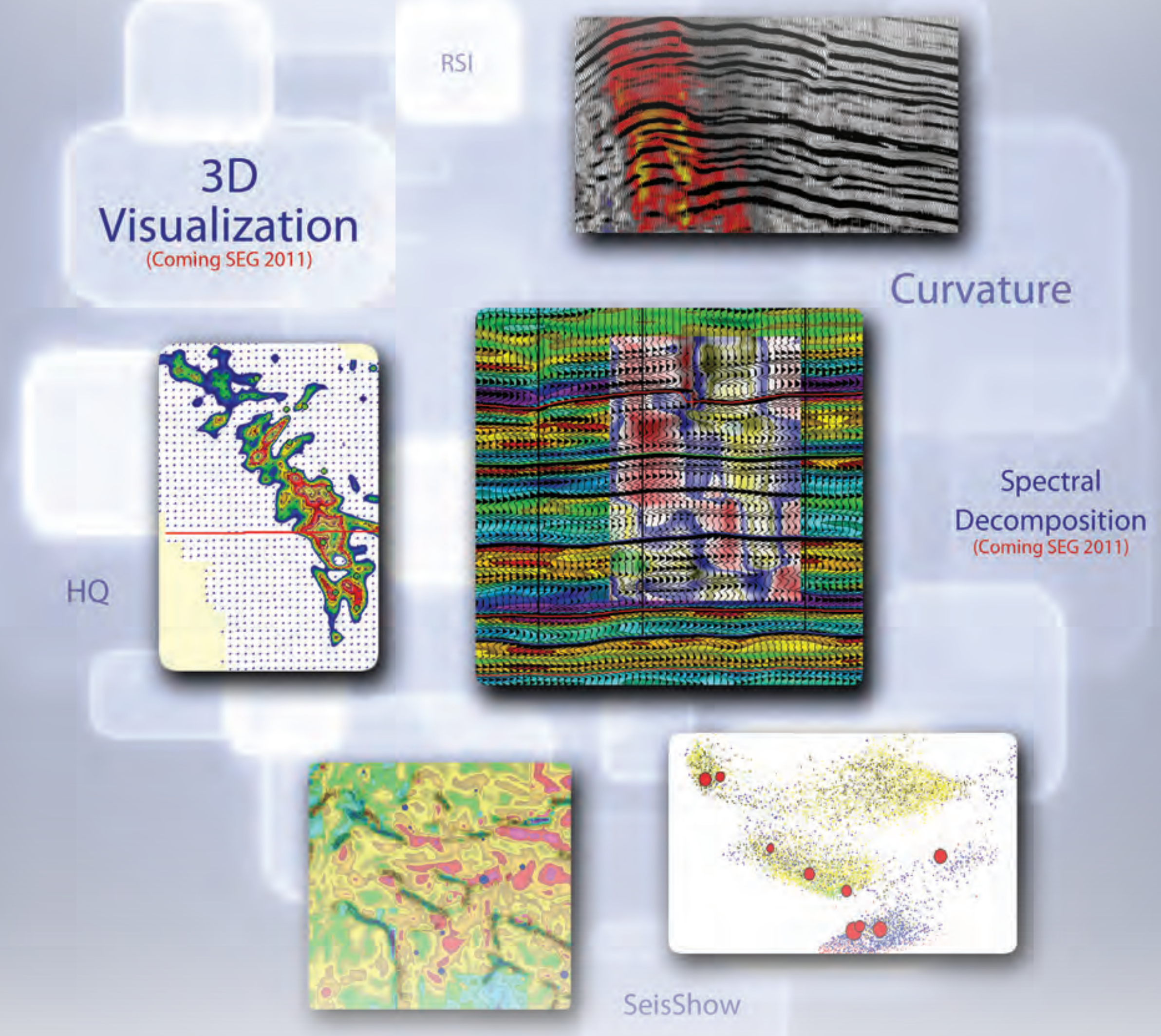

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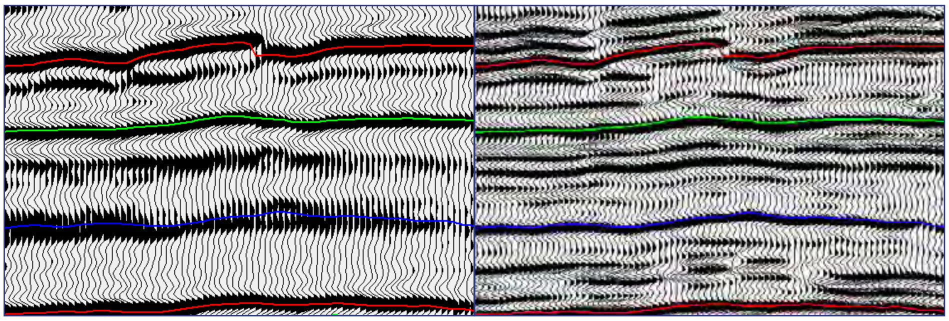

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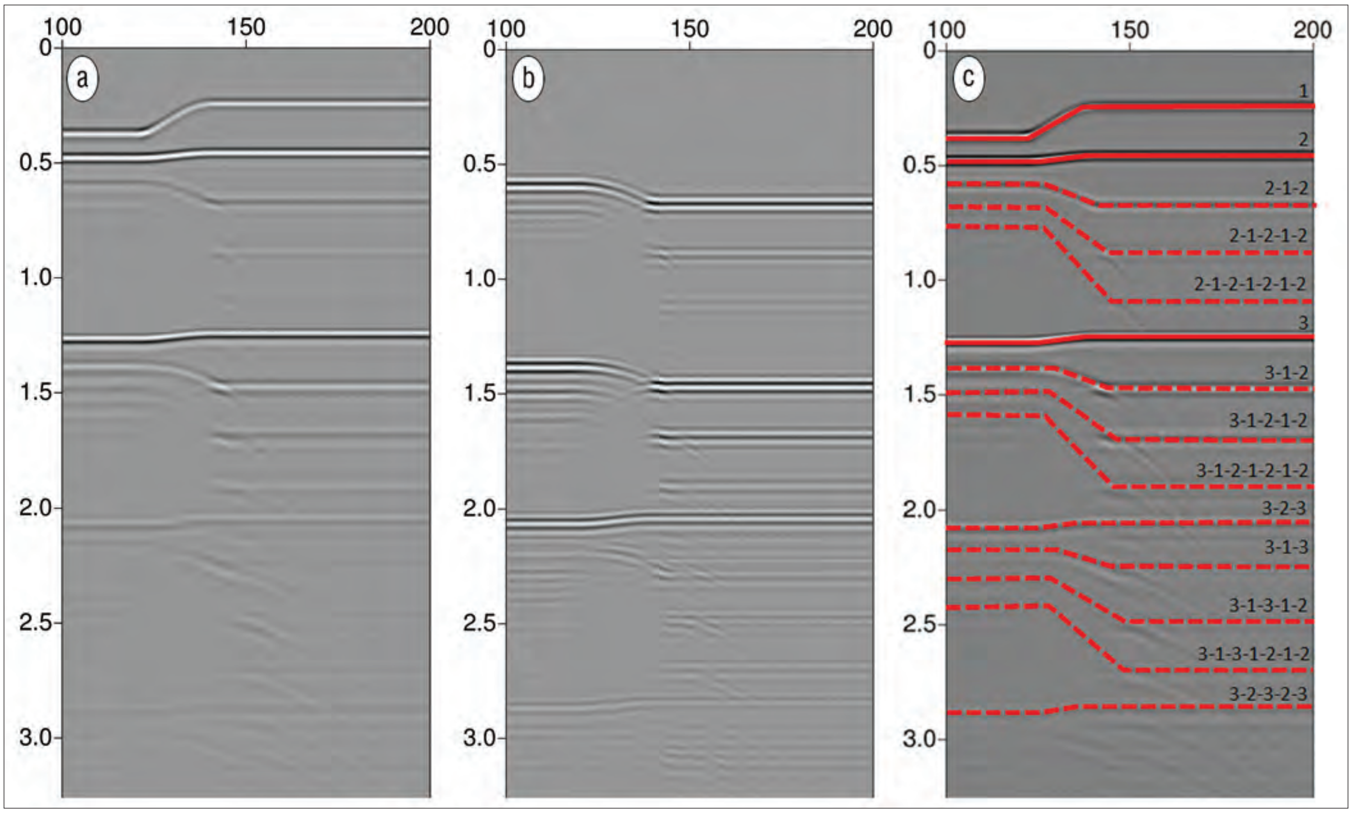

Figure 4. *Zero-offset sections: (a) input data, (b) predicted multiples, and (c) labeling of events.*

recorded at the surface is extremely large as a result of the various ways three reflectors can be combined to form internal multiples. The ISS internal multiple algorithm predicts all of them at once, without any interpretation required on the data, as shown in Figure 4b and Figure 4c.

Discussion

We observe that the layer-related approach (Berkhout and Vershuur, 2005; Vershuur and Berkhout, 2005) would not achieve the same result. Figure 5a shows the four types of first-order internal multiples generated within a three-reflector Earth. If the reference level (the lower boundary of the layer) that separates an internal multiple's upward and downward reflections were chosen between the first and the second reflectors, the layer-related method would predict the three types of first-order internal multiples shown in Figure 5b. If that strategy were applied in the example shown in Figure 4, all multiples characterized by type 3[23]n would be absent in the prediction. Figure 5c shows a different prediction produced by selecting the reference level between the second and third reflector. Similarly, in the example in Figure 4, if the downward-reflecting level were chosen between events (2) and (3), the 2[12]n event type would not be predicted. Notice that once the reference level is chosen, the events above this level can act only as downward reflectors; similarly, the events below this level can contribute only as upward reflectors. In Figure 5a, however, the second reflector contributes both as an upward reflector (for the two internal multiples in the middle) and as a downward reflector (for the rightmost internal multiple). Therefore, for any choice of downward-reflecting layer, there is at least one type of first-order internal multiple which cannot be predicted.

Conclusions

The inverse scattering series provides an approach to internal multiple attenuation with the potential to address the challenges of modern seismic exploration on land and in complex marine settings. That capability has recently been further delineated and demonstrated by complex synthetic and land field data tests (Fu et al.; Luo et al.). Through the analysis and the examples in this paper, we illustrate its inner workings and emphasize the key concepts at the base of its capability to provide (1) a comprehensive and accurate prediction of all internal multiples (2) in a purely data-driven manner, and (3) in an Earth with strong lateral variations. TLE

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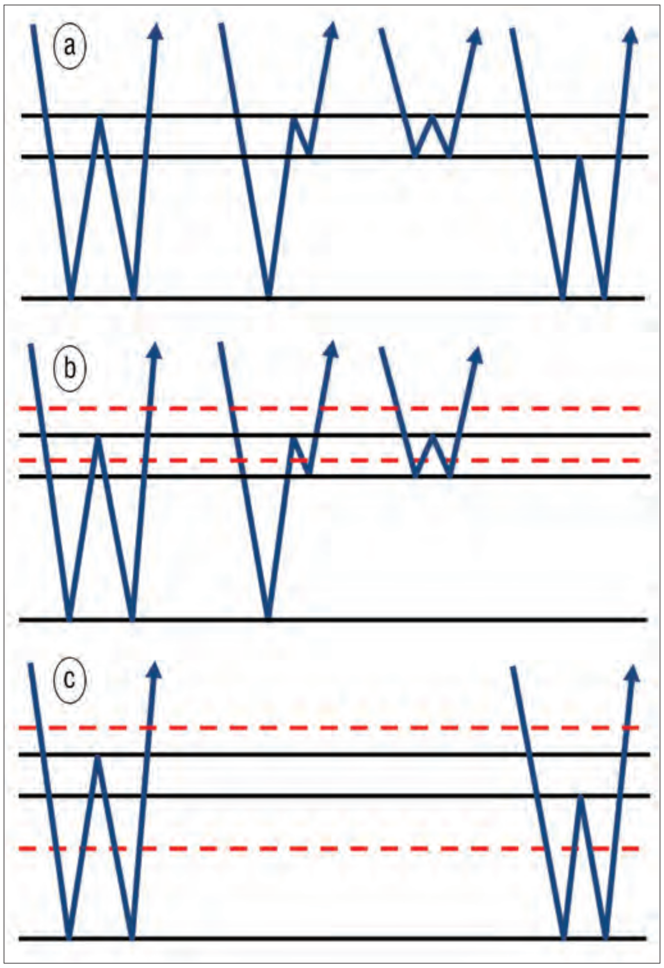

Figure 5. *(a) Four types of first-order internal multiples are generated by three reflectors; (b) and (c) the first-order internal multiples predicted by the feedback layer method using different definitions of the downward generator layer (red dashed lines).*

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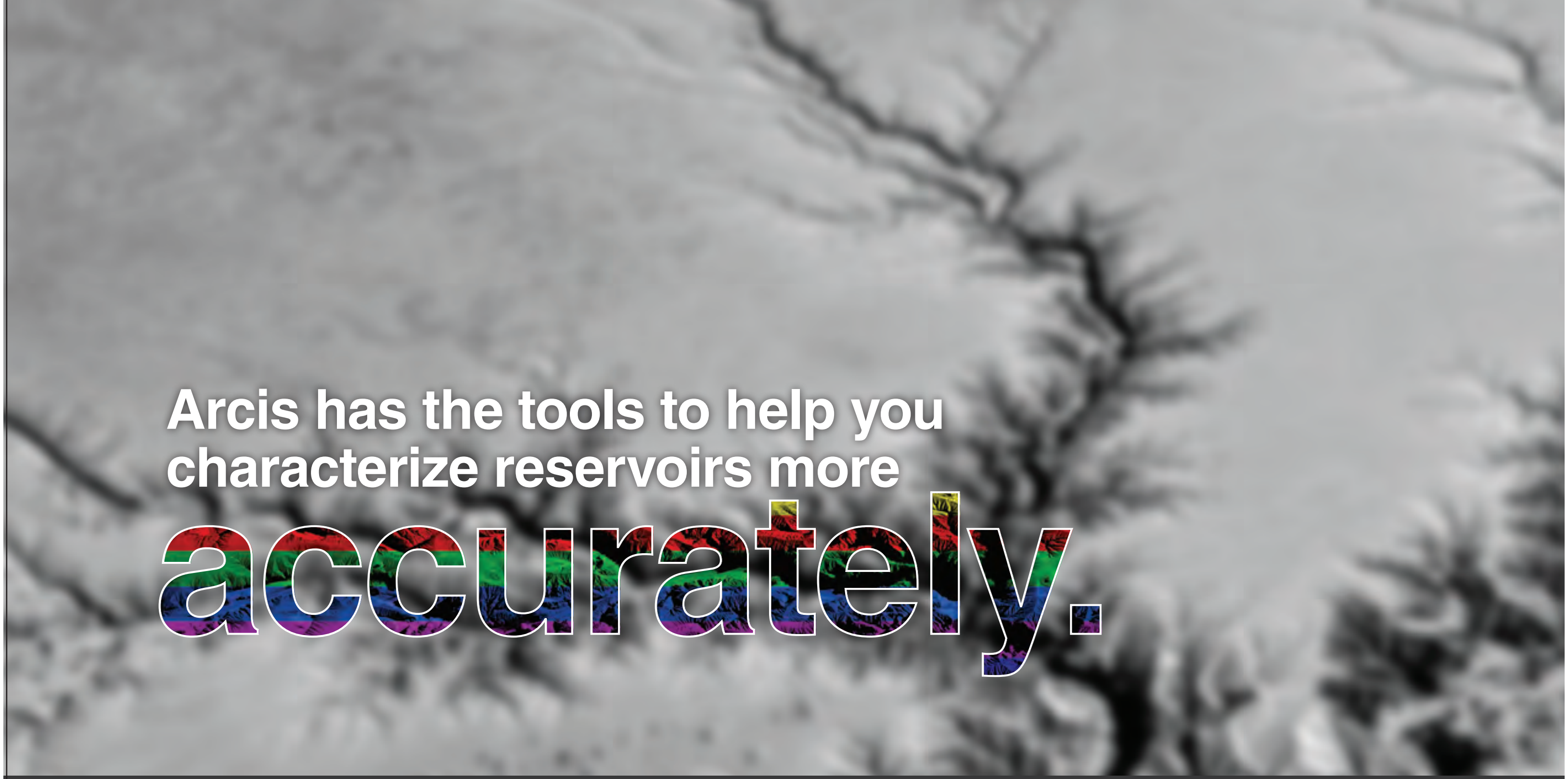

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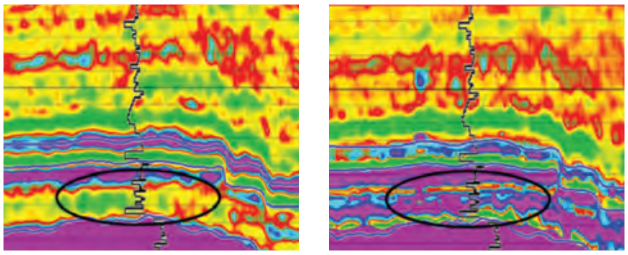
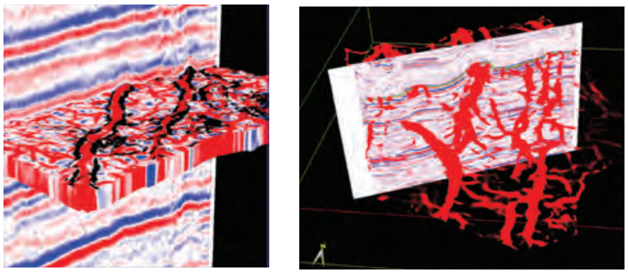
Left: P-impedance obtained from model-based, poststack inversion.
Right: P-impedance obtained from Probabilistic Neural Network Analysis.



Notice the detailed and accurate correlation of the impedance values with the impedance log curves as seen on the neural network estimated impedance.

Left: Strat-cube from the most positive curvature attribute co-rendered with coherence seen here in a 3D chair view.
Right: Strat-slice from k_1 curvature shows fault/fracture skeleton with transparency.



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