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## Wave Theoretic Approaches to Multiple Attenuation: Concepts, Status, Open Issues, and Plans: Part II

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### Abstract

The inverse scattering series represents the only direct multidimensional inversion procedure. The directness of the method (towards a single objective) implies a purposefulness and focus. If the objective is viewed as being achieved through an ordered sequence of steps, we can then imagine that these steps themselves reside in the algorithm. The logic behind the resulting free-surface and internal multiple attenuation algorithms is revisited and an informal comparison with the evolution of the feedback method is presented. The inverse scattering multiple attenuation algorithms are illustrated using field-data examples.

### Introduction

The inverse scattering method for attenuating free-surface and internal multiples (Ref. 1, Ref. 2, Ref. 3) provides a unique set of algorithms for the removal of all free-surface and internal multiples with absolutely no subsurface information, interpretive intervention, iteration, updating, muting, or velocity or event picking. These algorithms derive from identifying terms (and portions of terms) of the multidimensional inverse series for seismic data (Ref. 4) that carry out specific tasks, within the overall inversion process, in a purposeful and direct manner. This concept of associating certain terms (and subseries) with task-separated inverse processes allows great benefit to derive from reaching one (or more) of these goals under circumstances when all of these objectives are not achievable. Further, the fact that each term has a well-defined specific function, within this four distinct task separated inversion framework, allows the prediction of the effect of different portions of the series – independent of

the nature of the target. For example, the individual terms in the free-surface demultiple subseries each eliminate a different specific order of free-surface multiple – completely and totally independent of the nature of the earth. These terms carry out their assigned purpose not only independent of the nature of the earth's structure and lithology, but also independent of whether the earth is acoustic, elastic or anelastic.

A recent set of papers (Ref. 5, Ref. 6) provided synthetic data tests as an empirical comparison of these inverse scattering free-surface and internal multiple methods and the feedback method pioneered by Berkhout (Ref. 7) and developed by Verschuur *et al.* (Ref. 8). References (5) and (6) are comparison papers and mainly consist of numerical and synthetic data examples. One objective of the current paper is to continue this analysis and synthesis.

### Scattering theory

Scattering theory is a form of perturbation theory. It relates the actual impulse response,  $G$ , and the reference impulse response,  $G_0$ , to the difference between the actual and reference media, which is characterized by the operator,  $V$ .  $G_0$  and  $G$  satisfy the differential equations

$$L_0 G_0 = \delta \quad (1)$$

$$L G = \delta \quad (2)$$

where  $L_0$ ,  $L$  are the differential operators describing reference and actual propagation,  $\delta$  represents an impulsive source, and

$$L_0 - L = V \quad (3).$$

The fundamental relationship between  $G$ ,  $G_0$  and  $V$  is

$$G = G_0 + G_0 V G \quad (4).$$

The forward problem starts with  $G_0$  and  $V$  and produces  $G$ ; the inverse problem starts with  $G_0$  and measurements of  $G$  (on a surface outside of  $V$ ) to determine  $V$ .

The forward problem can be represented by a series from equation (4)

$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \dots \quad (5)$$

and the latter can be represented as a feedback process with a series of  $n$  repeated applications of  $(G_0 V)^n$  acting to the left of  $G_0$ .

The scattered field,  $\psi_s$ , is defined as the difference between  $G$  and  $G_0$ . The inverse series constructs  $V$  as a series in orders of the measured data,  $D$ , where  $D = (\psi_s)_m$  and  $(\psi_s)_m$  represents the values of the scattered field  $\psi_s$  on the measurement surface where the sources and receivers reside. The inverse series for  $V$  is

$$V = V_1 + V_2 + \dots \quad (6)$$

where  $V_n$  is the portion of  $V$  that is  $n$ -th order in  $D$ . Substitution of (6) into (5) evaluated on the measurement surface, and matching terms of equal order in the data gives

$$D = (G_0 V_1 G_0)_m \quad (7a)$$

$$0 = (G_0 V_2 G_0)_m + (G_0 V_1 G_0 V_1 G_0)_m \quad (7b)$$

$$0 = (G_0 V_3 G_0)_m + (G_0 V_1 G_0 V_2 G_0)_m + (G_0 V_2 G_0 V_1 G_0)_m \dots \\ + (G_0 V_1 G_0 V_1 G_0 V_1 G_0)_m \quad (7c) \\ \vdots$$

Equation (7a) allows us to solve for  $V_1$  from  $D$  and  $G_0$ ; (7b) allows us to solve for  $V_2$  from  $V_1$  and  $G_0$ ; and (7c) allows us to solve for  $V_3$  in terms of  $V_1$ ,  $V_2$  and  $G_0$ . Hence, the construction of the entire series is given in an explicit step-by-step manner directly in terms of  $D$  and  $G_0$ .

Consider the ordered sequence of tasks within the process of inversion as

- (1) eliminate free-surface multiples
- (2) eliminate internal multiples
- (3) transform primaries in time to the imaged reflectivity in space, and
- (4) invert these imaged primaries to predict the relative changes in earth mechanical properties at the reflector.

If we imagine that inversion consists of these tasks and that the construction of  $V$  is synonymous with inversion, then it follows that the four tasks reside within the construction of  $V$ . Since  $V$  is constructed from only measured data and  $G_0$  through equations (7), it then follows that each task is achievable from operations only involving the measured data and the reference Green's function,  $G_0$ . The specific subseries of equation (6) that attenuate free-surface and internal multiples are described in detail in Ref. 2 and the references contained therein.

## A priori information and the reference medium

The choice of reference medium (and the concomitant need for a priori information) depends on the particular inversion task you are considering, the level of reference information that allows that task-specific subseries to be useful (i.e., convergent or at least asymptotic), and the availability of reliable a priori information at that particular point in the sequence of inversion tasks. For example, prior to carrying out the tasks of multiple attenuation, it is more difficult to achieve a reliable estimate of background velocity than afterwards. In general, the simplest reference medium that satisfies the criteria listed above is the model of choice. For free-surface multiple elimination, the reference medium is a half-space of water bounded by a free-surface at the air-water boundary. The internal multiple subseries uses a whole-space of water as the reference medium. Hence, absolutely no a priori information below the measurement level is required for either the free-surface or internal multiple subseries.

The reference Green's function,  $G = G_0^d + G_0^{FS}$  for the half-space of water bounded by a free surface at the air-water boundary is illustrated in Fig. 1.  $G_0^d$  is the causal whole space Green's function and  $G_0^{FS}$  is the extra term in  $G_0$  due to the presence of the free surface.  $G_0^{FS}$  can be interpreted as the response of a negative mirror-image of the actual source across the free surface: the reference Green's function  $G = G_0^d + G_0^{FS}$  vanishes at the free surface.

The role of  $G_0^{FS}$  in the forward series (5) is to create all of the extra events that owe their existence to the presence of the free surface. Its role in the inverse series (6) (and (7)) is to perform all of the extra inversion tasks that arise due to reflection data containing free-surface generated events (ghosts and free-surface multiples).

## Free-surface algorithms

The feedback method for free-surface multiple attenuation describes a very similar algorithm as the inverse scattering series for free-surface multiples. The difference resides in the fact that the inverse scattering free-surface method (Refs. 1 and 2) accounts for the actual source in the water column whereas the feedback method corresponds to a vertical dipole of the actual source. However, the free-surface event generating and removing mechanism are identical and  $G_0^{FS} = W^+ R_0^- W^-$  where  $W^+$  is the upward propagation,  $R_0^-$  is the downward reflection at the free surface and  $W^-$  is the downward propagation. This relates key ingredients of the inverse scattering and feedback methods for free-surface multiples and explains the similarity of their respective free-surface algorithms.

## Internal multiple algorithms

The inverse scattering method for internal multiples corresponds to a subseries of the series for  $V$  that

automatically eliminates all internal multiples starting with data  $D$  (consisting of primaries and internal multiples) and the whole-space Green's function for water,  $G_0^d$ .

The first term in that elimination series represents the attenuation of all first order internal multiples, independent of (and oblivious to) the location of the reflections that generate the upward and downward reflections. It predicts the exact time of all internal multiples and well-approximates the amplitude of internal multiples of an entire p-wave history. The arrival time of internal multiples with one or more shear-wave leg is also predicted precisely but the predicted amplitude of these converted wave internal multiples is less accurate than for the corresponding p-waves.

The feedback method (Ref. 6 and Ref. 7) for internal multiples was originally formulated as basically a sequence that repeats the free-surface program by locating and defining the structure and reflection properties of the ocean bottom and subsequently reflectors below. It then removes those events that have their shallowest downward reflections at the specified reflector. This feedback program is in effect a stripping technique and requires accurate knowledge of the overburden above the reflector to allow for both precise spatial location and amplitudes of reflection (as a function of angle).

The development of common-focal-point (CFP) time imaging concepts (as an intermediate step preceding imaging at depth) caused the original depth imaging of the feedback internal multiple algorithm to be examined and to evolve into a CFP *time* image form, for the identified reflector where a shallowest downward reflector occurs. The latter time images do not require an accurate overburden model and the resulting forms begin to emulate characteristics of the first term of the inverse scattering series for internal multiple attenuation. A different implementation of the feedback method is described in Ref. 9. The analysis and comparison of this recently evolved form of the feedback method for internal multiples and its relationship to the first term of the inverse scattering procedure continues. For achieving goals significantly beyond current internal multiple attenuation capability and moving towards internal multiple elimination, the feedback and inverse scattering methodologies are currently on two totally different trajectories. The feedback method returns to its original program of determining the velocity model of the overburden (using, e.g., one-way tomography) and depth imaging, whereas the inverse series goes to higher terms in the internal multiple removal series that only depend on  $D$  and  $G_0$ , and, hence, avoids the need for the velocity or depth model. We anticipate that there will be circumstances for which either one or the other (or some combination) of these approaches for internal multiple elimination will be the method of choice.

### Field data example

In this section, we illustrate the application of free-surface and internal multiple attenuation to a field data example. These

data are acquired in an area with a relatively shallow water bottom and a number of high amplitude reflectors at depth. The first step is to remove the free-surface multiples in preparation for internal multiple attenuation. To do this, we first we applied predictive deconvolution in the tau-p domain to attenuate the short period water-bottom and peg-leg multiples. Since we were unable to adequately estimate the water-bottom reflection in the near-offset range, we did not use the inverse scattering free-surface approach to attenuate these multiples. However, we did use this latter method to attenuate the longer period multiples associated with the deeper high amplitude reflectors. A Radon demultiple filter was then applied to further attenuate any residual multiples. In Fig. 2, we show a near-offset gather of the data before and after the application of the multiple attenuation flow described above.

After the free-surface multiples have been sufficiently removed, the data are ready for internal multiple attenuation. Since the data requirements for internal multiple attenuation are the same as for free-surface multiples, no additional data interpolation or extrapolation is required at this stage. To get an idea of the location and severity of the internal multiples, we stack the data and compute a 1D post-stack internal multiple estimate. The first panel in Fig. 3 shows the stacked section after prestack free-surface multiple removal. The second panel in this figure shows the 1D post-stack internal multiple estimate. Note the correlation between the predicted dipping internal multiples and the dipping events in the input data. Usually the 1D estimate is not suitable for subtraction since the timing and amplitudes are in error. In this case, we were successful at attenuating the predicted internal multiples using an adaptive subtraction scheme. Judicious use of adaptive subtraction is required to avoid damaging primary reflections – particularly in the case of internal multiple attenuation.

The next step is to compute an internal multiple estimate using the 2D pre-stack algorithm. While more costly than the 1D estimate, the 2D estimate has the benefit of being able to attenuate internal multiples on pre-stack gathers in preparation for AVO analysis. In addition, the amplitude and timing of the events is more accurate in 2D thus enabling a more reliable subtraction. The first panel in Fig. 4 shows a common offset gather (offset = 1000 m) of the prestack data after free-surface demultiple. The second panel shows the 2D prestack internal multiple estimate for this same offset. Again we observe a good correlation between the predicted multiples and dipping events on the input data. The third panel shows the results after adaptively subtracting the predicted internal multiples from the input data.

### Conclusion

The inverse scattering series for attenuating free-surface and internal multiples are specifically designed for deep-water and/or complex multidimensional subsurfaces where one dimensional or moveout-trajectory assumptions are violated and where it would be prudent to avoid velocity picking, event

identification or interpretive intervention. In this paper, we revisited the logic-train behind these algorithms and continued our discussion and analysis – comparing the current and anticipated future directions of inverse scattering methods with the feedback approach. Field-data examples exemplify the inverse scattering algorithm.

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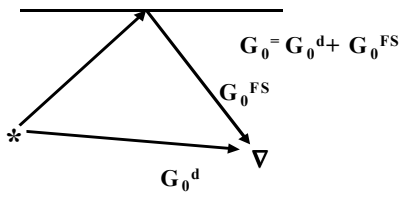


Fig. 1. The reference Green's function,  $G_0$  for the free-surface multiple attenuation subseries.  $G_0^d$  is the point to point propagation whole-space causal Green's function, and  $G_0^{FS}$  is the extra portion of  $G_0$  due to the presence of the free surface.

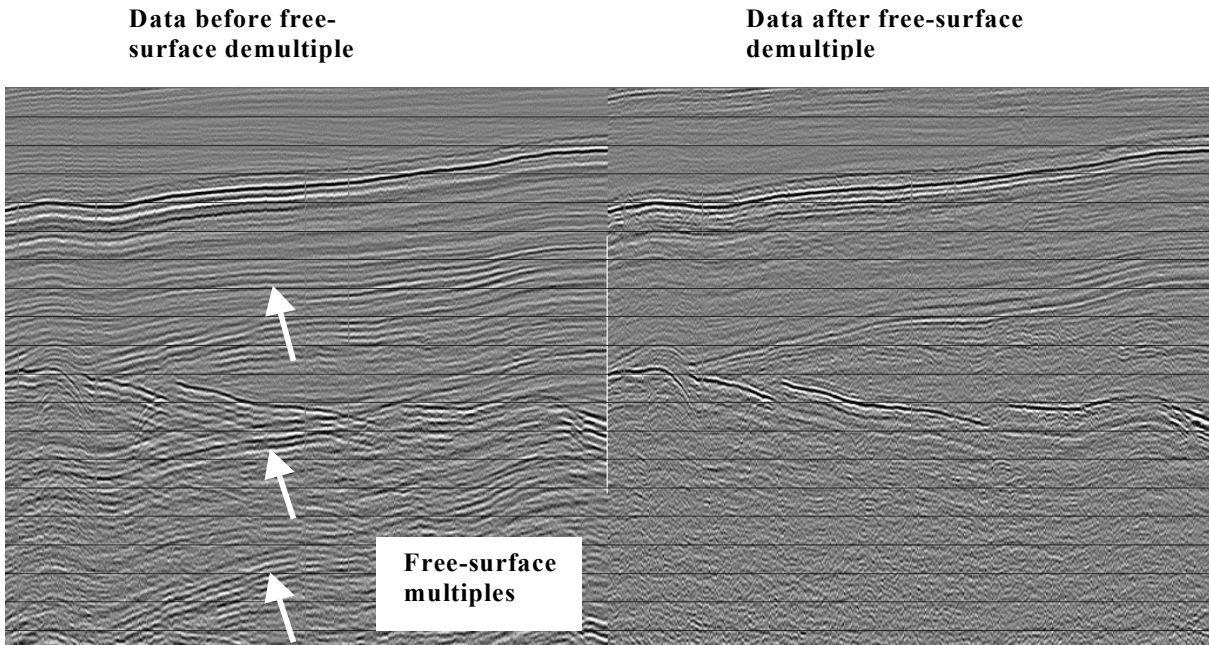


Fig 2. Pre-stack free-surface multiple attenuation example. The first panel shows a common-offset gather (offset=1000m) before free-surface multiple attenuation. The second panel shows these data after tau-p predictive deconvolution, Inverse Scattering free-surface demultiple, and Radon demultiple. (Seismic data come from a non-exclusive survey owned by Geco-Prakla. Permission to use these data is gratefully acknowledged.)

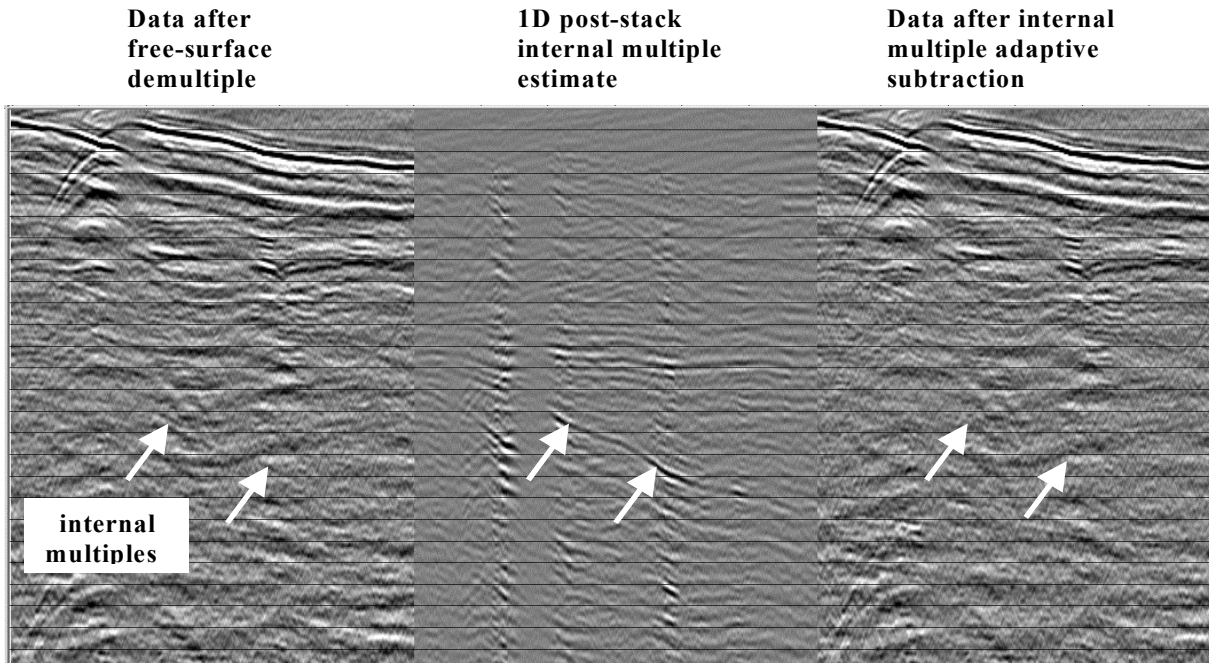


Fig. 3. Post-stack internal multiple attenuation example. The first panel shows a stacked section after pre-stack free-surface multiple attenuation and Radon demultiple. The second panel shows the 1D post-stack internal multiple estimate computed using the stack in the first panel. The third panel shows the stack in the first panel after adaptive subtraction of the estimated internal multiples in the second panel. (Seismic data come from a non-exclusive survey owned by Geco-Prakla. Permission to use these data is gratefully acknowledged.)

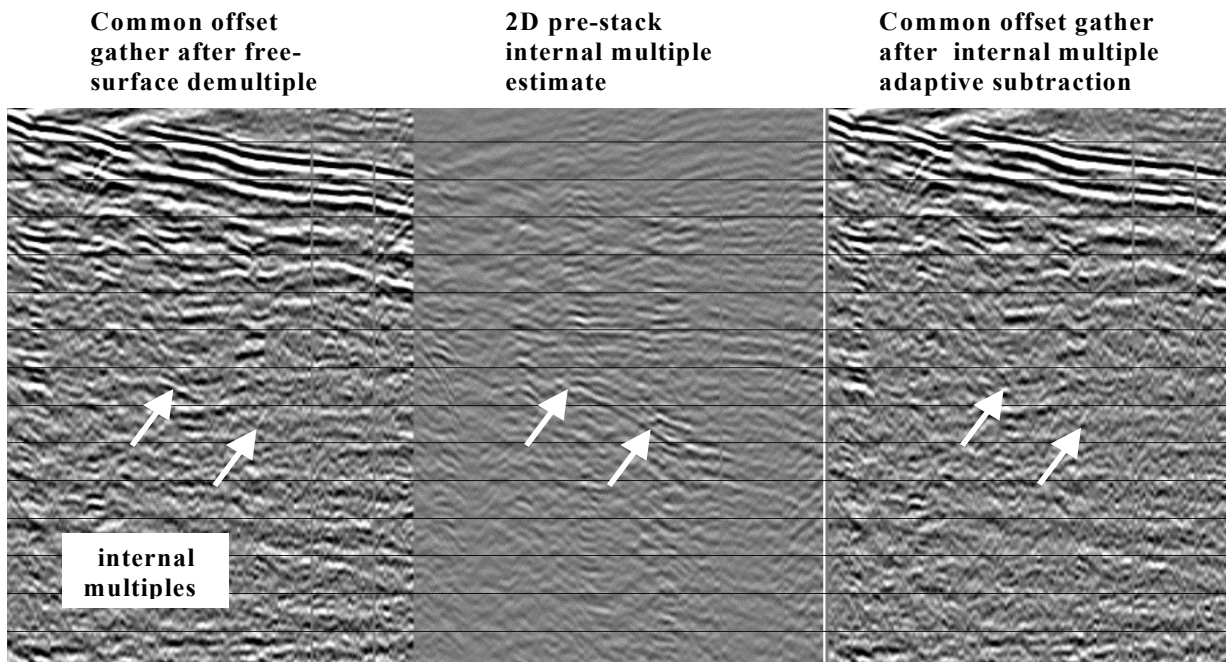


Fig. 4. Pre-stack internal multiple attenuation example. The first panel shows a common offset gather (offset = 1000 ft) after pre-stack free-surface multiple attenuation and Radon demultiple. The second panel shows the 2D pre-stack internal multiple for this same offset. The third panel shows the common offset data in the first panel after adaptive subtraction of the estimated internal multiples in the second panel. (Seismic data come from a non-exclusive survey owned by Geco-Prakla. Permission to use these data is gratefully acknowledged.)