Prediction of the wavefield anywhere above an ordinary towed streamer: application to source waveform estimation, demultiple, deghosting, data reconstruction and imaging

A.B. Weglein[†], T.H. Tan[*], S. A. Shaw[†], K. H. Matson[†], D. J. Foster[†]
[†]Arco, Plano, Texas, USA, [*]Shell International E&P, Rijswijk, The Netherlands

Abstract

In principle, it is not possible to compute the total two-way propagating pressure field above a cable from measurements of only the pressure field on a single typical towed streamer. It might appear that knowing the pressure field on the measurement surface together with the fact that the total field vanishes at the air-water "free-surface", would be sufficient information to compute the two-way field at all points between. However, the latter argument assumes knowledge of all medium properties and sources between the two levels where the pressure is known. The fact that the energy source lies between these two surfaces and that the source and its waveform are generally unknown, precludes computation of the two-way field between the cable and the free-surface. Weglein and Secrest (1990) describe how to compute the scattered field between the measurement surface and the free surface, and the source waveform below the measurement surface, given a cable (or in 3D, a surface) where both the pressure and its normal derivative are measured. Osen et al. (1998) and Tan (1992) show how the wavelet due to an isotropic source can be determined from pressure measured on a typical cable plus one extra phone between the cable and the free surface.

While in principle it is not possible to determine the field above the single towed streamer, it has recently been observed by Tan (1999) that this is possible in practice, for the frequencies and geometry corresponding to the typical marine seismic experiment. A typical depth of the towed streamer below the free-surface is ~10 m and the dominant seismic frequencies are less than ~125 Hz. It turns out that the term in the equation that blocks the ability to predict the field above the towed streamer is negligible due to the confluence of these depth and frequency factors. Hence, the typical depth of streamers and seismic frequencies conspire to make practice more accommodating than theory. Tan (1999) exploits this fact and then introduces a mathematically complex Wiener-Hopf Green's function to provide a stable wavelet estimation scheme from a single cable.

In this paper we review and further clarify these recent developments by placing them within the context of the general inverse-source problem. We also show that the ability to predict the field above the cable opens up a plethora of new seismic processing opportunities (in addition to the important application described by Tan, 1999). The new opportunities for progress include: the calculation of full source waveform both below and above the cable from single cable pressure measurements only; calculation of the scattered field between the cable and the free-surface, again with a single cable pressure measurements only; demultiple techniques based on up-down separation; creation of a vertical cable above the towed streamer; deghosting; data reconstruction; and two-way wave migration.

Introduction

Source signature estimation is one of the key outstanding problems in exploration seismology. There is a heightened interest in this topic due to the need for the wavelet in wave-theoretic multiple attenuation methods as well as for traditional structural and amplitude analyses at depth. For example, the energy-minimization criteria for estimating the wavelet (see, e.g., Verschuur et al., 1992, Carvalho and Weglein, 1994, Ikelle et al. 1997, and Matson, 2000) is often an adequate approach for free-surface multiple attenuation; however, it can be too blunt an instrument in some situations such as occurs with subtle subsalt internal multiples interfering with weak subsalt primaries (due to the transmission losses through salt). The latter problem, of current high priority and interest, is an important driver for developing methods for estimating the source waveform that are as theoretically complete and realistic as the seismic processing methods they are meant to serve. Something less can inhibit subsequent wave theoretic demultiple and imaging-inversion techniques from reaching their full potential.

A method that computes the entire source waveform was described in Weglein and Secrest (1990); it required the pressure and normal derivative on the measurement surface. Subsequent theory by Tan (1992) and Osen et al. (1998) provide the wavelet for an isotropic point source from the pressure on the measurement surface and an extra phone between the measurement surface and the free surface. These methods depend on a Green's function that vanishes on both the free and measurement surfaces. Tan (1999) points out that the latter Green's function will not provide a stable solution for the wavelet when the measurement surface is on the order of 10 m below the free surface and the source spectrum is less than ~125 Hz. The origin of this instability is the need to divide by the Green's function (to find the wavelet) that satisfies Dirichlet boundary conditions on those two surfaces. Under the normal depths and frequency range of the marine seismic experiment that Green's function corresponds to a waveguide and is vanishingly small. However, the origin of the instability provides a tremendous well of new opportunity that opens new doors for achieving not only the original source waveform goal, but also many other important seismic processing objectives.

Method

The source waveform method of Weglein and Secrest (1990)

$$\left.\begin{array}{l} p_o(\boldsymbol{r},\boldsymbol{r}_s,\omega) \\ \quad \boldsymbol{r}\text{ below S} \\ -p_s(\boldsymbol{r},\boldsymbol{r}_s,\omega) \\ \quad \boldsymbol{r}\text{ above S} \end{array}\right\} = \int_S \left\{ p(\boldsymbol{r}',\boldsymbol{r}_s,\omega)\frac{\partial G_o}{\partial n'}(\boldsymbol{r},\boldsymbol{r}',\omega) - G_o(\boldsymbol{r},\boldsymbol{r}',\omega)\frac{\partial p}{\partial n'}(\boldsymbol{r}',\boldsymbol{r}_s,\omega)\right\} dS' \quad (1)$$

produces the reference wave due to the actual source distribution, (or source waveform) p_o, from computing the total field p and dp/dn along the cable and evaluating the integral at any $\boldsymbol{r}$ below the measurement surface. G_o is the causal impulse response for a half-space of water bounded by a free surface at the air-water boundary. Evaluating the surface integral in Eq. (1) at any location above the measurement surface (and below the free surface) produces the scattered field, $p_s=p-p_o$, at that location. If you assume $p_o=A(\omega)G_o$, then the procedure provides an infinite number of estimates of $A(\omega)$; one for each $\boldsymbol{r}$ below S. The need for both measurements arises from the need to cancel the scattered field. This is a derivative procedure of the general extinction theorem (Weglein and Devaney, 1992 and Born and Wolf, 1959). Tests of the efficacy and robustness of this method for producing the wavelet and radiation pattern in the presence of aperture limited and sampled data are described in DeLima et al. (1990) and Keho et al. (1990).

Osen et al. (1998) and Tan (1992) were interested in eliminating the data requirement of the normal derivative. They achieve a compromise away from the generality of Weglein and Secrest (1990) for determining an arbitrary reference field, p_0 (without the need to know or determine the source character, e.g., individual gun response and array pattern) towards a lesser goal of determining the source wavelet (amplitude and phase) due to an isotropic source but with a requirement for considerably less data. They deduce that, in addition to the cable with pressure measurements, they require a single extra phone anywhere between the cable and the free surface. They achieve this by choosing a Green's function (in Green's Theorem) that vanishes on both the free and the measurement surfaces. Let G_0^D denote this two-surface Dirichlet Green's function (see Morse and Feshbach Vol I 1953, Tan 1992, and Osen et al. 1998), then

$$A(\omega)G_0^D(\boldsymbol{r},\boldsymbol{r}_s,\omega) = \int_{-\infty}^{+\infty} p(\boldsymbol{r}',\boldsymbol{r}_s,\omega)\frac{\partial G_0^D}{\partial n'}(\boldsymbol{r},\boldsymbol{r}',\omega)dS' - p(\boldsymbol{r}_I,\boldsymbol{r}_s,\omega) \quad (2)$$

where $\boldsymbol{r}$ is the evaluation point below the measurement surface and $\boldsymbol{r}_I$ is the mirror image of $\boldsymbol{r}$ across (and above) that surface. $\boldsymbol{r}_I$ is the location of the required extra phone. To find $A(\omega)$ from Eq. (2) requires division by G_0^D. Tan (1999) shows that for typical towing depths and seismic frequencies, that G_0^D is vanishingly small and the division is unstable. However, the smallness of the left hand member of the Eq. (2) compared to the terms of the right hand member, allows us to well approximate

$$p(\boldsymbol{r}_I,\boldsymbol{r}_s,\omega) \approx \int_{-\infty}^{+\infty} p(\boldsymbol{r}',\boldsymbol{r}_s,\omega)\frac{\partial G_0^D}{\partial n'}(\boldsymbol{r},\boldsymbol{r}',\omega)dS' \quad (3)$$

where $\boldsymbol{r}_I$ is any point between the measurement and free surfaces. Tan (1999) then introduces yet another Green's function that vanishes on the free surface and on the portion of the measurement surface that starts below the source and extends along the towed streamer to infinity. This more complex Green's function, $\overline{G}_0^D$, is stable under division at seismic acquisition depths and frequencies. In terms of $\overline{G}_0^D$, the wavelet $A(\omega)$ is given by

$$A(\omega) = \frac{\int p(\boldsymbol{r}',\boldsymbol{r}_s,\omega)\frac{\partial \overline{G}_0^D}{\partial n'}(\boldsymbol{r},\boldsymbol{r}',\omega)dS' - p(\boldsymbol{r}_I,\boldsymbol{r}_s,\omega)}{\overline{G}_0^D(\boldsymbol{r},\boldsymbol{r}_s,\omega)} \quad (4)$$

The scheme of Tan (1999) uses Eq. (3) to find $p(\boldsymbol{r}_I,\boldsymbol{r}_s,\omega)$ from measurements of p on the single towed streamer and then substitutes p into Eq. (4) to find $A(\omega)$. The effectiveness of this technique is demonstrated with synthetic and field data.

In this paper, we are proposing that in addition to the procedure that uses Eq. (3) and then Eq. (4) to find $A(\omega)$, one could use Eq. (3) to find dp/dn and then use Eq. (1) to find p_0. This has the potential to provide both the source waveform and its array characteristics, which have important applications to seismic processing methods such as multiple attenuation and AVO. Furthermore, the ability to compute the total wavefield at all points above an ordinary streamer from Eq. (3) (without knowing the source or its waveform) presents an enormous set of opportunities well beyond the original objectives of this research.

Prediction of the wavefield anywhere above an ordinary towed streamer

Conclusions

A method for predicting the total two-way wavefield anywhere above a typical towed streamer from measurements of only the pressure along the cable is placed in the broader context of the inverse-source problem and the extinction theorem. Methods for utilizing this new observation include: two-way imaging and migration-inversion, deghosting, up-down separation demultiple, and increasing aperture through creation of, e.g. a vertical cable above the actual. The generalization for elastic wavefields and multicomponent data follow from the elastic version of Eq. (1) in Weglein and Secrest (1990), designed for ocean-bottom and on-shore application. Issues under investigation include water depth, reference medium sensitivity and frequency ranges for the elastic generalization of Eq. (3).

Acknowledgments

The authors would like to thank Shell and ARCO for encouragement and support of this research. In particular, we are grateful to Fred Hoffman, James D. Robertson, Wim Walk, James K. O'Connell, Dodd DeCamp, Jon Sheiman, and Trilochan Padhi.

References

Born, M., and Wolf, E., 1959, Principles of Optics: Pergamon Press, p100.

Carvalho, P.M., and Weglein, A.B., 1994, Wavelet estimation for surface multiple attenuation using a simulated annealing algorithm: 64th Annual Internat. Mtg. Soc. Expl. Geophys., Expanded Abstract, 1481–1484.

DeLima, G.R., Weglein, A.B., Porsani, M.J., and Ulrych, T.J., 1990, Robustness of a new source-signature estimation method under realistic data conditions: A deterministic-statistical approach: 60th Ann. Internat. Mtg. Soc. Expl. Geophys. Expanded Abstracts, 1658–1660.

Ikelle, L.T., Roberts, G., and Weglein, A.B., 1997, Source signature estimation based on the removal of first-order multiples, Geophysics, 62, 1904–1920.

Keho, T.H., Weglein, A.B., and Rigsby, P.G., 1990, Marine source wavelet and radiation pattern estimation: 60th Ann. Internat. Mtg. Soc. Expl. Geophys. Expanded Abstracts, 1655–1657.

Matson, K.H., 2000, An overview of wavelet estimation using free-surface multiple removal: The Leading Edge, vol. 19, (1), p. 50.

Morse, P.M., and Feshbach, H., 1953, Methods of theoretical physics: McGraw-Hill.

Osen, A., Secrest, B.G., Admundsen, L., and Reitan, A., 1998, Wavelet estimation from marine pressure measurements: Geophysics, 63, 2108–2119.

Tan, T.H., 1999, Wavelet spectrum estimation, Geophysics, vol. 64, 6, 1836–1846.

Tan, T.H., 1999, Application of the Wiener-Hopf technique to the calculation of the diffraction of a cylindrical wave by a soft half-plane embedded in a fluid half-space, Geophysics, vol. 64, 6, 1847–1851.

Tan, T.H., 1992, Source signature estimation: Presented at the Internat. Conf. and Expo. of Expl. and Development Geophys., Moscow, Russia.

Verschuur, D.J., Berkhout, A.J., and Wapenaar, C.P.A., 1992, Adaptive surface-related multiple elimination: Geophysics, 57, 1166–1177.

Weglein, A.B., and Devaney, A.J., 1992, The inverse source problem in the presence of external sources: Proc. SPIE, 1767, 170–176.

Weglein, A.B., and Secrest, B.G., 1990, Wavelet estimation for a multidimensional acoustic or elastic earth. Geophysics, 55, 902–913.