Wavelet Estimation and Wavefield Prediction

– Notes –

30 November, 2001

Zhiqiang Guo

1 Introduction and Motivation

The objective of seismic exploration is to extract subsurface information from seismic data. The recorded data depends on both the source characteristics and subsurface. Therefore it is important to identify source characteristics and then remove its effects from seismic data. Also linear seismic processing such as migration and AVO benefits from knowledge of seismic source signature. Wave-theoretical multiple attenuation demands a good estimate of the wavelet. The current wavelet estimation for wave theoretic multiple attenuation is based on energy minimization, which relies on the total the energy of the wavefield being less when the multiples are removed. However, this criterion fails when both multiples and primaries are weak and when they destructively interfere. This motivates the search for a method that will directly determine the source wavelet so that these multiple attenuation methods can reach their full potential (see figure 1.1).

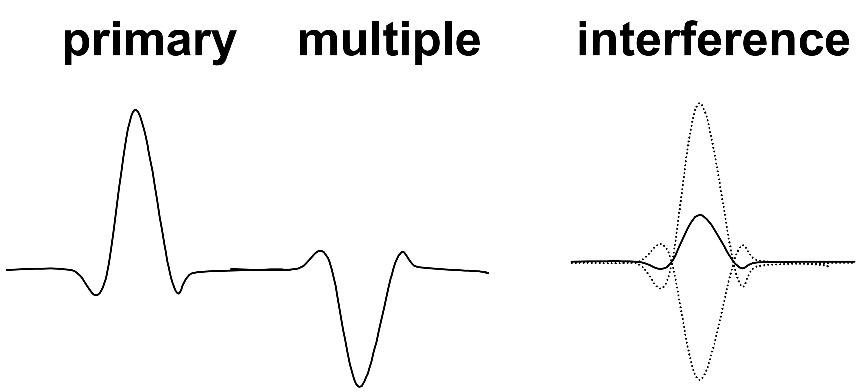

Figure 1.1 When primary destructively interferes with multiple, the total energy is weaker. After the multiple is removed from the data, the energy will be greater than it was prior to multiple removal. In this instance, the energy minimization criterion is invalid.

1.1 Wave theoretical multiple attenuation

In spite of recent progress in different approaches to remove multiples, multiples continue to be an important problem in seismic data processing. Only when multiples are identified or removed from the recorded data can imaging or AVO be done correctly.

There are currently three main categories of multiple attenuation methods: (1) those based on differential moveout between primaries and multiples, such as the Radon transform (Foster et al., 1992), (2) those based on differences on periodicity, such as deconvolution (Lokshtanov, 1999) which assumes that the multiples are periodic, while the primaries are not, and (3) those based on wave equation methods, such as the inverse scattering series approach (Carvalho et al., 1992, Weglein et al., 1997) and the wave equation recursive method (Verschuur et al., 1992), which first predict multiples and then subtract them from the original input data. In theory, the inverse scattering and recursive wave equation methods are more attractive than deconvolution and Radon transform, because wave equation methods assume no knowledge of the subsurface. These methods are therefore applicable to a complex 3-D Earth. However, they require an estimate of the source signature to predict multiples. In practice, the seismic source signature is not recorded so it is estimated from the data itself. The fundamental assumption for free-surface multiple attenuation is

$$\boldsymbol{M}_{i} = \frac{1}{\boldsymbol{A}} \boldsymbol{P} * \boldsymbol{M}_{i-1} \tag{1.1}$$

where $\boldsymbol{M}_i$ represents ith order surface multiples, $\boldsymbol{P}$ represents the primaries, * is convolution symbol, $\boldsymbol{A}$ represents acquisition wavelet, we need to remove $\boldsymbol{A}$ from the convolution $\boldsymbol{P} * \boldsymbol{M}_{i-1}$ because $\boldsymbol{A}$ appears twice, once in $\boldsymbol{P}$, once in $\boldsymbol{M}_{i-1}$ (figure 1.2).

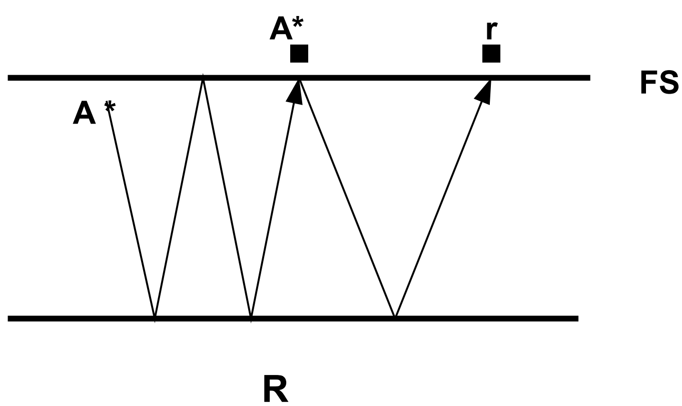

Figure 1.2 Free-surface multiple prediction in terms of sub-events: ith order multiples are predicted by convolving the (i–1)th order multiple with primary where the source is located at the position of the i^{th} order multiple event's receiver position.

1.1.2 1-D multiple attenuation

We illustrate the need for the source wavelet by working through a 1-D example of multiple attenuation (figure 1.3). Suppose we have a water layer overlying a reflector $\boldsymbol{R}$, and the free surface has reflection coefficient –1. $\boldsymbol{AR}$ represents the primary reflection, which is convolution of the source wavelet and the reflectivity in the frequency domain.

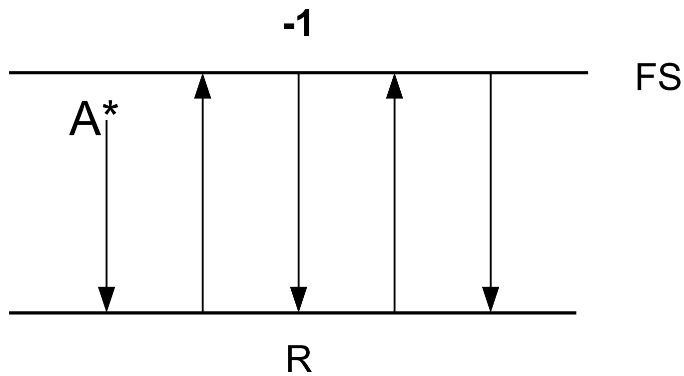

Figure 1.3 1-D construction of the wavefield when the free surface coefficient is –1. The subsurface reflectivity is $\boldsymbol{R}$ and the source wavelet is $\boldsymbol{A}$.

We can write the recorded wavefield $\boldsymbol{P}$ as

$$\boldsymbol{P} = \boldsymbol{AR} - \boldsymbol{AR}^2 + \boldsymbol{AR}^3 - \boldsymbol{AR}^4 + \ldots \qquad (1.2)$$

$$\boldsymbol{P} = \frac{\boldsymbol{AR}}{1+\boldsymbol{R}}$$

$$\boldsymbol{P}(1+\boldsymbol{R}) = \boldsymbol{AR}$$

$$\boldsymbol{R} = \frac{\boldsymbol{P}}{\boldsymbol{A}-\boldsymbol{P}} = \frac{\boldsymbol{P}/\boldsymbol{A}}{1-\boldsymbol{P}/\boldsymbol{A}} = \frac{\boldsymbol{P}}{\boldsymbol{A}}\left[\boldsymbol{I} + \left(\frac{\boldsymbol{P}}{\boldsymbol{A}}\right) + \left(\frac{\boldsymbol{P}}{\boldsymbol{A}}\right)^2 + \left(\frac{\boldsymbol{P}}{\boldsymbol{A}}\right)^3 + \ldots\right]$$

$$\boldsymbol{AR} = \boldsymbol{P} + \frac{1}{\boldsymbol{A}}\boldsymbol{P}^2 + \frac{1}{\boldsymbol{A}^2}\boldsymbol{P}^3 + \frac{1}{\boldsymbol{A}^3}\boldsymbol{P}^4 + \ldots \qquad (1.3)$$

This equation illustrates that the wavefield in the absence of free-surface multiples ($\boldsymbol{AR}$) can be obtained by a series of convolutions of the pressure data and spectral divisions involving the source wavelet ($\boldsymbol{A}$).

2 Wavelet Estimation

To illustrate the concepts we use the simple acoustic wave theory. However the resulting integral solutions are valid for an entire class of earth model types.

2.1 Scattering theory

Scattering theory describes the relationship between the physical properties of an actual medium and the physical properties of a reference medium in terms of the difference in the impulse responses for the actual and reference media. The latter wavefields, because of a localized source in space and time, are Green's functions. The difference between the actual and the reference media is characterized by the perturbation operator. The corresponding difference between the actual and reference wavefields is called the scattered field. In surface seismic exploration, the reference medium agrees with the actual medium at and above the measurement surface, hence the simplest medium is a half-space of water bounded by a free surface at the air-water interface.

Based on scattering theory, the actual earth can be parameterized as a homogeneous velocity reference medium with embedded reflectors (figure 2.1).

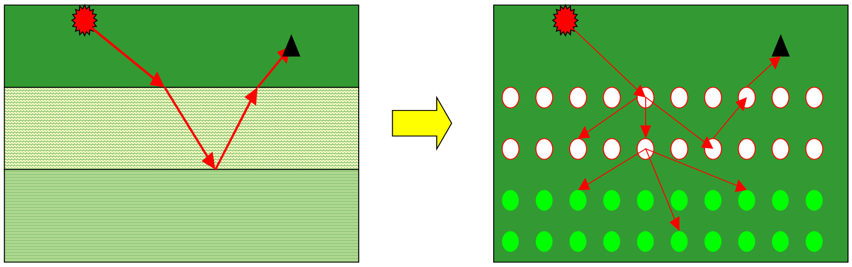

Figure 2.1 The actual heterogeneous medium can be parameterized as a homogeneous velocity reference medium with embedded scattering points.

Hence

$$\frac{1}{c^2(r)} = \frac{1}{c_0}[1-\alpha(r)] \tag{2.1}$$

Where c_0 is reference medium velocity, $\alpha(r)$ is called the index of refraction, which is used to characterize the difference between the actual and reference media, c is actual medium velocity. The variable velocity acoustic wave equation in an inhomogeneous medium, with constant density and localized source $A(t)$ in the time and spatial domain is

$$\nabla^2 P(r,r_0,t) - \frac{1}{c^2(r)}\frac{\partial^2 P(r,r_0,t)}{\partial t^2} = A(t)\delta(r-r_0)$$

Performing a temporal Fourier transform

$$P(r,r_0,\omega) = \int P(r,r_0,t)e^{i\omega t}dt$$

the wave equation above is re-written in frequency and spatial domain as

$$\nabla^2 P(r,r_0,\omega) + \frac{\omega^2}{c^2(r)}P(r,r_0,\omega) = A(\omega)\delta(r-r_0) \tag{2.2}$$

where r is any point in half space, r_0 is source location below free surface, $A(\omega)$ is the source signature, ω is angular frequency, P is the pressure field, $\delta(.)$ is a Dirac delta function, $c(r)$ is the actual medium velocity, we choose to characterize the velocity configuration $c(r)$ in term of a reference value c_0 and a variation in scattered index $\alpha(r)$ defined in equation (2.1).

Substituting equation (2.1) into equation (2.2) and reform it to

$$\nabla^2 P(r,r_0,\omega) + \frac{\omega^2}{c_0^{\ 2}}[1-\alpha(r)]P(r,r_0,\omega) = A(\omega)\delta(r-r_0)$$

$$\nabla^2 P(r,r_0,\omega) + \frac{\omega^2}{c_0^{\ 2}}P(r,r_0,\omega) = \frac{\omega^2}{c_0^{\ 2}}\alpha(r)P(r,r_0,\omega) + A(\omega)\delta(r-r_0) \tag{2.3}$$

We can regard the first term in right-hand side of equation (2.3) as the passive source due to scattering potential $\alpha(r)$. Taken together, the two terms on the right-hand side of equation (2.3) constitute the sources that produce the wavefield $P(r,r_0,\omega)$. Hence equation (2.3) can be written as an integral equation using the principle of superposition.

$$P(r,r_0,\omega)=\int_V\left[\frac{\omega^2}{c_0^{\ 2}}\alpha(r')P(r',r_0,\omega)+A(\omega)\delta(r'-r_0)\right]G_0(r,r',\omega)dr' \qquad (2.4)$$

where $G_0(r,r',\omega)$ is the Green's function for the constant reference medium, satisfying the following equation.

$$\nabla^2 G_0(r,r',\omega)+\frac{\omega^2}{c_0^{\ 2}}G_0(r,r',\omega)=\delta(r-r')$$

Furthermore equation (2.4) can be written as

$$P(r,r_0,\omega)=\int_V A(\omega)\delta(r'-r_0)G_0(r,r',\omega)dr'+\int_V\left[\frac{\omega^2}{c_0^{\ 2}}\alpha(r')P(r',r_0,\omega)\right]G_0(r,r',\omega)dr'$$

Using the delta function property

$$\int_V \delta(r-a)f(r)dr=f(a) \qquad , \ a\in V$$

we have

$$P(r,r_0,\omega)=A(\omega)G_0(r,r_0,\omega)+\int_V\frac{\omega^2}{c_0^{\ 2}}\alpha(r')P(r',r_0,\omega)G_0(r,r',\omega)dr' \qquad (2.5)$$

Equation (2.5) is called the Lippmann-Schwinger integral equation and is valid for all r . It can be also expressed as

$$P(r,r_0,\omega)=P_0(r,r_0,\omega)+P_s(r,r_0,\omega) \qquad (2.6)$$

where

$$P_0(r,r_0,\omega)=A(\omega)G_0(r,r_0,\omega) \qquad (2.7)$$

$$P_s(r,r_0,\omega)=\int G_0(r,r',\omega)\frac{\omega^2}{c_0^{\ 2}}\alpha(r')P(r',r_0,\omega)dr' \qquad (2.8)$$

$P_0(r,r_0,\omega)$ represents direct waves from source r_0 to point r, $P_s(r,r_0,\omega)$ represents scattered field.

The physical interpretation of equation (2.5 or 2.6) is that the total seismic wave field $P(r,r_0,\omega)$ can be expressed as the sum of reference wave-field P_0, the wavefield due to actual source in homogeneous velocity reference medium, and scattered field P_s, the wavefield due to scatters (deviation from the reference medium) in a (e.g., homogeneous) velocity reference medium.

Now we build the Green's function $G_0^D(r,r',\omega)$ in a homogenous medium with a supposed source at r and its corresponding mirror-imaged source r_I with opposite sign across free surface FS (figure 2.2), our goal is make the Green's function satisfy the Dirichlet boundary condition on free surface.

$$\nabla^2 G_0^D(r,r',\omega)+\frac{\omega^2}{c_0^{\ 2}}G_0^D(r,r',\omega)=\delta(r-r')-\delta(r_I-r') \qquad (2.9)$$

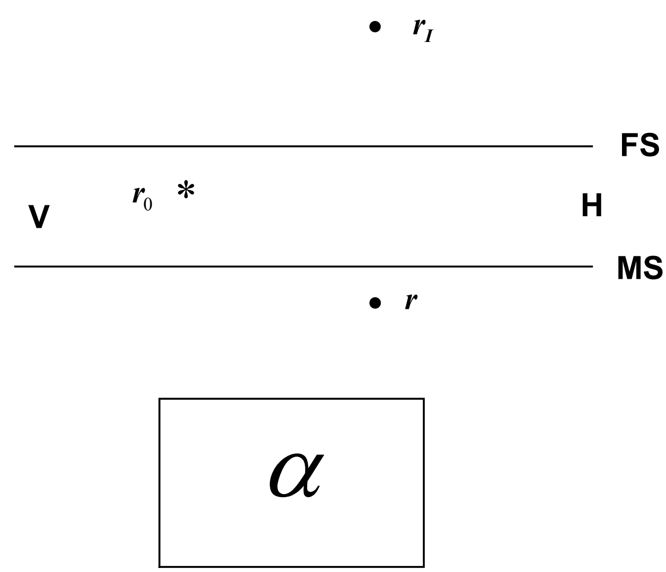

Figure 2.2 $\boldsymbol{r_I}$ is the mirror image point $\boldsymbol{r}$ across FS

Substituting the wavefield $\boldsymbol{P}(\boldsymbol{r},\boldsymbol{r}_0,\omega)$ in equation (2.3) and the Green's function $\boldsymbol{G}_0^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)$ in equation (2.9) to Green's theorem

$$\iiint_v d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\nabla^2\boldsymbol{G}_0{}^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)-\boldsymbol{G}_0{}^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)\nabla^2\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\right]=$$

$$\oint\!\!\oint_S d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\frac{\partial\boldsymbol{G}_0{}^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)}{\partial\boldsymbol{n}}-\boldsymbol{G}_0{}^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)\frac{\partial\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)}{\partial\boldsymbol{n}}\right] \tag{2.10}$$

Then we have

$$\iiint_v d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\nabla^2\boldsymbol{G}_0{}^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)-\boldsymbol{G}_0{}^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)\nabla^2\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\right]=$$

$$\iiint_v d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\delta(\boldsymbol{r}-\boldsymbol{r}')-\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\delta(\boldsymbol{r}_I-\boldsymbol{r}')-\frac{\omega^2}{\boldsymbol{c}_0^2}\boldsymbol{G}_0^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)\alpha(\boldsymbol{r}')\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\right]$$
$$+\iiint_v d\boldsymbol{r}'\left[-\boldsymbol{A}(\omega)\boldsymbol{G}_0^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)\delta(\boldsymbol{r}'-\boldsymbol{r}_0)\right] \tag{2.11}$$

We choose the integral volume $\boldsymbol{V}$ to be region between free surface (FS) and measurement surface (MS) (see figure 2.2). Furthermore, notice the third term in right hand side of equation (2.11) will be zero since the scattering potential $\alpha(\boldsymbol{r}')$ is outside of the integral $\boldsymbol{V}$.

Again, using the sifting property of the delta function, the first and second terms in right-hand side of equation (2.11) are both zero because **r** and its mirror image $\boldsymbol{r_I}$ are beyond $\boldsymbol{V}$. Hence equation (2.11) can be rewritten as

$$\iiint_v d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\nabla^2\boldsymbol{G}_0{}^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)-\boldsymbol{G}_0{}^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}',\omega)\nabla^2\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\right]=$$

$$-\boldsymbol{A}(\omega)\boldsymbol{G}_0^{\boldsymbol{D}}(\boldsymbol{r},\boldsymbol{r}_0,\omega)$$

Combining this equation with the right hand side of equation (2.10), we obtain

$$-A(\omega)G_0^D(r,r_0,\omega)=\oint\!\!\!\oint_S dr'\left[P(r',r_0,\omega)\frac{\partial G_0{}^D(r,r',\omega)}{\partial n}-G_0{}^D(r,r',\omega)\frac{\partial P(r',r_0,\omega)}{\partial n}\right] \quad (2.12)$$

Notice that both the Green's function $G_0^D(r,r',\omega)$ and P are zero at the free surface (FS). Then equation above becomes:

$$-A(\omega)G_0^D(r,r_0,\omega)=\iint_{MS} dr'\left[P(r',r_0,\omega)\frac{\partial G_0{}^D(r,r',\omega)}{\partial n}-G_0{}^D(r,r',\omega)\frac{\partial P(r',r_0,\omega)}{\partial n}\right]$$

$$A(\omega)=\frac{-1}{G_0^D(r,r_0,\omega)}\iint_{MS} dr'\left[P(r',r_0,\omega)\frac{\partial G_0{}^D(r,r',\omega)}{\partial n}-G_0{}^D(r,r',\omega)\frac{\partial P(r',r_0,\omega)}{\partial n}\right] \quad (2.13)$$

Equation (2.13) will be used to compute the wavelet. The expression $-A(\omega)G_0^D(r,r_0,\omega)$ is the reference wavefield $P_0(r,r_0,\omega)$ and in the case where the actual source is in fact a superposition of notional point sources, this expression yields the source wavelet radiation pattern. Next we will describe a Green's function $G_0^{DD}(r,r_0,\omega)$ that vanishes at the free surface and the measurement surface.

3 Wavefield Prediction

3.1 Wavefield prediction above MS

To remove the need for the normal derivative of the pressure at the measurement surface (MS) in equation (2.13), we write the wave equation in the frequency domain

$$\nabla^2 P(r',r_0,\omega)+\frac{\omega^2}{c^2(r')}P(r',r_0,\omega)=A(\omega)\delta(r'-r_0) \quad (3.1)$$

Take Green's function as following expression

$$\nabla^2 G_0^{DD}(r,r',\omega)+\frac{\omega^2}{c_0{}^2}G_0^{DD}(r,r',\omega)=\delta(r-r')-\delta(r_I-r') \quad (3.2)$$

where r_I is chosen to be inside the region between free surface (FS) and measurement surface (MS), it is the mirror image of r across MS (Figure 3.1).

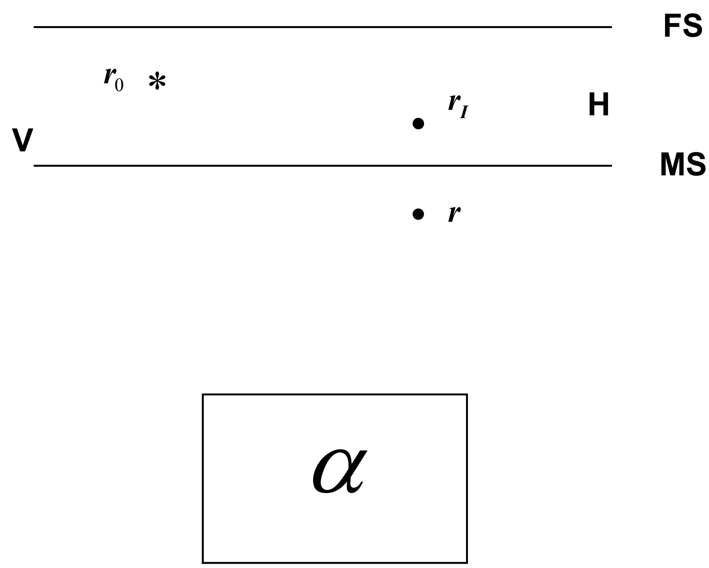

Figure 3.1 Mirror image $\boldsymbol{r_I}$ of $\boldsymbol{r}$ across the measurement surface MS inside. The volume $\boldsymbol{V}$ is bounded by the free surface and the measurement surface, and extends to infinity laterally.

Apply Green's theorem

$$\iiint_v d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\nabla^2\boldsymbol{G}_0^{\,DD}(\boldsymbol{r},\boldsymbol{r}',\omega)-\boldsymbol{G}_0^{\,DD}(\boldsymbol{r},\boldsymbol{r}',\omega)\nabla^2\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\right]=$$

$$\oiint_S d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\frac{\partial\boldsymbol{G}_0^{\,DD}(\boldsymbol{r},\boldsymbol{r}',\omega)}{\partial\boldsymbol{n}}-\boldsymbol{G}_0^{\,DD}(\boldsymbol{r},\boldsymbol{r}',\omega)\frac{\partial\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)}{\partial\boldsymbol{n}}\right]$$

Based on equation (2.1)

$$\frac{1}{\boldsymbol{c}^2(\boldsymbol{r})}=\frac{1}{\boldsymbol{c}_0}\left[1-\alpha(\boldsymbol{r})\right]$$

Substituting, we have

$$\iiint_v d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\delta(\boldsymbol{r}-\boldsymbol{r}')-\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\delta(\boldsymbol{r}_I-\boldsymbol{r}')-\frac{\omega^2}{\boldsymbol{c}_0^2}\boldsymbol{G}_0^{DD}(\boldsymbol{r},\boldsymbol{r}',\omega)\alpha(\boldsymbol{r}')\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\right]+$$

$$\iiint_v d\boldsymbol{r}'\left[-\boldsymbol{A}(\omega)\boldsymbol{G}_0^{DD}(\boldsymbol{r},\boldsymbol{r}',\omega)\delta(\boldsymbol{r}'-\boldsymbol{r}_0)\right]$$

$$=\oiint_S d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\frac{\partial\boldsymbol{G}_0^{\,DD}(\boldsymbol{r},\boldsymbol{r}',\omega)}{\partial\boldsymbol{n}}-\boldsymbol{G}_0^{\,DD}(\boldsymbol{r},\boldsymbol{r}',\omega)\frac{\partial\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)}{\partial\boldsymbol{n}}\right] \qquad (3.3)$$

If we choose $\boldsymbol{r}$ and scatterers $\alpha(\boldsymbol{r}')$ to be outside of the volume integral, then using the delta function property

$$\iiint_v d\boldsymbol{r}'\left[\delta(\boldsymbol{r}-\boldsymbol{r}')\boldsymbol{f}(\boldsymbol{r}')\right]=\boldsymbol{f}(\boldsymbol{r})$$

the first term and second term in left hand side of equation (3.3) become

$$\iiint_v d\boldsymbol{r}'\left[\boldsymbol{P}(\boldsymbol{r}',\boldsymbol{r}_0,\omega)\delta(\boldsymbol{r}-\boldsymbol{r}')\right]=0$$

$$\iiint_{v} dr' [P(r', r_0, \omega)\delta(r_I - r')] = P(r_I, r_0, \omega) \tag{3.4}$$

Also notice the third term on the left hand side of equation (3.3) will be zero due to the fact that $\alpha(r')$ is outside of the volume integral. Hence we have

$$-P(r_I, r_0, \omega) - A(\omega)G_0^{DD}(r, r_0, \omega) =$$
$$\oiint_S dr' \left[P(r', r_0, \omega)\frac{\partial G_0{}^{DD}(r, r', \omega)}{\partial n} - G_0{}^{DD}(r, r', \omega)\frac{\partial P(r', r_0, \omega)}{\partial n} \right]$$

We have selected the Green's function $G_0^{DD}(r, r', \omega)$ to be zero on both FS and MS (see Morse and Feshbach, Chapter 7). It is assumed that $P(r', r_0, \omega)_{|r'=FS} = 0$, then

$$-P(r_I, r_0, \omega) - A(\omega)G_0^{DD}(r, r_0, \omega) = \iint_{MS} dr' \left[P(r', r_0, \omega)\frac{\partial G_0^{DD}(r, r', \omega)}{\partial n} \right] \tag{3.5}$$

Tan (1999) points out that $G_0^{DD}(r, r', \omega)$ is vanishingly small for typical marine survey depths of approximately 6 m and seismic frequency less than 125 Hz. Therefore, the second term on the right hand side of the equation (3.4) can be ignored in comparison with the other terms. This results in the key observation:

$$-P(r_I, r_0, \omega) \approx \iint_{MS} dr' \left[P(r', r_0, \omega)\frac{\partial G_0^{DD}(r, r', \omega)}{\partial n} \right] \tag{3.6}$$

which can be used to predict wavefield at any point between the free surface and measurement surface using the Green's function that satisfies the Dirichlet boundary condition on the free surface and the measurement surface.

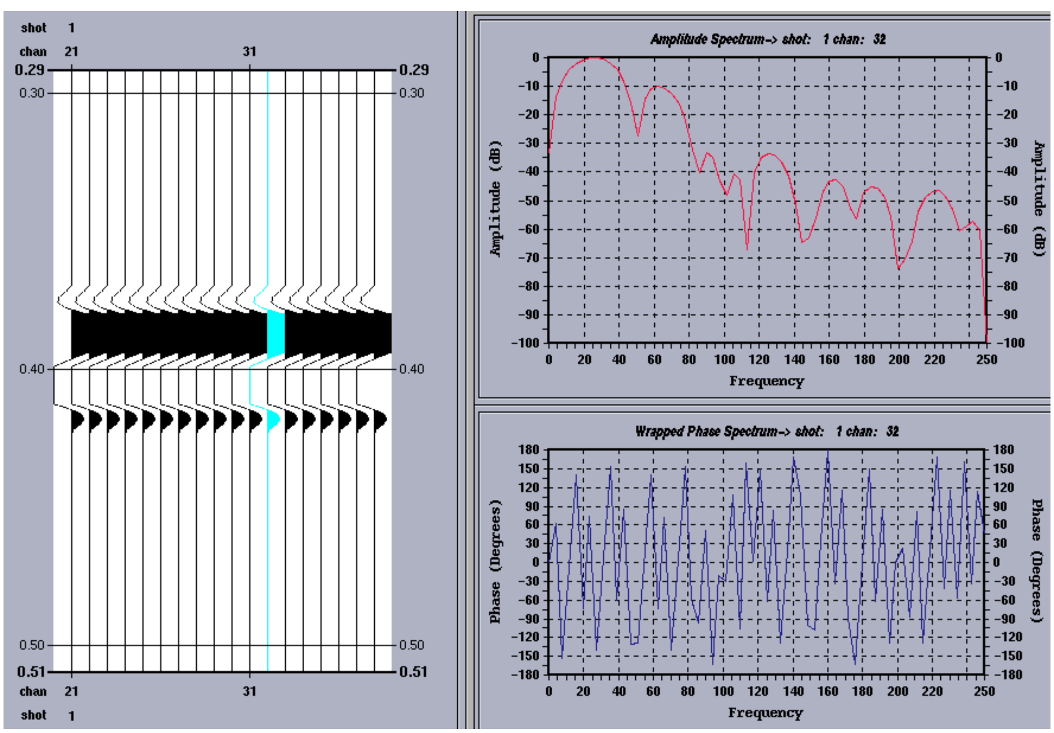

Figure 3. 2 Synthetic shot record for a source depth of 5 m and a cable depth of 15 m. The amplitude spectrum shows the receiver ghost notch at about 50 Hz at a simulated offset of about 2 m.

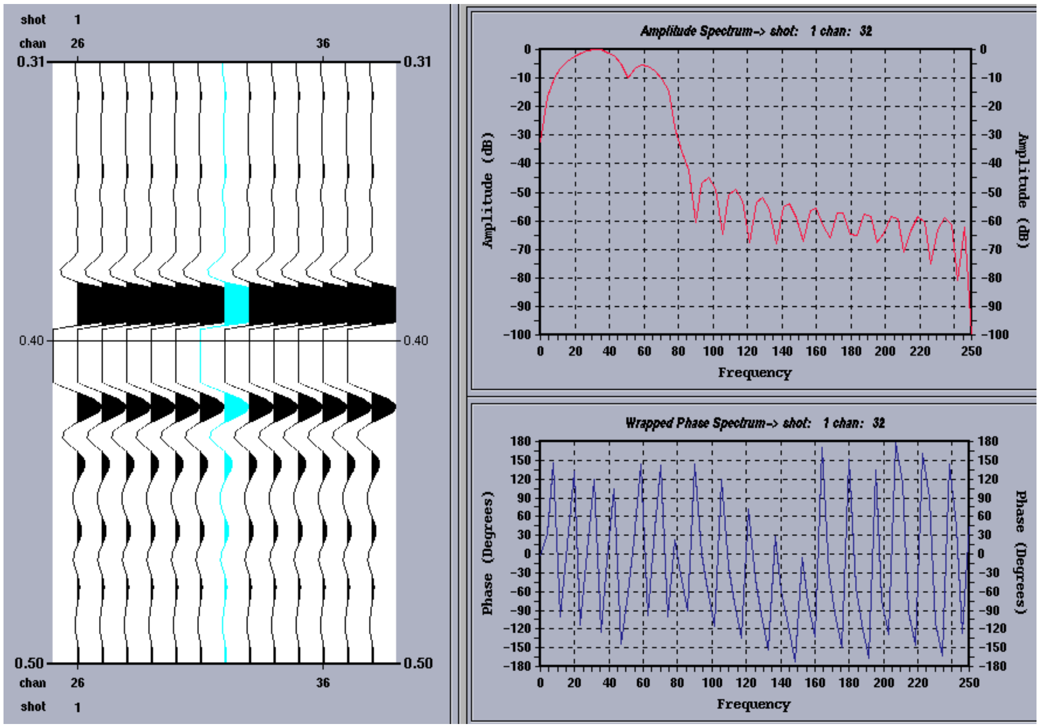

Figure 3.3 Predicted wavefield above the cable using equation (3.6) and the data in Figure 3.2 as input. The new cable depth is now 7 m below the free surface. The receiver ghost notch of the input data has been filled in (moved to higher frequency), and hence the bandwidth is improved. This is a preliminary but encouraging result.

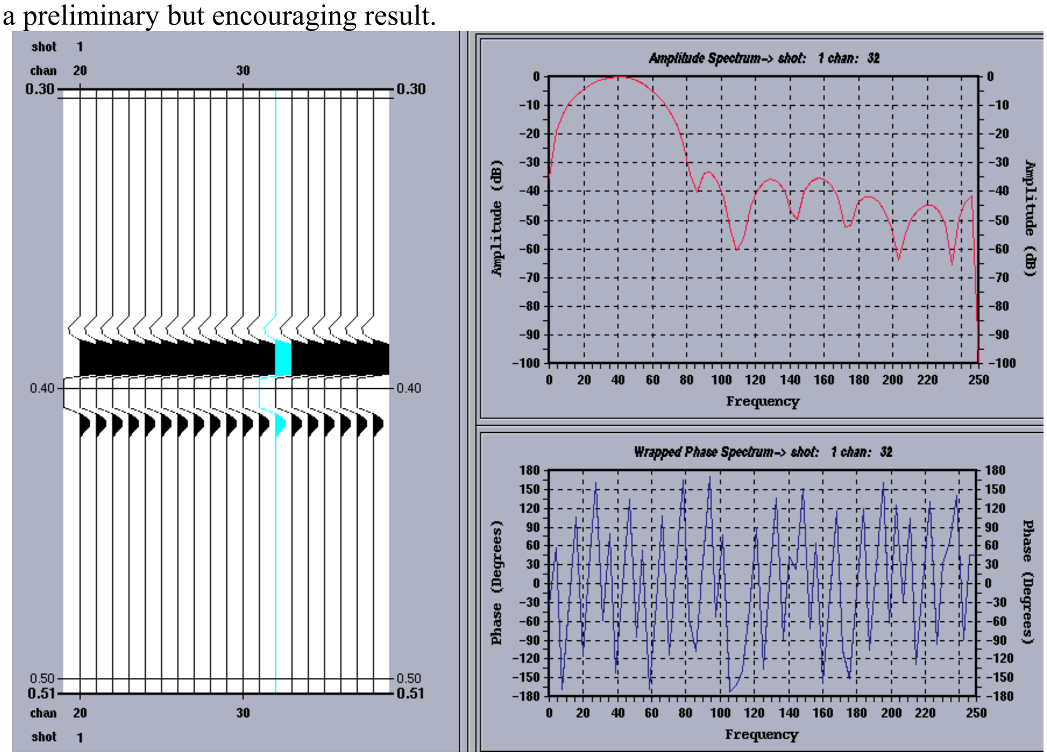

Figure 3.4 Synthetic shot record for a source depth of 5 m and a cable depth of 7 m. Compare with the predicted wavefield (Figure 3.3).