Mission - Oriented Seismic Research Program

Progressing the development of inverse scattering series direct, non-linear Q compensation and estimation procedures

Extending linear inversion to multidimensional media

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M-OSRP Annual Meeting, June 5-7, 2007

Annual Report pgs. 269-278

Key Points

The inverse scattering series can in principle (and in simple examples) correct for Q, directly and non-linearly in terms of data and a non-absorptive homogeneous reference medium

Non-linear theory/examples/methods can only be progressed to the level of completeness and complexity present in the associated linear inverse output

Arbitrary multi-dimensional distributions of c/Q may be determined to first order from multiple shot records of reflection seismic primary data

Recently Reported

The inverse scattering series is in principle accommodating of anelastic media (Weglein et al., 2003)

Forward scattering series studies have suggested that Q compensation of primaries is likely an (at least partially) separable inverse scattering series task

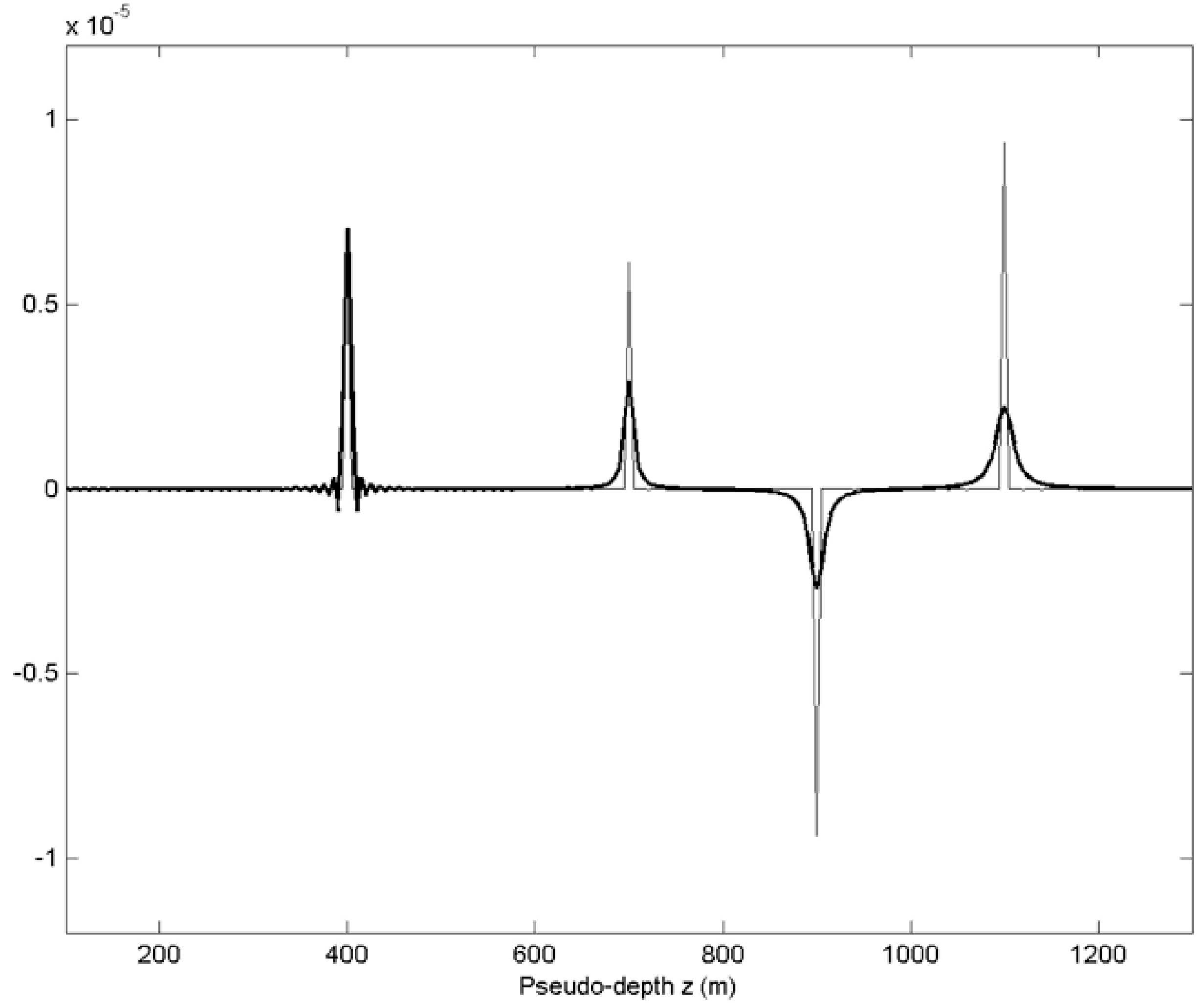
1D normal incidence tests of a candidate subseries are encouraging and reveal new inverse series activity and capability, but are within a highly idealized/simplified environment

Recently Reported

Summing an infinite number of primary processing terms that are judged to correct for absorptive propagation effects:

$$\beta_{LOQC}(z) = \sum_{n=0}^{\infty} \frac{(-1/2)^n}{n!} \beta_1^{(n)}(z) \left(\int_0^z \beta_1(z')dz' \right)^n$$
$$= \int_{-\infty}^{\infty} e^{ik\left[z - \frac{1}{2}\int_0^z \beta_1(z')dz'\right]} \beta_1(k)dk,$$

Recently Reported



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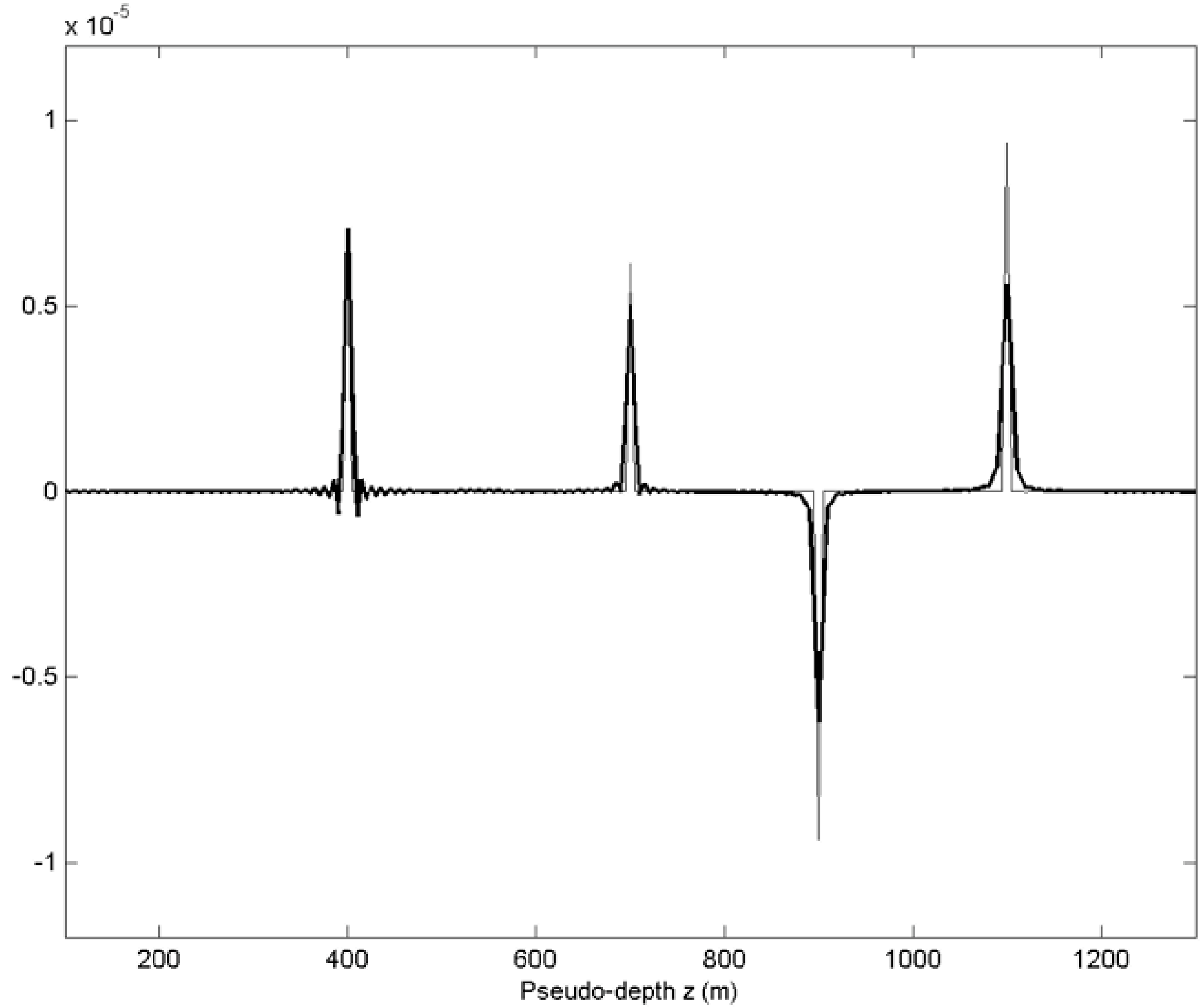


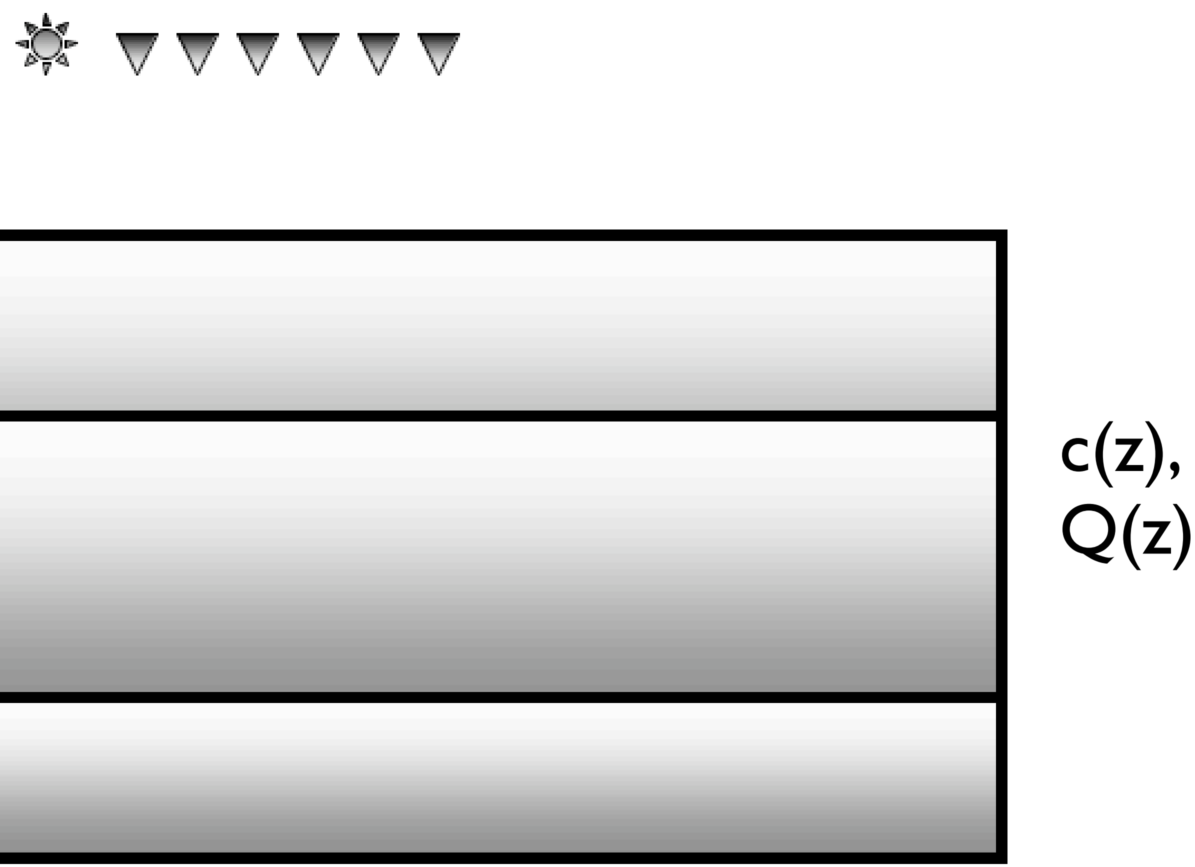
Non-linear and linear complexity/completeness

$$
\begin{aligned}
\psi &= \mathbf{G_0 V_1 G_0}, \\
0 &= \mathbf{G_0 V_2 G_0} + \mathbf{G_0 V_1 G_0 V_1 G_0}, \\
0 &= \mathbf{G_0 V_3 G_0} + \mathbf{G_0 V_1 G_0 V_2 G_0} + \mathbf{G_0 V_2 G_0 V_1 G_0} \\
&\quad + \mathbf{G_0 V_1 G_0 V_1 G_0 V_1 G_0},
\end{aligned}
$$

The linear inverse is the input to all higher order terms in the non-linear series and subseries. To attain a non-linear term or set of terms at a given level of complexity/completeness requires the linear term to be at at least that level.

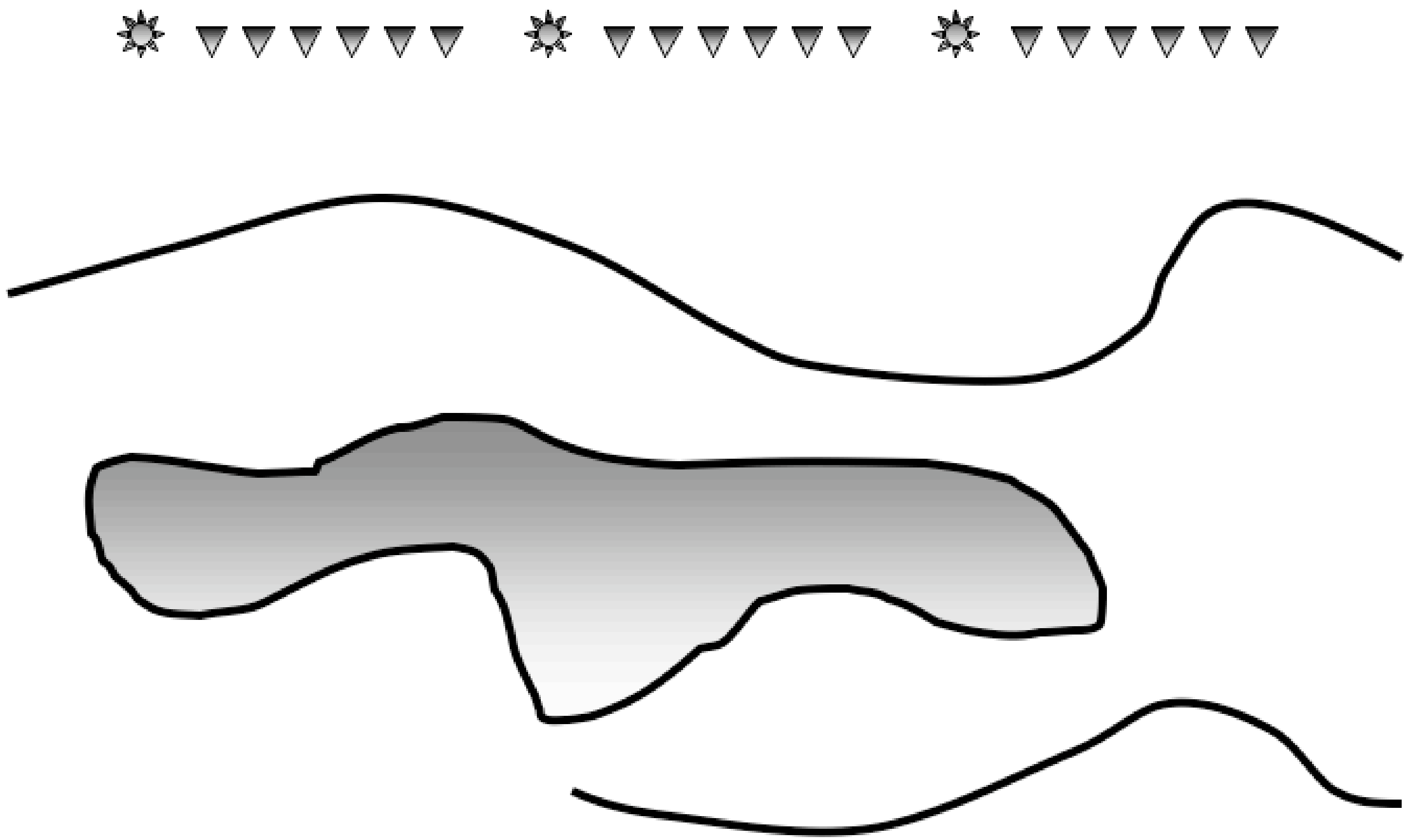
Previously

2-parameter, 1.5D medium, output provided by, e.g., a single shot record of reflected primary data



Currently

2-parameter, 2D medium (generalizing to 3D), output provided by multiple shot records of reflected primary data



c(x,z),
Q(x,z)

Approach

Linear inversion of primaries:

Cohen & Bleistein, Raz, Clayton & Stolt,
Stolt & Weglein, Beylkin, etc.

This development: follow the approach of Clayton and Stolt (1981).

Absorptive-dispersive scattering

Choose two equations, a reference and an actual:

$$\left[\nabla^2 + \frac{\omega^2}{c_0^2}\right] G_0(\mathbf{x}|\mathbf{x_s};\omega) = \delta(\mathbf{x} - \mathbf{x_s})$$

$$\left[\nabla^2 + K^2\right] G(\mathbf{x}|\mathbf{x_s};\omega) = \delta(\mathbf{x} - \mathbf{x_s})$$

In which the actual medium involves propagation in a constant Q medium (e.g., Aki and Richards, Kjartansson):

$$K \equiv \frac{\omega}{c(\mathbf{x})}\left[1 + \frac{F(\omega)}{Q(\mathbf{x})}\right] \qquad F(\omega) = \frac{i}{2} - \frac{1}{\pi}\ln\left(\frac{\omega}{\omega_r}\right)$$

IMPORTANT: we assume Q to be unknown, and F to be known.

Aside: review of Q

Q models arise from an attempt to quantitatively account for the transformation of wave energy into heat.

Typically developed through an alteration of the elastic constitutive (i.e., stress-strain) relations.

Three requirements:

- linear constitutive relations
- Q that is maximally independent of frequency
- causal wave response

underlie the best-accepted seismic Q models

Aside: review of Q

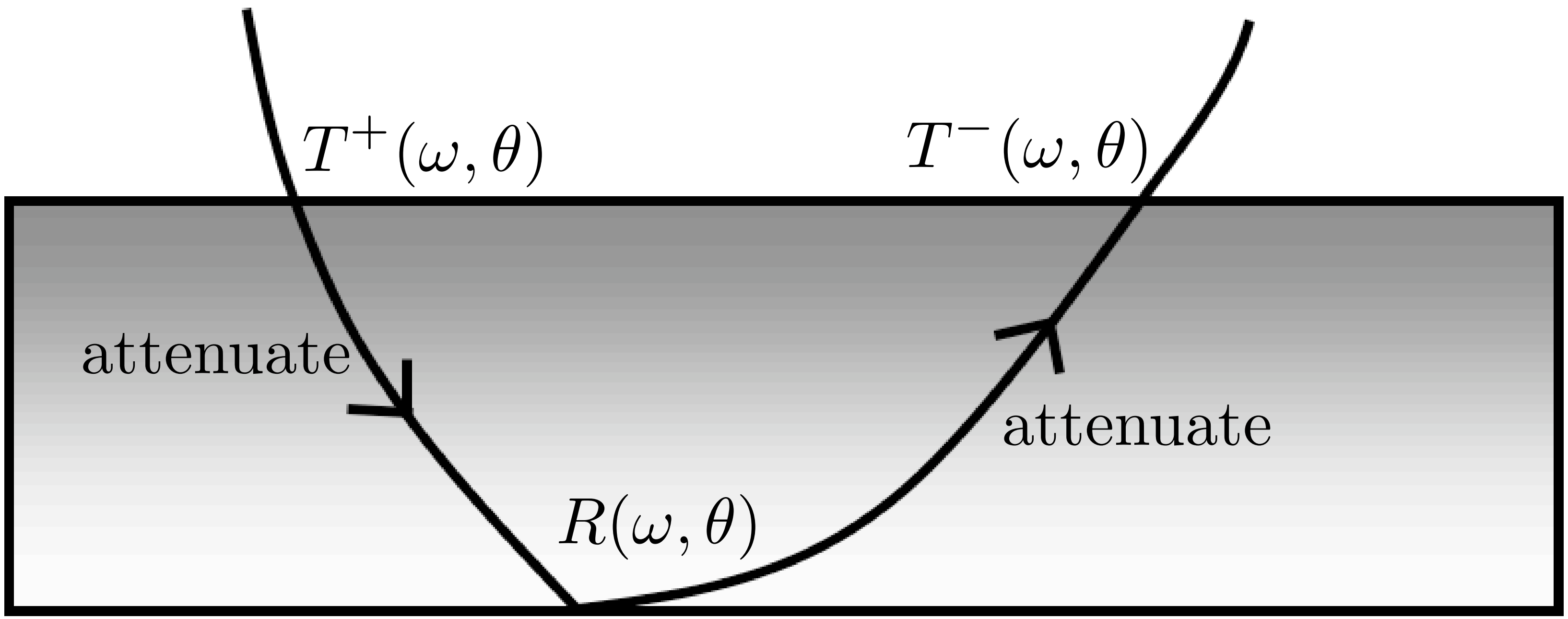
If the constitutive relations change, the elastic moduli and wave speeds change, becoming complex and frequency dependent.

$$k = \frac{\omega}{c} \rightarrow K = \frac{\omega}{c}\left[1 + \frac{i}{2Q} - \frac{1}{\pi Q}\log\left(\frac{\omega}{\omega_r}\right)\right]$$

propagation absorption dispersion

This in turn leads to a number of changes to our interpretation of an event.

Aside: review of Q



Plane wave transmission is generalized, non-local; amplitude and phase are changed by propagation, reflection coefficients are complex, frequency dependent.

Back to absorptive-dispersive scattering

Choose two equations, a reference and an actual:

$$\left[\nabla^2 + \frac{\omega^2}{c_0^2}\right] G_0(\mathbf{x}|\mathbf{x_s};\omega) = \delta(\mathbf{x} - \mathbf{x_s})$$

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IMPORTANT: we assume Q to be unknown, and F to be known.

Absorptive-dispersive scattering

The scattering potential, or perturbation is then:

$$V = L_0 - L$$

$$\equiv \frac{\omega^2}{c_0^2}\left[\alpha(\mathbf{x}) - 2F(\omega)\beta(\mathbf{x})\right]$$

where $\alpha(\mathbf{x}) = 1 - \dfrac{c_0^2}{c^2(\mathbf{x})}$ and $\beta(\mathbf{x}) = \dfrac{1}{Q(\mathbf{x})}$

And we work to compute the linear component of V from the data, using

$$D = G_0 V_1 G_0$$

Linear inversion

The full linear data equations are

$$D''(k_g, k_s, \omega) = S(\omega) \int d\mathbf{x}' G_0(k_g, z_g | \mathbf{x}'; \omega) V_1(\mathbf{x}') G_0(\mathbf{x}' | k_s, z_s; \omega)$$

or, after substituting in the form of the perturbation and the (homogeneous acoustic) Green's functions:

$$D'(k_g, k_s, \omega) = -\frac{\omega^2}{4c_0^2 q_g q_s} \left[\alpha_1(k_g - k_s, -q_g - q_s) - 2F(\omega)\beta_1(k_g - k_s, -q_g - q_s)\right]$$

where D' is the primary data D'' corrected for s/r depths and the wavelet:

$$D'(k_g, k_s, \omega) = e^{i(q_g z_g + q_s z_s)} \frac{D''(k_g, k_s, \omega)}{S(\omega)}$$

Linear inversion

The C&S (1981) approach involves a change of variables:

$$k_m = k_g - k_s, \quad k_h = k_g + k_s, \quad q_z = -q_g - q_s$$

when this change is made, the data equations finally become:

$$\alpha_1(k_m, q_z) - 2F(k_m, k_h, q_z)\beta_1(k_m, q_z) = D(k_m, k_h, q_z)$$

where

$$F(k_m, k_h, q_z) = \frac{i}{2} - \frac{1}{\pi}\ln\left(\frac{\omega(k_m, k_h, q_z)}{\omega_r}\right)$$

and we have implemented one last "preprocessing" alteration to the data:

$$D(k_m, k_h, q_z) = -4\frac{q_z^4 - k_m^2 k_h^2}{(q_z^2 + k_m^2)(q_z^2 + k_h^2)} D'(k_m, k_h, q_z)$$

The requirement

$$\alpha_1(k_m, q_z) - 2F(k_m, k_h, q_z)\beta_1(k_m, q_z) = D(k_m, k_h, q_z)$$

The linear relationship must produce, for each k_h, and each required pair of model spectrum values (q_z,k_m), an independent equation.

In matrix form

Let us express this requirement in terms of a matrix equation. For N discrete values of k_h, we have:

$$\mathbf{W_h}(k_m, q_z) \left[\begin{array}{c} \alpha_1(k_m, q_z) \\ \beta_1(k_m, q_z) \end{array} \right] = \mathbf{d}(k_m, q_z)$$

where

$$\mathbf{W_h}(k_m, q_z) = \left[\begin{array}{cc} 1 & -2F(k_m, k_{h_1}, q_z) \\ 1 & -2F(k_m, k_{h_2}, q_z) \\ \vdots & \vdots \\ 1 & -2F(k_m, k_{h_N}, q_z) \end{array} \right] \qquad \mathbf{d}(k_m, q_z) = \left[\begin{array}{c} D(k_m, k_{h_1}, q_z) \\ D(k_m, k_{h_2}, q_z) \\ \vdots \\ D(k_m, k_{h_N}, q_z) \end{array} \right]$$

In matrix form

Assuming N>2, a least-squares solution is

$$\left[\begin{array}{c}\alpha_1\\ \beta_1\end{array}\right]_{\mathrm{lsq}} = \left(\mathbf{W_h}^T\mathbf{W_h}\right)^{-1}\mathbf{W_h}^T\mathbf{d}$$

The term in brackets is invertible if the columns of W_h are linearly independent. The first column is constant, so the second may not be.

Therefore, $F(k_m, k_h, q_z)$ must vary with k_h.

Issues

The function F does vary with k_h, so in principle the 2-parameter linear inversion is well-posed for a multidimensional medium.

However:

The degree of variability at any particular q_z, k_m is important, and currently under-examined.

Leakage: important if giving thought to stand-alone use of this as a Q estimation procedure.

Detectability: where is the c/Q information coming from?

Detectability

Linear c/Q inversion uses information from the event amplitude. This is a major departure from standard approaches to Q estimation.

Recently and independently suggested as a means for standard inversion for A-D medium parameters by Lam, A.T. de Hoop et al.

Raises practical issues. Can the absorptive-dispersive $R(q_z, \theta)$ be detected in the data?

Summary

Non-linear inverse scattering series methods extendable only to the level achieved by the corresponding linear step.

1.5D case has been extended to accommodate 2D (3D) media.

Data interrogation: highly unusual form of Q estimation... detectability?

Ongoing research:

- Numerical testing with synthetics: 1.5D, 2D + data prep.
- Theoretical development of candidate subseries
- Determine data requirements in theory + reconcile with current and/or near term available data

Acknowledgments

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Acknowledgments

Landmark / Halliburton

Petrobras

PetroChina

PGS
Petroleum Geo-Services

Saudi Aramco

Shell

Statoil

WesternGeco

U.S. Federal Government Research Support

B.E.S.

United States Department of Energy
Basic Energy Sciences

C.M.G.

National Science Foundation
Collaboration in Mathematical Geosciences