M-OSRP
Mission - Oriented Seismic Research Program

Towards velocity independent collapsing of diffractions

Early stage concepts and approximations

Kris Innanen

M-OSRP Annual Meeting, June 5-7, 2007

Annual Report pgs. 279-303

Key Points

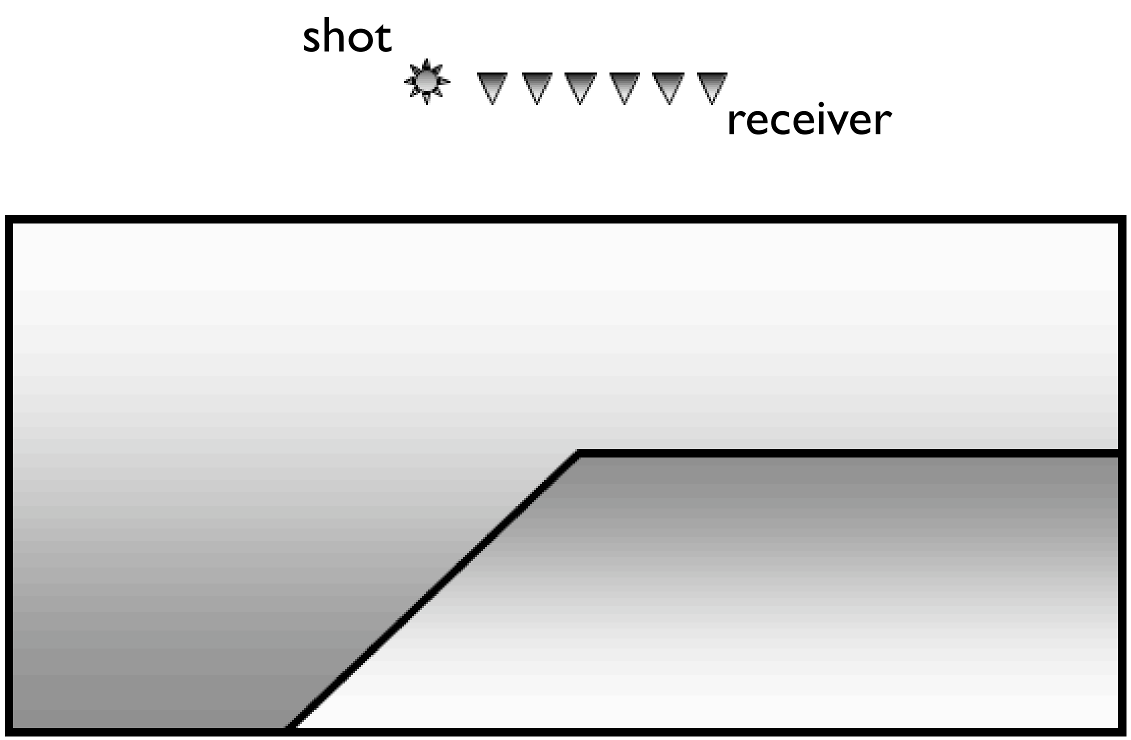
Liu et al., (2005-2007) present a 2D algorithm for non-linear inverse scattering series imaging that addresses imaging issues that have a 1D analog

Concurrent to this, we continue to attempt more complete treatment of multidimensional phenomena in wave data; diffractions are an important example

Investigate approaches, approximations, and trade-offs leading to (1) direct, non-linear imaging methods which collapse diffractions, and potentially (2) the ability to retain closed-forms

Diffractions

Distinct in reflected data in the presence of a discontinuity, i.e., here

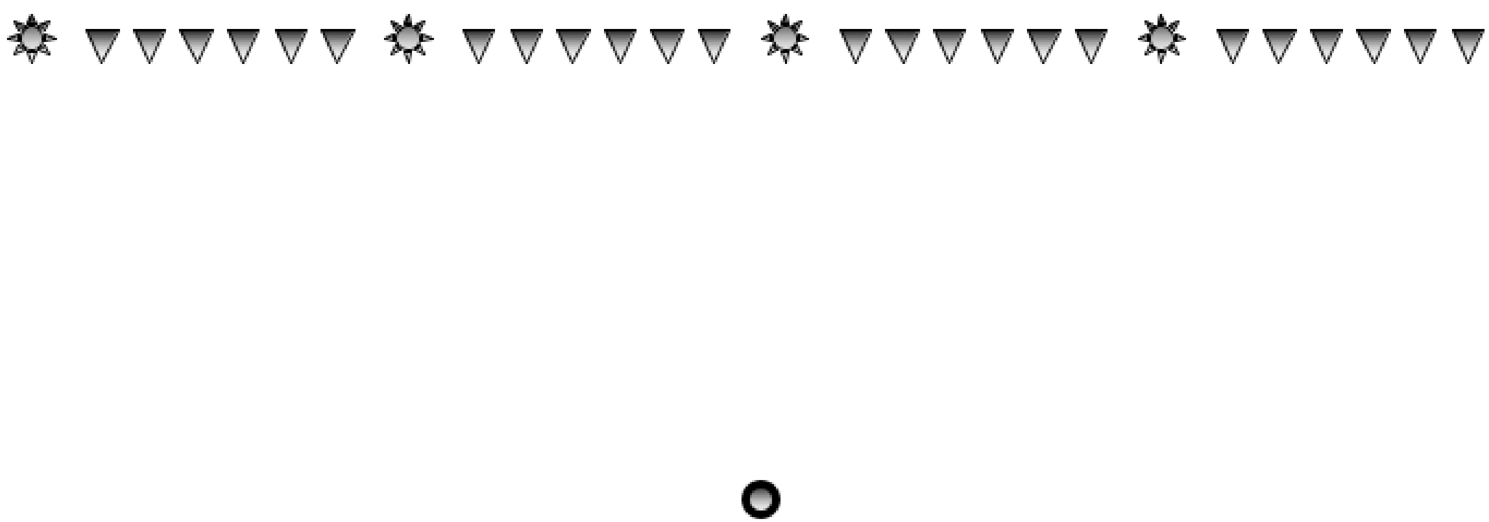


Diffractions

Distinct in reflected data in the presence of a discontinuity, i.e., but not here

Diffractions

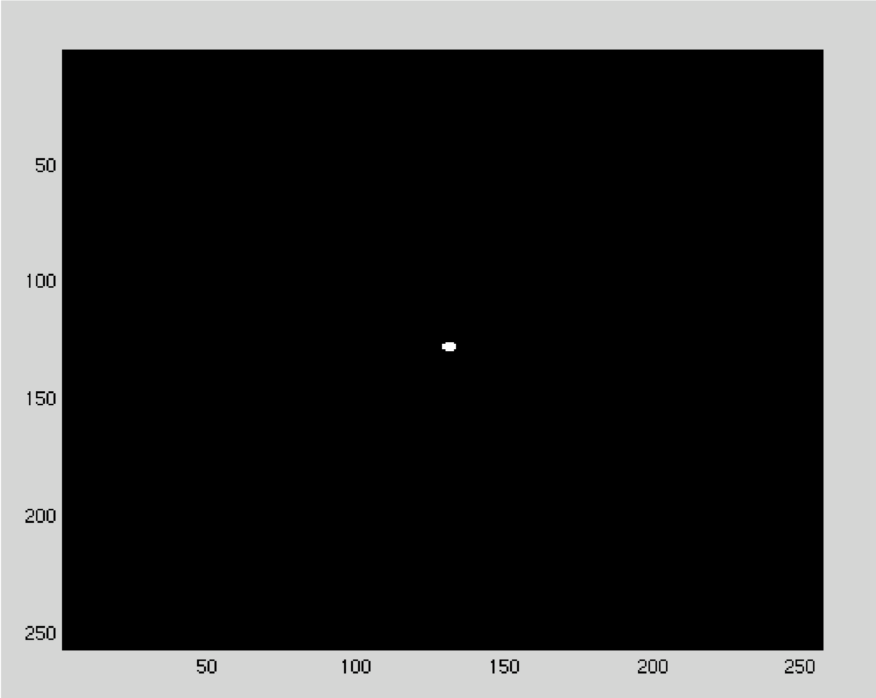
Consider the point scatterer as a basic model for making a diffraction.

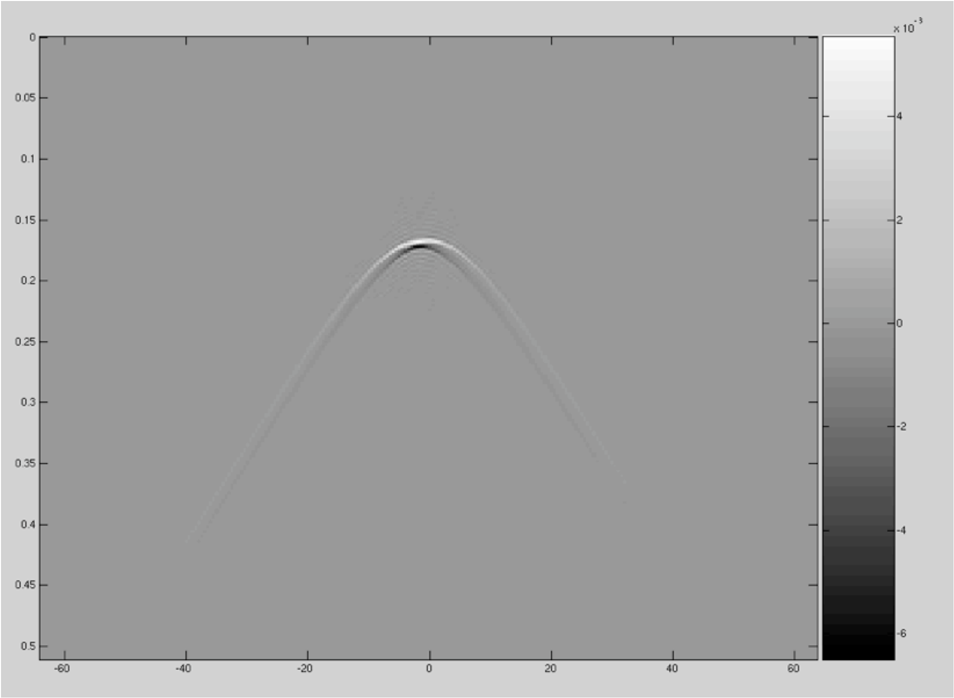


$$A\delta(\mathbf{x} - \mathbf{x}_A)$$

Diffractions

Numerically straightforward to examine:





Diffractions

Additionally permits an approximate but useful math model. Born approximation of the response of a single point scatterer embedded in a homogeneous wholespace:

$$D(k_g, z_g, k_s, z_s, \omega) \approx \int dx' \int dz' G_0(k_g, z_g, x', z', \omega)\omega^2/c_0^2 A\delta(x' - x_A, z' - z_A) G_0(x', z', k_s, z_s, \omega)$$

or

$$D(k_g, z_g, k_s, z_s, \omega) \approx -A\frac{k^2}{4q_g q_s} e^{-iq_g(z_g - z_A)} e^{-iq_s(z_s - z_A)}$$

where $q_g = \frac{\omega}{c_0}\sqrt{1 - \frac{k_g^2 c_0^2}{\omega^2}}$ and $q_s = \frac{\omega}{c_0}\sqrt{1 - \frac{k_s^2 c_0^2}{\omega^2}}$

Questions

Suppose the actual medium and the reference medium differ significantly? How does the forward scattering series act to construct diffracted primaries for large, sustained perturbations?

How do we translate any such insight into a capture of an inverse scattering series non-linear, direct processing procedure with the wherewithal to collapse diffractions?

What are the opportunities for, and costs of, retaining closed-forms?

Forward series and diffracted primaries

For perturbations of arbitrary heterogeneity, the separate construction of individual events can be approximately carried out.

For instance, a proposed approximation of the wave transmitting directly through an extended 3D perturbation reduces to the eikonal approximation when the perturbation varies in 1D only.

The fully 2D and 3D versions of such event approximations require a series.

Forward series and diffracted primaries

A proposed compromise (MOSRP06): suppose the perturbation to consist of a small transient 2D part and a large, sustained 1D part:

$$\alpha(x,z) = 1 - \frac{c_0^2}{c^2(x,z)}$$

Forward series and diffracted primaries

A proposed compromise (MOSRP06): suppose the perturbation to consist of a small transient 2D part and a large, sustained 1D part:

$$\alpha(x,z) = \underbrace{A(z)}_{\text{large, sustained}} + \underbrace{B(x,z)}_{\text{small, transient}}$$

Forward series and diffracted primaries

Schematically, such perturbations act within the series like this:

$$A + B$$

$$A^2 + AB + B^2$$

$$A^3 + A^2B + AB^2 + B^3$$

$$A^4 + A^3B + A^2B^2 + AB^3 + B^4$$

In the fully 1D case, the leftmost column speaks of A acting on itself non-linearly: $A + A^2 + A^3 + \ldots = (1+A+A^2+\ldots)A\ldots$ closed forms arise.

By extracting the second column, can we (1) approximate 2D wave phenomena while (2) maintaining 1D closed-forms?

Forward series and diffracted primaries

A program for approximating diffractive primaries:

1. Set up forward scattering series for a fully 2D scalar medium
2. Divide perturbation into A and B parts.
3. Retain terms which, by argument from scattering geometry, contribute to primaries only.
4. Retain terms linear in B(x,z) and non-linear in A(z).

Use homogeneous acoustic reference medium with wavespeed c_0, with associated analytic Green's functions and the previously defined perturbation...

Forward series and diffracted primaries

First few terms:

$$R_1^P(k_g, z_g, k_s, z_s, \omega) = -\frac{e^{-iq_g z_g - iq_s z_s}}{4}\frac{k^2}{q_g q_s}\int dz' e^{i(q_g+q_s)z'}\alpha(k_g - k_s, z')$$

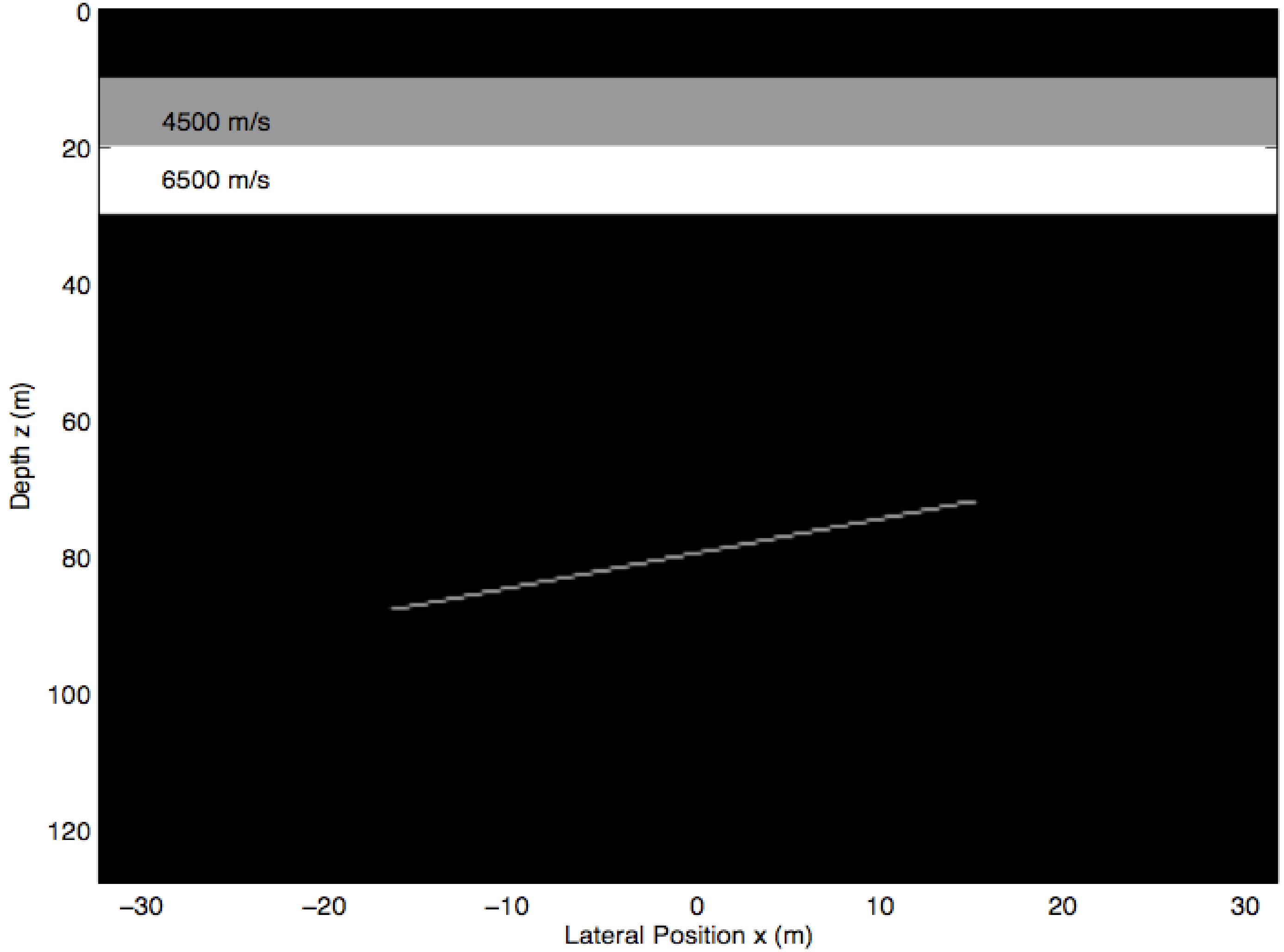
$$R_2^P(k_g, z_g, k_s, z_s, \omega) = -\frac{e^{-iq_g z_g - iq_s z_s}}{4}\frac{k^2}{q_g q_s}\int dz' e^{i(q_g+q_s)z'}\left(\frac{-iq_g}{2\cos^2\theta_g}\int_{z_g}^{z'} A(z'')dz''\right)\alpha(k_g - k_s, z')$$
$$-\frac{e^{-iq_g z_g - iq_s z_s}}{4}\frac{k^2}{q_g q_s}\int dz' e^{i(q_g+q_s)z'}\alpha(k_g - k_s, z')\left(\frac{-iq_s}{2\cos^2\theta_s}\int_{z_s}^{z'} A(z'')dz''\right)$$

Summing:

$$R^P(k_g, z_g, k_s, z_s, \omega) = \int dz' \frac{e^{iq_g\left[z' - z_g - \frac{1}{2\cos^2\theta_g}\int_{z_g}^{z'} A(z'')dz''\right]}}{i2q_g} k^2\alpha(k_g - k_s, z')\frac{e^{iq_g\left[z' - z_s - \frac{1}{2\cos^2\theta_s}\int_{z_s}^{z'} A(z'')dz''\right]}}{i2q_s}$$

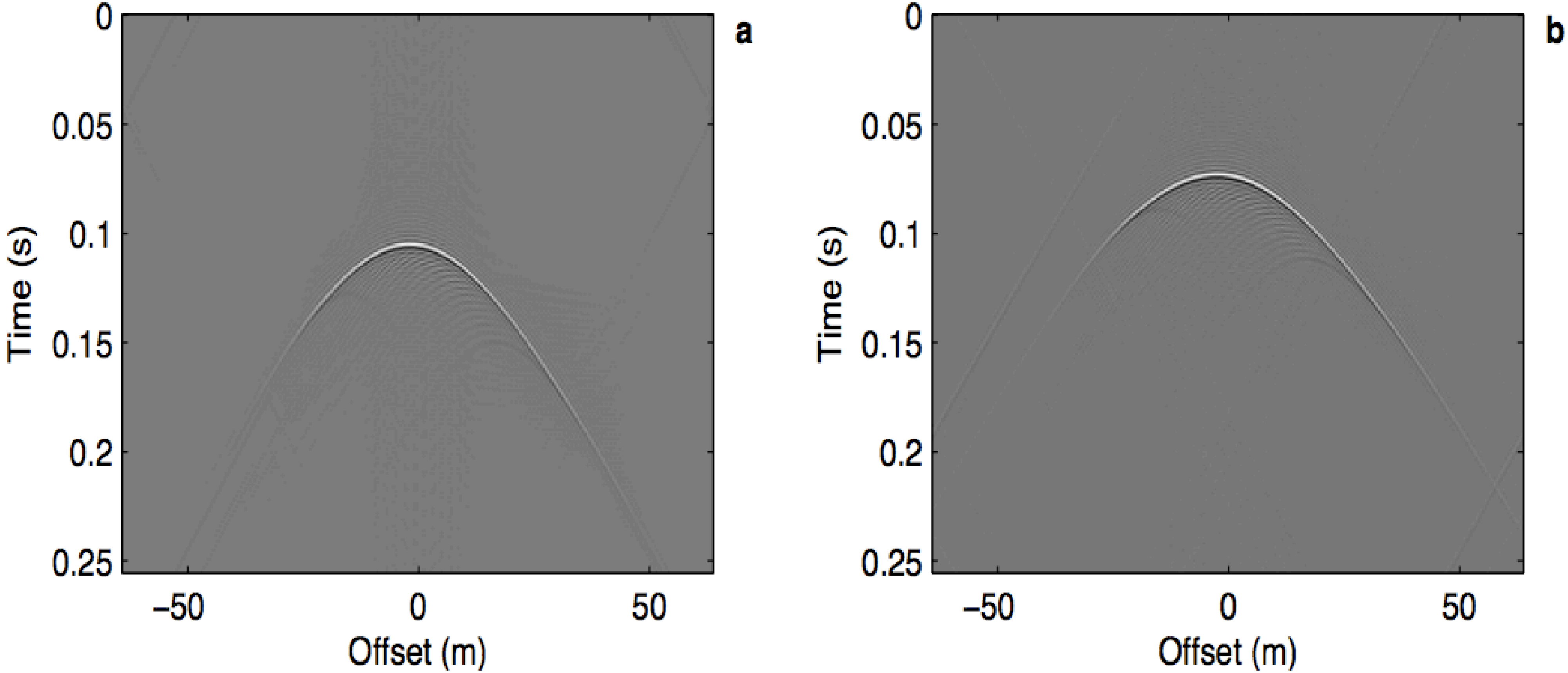
Forward series and diffracted primaries

Numerically appears to correctly incorporate a depth-dependent overburden in the perturbation:



Forward series and diffracted primaries

Numerically appears to correctly incorporate a depth-dependent overburden in the perturbation:



Born approximation

Nonlinear R^P approximation

A 1.5D inverse series approach

Using a forward series approximation of primaries as a framework to launch to inverse methods

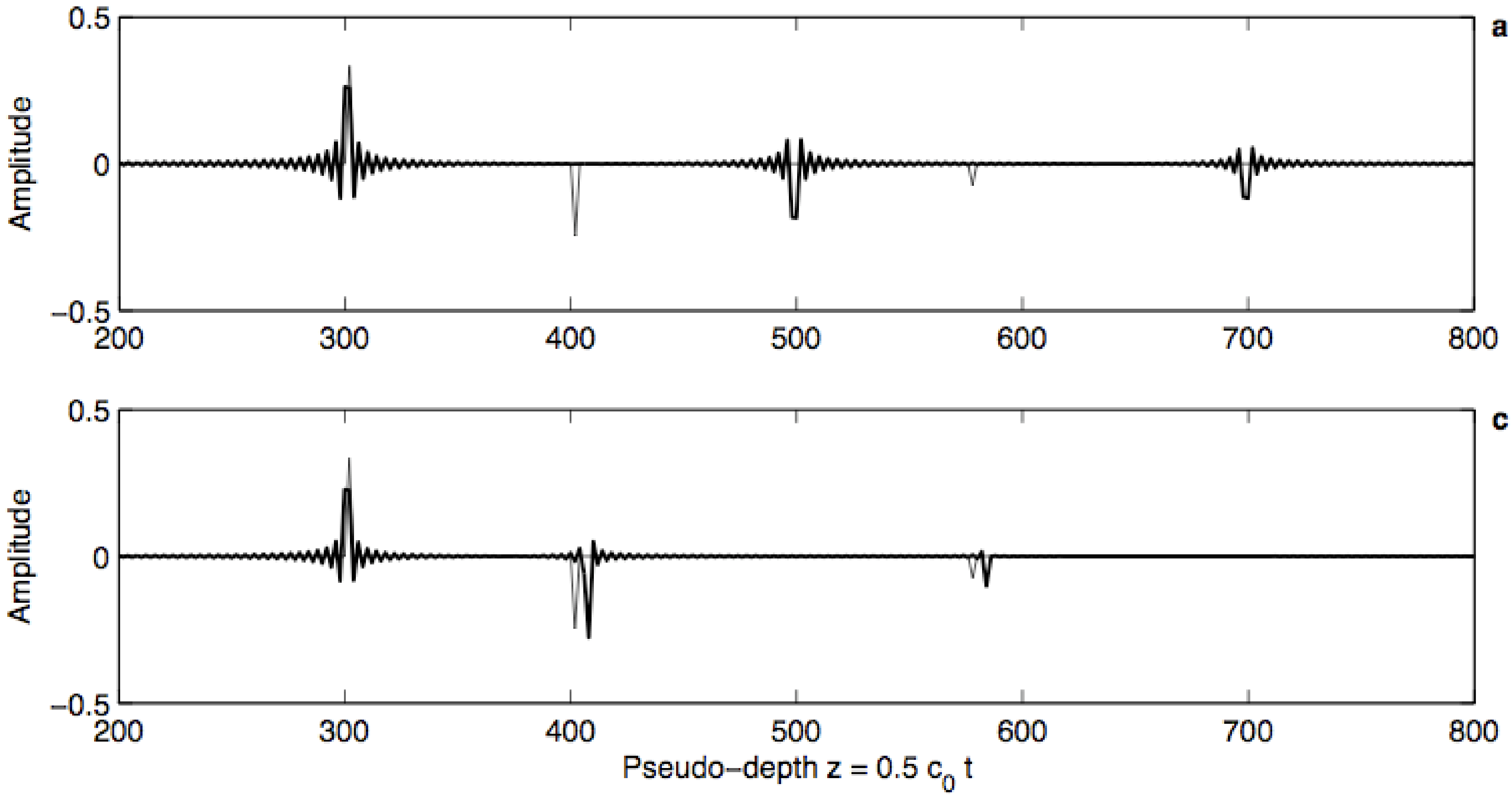
We can constrain the FSS in 1.5D to create approximations of primaries, and invert these approximations order-by-order (SEG 2005):

$$D_P(k_s, q_s, \theta) = -\frac{e^{ik_s x_g}}{4\cos^2\theta}\int dz' e^{i2q_s z'} \left\{ \sum_{n=0}^{\infty} C_n^{+} \left[\alpha(z) \left(\sum_{k=1}^{\infty} \frac{1}{4^{k-1}} \int_0^z dz' \alpha^k(z') \right)^n \right]^{(n)} \right\}$$

$$\alpha_L(z,\theta) = \sum_{n=0}^{\infty} \frac{(-1/2)^n}{n! \cos^{2n}\theta} \left[\alpha_1(z,\theta) \left(\sum_{k=1}^{\infty} \frac{1}{4^{k-1}} \int_0^z dz' \alpha_1^k(z',\theta) \right)^n \right]^{(n)}$$

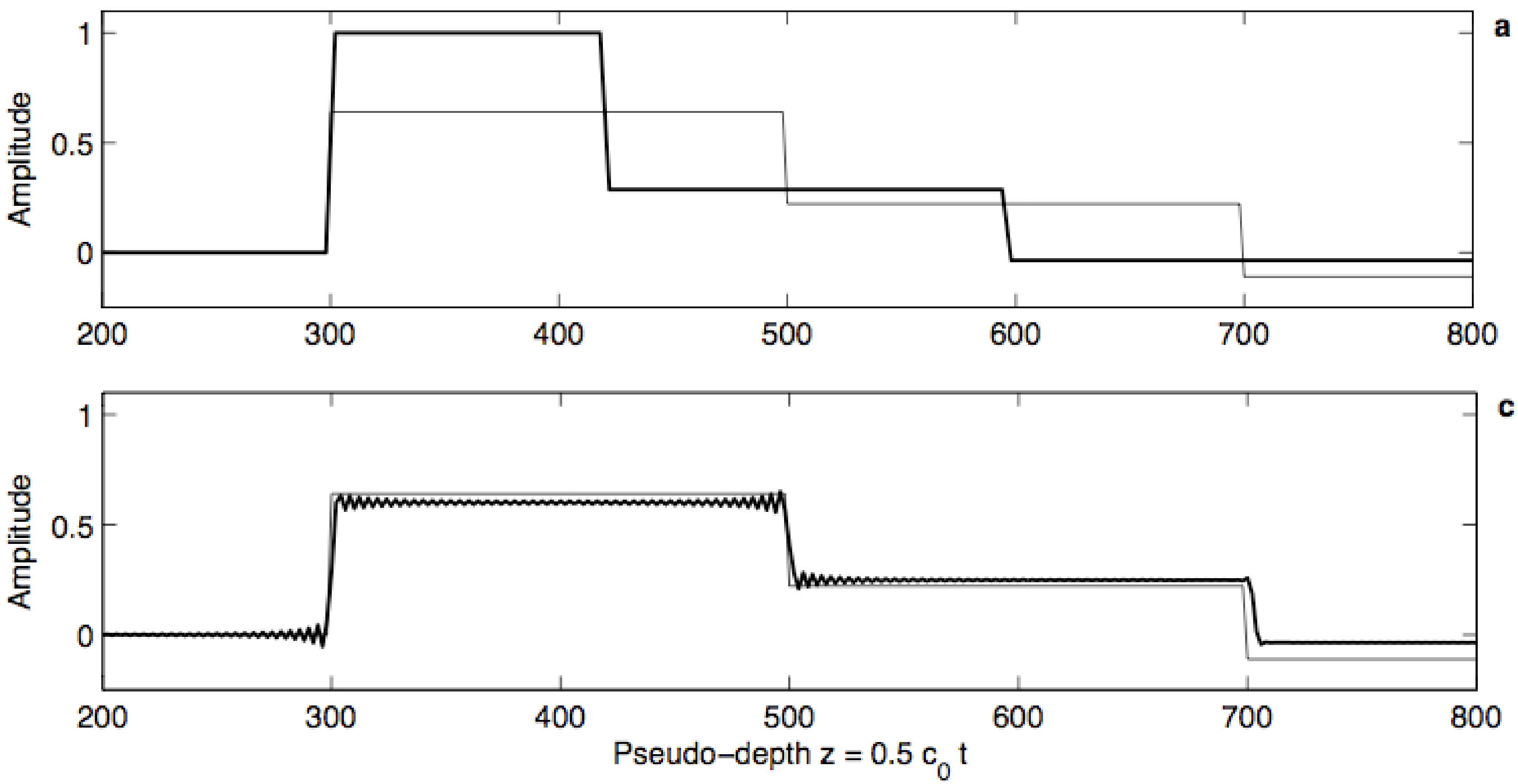
A 1.5D inverse series approach

$$D_P(k_s, q_s, \theta) = -\frac{e^{ik_s x_g}}{4\cos^2\theta}\int dz' e^{i2q_s z'}\left\{\sum_{n=0}^{\infty} C_n^+ \left[\alpha(z)\left(\sum_{k=1}^{\infty}\frac{1}{4^{k-1}}\int_0^z dz'\alpha^k(z')\right)^n\right]^{(n)}\right\}$$



A 1.5D inverse series approach

$$\alpha_L(z,\theta) = \sum_{n=0}^{\infty} \frac{(-1/2)^n}{n! \cos^{2n}\theta} \left[\alpha_1(z,\theta) \left(\sum_{k=1}^{\infty} \frac{1}{4^{k-1}} \int_0^z dz' \alpha_1^k(z',\theta) \right)^n \right]^{(n)}$$



Approach

1.5D forward scattering approximation of primaries linked to 1.5D non-linear series for target location and amplitude.

Link multidimensional forward scattering series approximations of primaries to non-linear subseries for target location and amplitude?

In particular at this stage, do so for the approximate form that has permitted closed-forms to be retained?

Inverting the diffraction approximation

We have the series for primaries diffracting from small, transient 2D target perturbation embedded within a large, sustained 1D overburden perturbation. Expand:

$$D(k_g, z_g, k_s, z_s, \omega) = -\frac{e^{-iq_g z_g - iq_s z_s}}{4} \frac{k^2}{q_g q_s} \int dz' e^{i(q_g+q_s)z'} \alpha(k_g - k_s, z')$$

$$\times \left[1 + \left(\frac{-iq_g}{2\cos^2\theta_g} \int_{z_g}^{z'} A(z'')dz'' \right) + \left(\frac{-iq_s}{2\cos^2\theta_s} \int_{z_s}^{z'} A(z'')dz'' \right) \right.$$

$$+ \frac{1}{2!} \left(\frac{-iq_g}{2\cos^2\theta_g} \int_{z_g}^{z'} A(z'')dz'' \right)^2 + \left(\frac{-iq_g}{2\cos^2\theta_g} \int_{z_g}^{z'} A(z'')dz'' \right) \left(\frac{-iq_s}{2\cos^2\theta_s} \int_{z_s}^{z'} A(z'')dz'' \right)$$

$$\left. + \frac{1}{2!} \left(\frac{-iq_s}{2\cos^2\theta_s} \int_{z_s}^{z'} A(z'')dz'' \right)^2 + \ldots \right]$$

Inverting the diffraction approximation

Setting the perturbation equal to a series in orders of the primary data, we equate like orders in the normal fashion.

The linear relationship gives rise to the linear input to all higher order terms (Liu, this meeting):

$$\alpha_1(k_m, k_z|k_h) = -4c_0^2 e^{iq_g z_g + iq_s z_s} \frac{q_g q_s}{\omega^2(k_m, k_h, k_z)} D(k_m, k_h, k_z)$$

...which is then inverse Fourier transformed to the z, k_m, domain, and treated for fixed values of k_h.

Inverting the diffraction approximation

Expressing the higher order terms as operations on the linear term, and summing several of them produces:

$$\sum_{n=0}^{\infty} \alpha_{n+1}(k_m, k_z|k_h) = \int dz' e^{i(q_g+q_s)z'} \alpha_1(k_m, z'|k_h) \left[\sum_{l=0}^{\infty} \frac{1}{l!} \left(\frac{iq_g}{2\cos^2\theta_g} \int_{z_g}^{z'} \alpha_1(k_m, z''|k_h) dz'' \right)^l \right]$$
$$\times \left[\sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{iq_s}{2\cos^2\theta_s} \int_{z_s}^{z'} \alpha_1(k_m, z''|k_h) dz'' \right)^m \right]$$

Inverting the diffraction approximation

Calling the sum on the LHS the output, and summing the two series on the RHS (which follow patterns similar to the 1.5D case):

$$\alpha_{II}(k_m, k_z|k_h) = \int dz' e^{iq_g\left[z'+Z(z_g,z',\theta_g,\alpha_1)\right]}\alpha_1(k_m, z'|k_h)e^{iq_s\left[z'+Z(z_s,z',\theta_s,\alpha_1)\right]}$$

where

$$Z(z, z', \theta, \alpha_1) = \frac{1}{2\cos^2\theta}\int_z^{z'} \frac{\alpha_1(k_m, z''|k_h)}{1 - 0.25\alpha_1(k_m, z''|k_h)}dz''$$

Plan

Synthetic testing

Dovetail with Liu's expertise in fast/accurate linear inversion

Parametrization: k_h? p_h?

Extension to the 2D overburden case

closed forms no longer likely

Connection to non 1D-analog terms at second order (Liu, 2006)

Key Points

Liu et al., (2005-2007) present a 2D algorithm for non-linear inverse scattering series imaging that addresses imaging issues that have a 1D analog

Concurrent to this, we continue to attempt more complete treatment of multidimensional phenomena in wave data; diffractions are an important example

Investigate approaches, approximations, and trade-offs leading to (1) direct, non-linear imaging methods which collapse diffractions, and potentially (2) the ability to retain closed-forms

Acknowledgments

SPONSORS

Anadarko Petroleum Corporation

bhpbilliton

British Petroleum

Chevron

ConocoPhillips

devon

Eni

ExxonMobil

Geotrace

GX Technology

Hess Corporation

IBM
International Business Machines Corporation

Acknowledgments

Landmark / Halliburton

Petrobras

PetroChina

PGS
Petroleum Geo-Services

Saudi Aramco

Shell

Statoil

WesternGeco

U.S. Federal Government Research Support

B.E.S.

United States Department of Energy
Basic Energy Sciences

C.M.G.

National Science Foundation
Collaboration in Mathematical Geosciences