Adaptive subtraction for free surface multiples using ICA: Review, updates and example

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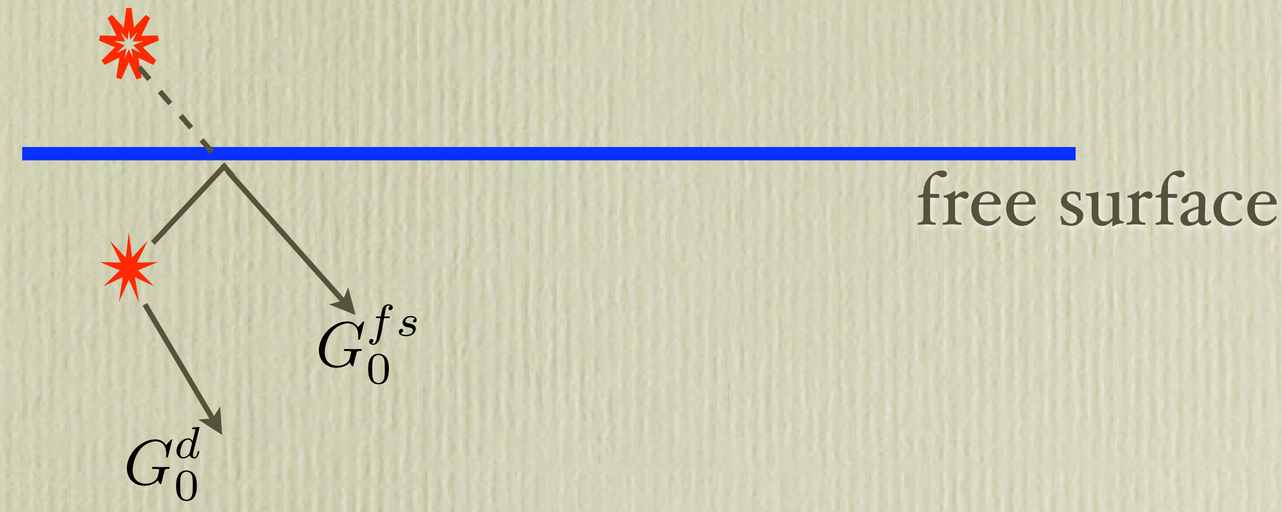
Topics

- Free surface multiple elimination (FSME)
- Adaptive subtraction
  - Matching filters
  - Independent component analysis
- Gulf of Mexico example, comparison to two-pass matching filter algorithms.

Since last year

- Reference filters: constraining matching filters using adjacent traces.
- Apply matching filters to FSME predictions, and not to the data.
- Gulf of Mexico example.

The free surface



$$\mathcal{L}\{G_0, V_0\} = f(t)\delta(t)\delta(x - x_s)[\delta(z - z_s) - \delta(z + z_s)]$$

$$G_0 = G_0^d + G_0^{fs}$$

Free surface multiple elimination

$$D_1 = G_0^d V_1 G_0^d$$

$$\psi_s^{fs} = G_0^d V_1 G_0^{fs} V_1 G_0^d + G_0^d V_1 G_0^{fs} V_2^{fs} G_0^d + \cdots$$

D_2 ← data-data interaction

D_3 ← data-data-data interaction

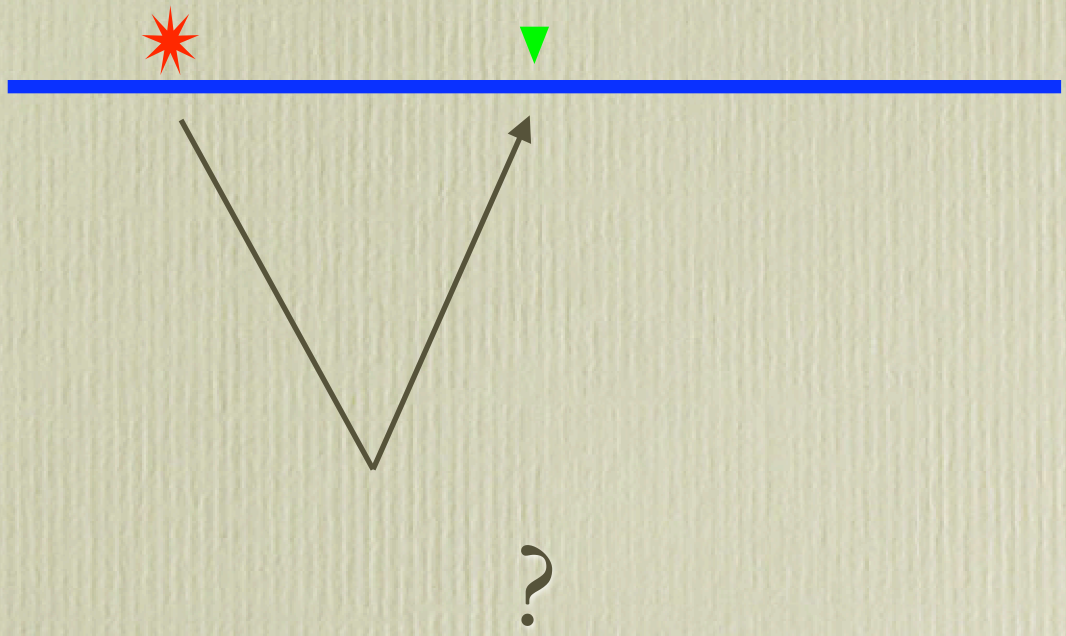
(Carvalho and Weglein)

First order

?

First order multiple is a combination of two primary events.
(data-data interaction, i.e. autocorrelation)

First order



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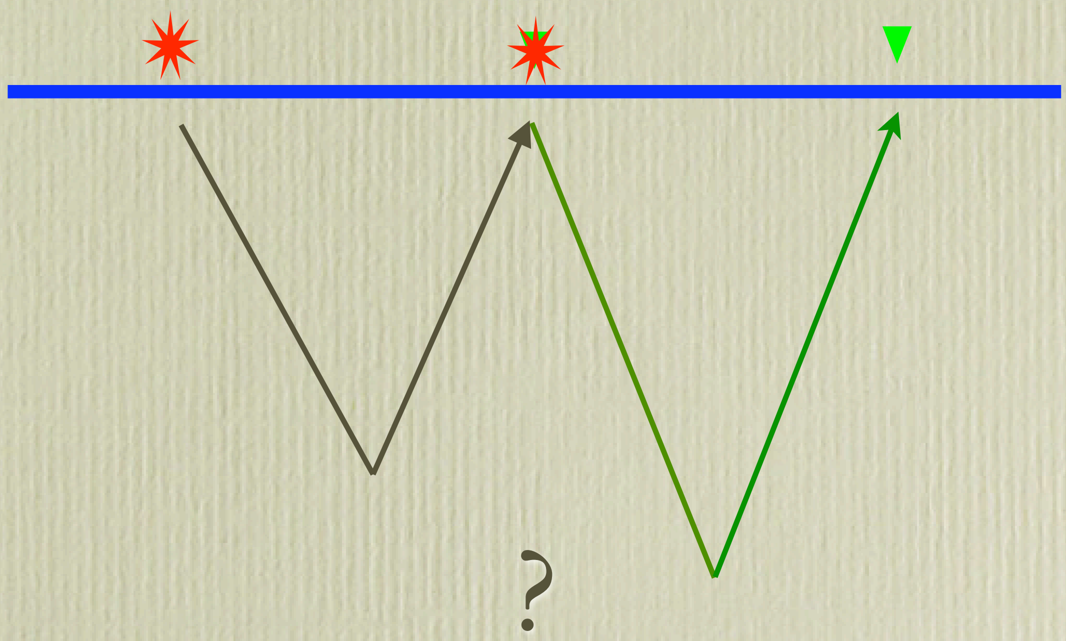
First order



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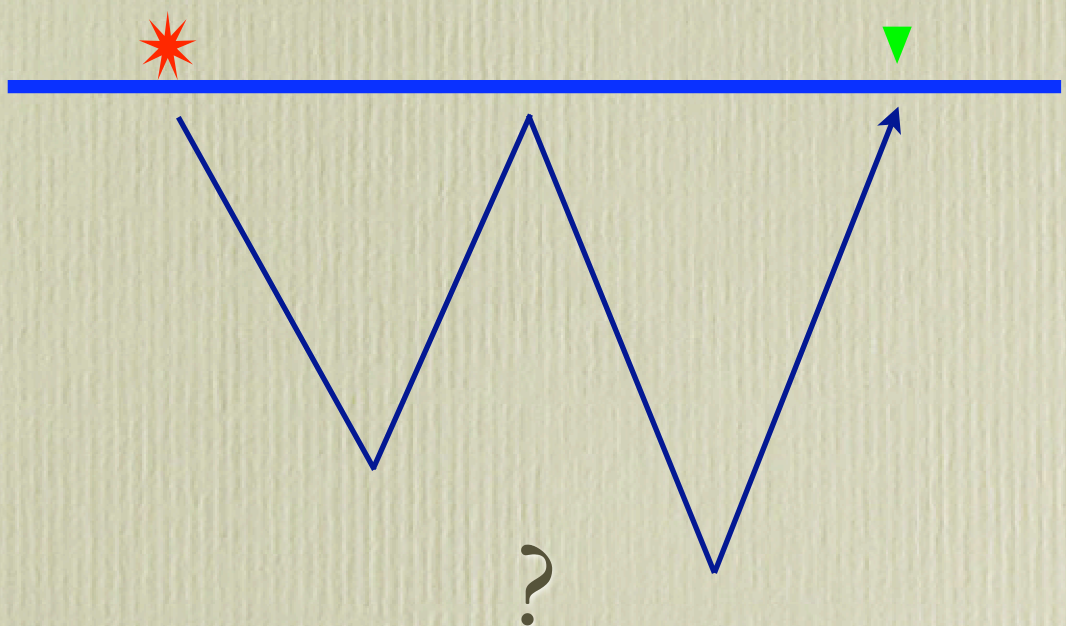

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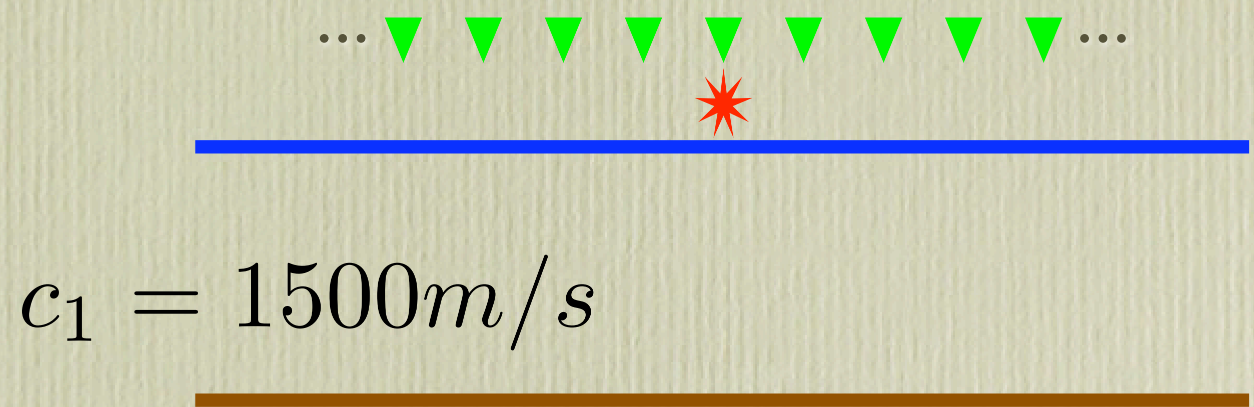
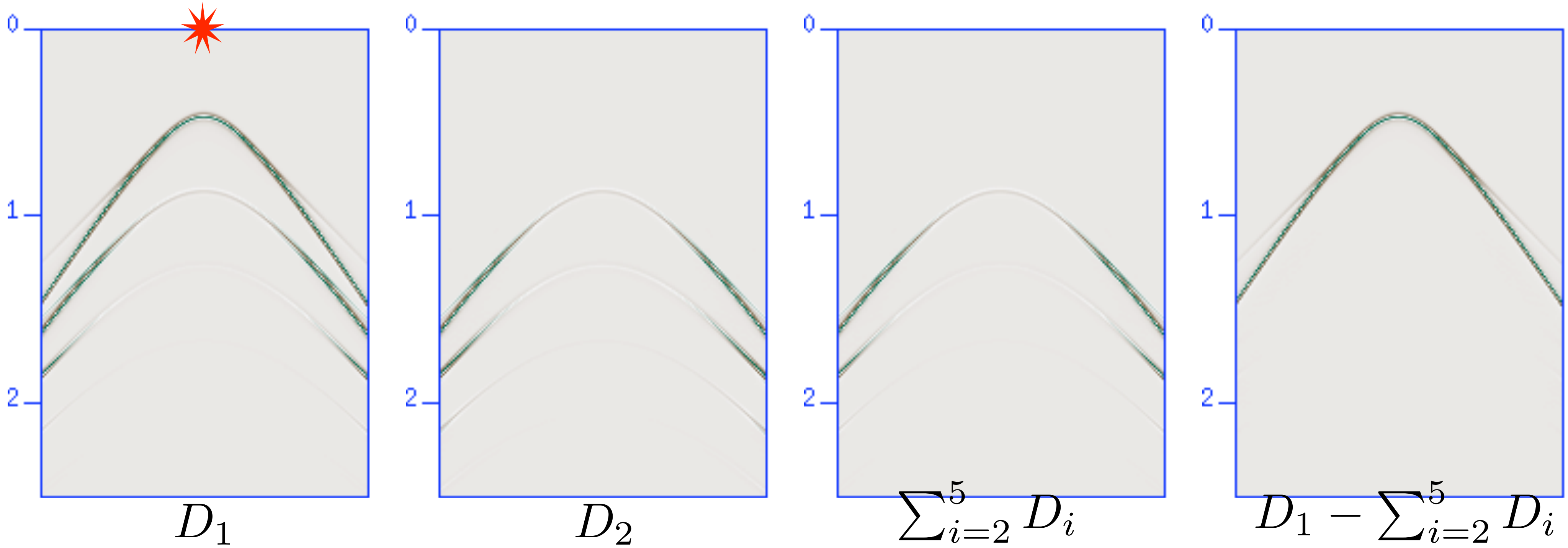
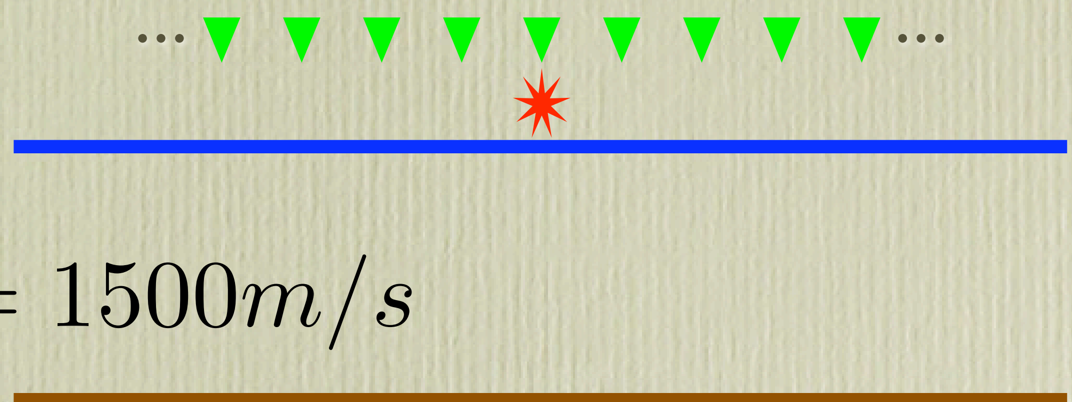
Toy example

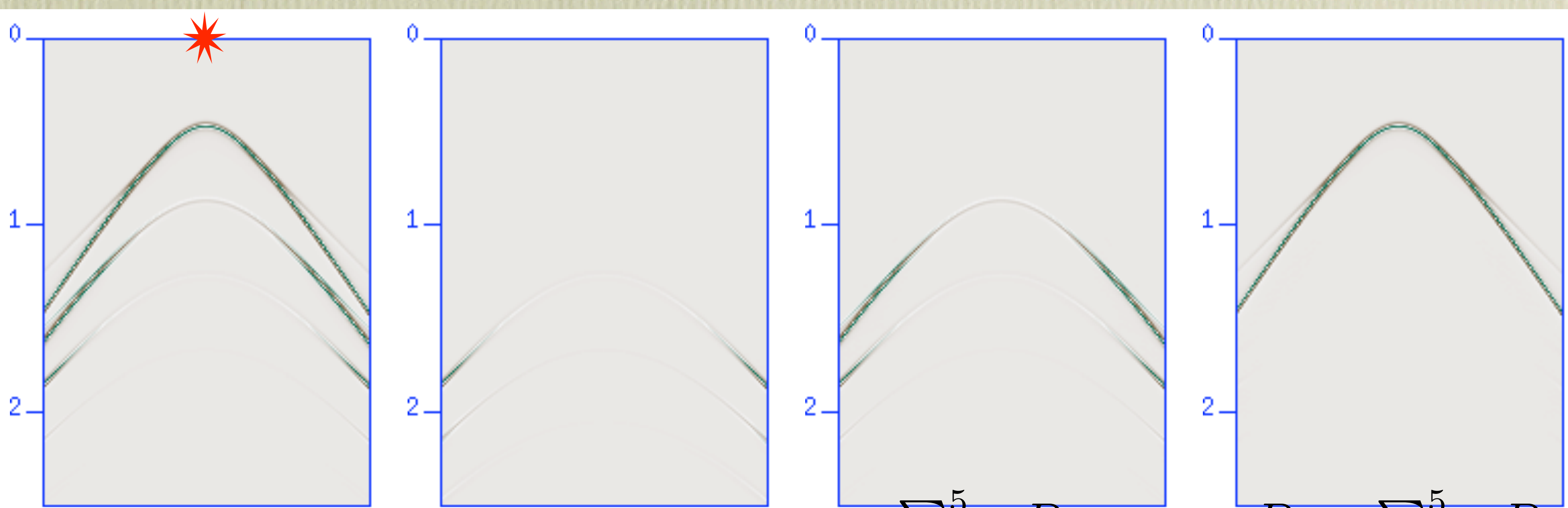




Toy example

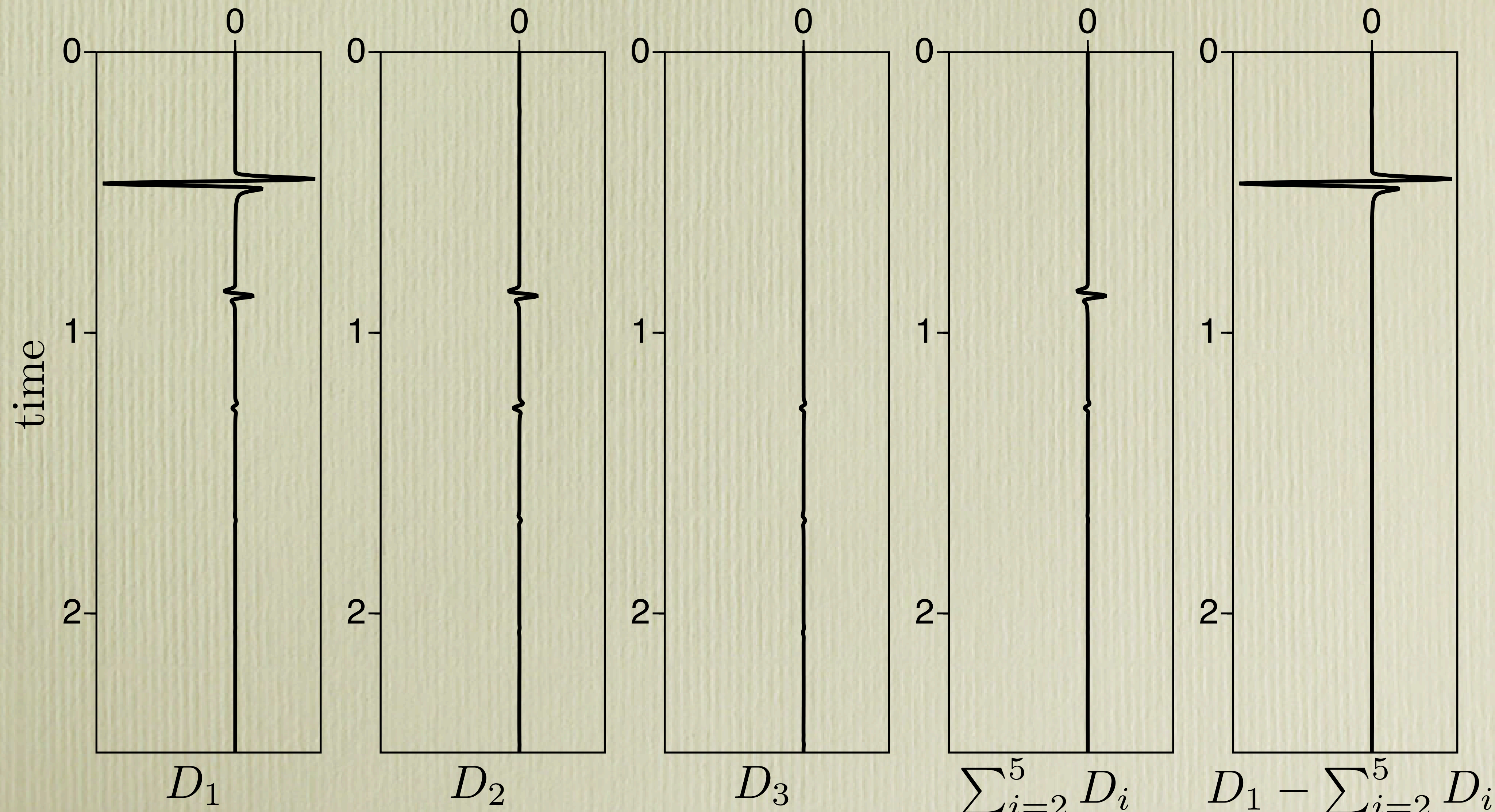


$c_1 = 1500m/s$



D_1 $\quad$ D_3 $\quad$ $\sum_{i=2}^{5} D_i$ $\quad$ $D_1 - \sum_{i=2}^{5} D_i$

Zero offset trace



But,

The preceding result required information:

ghosts

source wavelet

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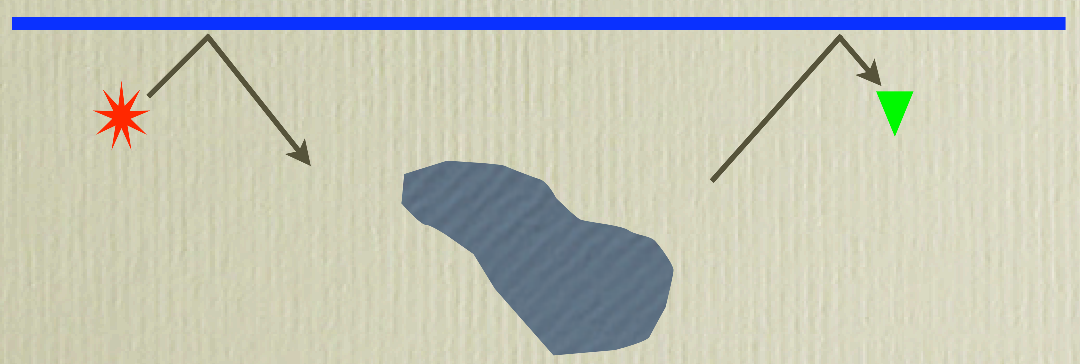

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The preceding result required information:

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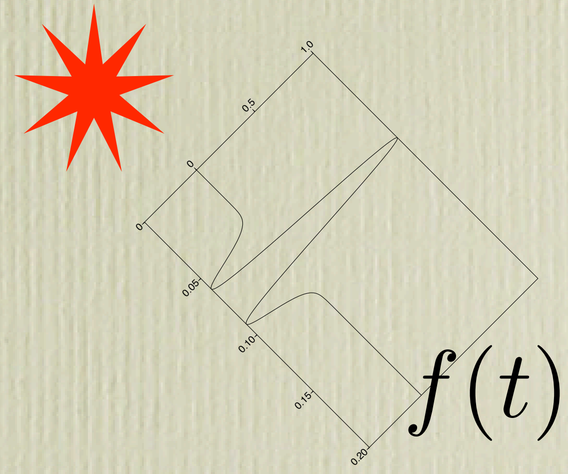
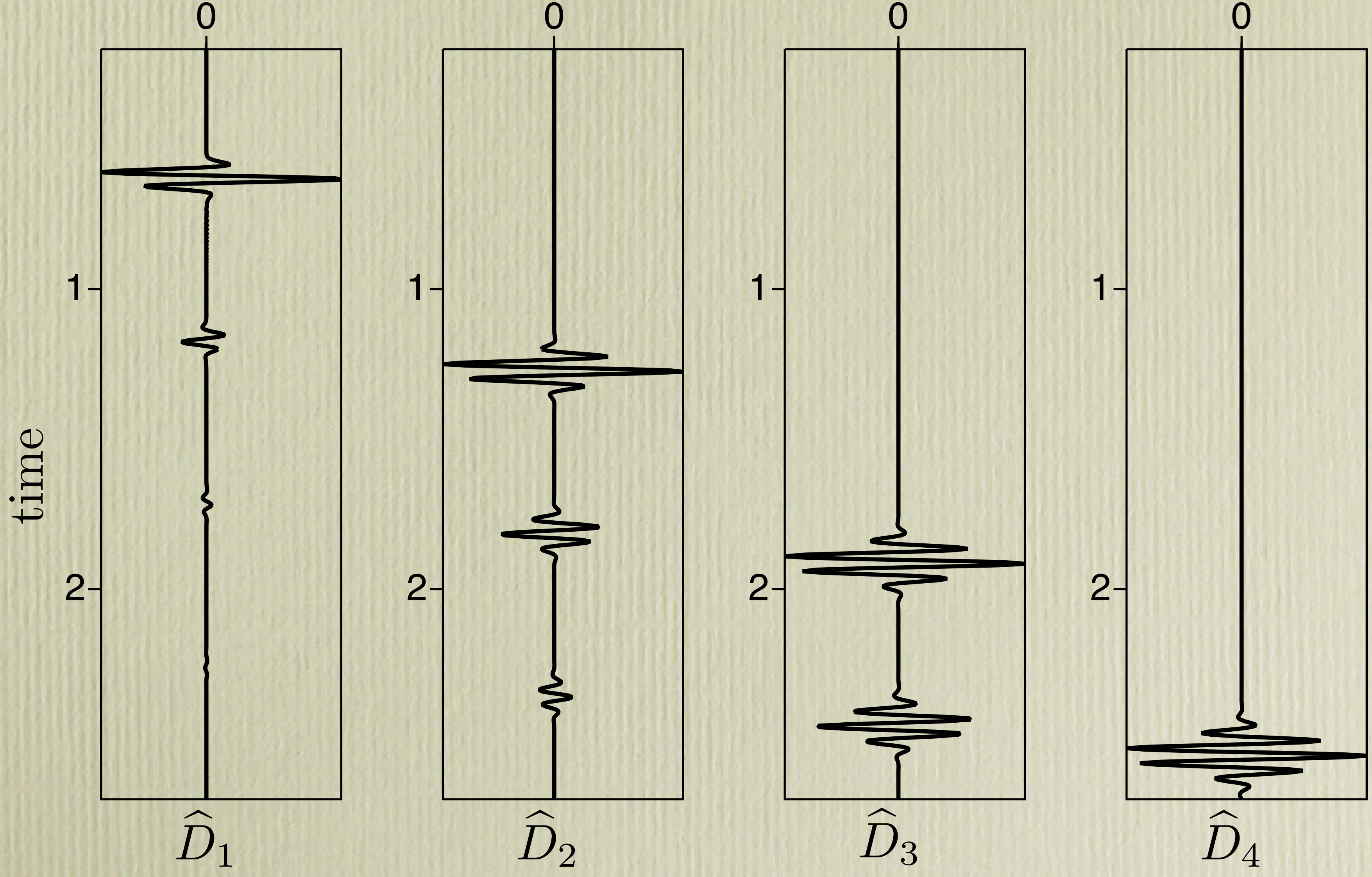


Additionally,

- we assume,
    - perfectly reflecting and horizontal free surface.
    - data with adequate recording aperture.
    - no cable feathering.
    - deconvolution of the instrument response.
    - etc.

Without deghosting, wavelet

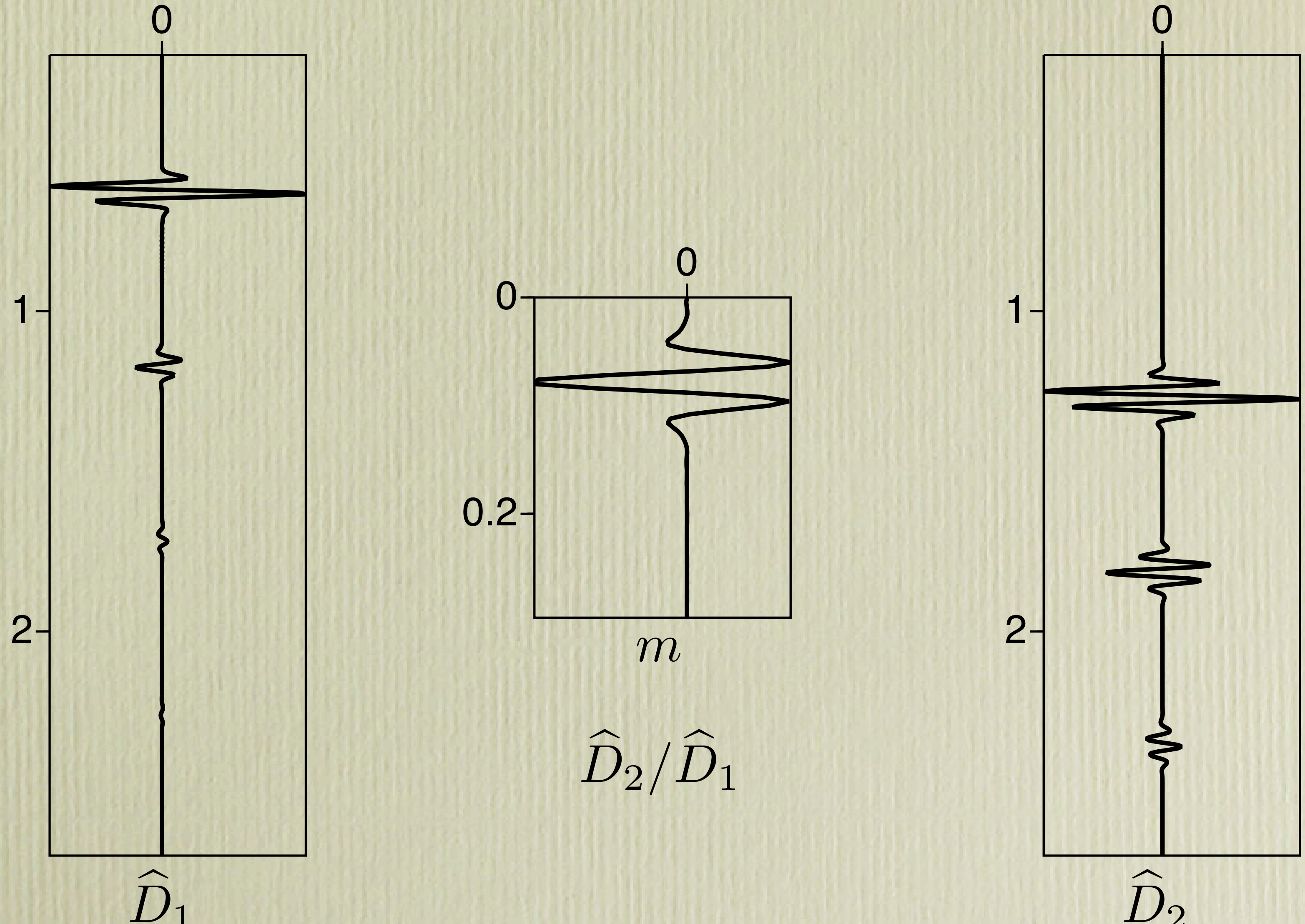


please note, normalized amplitudes

Round-a-bout solution

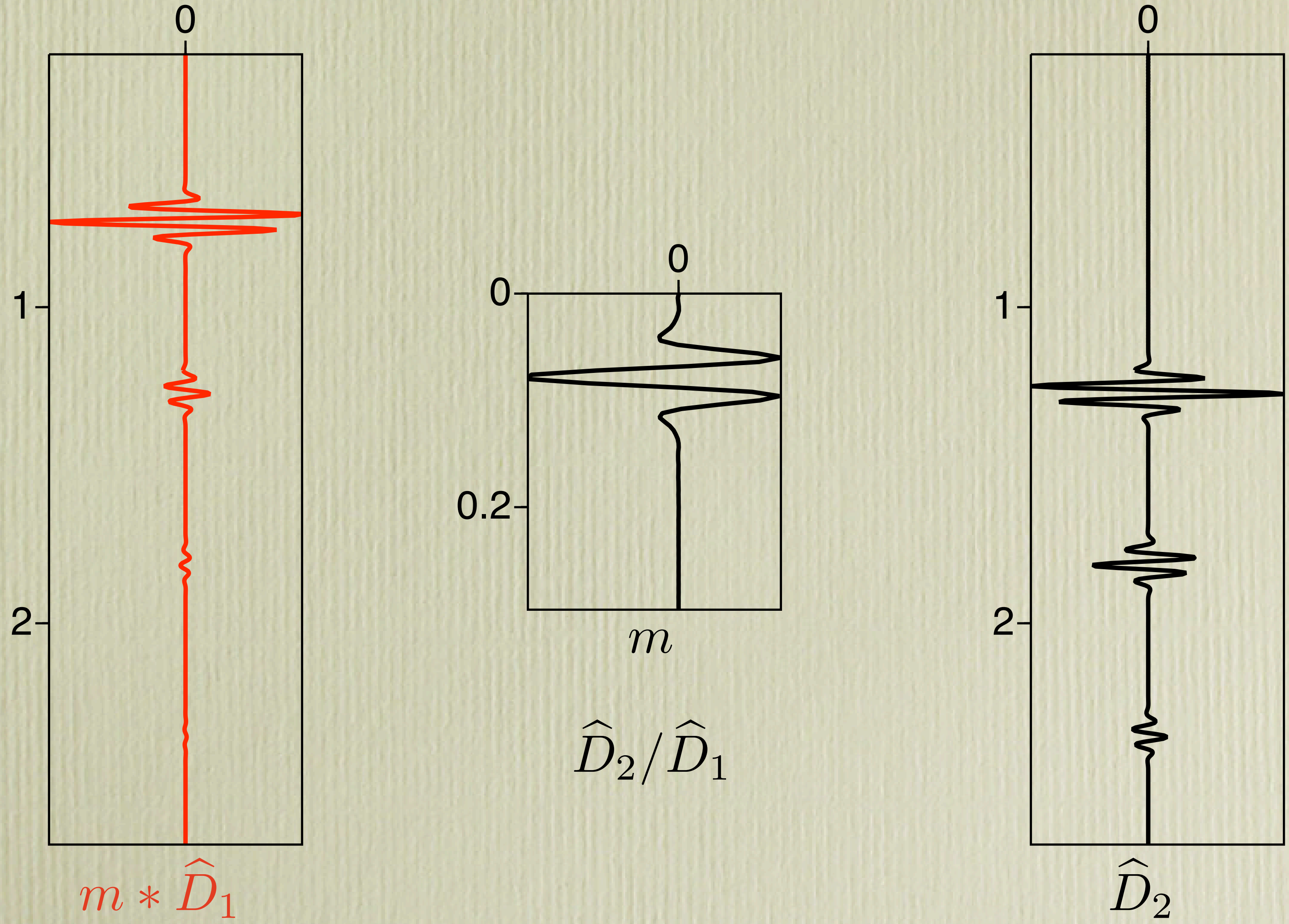
1. Matching (shaping) filters.
2. Blind source seperation.
    a. Independent component analysis.

Match data to prediction



match, using a least squares criteria

Match data to prediction



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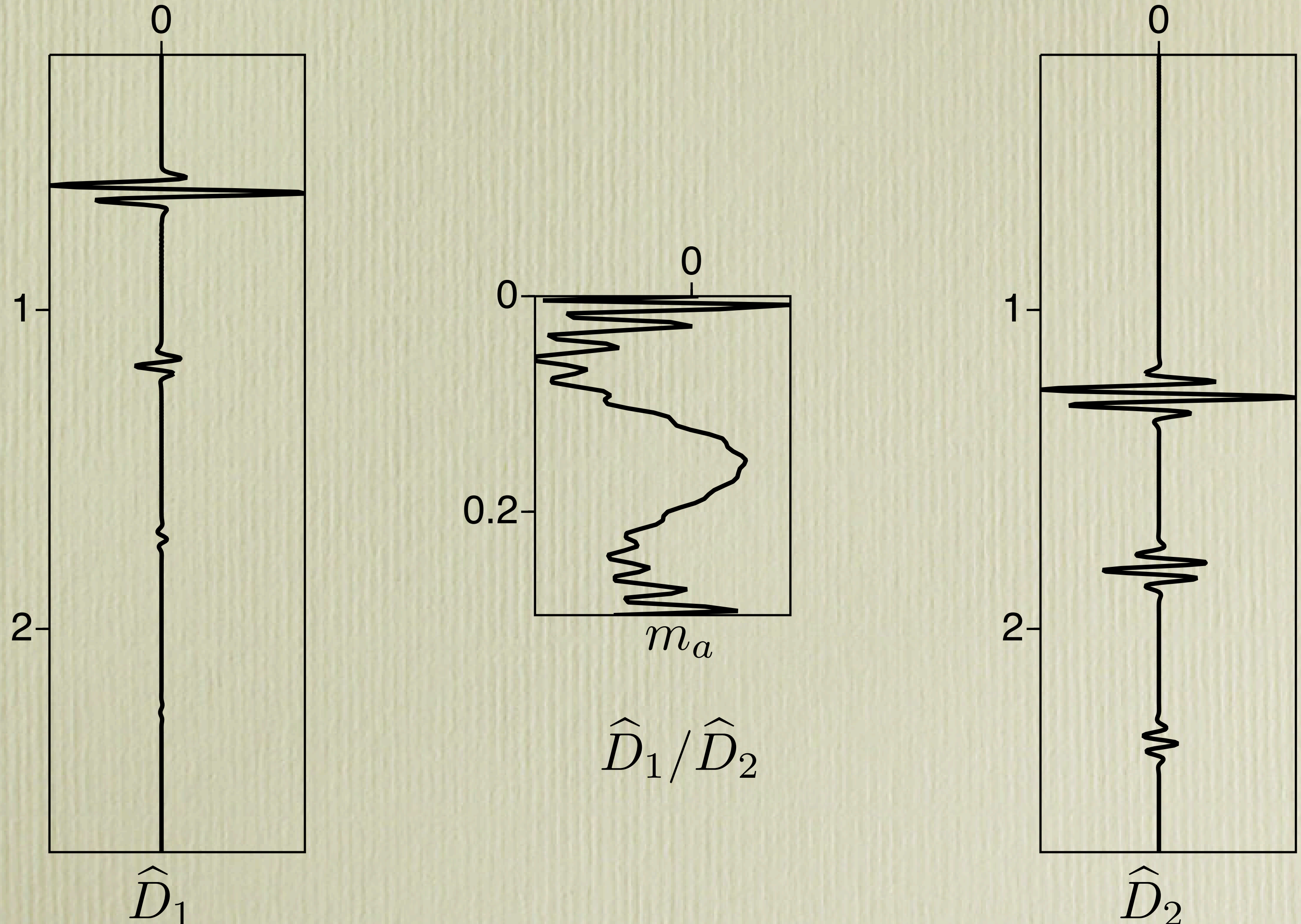

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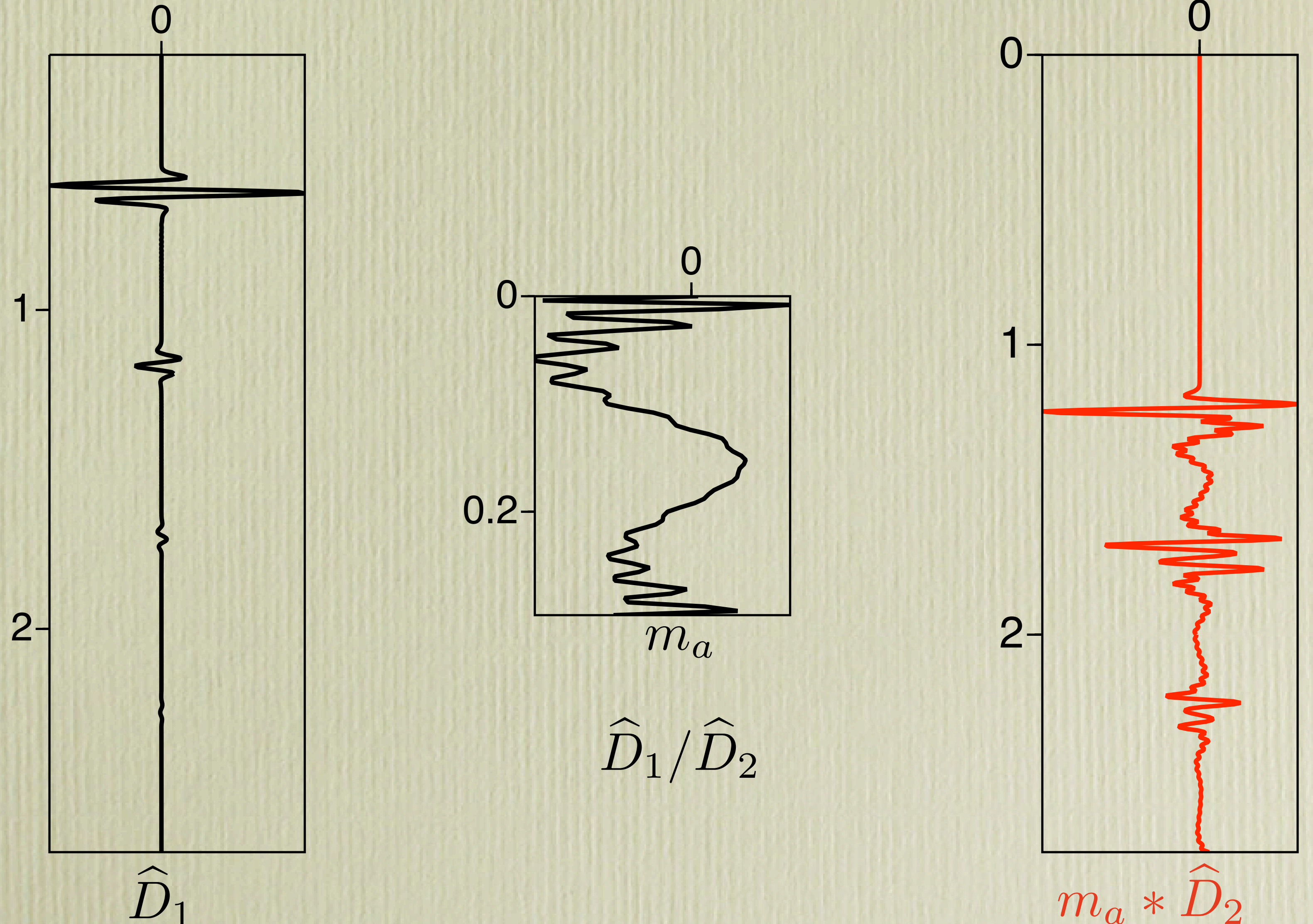

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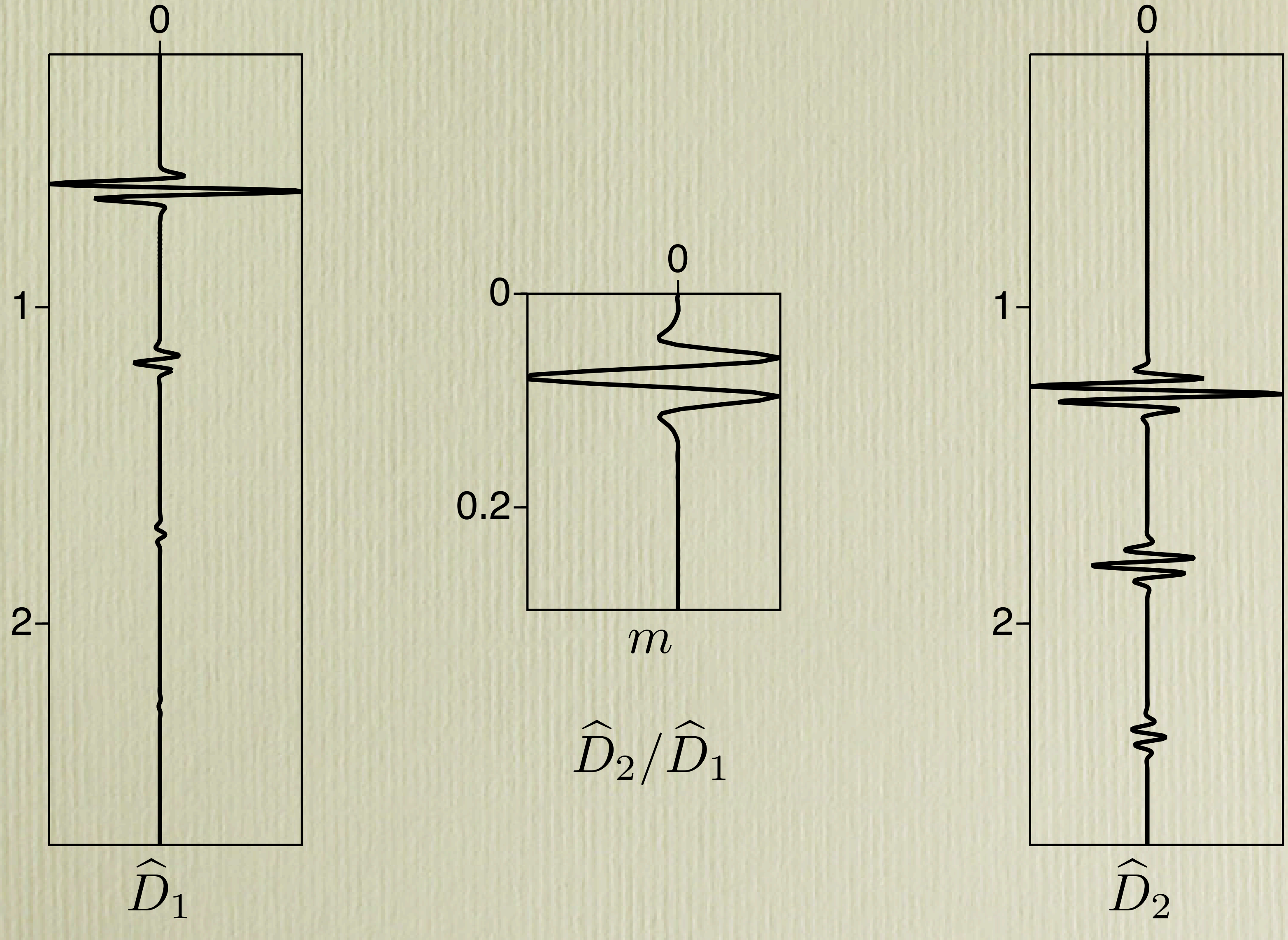
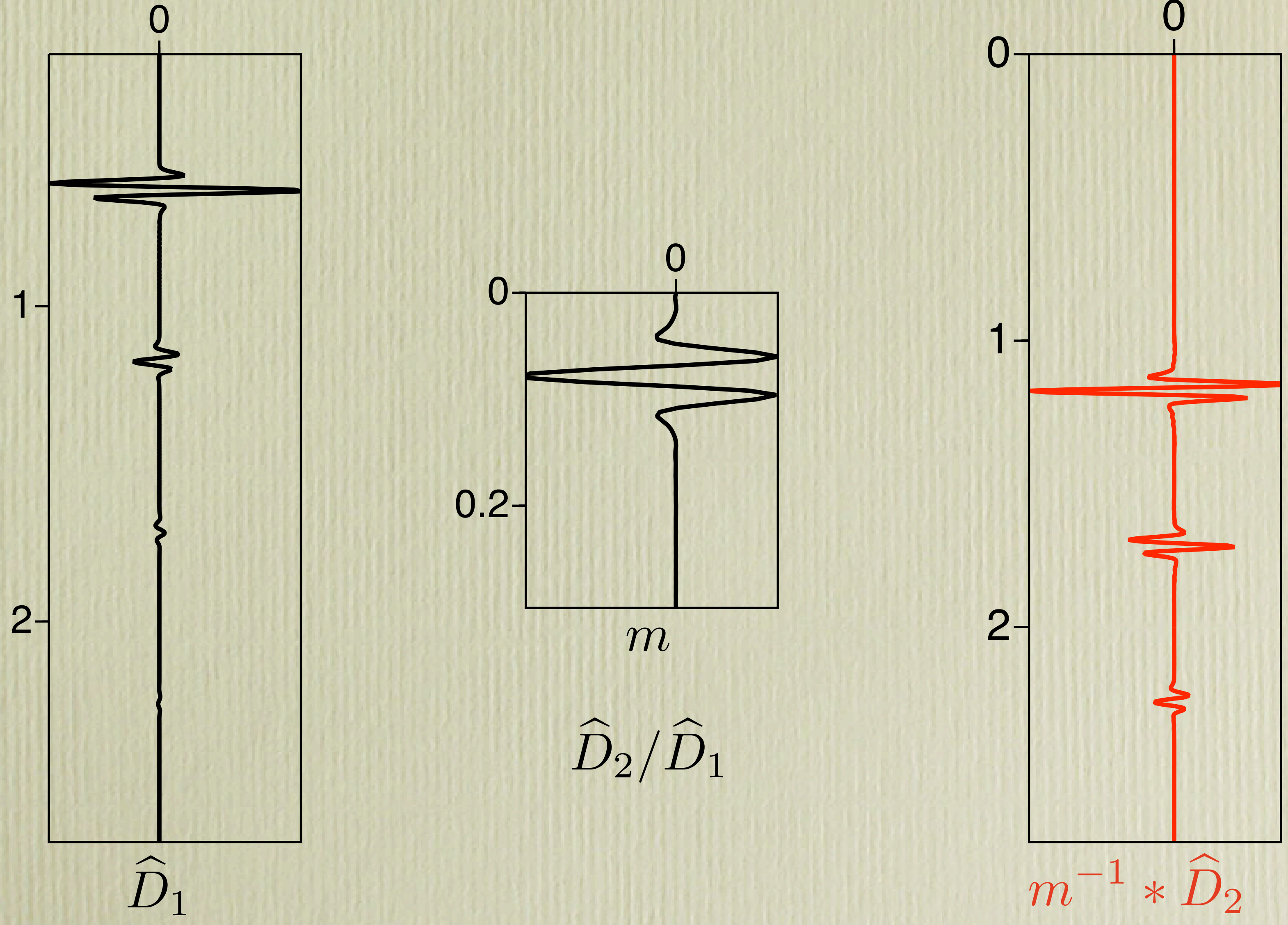
Match prediction to data



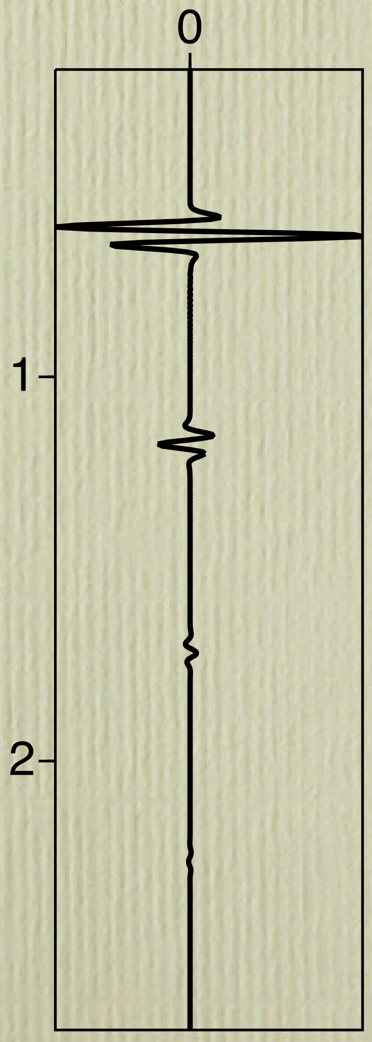
application of the inverse matching filter

Match prediction to data

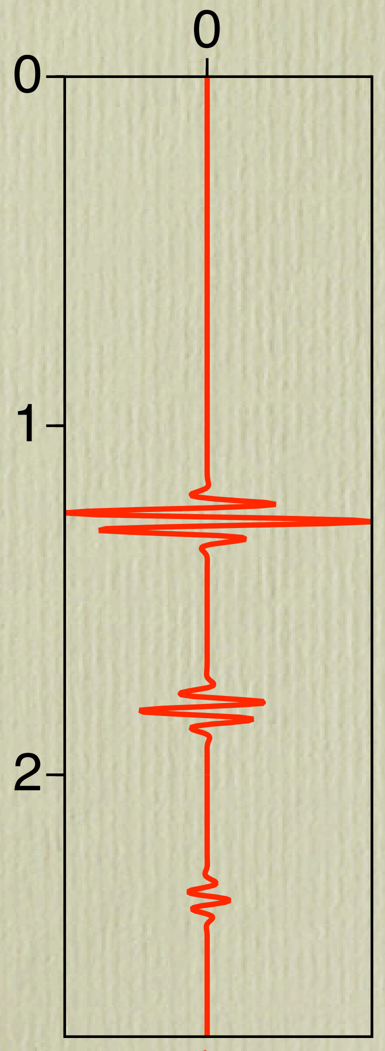


application of the inverse matching filter

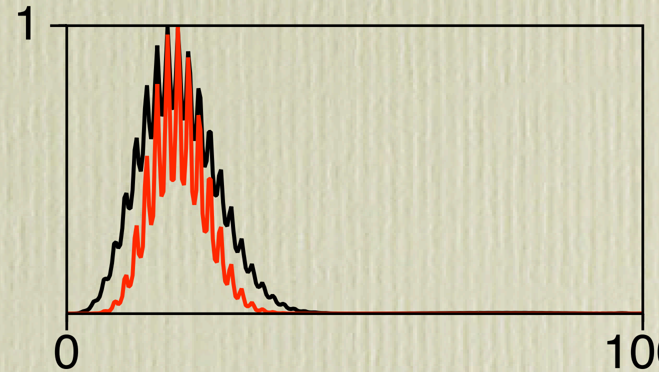
Analysis



$\widehat{D}_1$



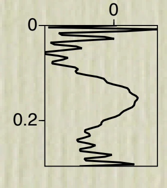
$\widehat{D}_2$



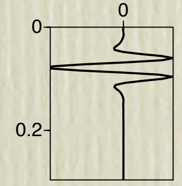
normalized amplitude spectra

$$f_1 > f_2$$

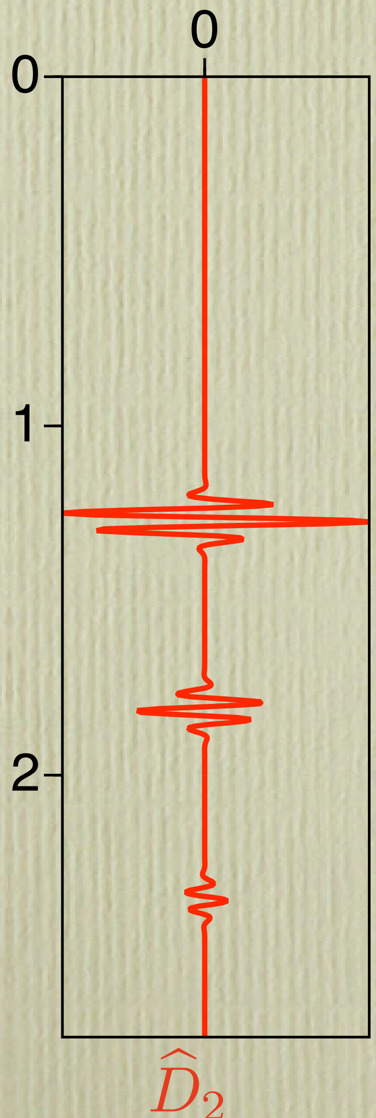
$\widehat{D}_1/\widehat{D}_2$

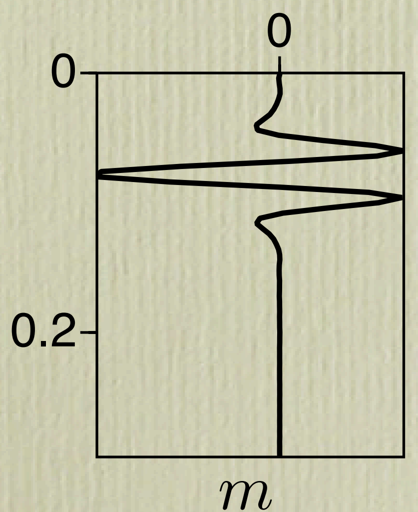


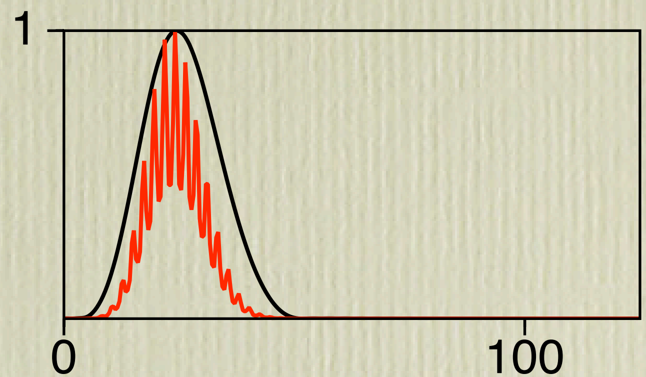
$\widehat{D}_2/\widehat{D}_1$



Analysis







normalized amplitude spectra

$f_m = f_2$

stable

D_2/m

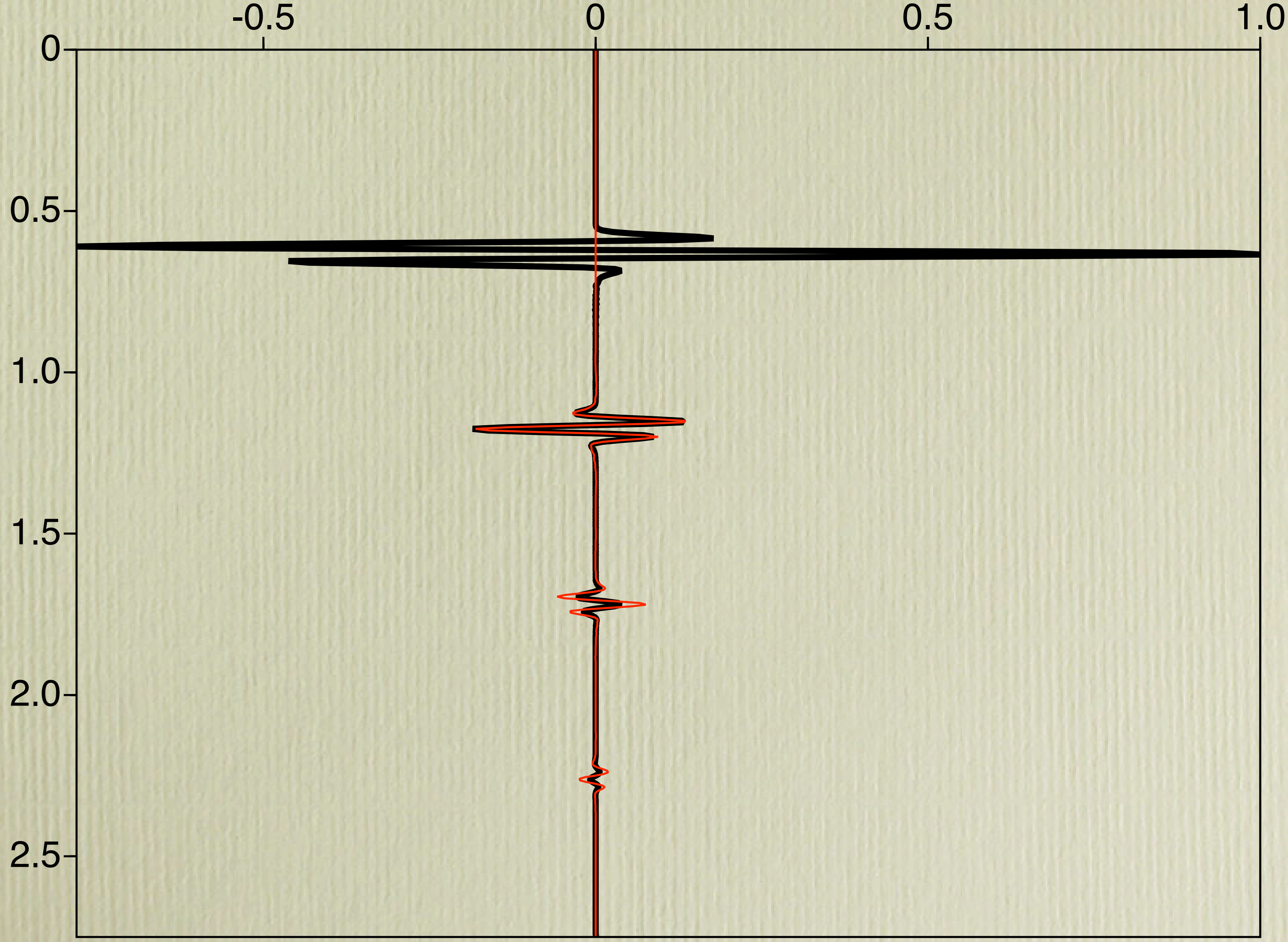
Analysis

Matching the data to the prediction, limits the frequency content of the data to that of the prediction.

Therefore, the bandwidth of the filter is equal to the bandwidth of the prediction,

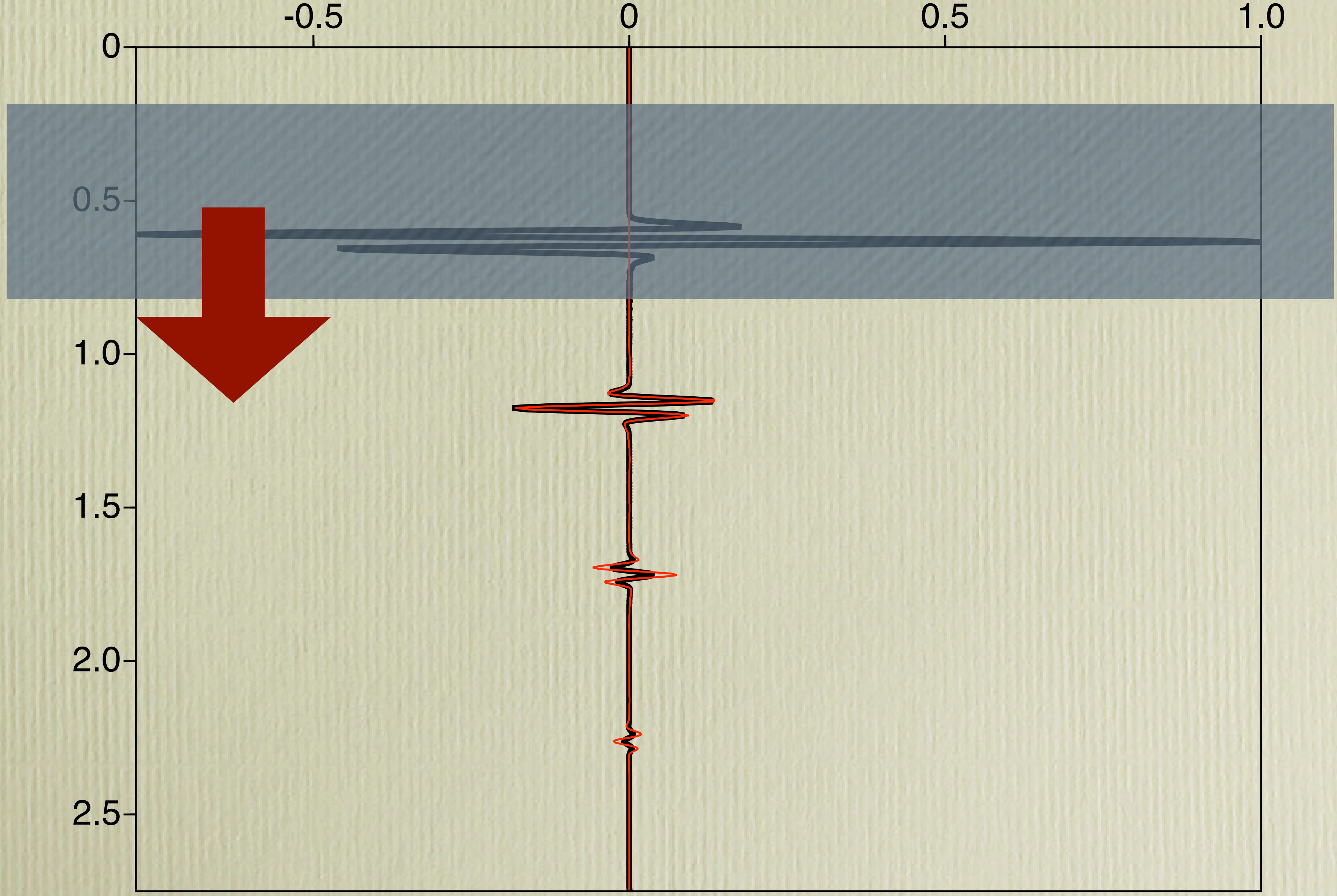
and the inverse matching filter applied to the prediction works.

Matched?



Find/Apply filters within a sliding window

Matched?



Find/Apply filters within a sliding window

Blind source separation (mixing)

source 1

s_1

source 2

s_2

$$
\begin{aligned}
x_1 &= 1.0 s_1 + 1.1 s_2 \\
x_2 &= 1.3 s_1 + 1.2 s_2
\end{aligned}
$$

Blind source separation (mixing)

mixture 1

x_1

mixture 2

x_2

$$\begin{aligned} x_1 &= 1.0 s_1 + 1.1 s_2 \\ x_2 &= 1.3 s_1 + 1.2 s_2 \end{aligned}$$

Blind source separation (de-mixing)

mixture 1

x_1

mixture 2

x_2

$$
\begin{aligned}
y_1 &= b_{11}x_1 + b_{12}x_2 \\
y_2 &= b_{21}x_1 + b_{22}x_2
\end{aligned}
$$

$$b_{ij} = ?$$

Blind source separation (de-mixing)

independent component 1

y_1

independent component 2

y_2

$$y_1 = b_{11}x_1 + b_{12}x_2$$
$$y_2 = b_{21}x_1 + b_{22}x_2$$

$$b_{ij} = ?$$

Independent component analysis (ICA)

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ & & \vdots & \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_m \end{bmatrix}$$

mixtures mixing matrix sources

$$\mathbf{x} = \mathbf{A}\mathbf{s}$$

ICA de-mixing

$$\mathbf{x} = \mathbf{A}\mathbf{s}$$

$$\mathbf{y} = \mathbf{B}\mathbf{x}$$

$$\mathbf{y} = \mathbf{B}\mathbf{x} = \mathbf{B}\mathbf{A}\mathbf{s} = \mathbf{D}\mathbf{s}$$

y_j is an independent component when:

$$y_j \propto s_i$$

ICA de-mixing

$$\mathbf{x} = \mathbf{A}\mathbf{s} \qquad \mathbf{y} = \mathbf{B}\mathbf{x}$$

$$y_j = d_{j1}s_1 + d_{j2}s_2 + \cdots + d_{jm}s_m$$

y_j is an independent component when:

$$y_j \propto s_i$$

Central limit theorem

$$y_j = d_{j1}s_1 + d_{j2}s_2 + \cdots + d_{jm}s_m$$

The central limit theorem says that a sum of random variables is more Gaussian than its parts.

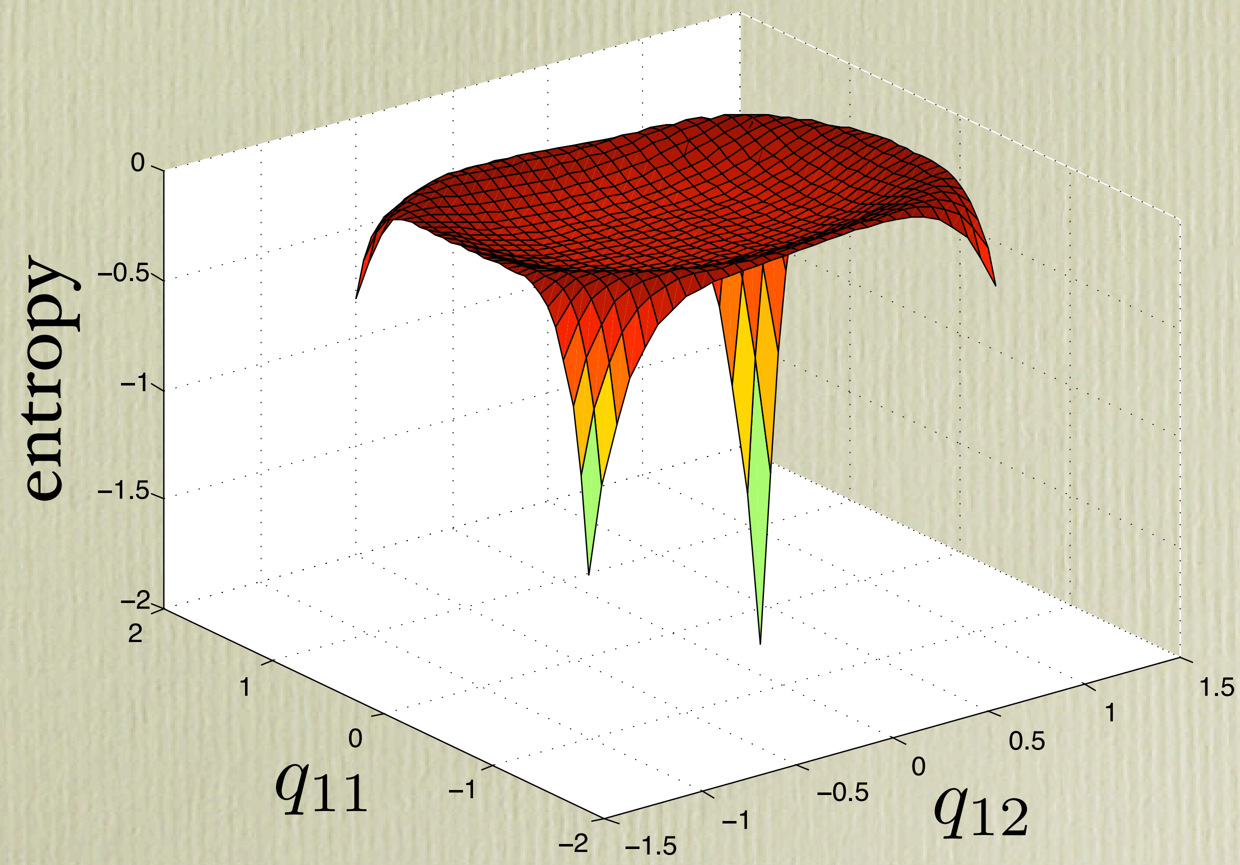
- y_j is least Gaussian when at most one coefficient is non-zero.
- y_j is an independent component when only one coefficient is non zero.
- y_j is an independent component when it is least Gaussian.

Entropy and Gaussianity

minimum entropy = least Gaussian
least Gaussian = independent component

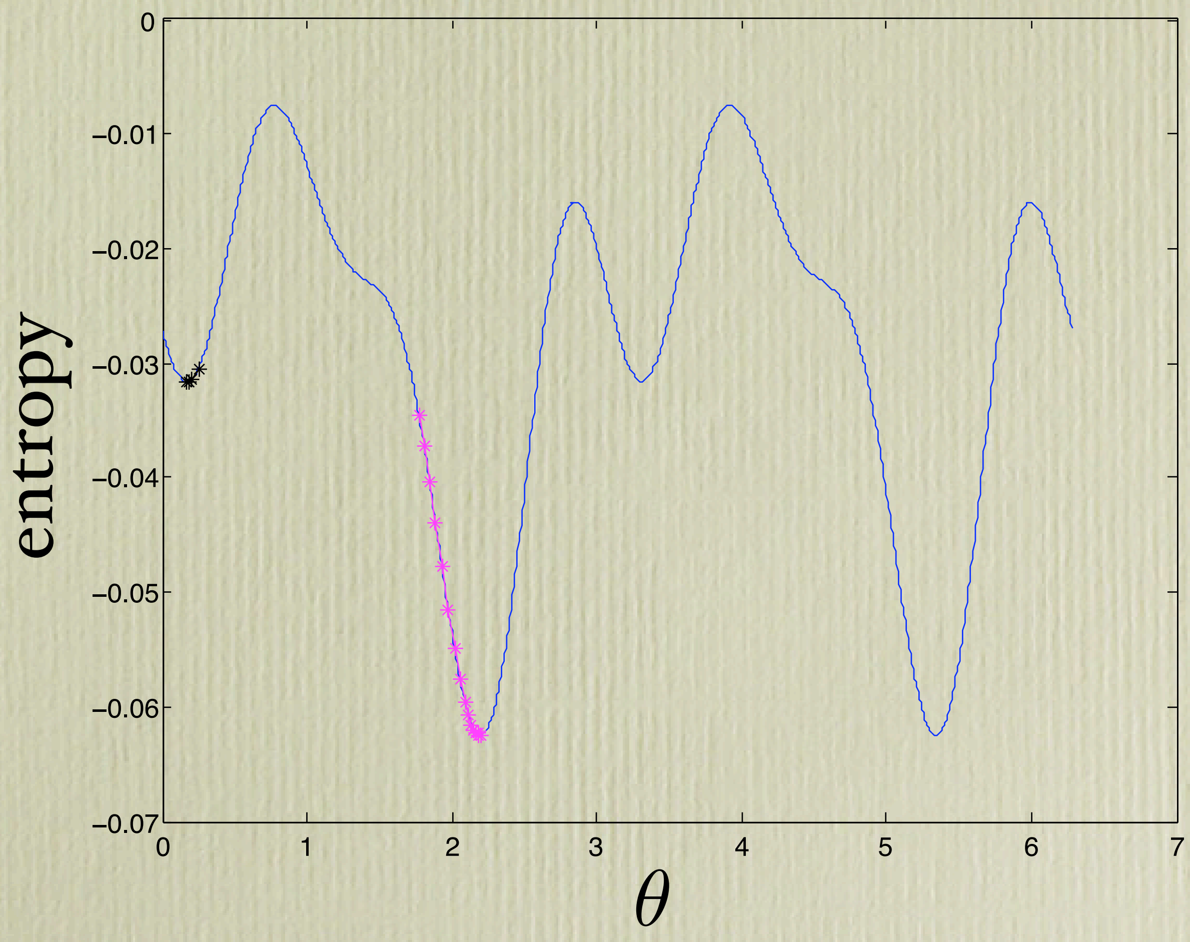
$$y_j$$

Minimizing Entropy



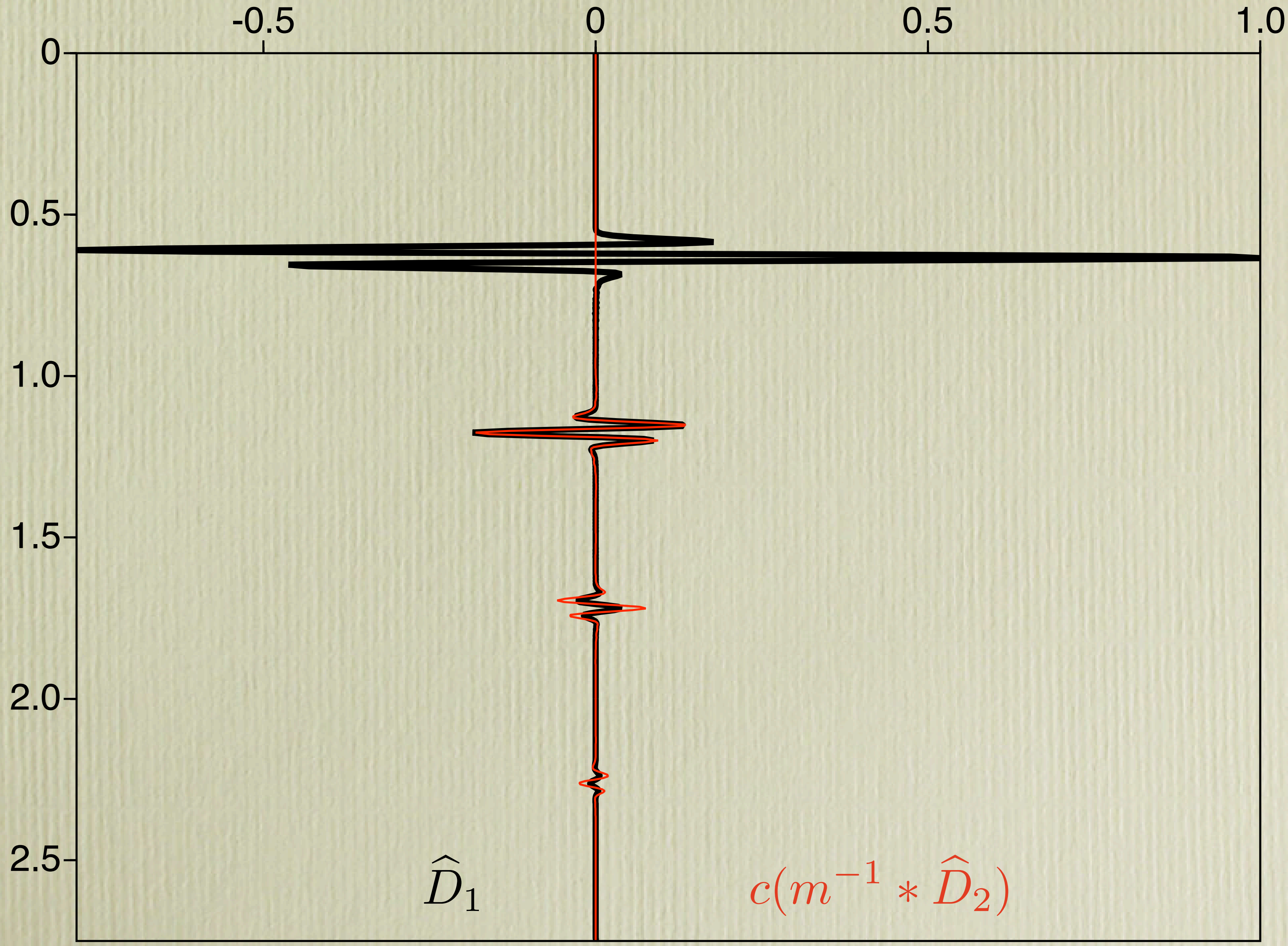
making 'almost' independent graduate students

Constrained minimization

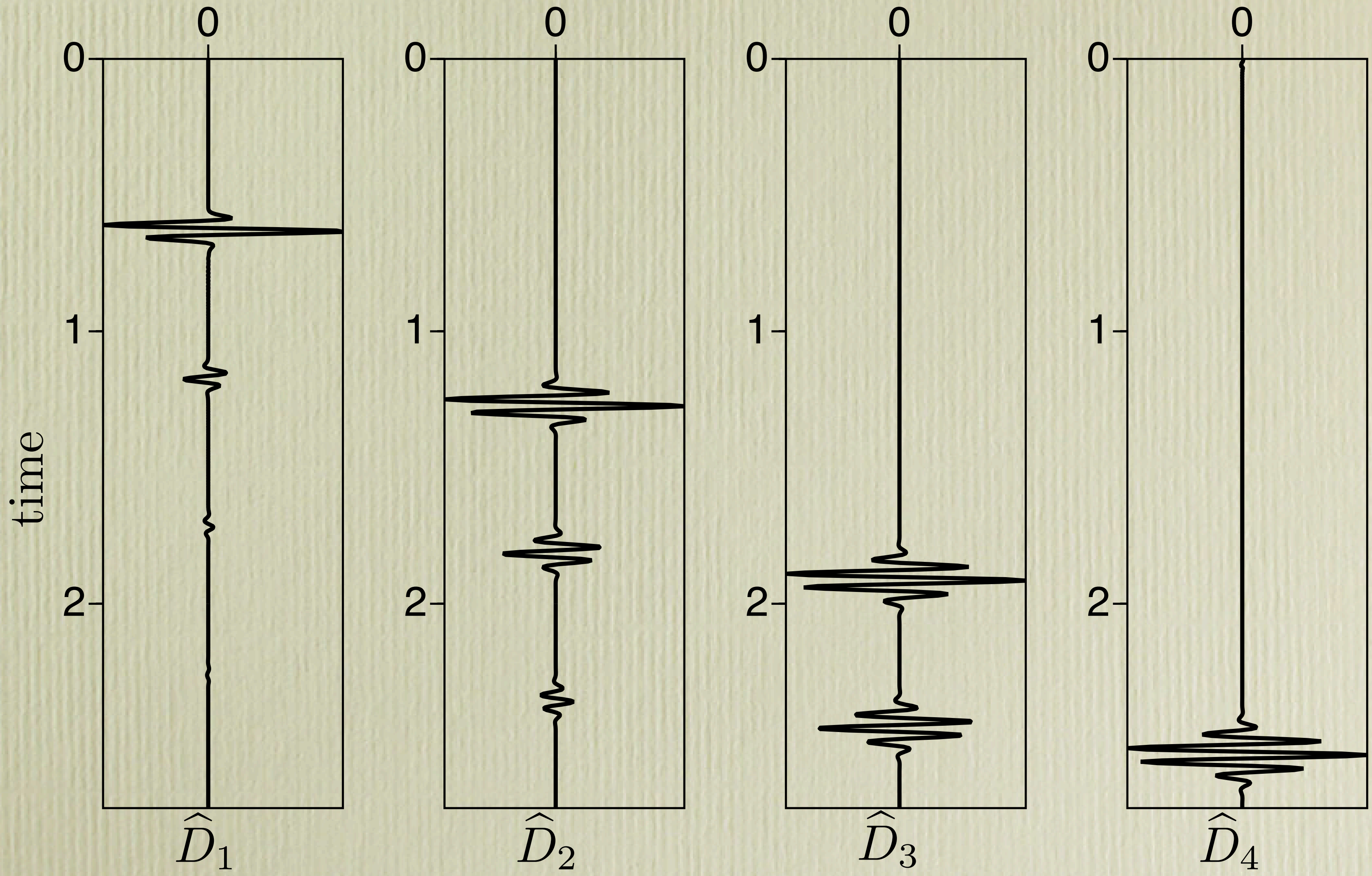


(unit variance constraint)

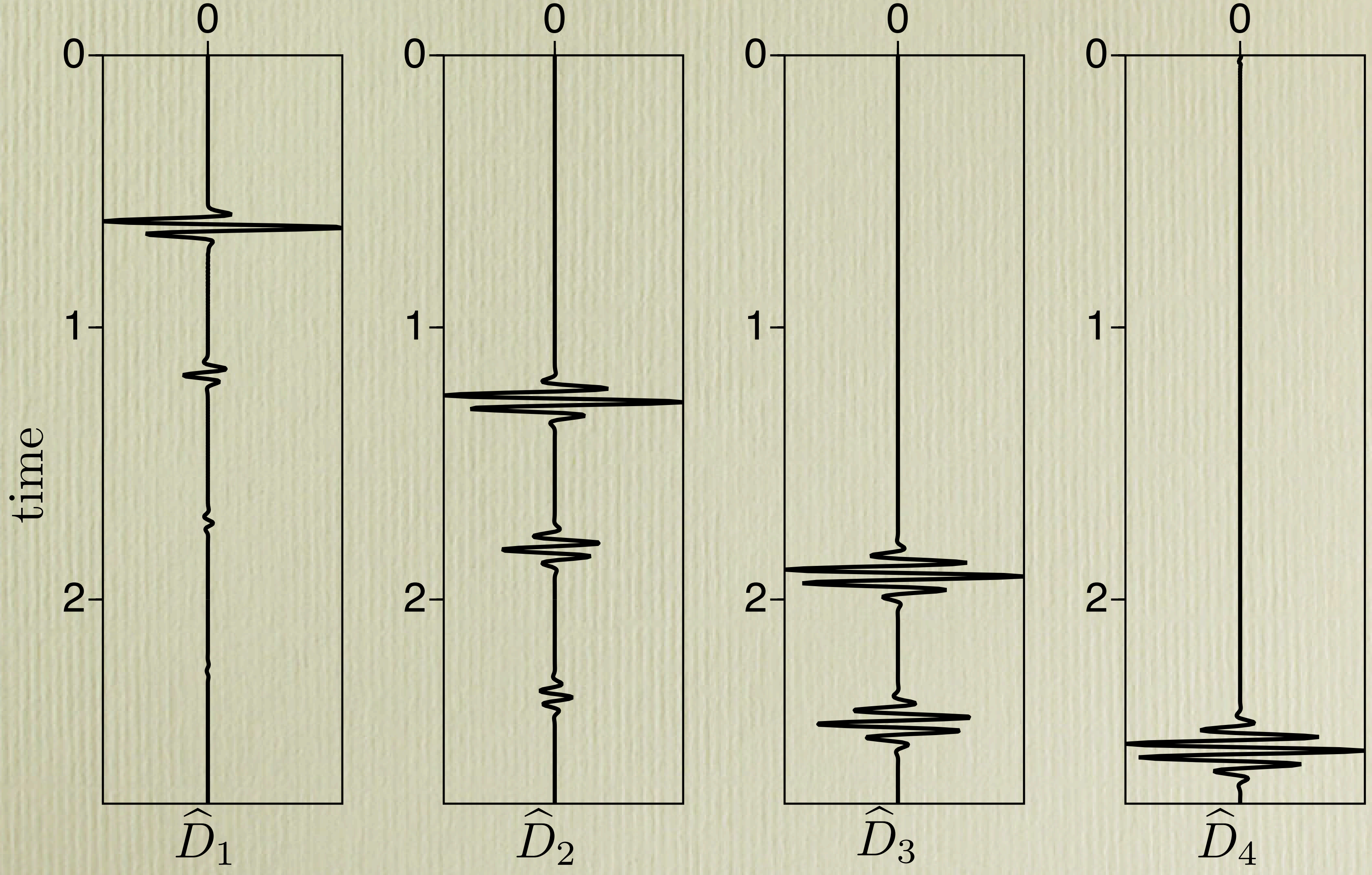
Remember this...



...and this?



Apply matching filters



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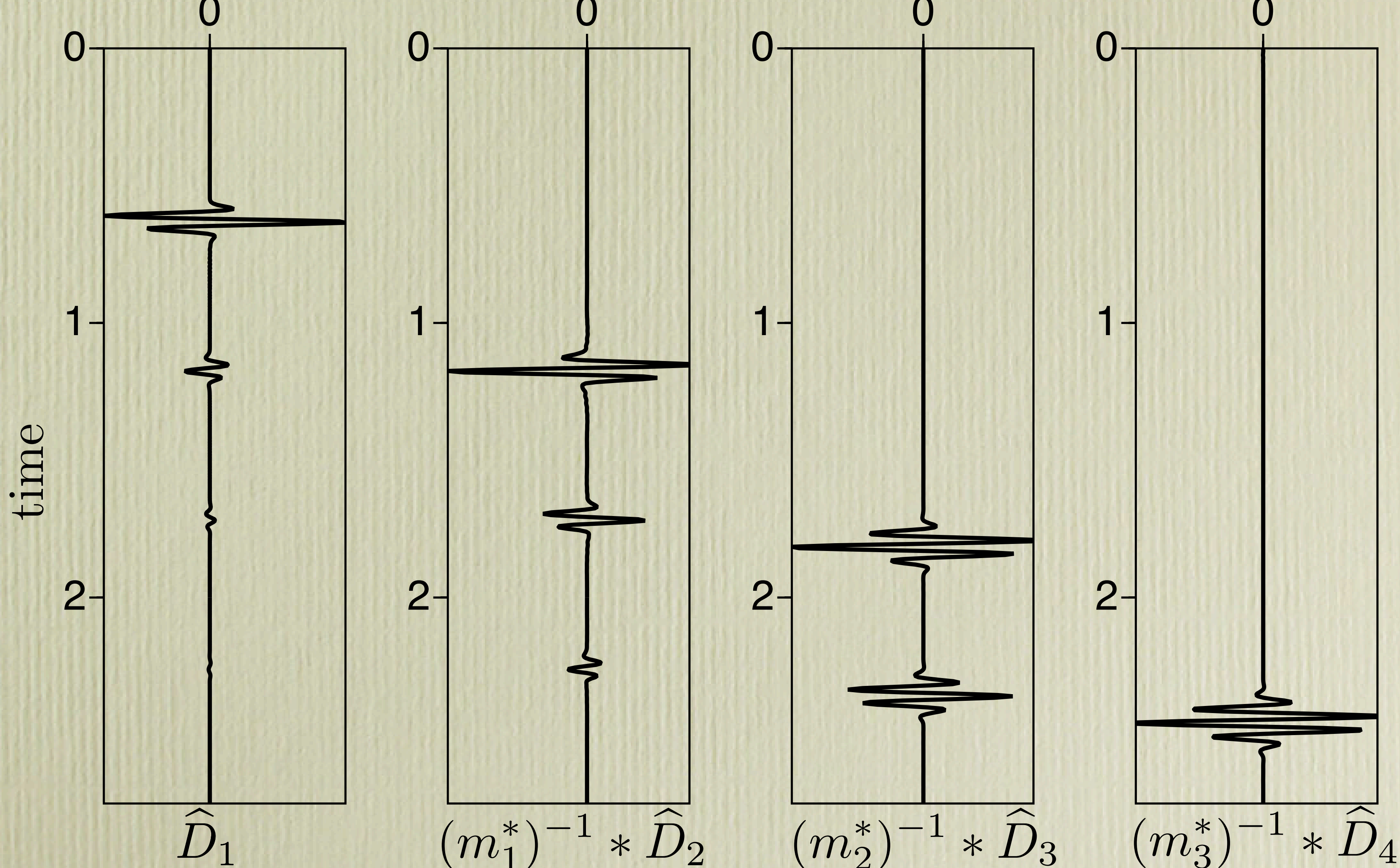

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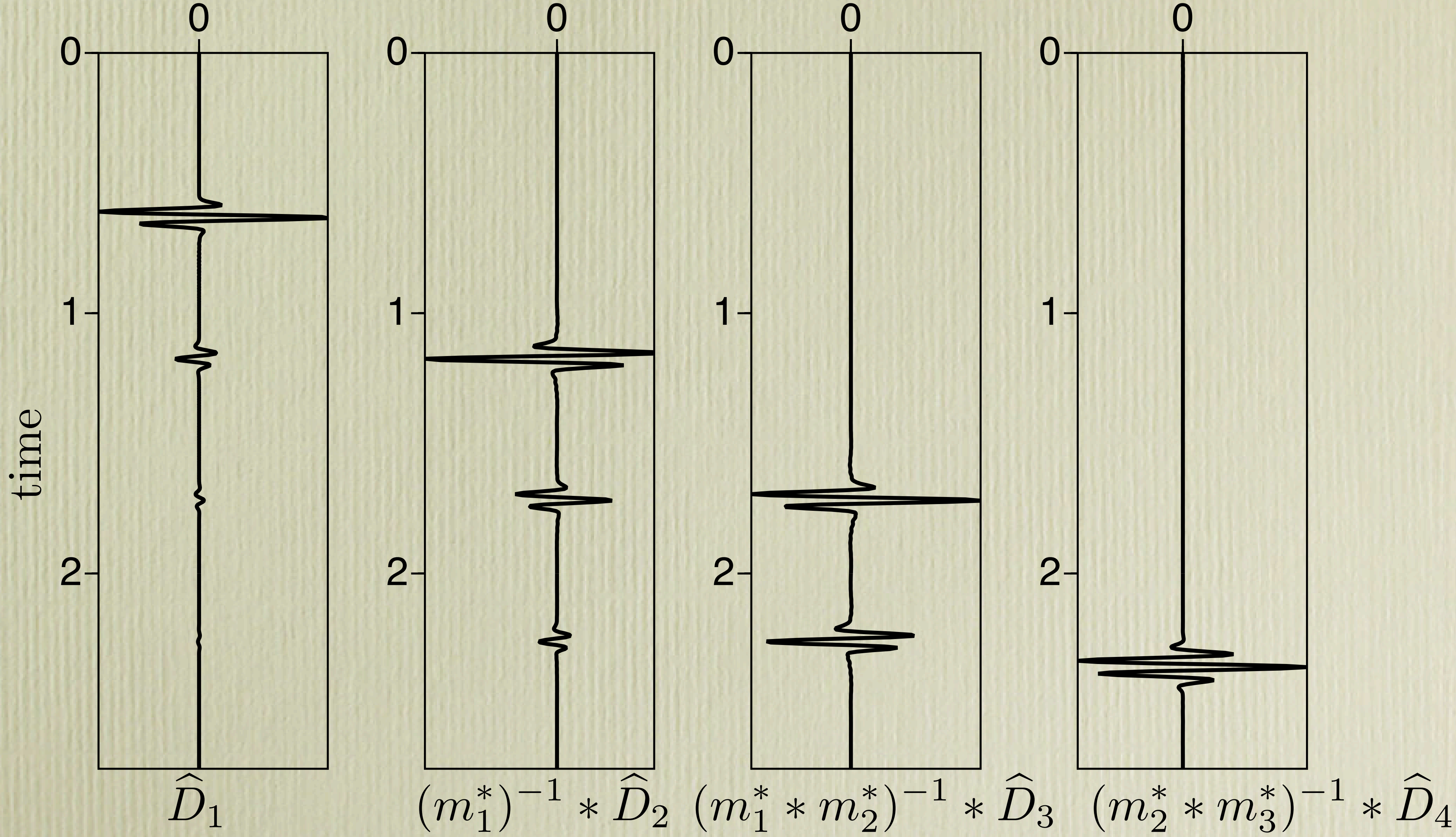

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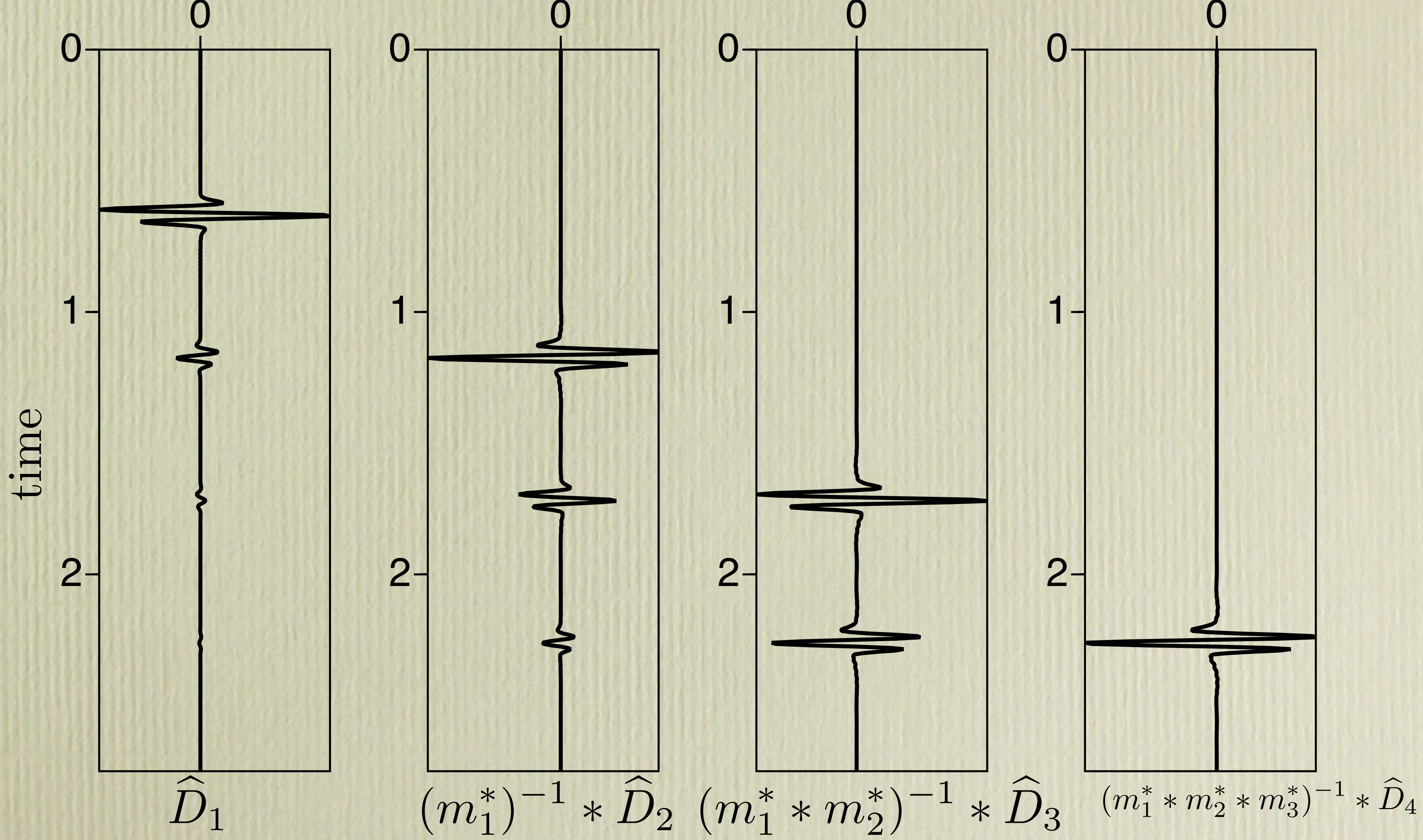

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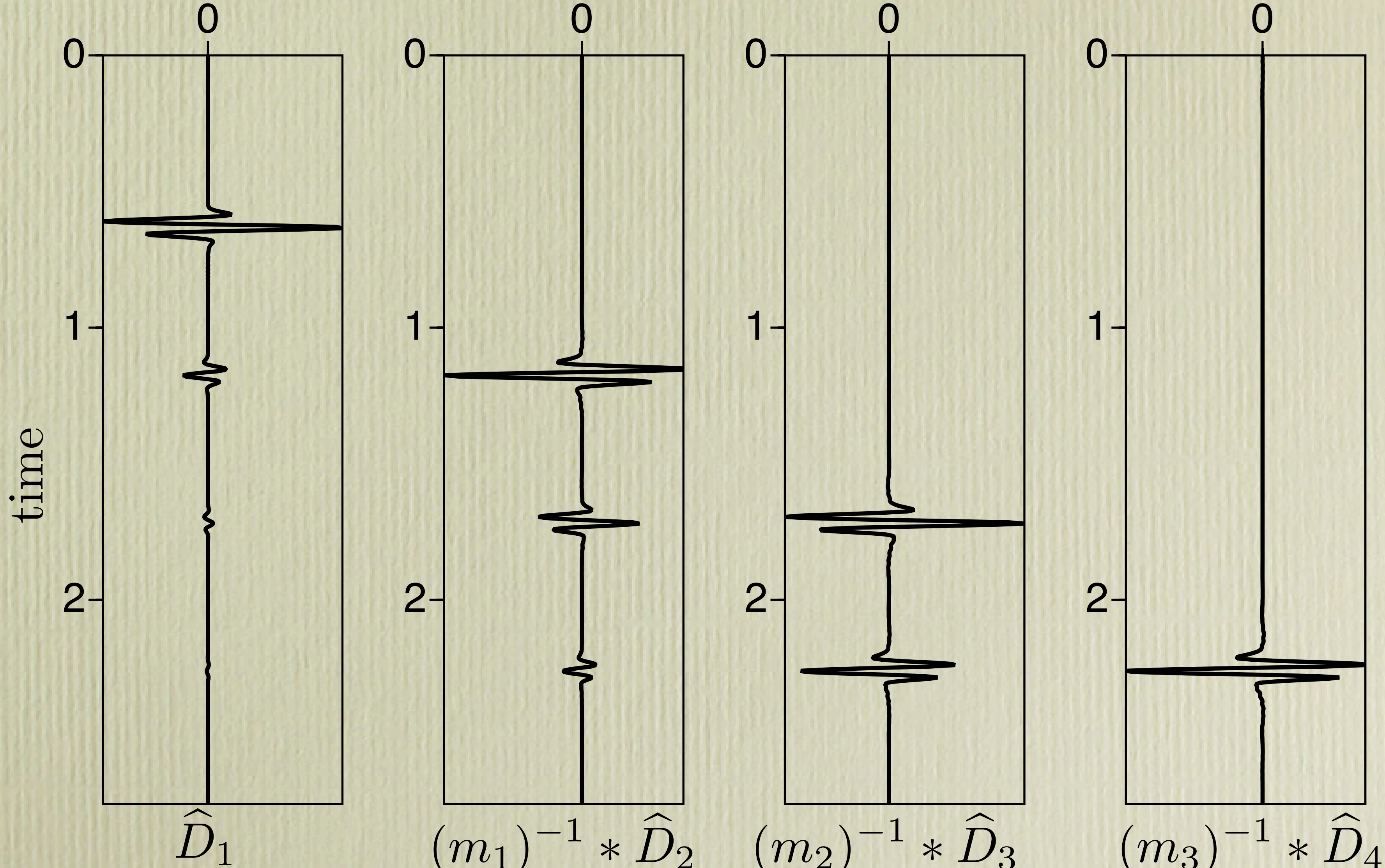

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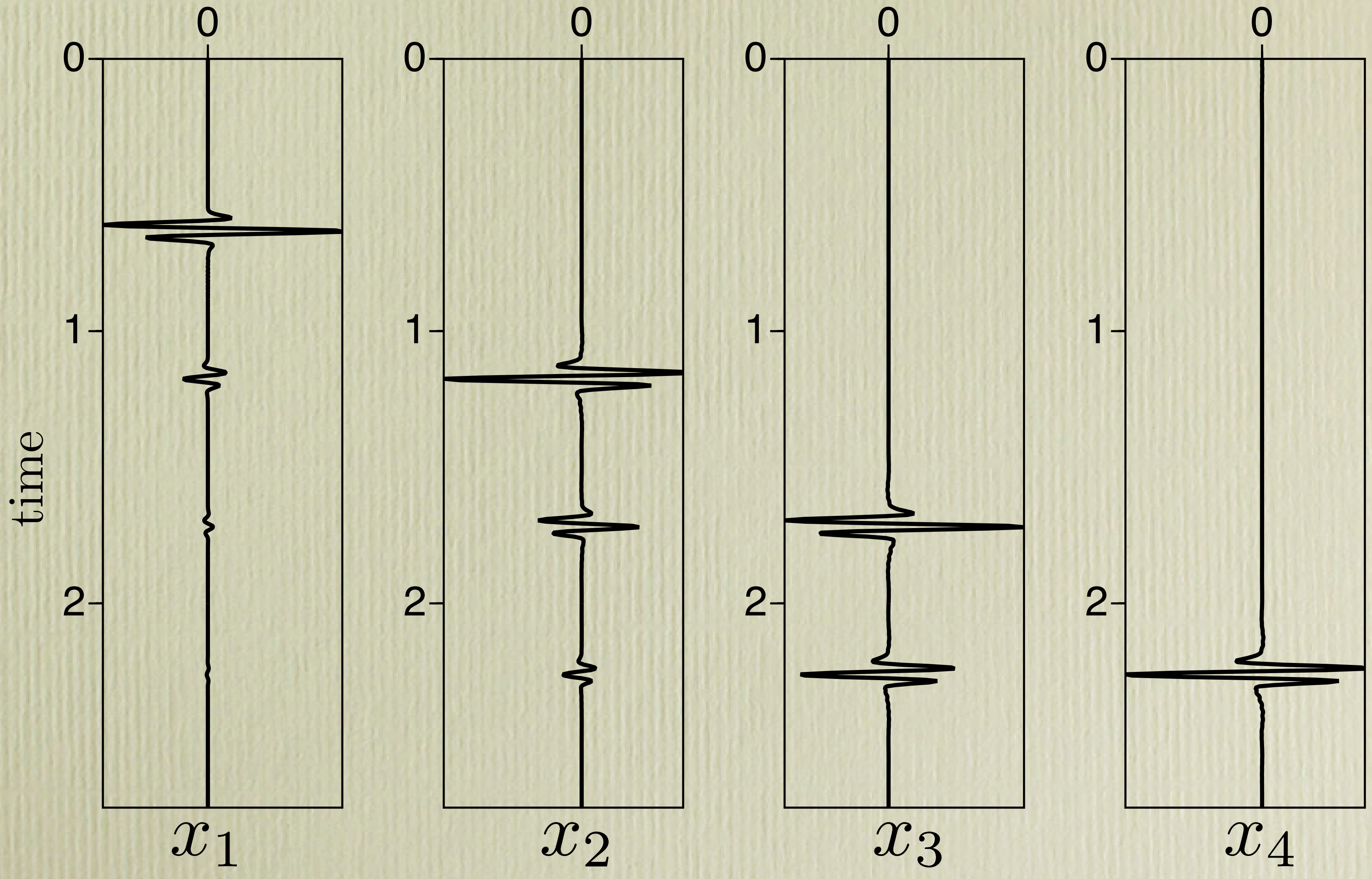


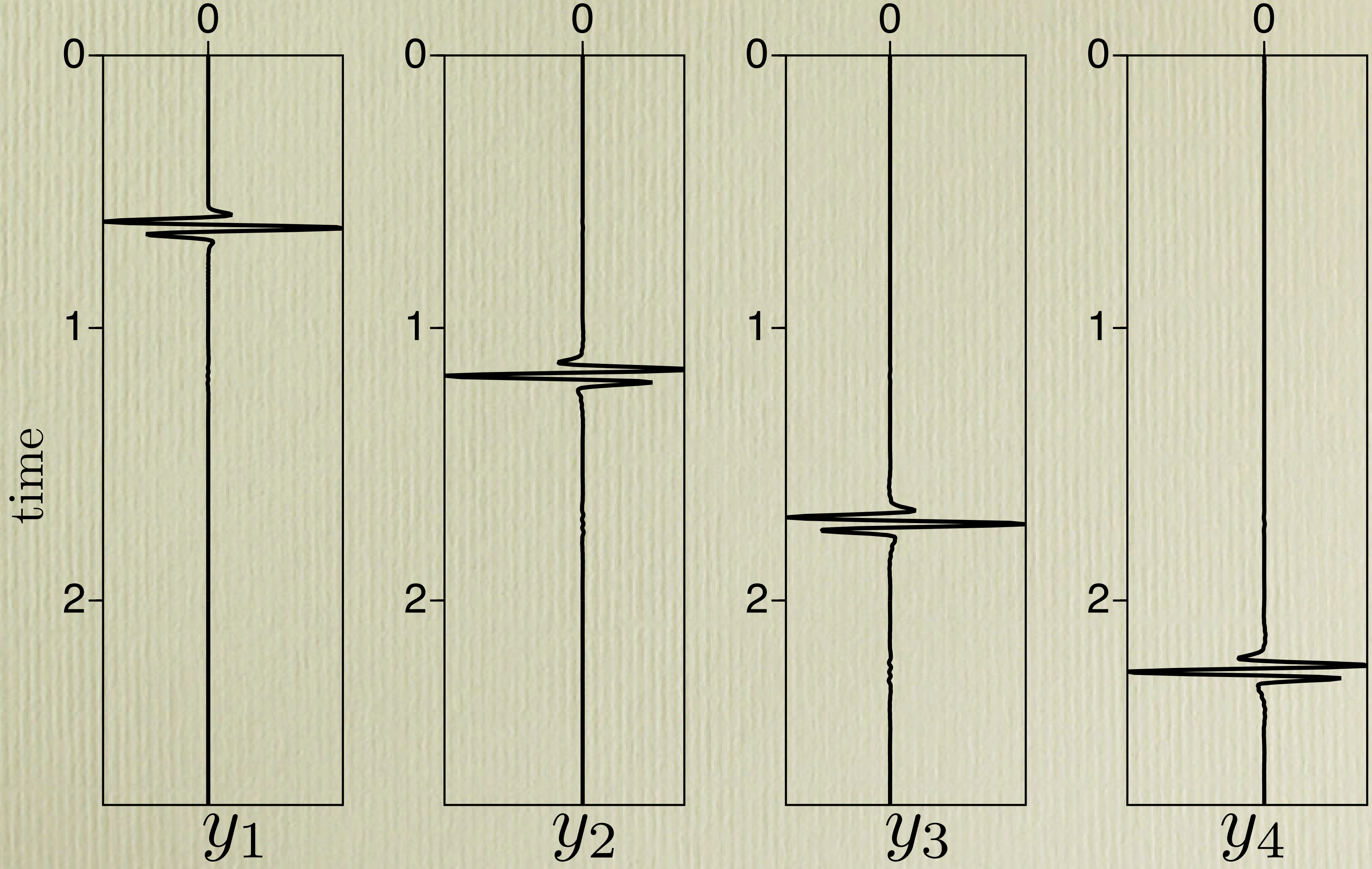
Matched data and ICA

After applying the matching filter, the data follows the ICA mixing model

$$\mathbf{x} = \mathbf{A}\mathbf{s}$$

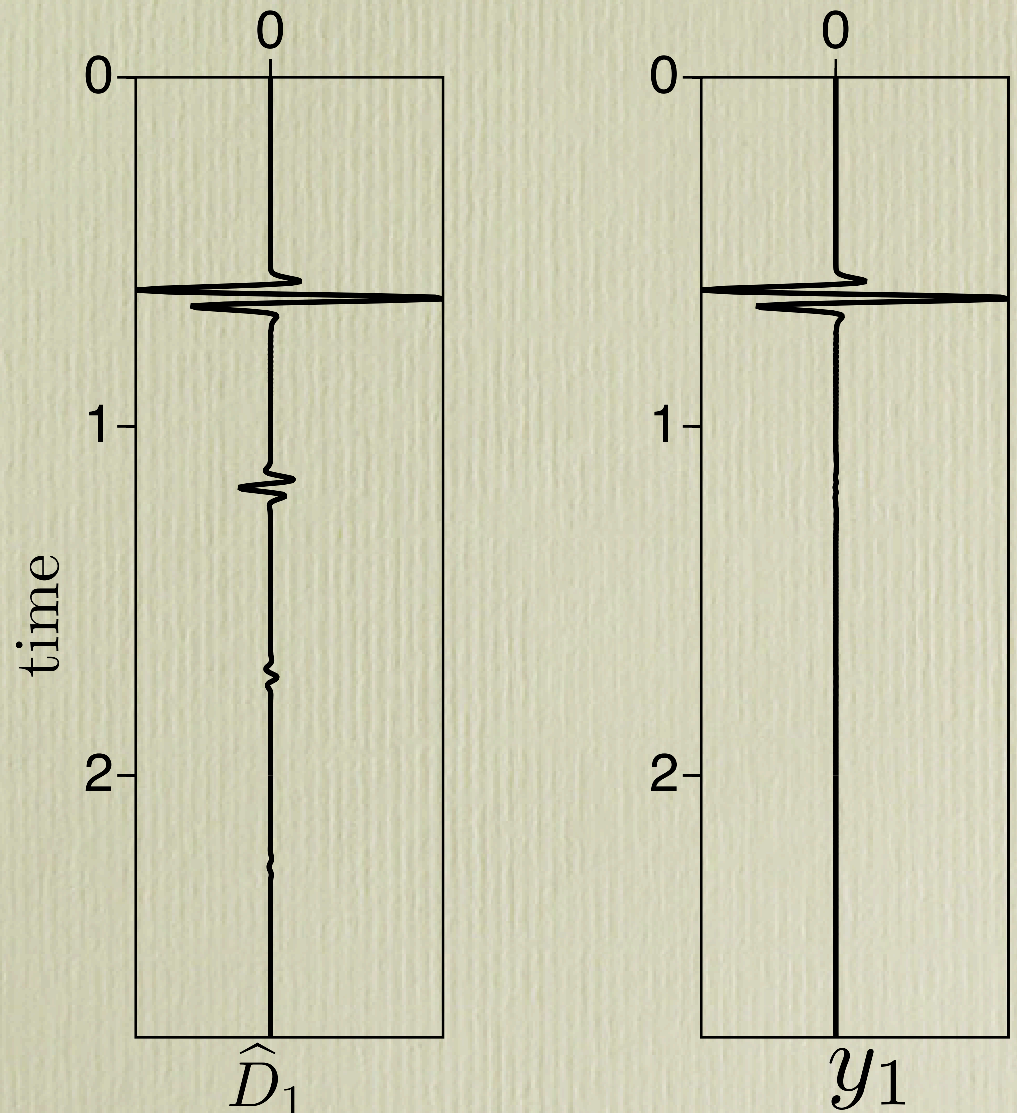
$$\begin{bmatrix} \widehat{D}_1 \\ m_1^{-1} * \widehat{D}_2 \\ \vdots \\ m_{m-1}^{-1} * \widehat{D}_m \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & c_{22} & \cdots & c_{2m} \\ & & \vdots & \\ 0 & 0 & \cdots & c_{mm} \end{bmatrix} \begin{bmatrix} P \\ M_1 \\ \vdots \\ M_{m-1} \end{bmatrix}$$

Apply ICA

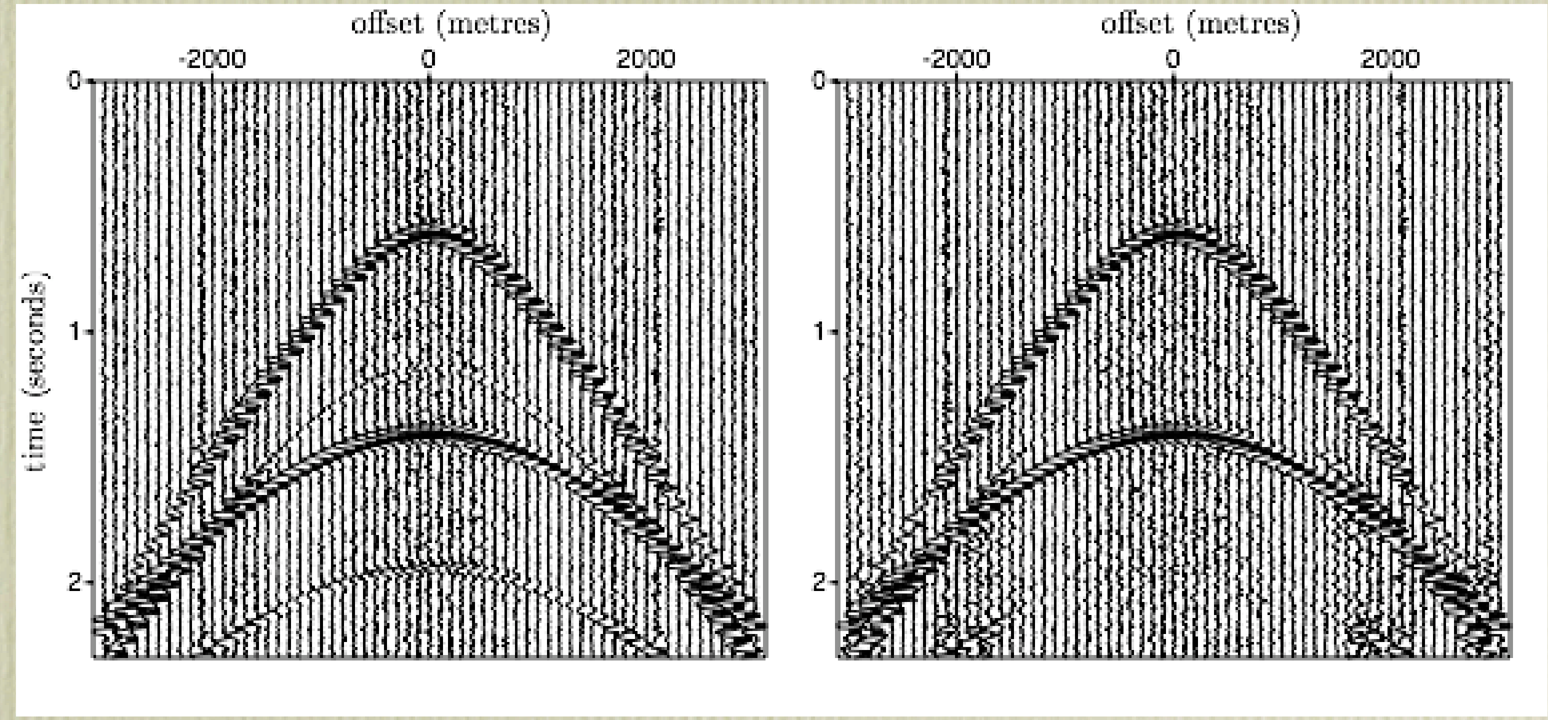


(independent components)

Result



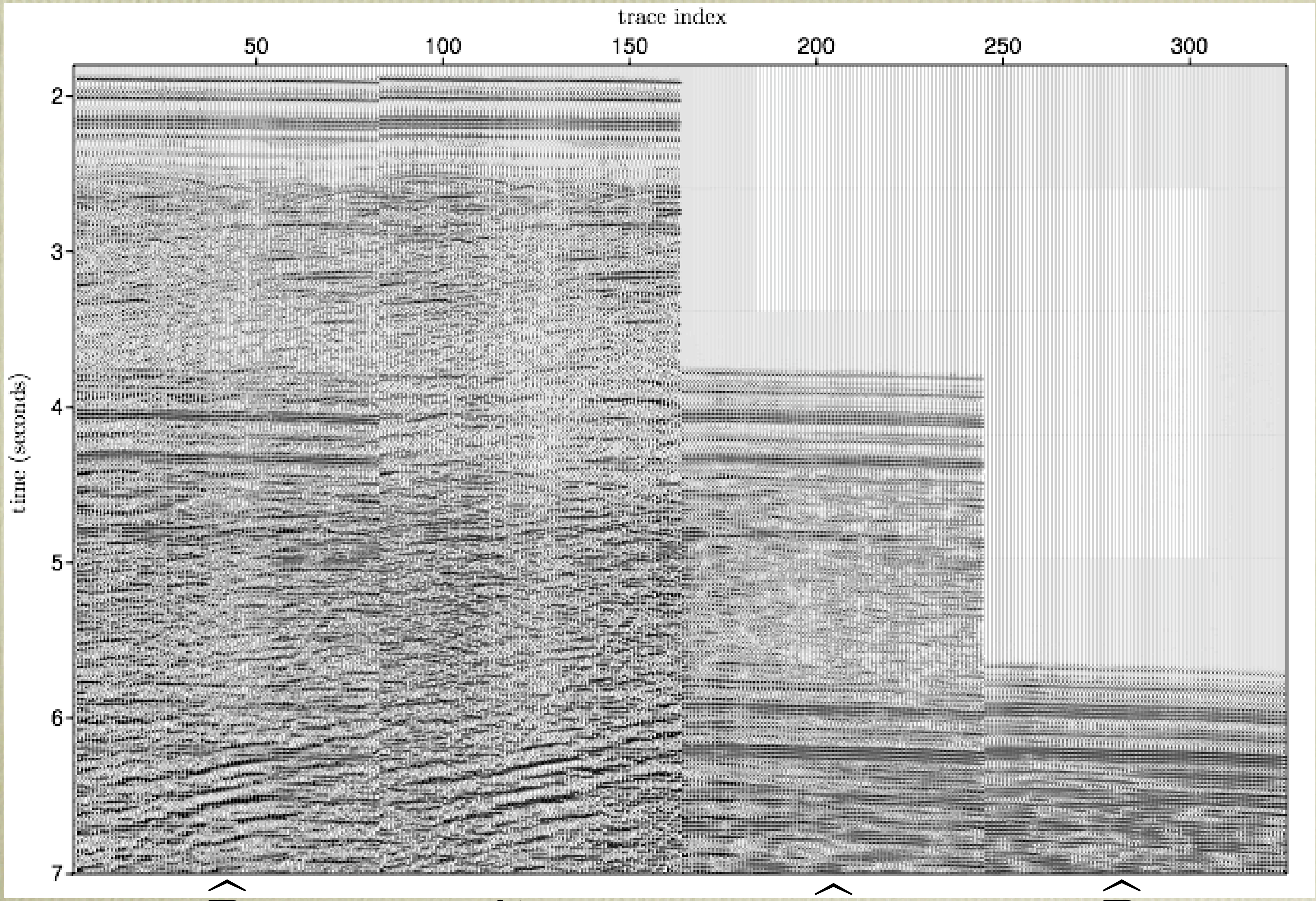
Reference matching filter



$$\phi(\mathbf{m}) = ||\hat{\mathbf{d}}_{n+1} - \mathbf{H}_n\mathbf{L}\mathbf{m}||_2^2 + \mu||\mathbf{m} - \mathbf{m}_0||_2^2,$$

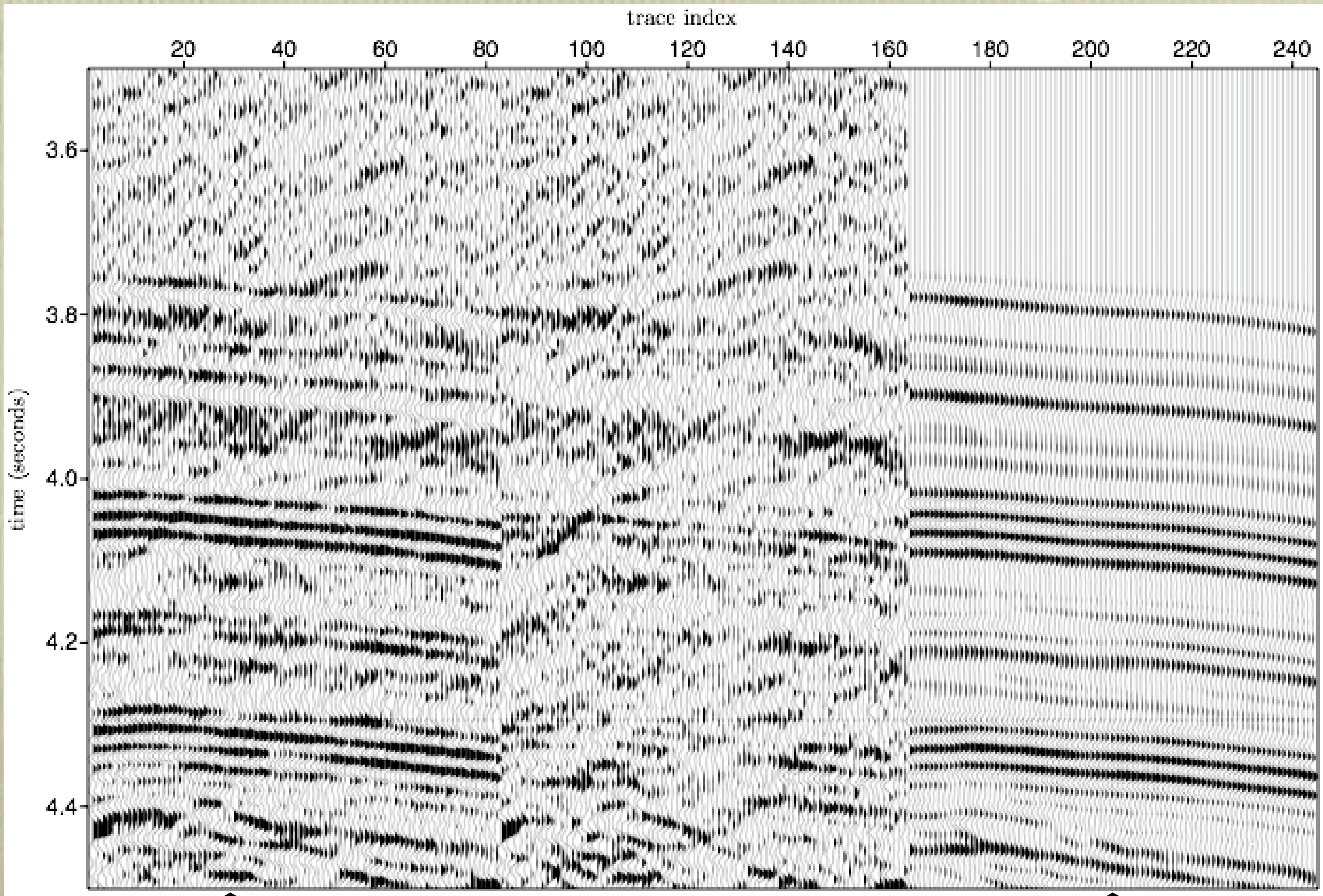
(reference matching filter from the zero offset trace)

Gulf of Mexico example



$\widehat{D}_1$ y_1 $\widehat{D}_2$ $\widehat{D}_3$

Gulf of Mexico example



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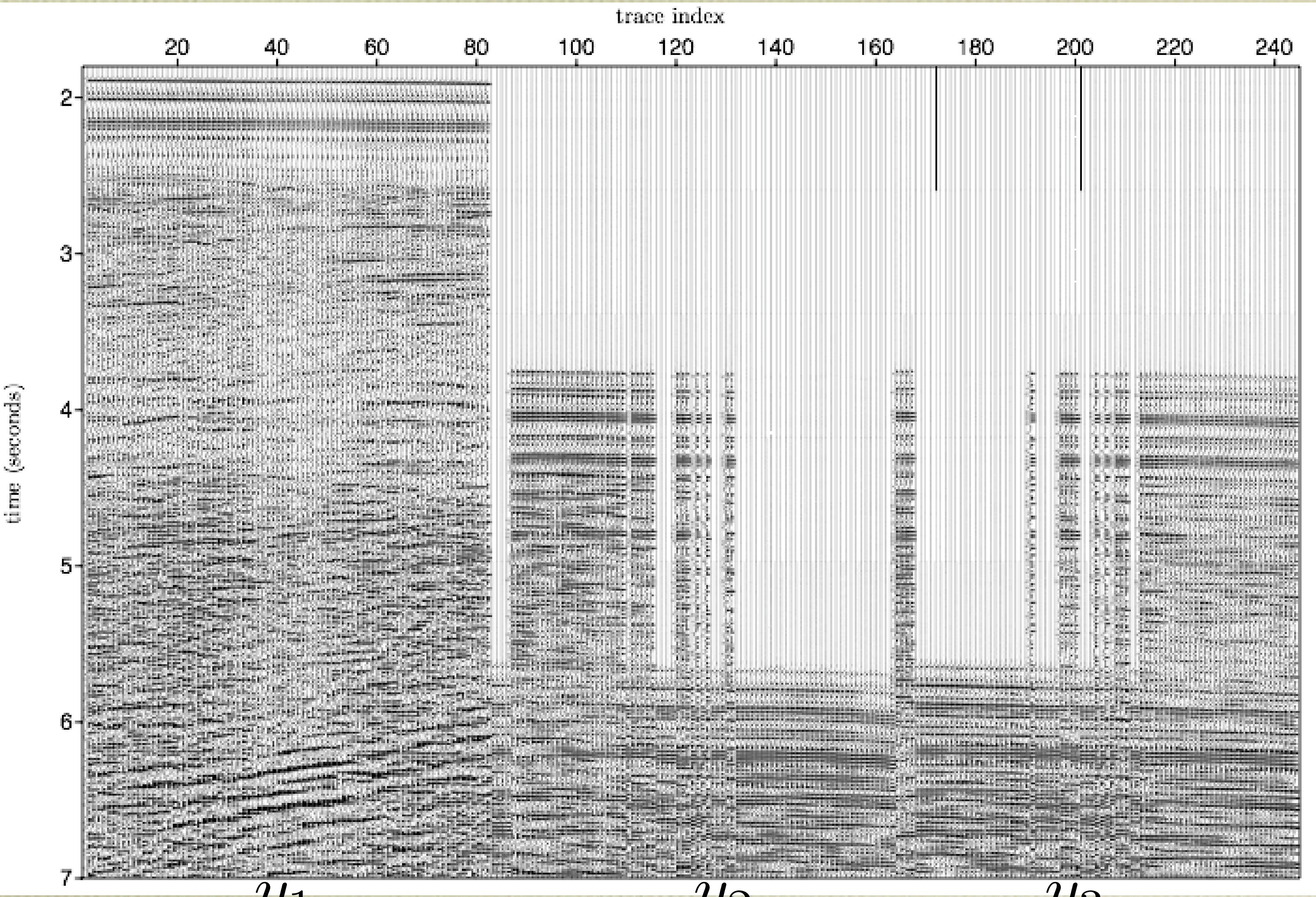
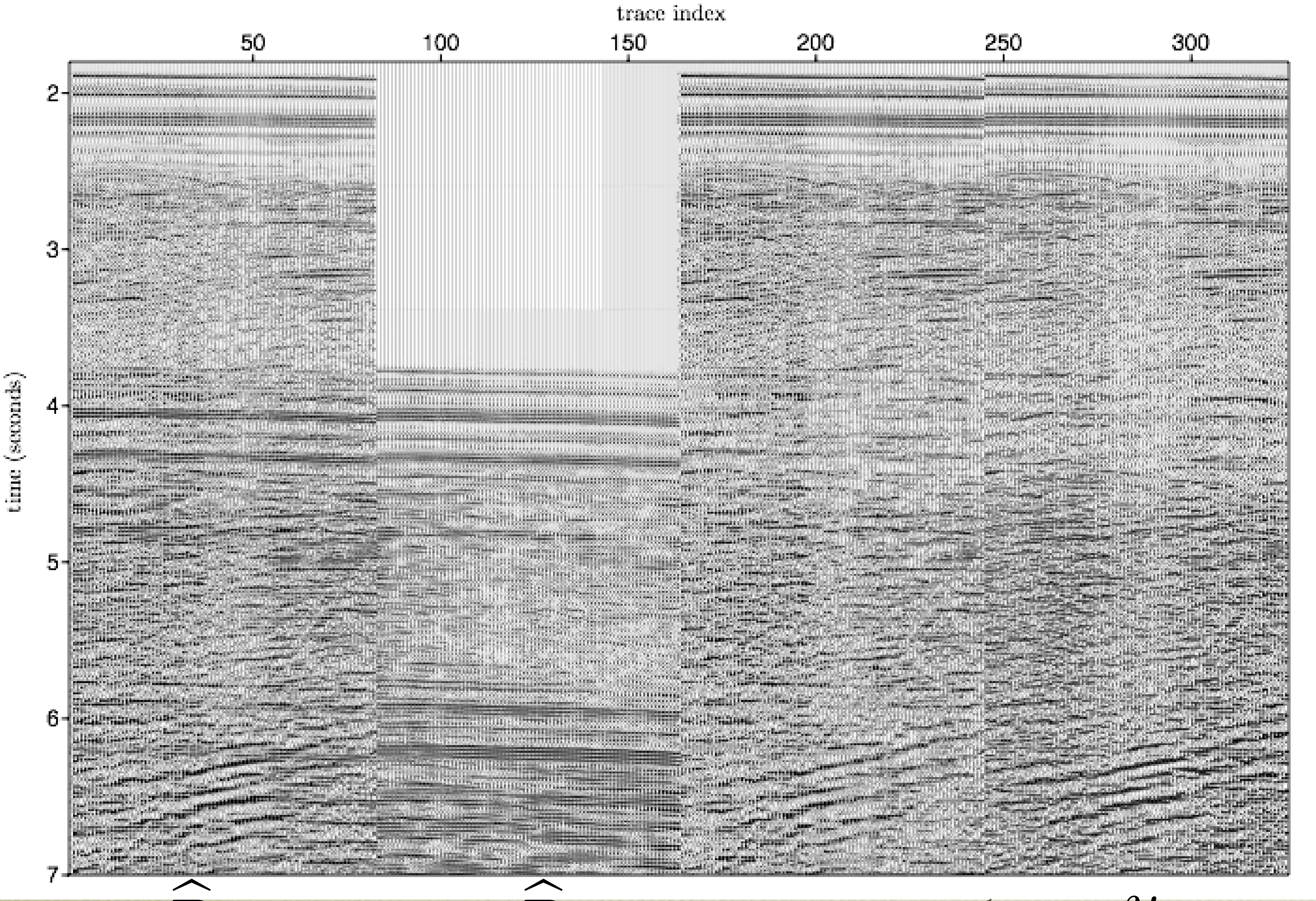
Gulf of Mexico example



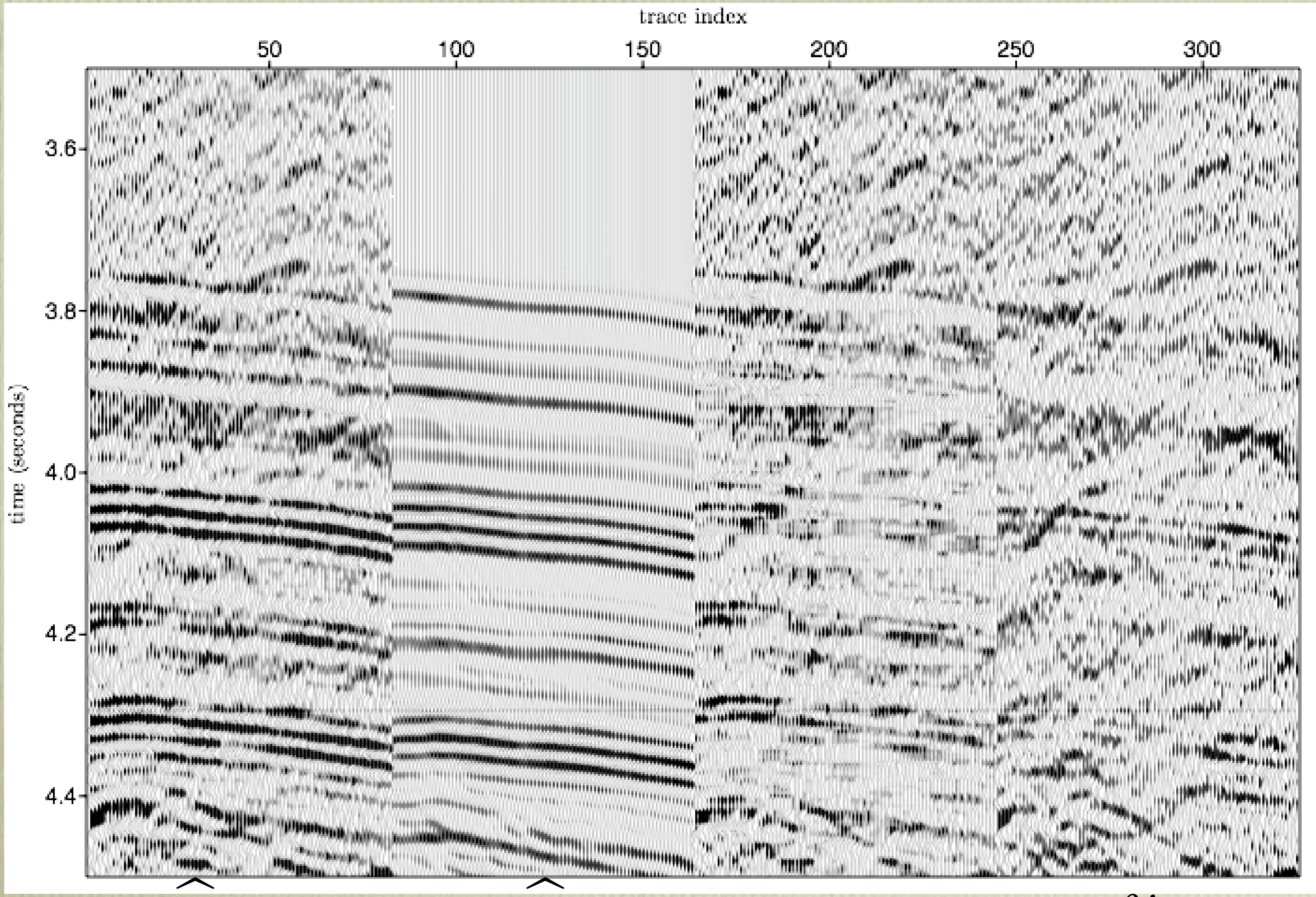
y_1 y_2 y_3

Gulf of Mexico Example



$\widehat{D}_1$ $\widehat{D}_2$ 2 pass match y_1

Gulf of Mexico Example



$\widehat{D}_1$ $\widehat{D}_2$ 2 pass match y_1

Discussion

- Adaptive subtraction for FSME
  - matching filters
  - independent component analysis (ICA)
- inverse matching filters, reference matching filters, Gulf of Mexico example