Mission - Oriented Seismic Research Program

Modeling the phase and amplitude of P waves in a heterogeneous elastic medium

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Motivation

For mainstream seismic processing objectives

- Ray-theory modeling can select events to model, but cannot arrange wave illumination, diffractions, and amplitude fidelity
- Wave-theory modeling in multi-dimensions, e.g., finite-difference elastic wave methods, models all wave types and histories

Motivation (continued)

- In this paper we provide a wave-theory method with the selectivity of ray theory combined with the illumination, diffractions, and amplitude fidelity of wave theory
- For ISS depth imaging this paper is a step in a strategy to take us from "it's possible" to "it's an addition to the seismic toolbox"

Motivation (continued)

- Currently, our elastic ISS depth-imaging formulation seeks the location of a reflector in the earth where the history of waves moving from source to the reflector and then up from the reflector to the receiver can contain P and S experiences on the down and up propagation.
- That requires three unknown physical properties $v_P(x, y, z)$, $v_S(x, y, z)$, and $\rho(x, y, z)$.

Motivation (continued)

- ISS depth-imaging algorithms have a rapid increase in the number and types of terms they need to include as the number of unknown physical properties increases.
- With the P-only history elastic modeling, we expect to reduce the number of unknown physical quantities, and the effort ISS has to put forth to provide an accurate depth image.

Motivation (continued)

The test will be:

- With current best-available methods to approximate the velocity, and the approximate methods (finite-difference RTM) to image through the velocity, can the state of ISS imaging at some point beat that combined product of approximations
- Finite-difference modeling and RTM don't like rapid changes
- ISS imaging feeds on rapid changes

2D Isotropic Heterogeneous Elastic Media

- We choose (here for convenience, although it is unnecessary) to describe the medium as a homogeneous background plus a perturbation in background material properties that results in actual medium properties
- For this specific case, the coupled equations for P- and S-waves, ϕ_P and ϕ_S respectively, can be written as

2D Isotropic Heterogeneous Elastic Media (continued)

$$\left.\begin{aligned} \left[\nabla^2 + \frac{\omega^2}{{\alpha_0}^2}\right]\phi_P &= V_{PP}\phi_P + V_{PS}\phi_S + f_P \\ \left[\nabla^2 + \frac{\omega^2}{{\beta_0}^2}\right]\phi_S &= V_{SS}\phi_S + V_{SP}\phi_P + f_S \end{aligned}\right\} \qquad (1)$$

where α_0 is the reference P-wave velocity, β_0 is the reference S-wave velocity, V_{PP}, V_{PS}, V_{SP}, V_{SS} are the matrix elements of the perturbation operator in the P-S representation, and f_P, f_S are the components of the source function in the P-S representation

2D Isotropic Heterogeneous Elastic Media (continued)

Following the logic in the Short note in the Annual Report, the causal solution to the second equation in equation 1 can be written as

$$\phi_S = \int G_S \left(V_{SP} \phi_P + f_S \right) \tag{2}$$

where G_S is chosen to be the causal whole-space solution of

$$\left[\nabla^2 + \frac{\omega^2}{{\beta_0}^2} - V_{SS} \right] G_S = \delta \tag{3}$$

2D Isotropic Heterogeneous Elastic Media (continued)

Substituting this result back into the first equation of equation 1, we obtain

$$\left[\nabla^2 + \frac{\omega^2}{{\alpha_0}^2}\right]\phi_P$$
$$= V_{PP}\phi_P + V_{PS}\int G_S V_{SP}\phi_P + V_{PS}\int G_S f_S + f_P \qquad (4)$$

For the case where the source generates only P-waves, $f_S = 0$, we define a new form of the perturbation, responsible for self-coupling and inter-channel coupling of the propagating P-wave, as

2D Isotropic Heterogeneous Elastic Media (continued)

$$\mathbf{V} \equiv V_{PP} + V_{PS} \int G_S V_{SP}, \tag{5}$$

make the identification that the reference wave field results from propagating the source with the reference P-wave Green's function, i.e., $\phi_P^0 = G_P^0 f_P$, and obtain

$$\phi_P = \phi_P^0 + G_P^0 \mathbf{V} \phi_P, \tag{6}$$

a single-channel equation for the total-pressure wave field, ϕ_P, which builds in all of the S-channel interactions that influence the P-channel, without solving for ϕ_S

2D Isotropic Heterogeneous Elastic Media (continued)

The equation for ϕ_P is an integral equation for which the solution can be expanded, using the Born series, as

$$\phi_P = \phi_P^0 + G_P^0 \mathbf{V} \phi_P^0 + G_P^0 \mathbf{V} G_P^0 \mathbf{V} \phi_P^0 + \ldots \ . \tag{7}$$

which is a direct modeling equation for a P-wave in an inhomogeneous elastic medium

3D Isotropic Heterogeneous Elastic Media

For a 3D inhomogeneous elastic medium, the 3D elastic wave equation can be written

$$\hat{L}\vec{\phi} = (\hat{L}_0 - \hat{V})\vec{\phi} = \vec{f}$$

$$\begin{pmatrix} {L_0}^P - V_{PP} & -V_{PS_H} & -V_{PS_V} \\ -V_{S_H P} & {L_0}^{S_H} - V_{S_H S_H} & -V_{S_H S_V} \\ -V_{S_V P} & -V_{S_V S_H} & {L_0}^{S_V} - V_{S_V S_V} \end{pmatrix} \begin{pmatrix} \phi_P \\ \phi_{S_H} \\ \phi_{S_V} \end{pmatrix} = \begin{pmatrix} f_P \\ f_{S_H} \\ f_{S_V} \end{pmatrix} \quad (8)$$

3D Isotropic Heterogeneous Elastic Media (continued)

Again following the logic in the Short note, we assume that our source generates only P-waves and write the system of three coupled equations for the components of the wave field:

$$\left[\nabla^2 + \frac{\omega^2}{{\alpha_0}^2} - V_{PP}\right]\phi_P = V_{PS_H}\phi_{S_H} + V_{PS_V}\phi_{S_V} + f_P \quad (9)$$

$$\left[\nabla^2 + \frac{\omega^2}{{\beta_0}^2} - V_{S_H S_H}\right]\phi_{S_H} = V_{S_H P}\phi_P + V_{S_H S_V}\phi_{S_V} \quad (10)$$

$$\left[\nabla^2 + \frac{\omega^2}{{\beta_0}^2} - V_{S_V S_V}\right]\phi_{S_V} = V_{S_V P}\phi_P + V_{S_V S_H}\phi_{S_H} \quad (11)$$

3D Isotropic Heterogeneous Elastic Media (continued)

Introduce the Green's functions G_{S_H} and G_{S_V} defined as the whole-space causal solutions of

$$\left[\nabla^2 + \frac{\omega^2}{{\beta_0}^2} - V_{S_H S_H}\right] G_{S_H} = \delta \tag{12}$$

$$\left[\nabla^2 + \frac{\omega^2}{{\beta_0}^2} - V_{S_V S_V}\right] G_{S_V} = \delta \tag{13}$$

Therefore, the SV component of the wave field is

$$\phi_{S_V} = G_{S_V}\left(V_{S_V P}\phi_P + V_{S_V S_H}\phi_{S_H}\right) \tag{14}$$

3D Isotropic Heterogeneous Elastic Media (continued)

Substituting the right-hand side of equation 14 into equation 10 gives

$$\begin{aligned}\phi_{S_H} =& G_{S_H}\left(V_{S_H P} + V_{S_H S_V} G_{S_V} V_{S_V P}\right)\phi_P \\ &+ G_{S_H} V_{S_H S_V} G_{S_V} V_{S_V S_H}\phi_{S_H} \qquad (15)\\ &\left(1 - G_{S_H} V_{S_H S_V} G_{S_V} V_{S_V S_H}\right)\phi_{S_H} \\ =& G_{S_H}\left(V_{S_H P} + V_{S_H S_V} G_{S_V} V_{S_V P}\right)\phi_P \qquad (16)\end{aligned}$$

Inverting the operator acting on ϕ_{S_H} gives

3D Isotropic Heterogeneous Elastic Media (continued)

$$\left(1 - G_{S_H} V_{S_H S_V} G_{S_V} V_{S_V S_H}\right)^{-1} = \sum_{k=0}^{+\infty} \left(G_{S_H} V_{S_H S_V} G_{S_V} V_{S_V S_H}\right)^k \tag{17}$$

$$\phi_{S_H} = \left[\sum_{k=0}^{+\infty} \left(G_{S_H} V_{S_H S_V} G_{S_V} V_{S_V S_H}\right)^k \right] G_{S_H} \left(V_{S_H P} + V_{S_H S_V} G_{S_V} V_{S_V P}\right) \phi_P \tag{18}$$

an expression for ϕ_{S_H} in terms of ϕ_P

3D Isotropic Heterogeneous Elastic Media (continued)

Substituting this expression into equation 9, we find that the P component of the wave field, as it propagates through a 3D heterogeneous elastic medium, is governed by the following scalar equation

3D Isotropic Heterogeneous Elastic Media (continued)

$$
\begin{aligned}
\phi_P = & \phi_P^0 + G_P^0 \Bigg\{ V_{PP}\phi_P \\
& + G_P^0 V_{PS_H} \left[\sum_{k=0}^{+\infty} \left(G_{S_H} V_{S_H S_V} G_{S_V} V_{S_V S_H} \right)^k \right] \\
& \times G_{S_H} \left(V_{S_H P} + V_{S_H S_V} G_{S_V} V_{S_V P} \right) \phi_P \\
& + G_P^0 V_{PS_V} G_{S_V} \Big(V_{S_V P} + V_{S_V S_H} \\
& \times \left[\sum_{k=0}^{+\infty} \left(G_{S_H} V_{S_H S_V} G_{S_V} V_{S_V S_H} \right)^k \right] G_{S_H} \big(V_{S_H P} \\
& + V_{S_H S_V} G_{S_V} V_{S_V P} \big) \Big) \Bigg\} \phi_P
\end{aligned} \tag{19}
$$

3D Isotropic Heterogeneous Elastic Media (continued)

Rewriting equation 19 gives the form

$$\phi_P = \phi_P^0 + G_P^0 \mathcal{V}_{PP} \phi_P \tag{20}$$

where $\mathcal{V}_{PP}$ is everything inside the curly brackets in equation 19

3D Isotropic Heterogeneous Elastic Media (continued)

A Born or Neumann series form of equation 20 provides a modeling equation for ϕ_P:

$$\phi_P = \phi_P^0 + G_P^0 \mathcal{V}_{PP} \phi_P^0 + G_P^0 \mathcal{V}_{PP} G_P^0 \mathcal{V}_{PP} \phi_P^0 + \dots \qquad (21)$$

which models P-waves directly in a heterogeneous elastic medium with all intermediate P, SH, and SV episodes in the predicted P-wave's experience and history included

3D Isotropic Heterogeneous Elastic Media (continued)

$$\phi_P = \phi_P^0 + G_P^0 V_{PP} \phi_P^0 + G_P^0 V_{PP} G_P^0 V_{PP} \phi_P^0 + \dots \qquad (22)$$

Equation 22, with $\mathcal{V}_{PP}$ replaced by V_{PP}, will predict a P-wave and P-wave events in measured data, when those P-waves and events only have intermediate P-wave episodes in their history

Conclusions

- We have provided: (a) a formalism for modeling the amplitude and phase of P-waves in a heterogeneous elastic medium, and (b) a procedure for selecting and modeling the amplitude and phase of P-waves (and P-wave events in measured data) that have only intermediate P-wave episodes in their experiences and history
- The latter could be a useful compromise and timely interim practical response to the daunting full 3D elastic modeling problem

Conclusions (continued)

- Analytic forward-series calculations of equation 7 in 2D and equation 22 in 3D, for example, with an oblique incident plane wave reflecting off a single reflector (and then two reflectors) where elastic properties change, will allow us to examine the way different actual converted and un-converted wave events are constructed from reference P and S properties and contributions within the series

Conclusions (continued)

- The implications and impact of the ideas and formalism put forth in this paper for: (a) large-scale modeling, such as SEAM, (b) RTM, (c) so-called "FWI", and (d) moving the inverse scattering series imaging without the velocity from viable (Weglein et al., 2012) to delivering differential added value — will be examined and reported in future correspondence

Conclusions (continued)

- The Born series form described for P-wave-only-history P-waves in a heterogeneous isotropic elastic medium could be well-suited for a modeling project (such as SEAM modeling) with a smooth background with small, rapid perturbations generating reflections and where events with converted wave episodes are often treated as noise
- Also, methods to directly solve the Fredholm integral equation of the second kind, equations 6 and 21, will be investigated

References I

Weglein, Arthur B., Fang Liu, Xu Li, Paolo Terenghi, Ed Kragh, James D. Mayhan, Zhiqiang Wang, Joachim Mispel, Lasse Amundsen, Hong Liang, Lin Tang, and Shih-Ying Hsu. "Inverse scattering series direct depth imaging without the velocity model: first field data examples." *Journal of Seismic Exploration* 21 (March 2012): 1–28.