A new method to eliminate first order internal multiples for a normal incidence plane wave on a 1D earth

Yanglei Zou, Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston

SUMMARY

A new method to remove internal multiples has been derived under 1D normal incidence. This new method is a step further from the inverse scattering series(ISS) internal-multiple attenuator(IMA) to an eliminator under 1D normal incidence. In the procedure of the method, it constructs the reflection coefficients in order to remove the extra transmission coefficients of the events and then constructs a new function based on the reflection coefficients. This method may be relevant and provide value when primaries and internal multiples interfere with each other in both on-shore and off-shore data under near 1D circumstances. This method does not seek higher order terms in the ISS to construct an algorithm that eliminate first order internal multiples generated by all reflectors.

INTRODUCTION

The inverse scattering series(ISS) allows specific seismic processing objectives, such as free-surface-multiple removal and internal-multiple removal to be achieved directly in terms of data, without any subsurface estimation of the earths properties.

For internal-multiple removal, the Inverse Scattering Series Internal-Multiple Attenuator(IMA) can predict correct time and well-understood amplitude for all internal multiples without any subsurface information. The IMA can remove internal multiples more effectively by using energy minimization adaptive subtraction (EMAS). However, events may interfere with each other in both on-shore and off-shore seismic data. In these cases, the EMAS criteria may fail. For example, when a primary destructively interferes with an internal multiple and the real energy of the primary is greater than the interfering event, the EMAS will not only remove the internal multiple but also touch the primary. The EMAS criteria is to remove one event in the interfering events and obtain the minimum data energy. However, in this example, the criteria fails as the real primary has greater energy.

Predicting the correct amplitude of the internal multiples is an effective way of avoiding the limitations of EMAS. W.Herrera and A.B.Weglein(2012) and has derived a subseries that can eliminate all first order internal multiples generated at the shallowest reflector and can further attenuate deeper internal multiples. The present work is a step further from the IMA to an eliminator under 1D normal incidence. The method is derived based on the analytic expressions of the data under 1D normal incidence. And in the procedure of the method, it determines the reflection coefficients in order to remove the extra transmission coefficients of the events and constructing a new function based on the reflection coefficients. This method may be relevant and provide value when primaries and internal multiples destructively interfering with each other in 1D normal incidence data.

INTERNAL MULTIPLE ATTENUATOR(IMA) AND ATTENUATION FACTOR(AF) UNDER 1D NORMAL INCIDENCE

The 1D normal incidence version of IMA given by Araújo (1994) Weglein et al. (1997) is presented as follows:

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \times \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z''). \quad (1)$$

To demonstrate the mechanism of the internal multiple attenuation algorithm and to examine its properties, Weglein et al. (2003) considered the simplest two-layer model that can produce an internal multiple. For this model, the reflection data caused by an impulsive incident wave $\delta(t-\frac{z}{c})$ is:

$$D'(t) = R_1\delta(t-t_1) + T_{01}R_2T_{10}\delta(t-t_2) + \cdots \quad (2)$$

where t_1, t_2 and R_1, R_2 are the two way times and reflection coefficients from the two reflectors,respectively; and T_{01} and T_{10} are the coefficients of transmission between model layers 0 and 1 and 1 and 0, respectively.

$$D'(\omega) = R_1 e^{i\omega t_1} + T_{01}R_2T_{10}e^{i\omega t_2} + \cdots. \quad (3)$$

where$D'(\omega)$ is the temporal Fourier transform of $D'(t)$. Make a water speed migration with:$z_1 = \frac{c_0 t_1}{2}$ $z_2 = \frac{c_0 t_2}{2}$.

The input data can now be expressed in terms of $k = k_z$, z_1 and z_2:

$$b(k) = R_1 e^{ikz_1} + T_{01}R_2T_{10}e^{ikz_2} + \cdots \quad (4)$$

The date is now ready for the internal multiple algorithm. Substituting $b(k)$ into the algorithm, we derive the prediction in the time domain:

$$b_3t = R_1R_2^2T_{01}^2T_{10}^2\delta(t-(2t_2-t_1)) \quad (5)$$

From the example it is easy to compute the actual first order internal multiple precisely:

$$-R_1R_2^2T_{01}T_{10}\delta(t-(2t_2-t_1)) \quad (6)$$

Therefore, the time prediction is precise, and the amplitude of the prediction has an extra power of $T_{01}T_{10}$ which is called the Attenuation Factor(AF), thus defining exactly the difference between the attenuation represented by b_3 and elimination.

To derive a general formula for the amplitude prediction of the algorithm, A.C.Ramírez and A.B.Weglein (2005) analyzed a model with n layers and respective velocities C_n, n is an integer. By using the definitions $R_1 = R_1'$, $R_N' = R_N\prod_{i=1}^{N-1}(T_{i-1,i}T_{i,i-1})$

DOI http://dx.doi.org/10.1190/segam2013-1172.1

and Einsteins summation, the reflection data from a normal incident spike wave we obtain the following:

$$D(t) = R'_n \delta(t - t_n) + internal\ multiples \quad (7)$$

The generalized prediction of the attenuator is obtained by the following:

$$b_3^{IM}(k) = R'_i R'_j R'_k e^{ikz_i} e^{ikz_j} e^{ikz_k} \quad (8)$$

which in the time domain becomes

$$b_3^{IM}(k) = R'_i R'_j R'_k \delta(t - (t_i + t_k - t_j)) \quad (9)$$

By evaluating Equation (9) for different values of i, j and k the amplitude prediction of first order internal multiples is obtained and can be generalized for any amount of layers in a 1D model. Compared with the real amplitude of internal multiples in the data, we can obtain the AF(Figure 1 shows an example of the Attenuation Factor).

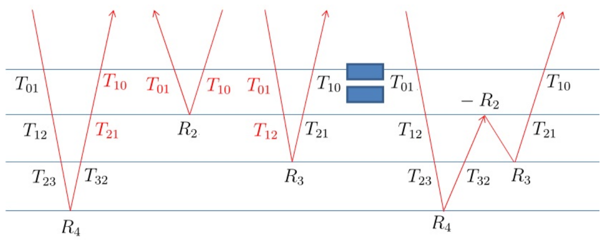

Figure 1: an example of the Attenuation Factor of a first order internal multiple generated at the second reflector. The red terms in this figure show the extra transmission coefficients. The Attenuation Factor in this example is $AF_2 = (T_{01}T_{10})^2 T_{12} T_{21}$

The attenuation factor, AF_j, in the prediction of internal multiples is given by the following:

$$AF_j = \begin{cases} T_{0,1}T_{1,0} & (j=1) \\ \prod_{i=1}^{N-1} (T_{i-1,i}^2 T_{i,i-1}^2) T_{j,j-1} T_{j-1,j} & (1 < j < J) \end{cases} \quad (10)$$

The attenuation factor AF_j can also be performed by using reflection coefficients:

$$AF_j = \begin{cases} 1 - R_1^2 & (j=1) \\ (1-R_1^2)^2(1-R_2^2)^2 \cdots (1-R_{j-1}^2)^2(1-R_j^2) & \\ & (1 < j < J) \end{cases} \quad (11)$$

The subscript j represents the generating reflector, and J is the total number of interfaces in the model. The interfaces are numbered starting with the shallowest location.

A NEW IDEA TO ELIMINATE INTERNAL MULTIPLES UNDER 1D NORMAL INCIDENCE

The discussion above demonstrates that all first order internal multiples generated at the same reflector have the same AF. In order to predict correct amplitude of first order internal multiples directly in terms of data, a new term in the second integral of IMA can be developed to remove the AF and make the function an eliminator. The function must be developed from

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \times \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \quad (12)$$

to

$$b_E^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \times \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \quad (13)$$

For the 1D normal incidence, $b_1(z)$ is the water speed migration of the data. It is expressed as follows:

$$b_1(z) = R_1\delta(z-z_1) + R'_2\delta(z-z_2) + R'_3\delta(z-z_3) + \cdots + R'_n\delta(z-z_n) + \cdots \quad (14)$$

The $F[b_1(z)]$ should have the form as the following:

$$\begin{aligned} F[b_1(z')] &= \frac{R_1}{AF_{j=1}}\delta(z'-z_1) + \frac{R'_2}{AF_{j=2}}\delta(z'-z_2) + \cdots \\ &+ \frac{R'_n}{AF_{j=n}}\delta(z'-z_n) + \cdots \\ &= \frac{R_1\delta(z'-z_1)}{1-R_1^2} + \frac{R'_2\delta(z'-z_2)}{(1-R_1^2)^2(1-R_2^2)} + \cdots \\ &+ \frac{R'_n\delta(z'-z_n)}{(1-R_1^2)^2(1-R_2^2)^2\cdots(1-R_{n-1}^2)^2(1-R_n^2)} \\ &+ \cdots \end{aligned} \quad (15)$$

By using reverse engineering,Y.Zou(2013) derived the $F[b_1(z)]$ directly in terms of data:

$$F[b_1(z)] = \lim_{\varepsilon' \to 0} \frac{c(z)\int_{z-\varepsilon}^{z+\varepsilon} c(z'')dz''}{\int_{z-\varepsilon}^{z+\varepsilon} b_1(z')dz' \{1 - [\int_{z-\varepsilon}^{z+\varepsilon} c(z'')dz'']^2\} + \varepsilon'} \quad (16)$$

$$c(z) = \frac{b_1(z)}{1 - \int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' c(z'')} \quad (17)$$

To derive the $F[b_1(z)]$ function from $b_1(z)$, c(z) must first be solved in equation (17). Thereafter, c(z) is integrated into Equation (16). And take $F[b_1(z)]$ into equation (13), we will get the new equation.

First type of equation approximation

Equation (17) is an integral equation:

$$c(z) = \frac{b_1(z)}{1 - \int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' c(z'')}$$

Generally, this kind of equation does not have analytical solutions; hence, an approximation must be made for equation

DOI http://dx.doi.org/10.1190/segam2013-1172.1

(17). The simplest approximation is presented as follows:

$$\begin{aligned} c(z)_{1T} =& \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' c(z'')} \\ \approx& \frac{b_1(z)}{1-0} \\ \approx& b_1(z) \end{aligned} \quad (18)$$

Integrate $c(z)_{1T}$ into equation (16). And take $F[b_1(z)]$ into equation (13), we will get the first type of equation approximation. It can be shown that this first kind approximation can predict correct amplitude for all first order internal multiples generated at the shallowest reflector and can further attenuate deeper internal multiples.

Second type of equation approximation

A more accurate approximation is presented as follows:

$$\begin{aligned} c(z)_{2T} =& \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' c(z'')} \\ \approx& \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' b_1(z'')} \end{aligned} \quad (19)$$

Integrate $c(z)_{2T}$ into Equation (16). And take $F[b_1(z)]$ into equation (13), we will get the second type of equation approximation. This type of approximation can predict the correct amplitude for all first order internal multiples generated at the shallowest and next shallowest reflectors and can further attenuate deeper internal multiples.

And we can again replace the c(z) in the denominator by equation(19) to get a better approximation, for 1D normal incidence data, all these three approximations are presented as follows:

$$\begin{aligned} c(z)_{1T} =& R_1\delta(z-z_1) + R'_2\delta(z-z_2) + R'_3\delta(z-z_3) \\ &+ R'_4\delta(z-z_4) + \cdots \end{aligned} \quad (20)$$

$$\begin{aligned} c(z)_{2T} =& R_1\delta(z-z_1) + \frac{R'_2}{1-R_1^2}\delta(z-z_2) + \frac{R'_3}{1-R_1^2-R_2'^2}\delta(z-z_2) \\ &+ \frac{R'_4}{1-R_1^2-R_2'^2-R_3'^2}\delta(z-z_4) + \cdots \\ =& R_1\delta(z-z_1) + R_2\delta(z-z_2) + \frac{R'_3}{1-R_1^2-R_2'^2}\delta(z-z_2) \\ &+ \frac{R'_4}{1-R_1^2-R_2'^2-R_3'^2}\delta(z-z_4) + \cdots \end{aligned} \quad (21)$$

$$\begin{aligned} c(z)_{3T} =& R_1\delta(z-z_1) + R_2\delta(z-z_2) + \frac{R'_3}{1-R_1^2-R'_2\frac{R'_2}{1-R_1^2}}\delta(z-z_3) \\ &+ \frac{R'_4}{1-R_1^2-R'_2\frac{R'_2}{1-R_1^2}-R'_3\frac{R'_3}{1-R_1^2-R_2'^2}}\delta(z-z_4) + \cdots \\ =& R_1\delta(z-z_1) + R_2\delta(z-z_2) + R_3\delta(z-z_3) \\ &+ \frac{R'_4}{1-R_1^2-R'_2\frac{R'_2}{1-R_1^2}-R'_3\frac{R'_3}{1-R_1^2-R_2'^2}}\delta(z-z_4) + \cdots \end{aligned} \quad (22)$$

We can see the first event has correct amplitude in $c(z)_{1T}$, the first two events have correct amplitude in $c(z)_{2T}$ and the first three events have correct amplitude in $c(z)_{3T}$. In those equations, the reflection coefficients of first several layers have been correctly constructed and reflection coefficients of deeper layers have been better constructed. That means in the procedure, the reflection coefficients have been constructed in order to remove extra transmission coefficients from the middle integral.

Only primaries are considered as the input in deriving all these equations. However, for these two types of approximations, the conclusion is still valid when we consider real data which contains both primaries and internal multiples as input. By using these approximations to predict the amplitude of internal multiples generated at the shallowest and next shallowest reflectors, in $F[b_1(z)]$, only the part of the data preceding the second primary is used. Considering that the internal multiples do not arrive prior to the second primary, that part of the data remains the same when only primaries or both primaries and internal multiples are considered.

NUMERICAL EXAMPLES

This section presents a numerical example that shows the result of the original IMA and the two types of equation approximation of the new equation.

Figure 2 and 3 show the model used in this study and the 1D normal incidence input data, respectively. We will do following comparison of the part of the data in the red rectangular shown in figure 3.

The output of IMA in Figure 4 clearly shows that all multiples are predicted with the correct time and approximate amplitude. Figure 5, which displays the first type of equation approximation of the new function, shows that all internal multiples with a downward reflection at the shallowest reflector (IM_{212},IM_{312} and IM_{213})are removed. And in Figure 6,we can see all internal multiples generated at the shallowest and the next shallowest reflectors (IM_{212},IM_{312},IM_{213} and IM_{323}) are removed by the second type of equation approximation.

In the figure 4,5 and 6:

P_3 is the Third primary. IM_{212},IM_{213},IM_{312} are internal multiples with a downward reflection at the shallowest reflector.The three numbers in the subscript refer to the historical number of reflectors in the internal multiples. For example, IM_{212} is a first order internal multiple with two upward reflection at the second reflector and a downward reflection at the first(shallowest) reflector. IM_{323} is a internal multiple with a downward reflection at the next shallowest reflector. The spurious event is an false event generated by IM_{212},P_3 and IM_{212}, which exist in every figure. (A method for removing the spurious events have been discovered by Ma et al. (2012) Liang et al. (2012)).

DOI http://dx.doi.org/10.1190/segam2013-1172.1

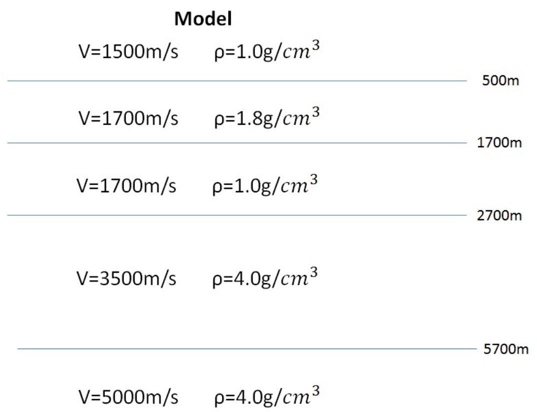

Figure 2: Model

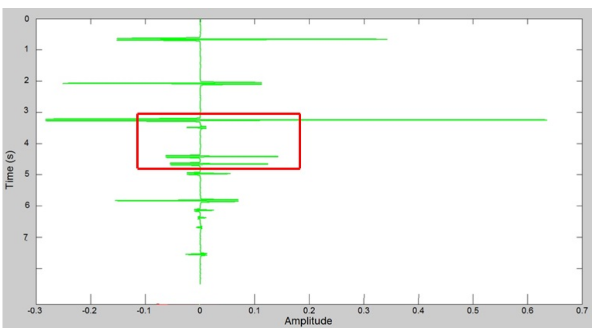

Figure 3: Input data(1D normal incidence)

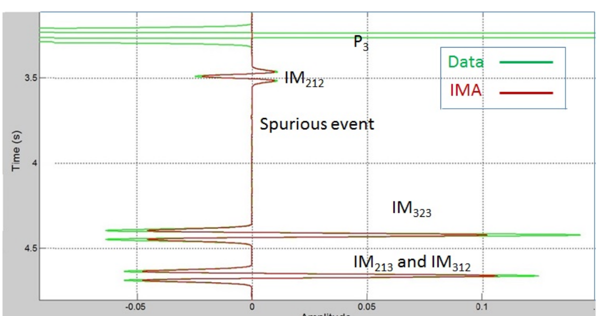

Figure 4: Output of the ISS-IMA

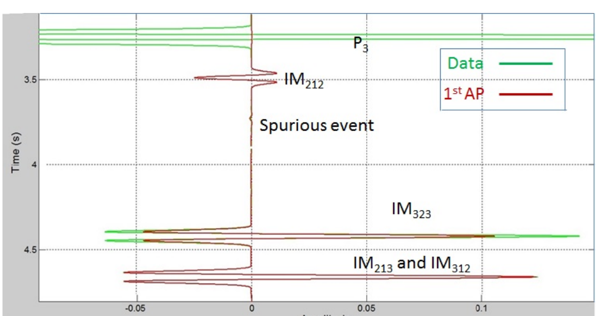

Figure 5: Output of the first type of equation approximation

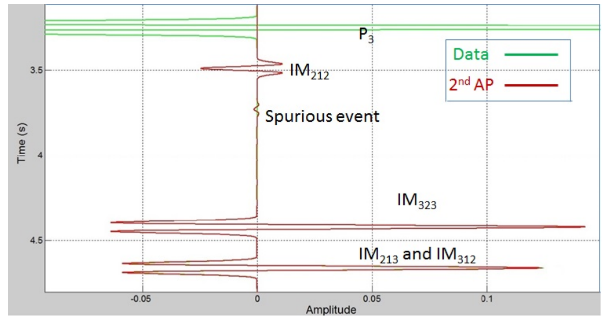

Figure 6: Output of the second type of equation approximation

CONCLUSION

1.A new method to remove internal multiples has been derived under 1D normal incidence. In the procedure of the method, it constructs the reflection coefficients in order to remove the extra transmission coefficients and then constructs a new function based on the reflection coefficients.

2.Two different types of equation approximation are also presented: (1) The first type of equation approximation can predict the correct amplitude of all first order internal multiples generated at the shallowest reflector. (2)The second type of equation approximation can predict the correct amplitude of all first order internal multiples generated at the shallowest and next shallowest reflectors. Depending on the goals, different types of approximation can be made, and we can achieve each specific goal by using the corresponding equation approximations. In practise, the elimination (not attenuation) method of internal multiples may be relevant and provide value when primaries and internal multiples interfere with each other in both on-shore and off-shore data under near 1D circumstances.

3.This equation and its approximations:
(a)not generate any more events than IMA.
(c)not touch primaries.

ACKNOWLEDGMENTS

We are grateful to all M-OSRP sponsors for long-term encouragement and support in this research. We would like to thank all our colleagues for the help of finishing this abstract and valuable discussions in this research program.

DOI http://dx.doi.org/10.1190/segam2013-1172.1

http://dx.doi.org/10.1190/segam2013-1172.1

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2013 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Araújo, F. V., 1994, Linear and nonlinear methods derived from scattering theory: Backscattered tomography and internal multiple attenuation: Ph.D. thesis, Universidade Federal da Bahia.

Liang, H., C. Ma, and A. B. Weglein, 2012, A further general modification of the leading order ISS attenuator of first order internal multiples to accommodate primaries and internal multiples when an arbitrary number of reflectors generate the data: Theory, development, and examples: Mission Oriented Seismic Research Program Annual Report, 148–165.

Ma, C., H. Liang, and A. B. Weglein, 2012, Modifying the leading order ISS attenuator of first-order internal multiples to accommodate primaries and internal multiples: fundamental concept and theory, development, and examples exemplified when three reflectors generate the data: Mission Oriented Seismic Research Program Annual Report, 133–147.

Ramírez, A. C., and A. B. Weglein, 2005, Progressing the analysis of the phase and amplitude prediction properties of the inverse scattering internal multiple attenuation algorithm: Journal of Seismic Exploration, **13**, 283–301.

Weglein, A. B., F. V. Araújo, P. M. Carvalho, R. H. Stolt, K. H. Matson, R. T. Coates, D. Corrigan, D. J. Foster, S. A. Shaw, and H. Zhang, 2003, Inverse scattering series and seismic exploration: Inverse Problems, **19**, no. 6, R27–R83, http://dx.doi.org/10.1088/0266-5611/19/6/R01.

Weglein, A. B., F. A. Gasparotto, P. M. Carvalho, and R. H. Stolt, 1997, An inverse-scattering series method for attenuating multiples in seismic reflection data: Geophysics, **62**, 1975–1989, http://dx.doi.org/10.1190/1.1444298.