# Including higher-order Inverse Scattering Series terms to address a serious shortcoming/problem of the internal-multiple attenuator: exemplifying the problem and its resolution

*Chao Ma * and Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston*



## SUMMARY

The Inverse Scattering Series (ISS) internal-multiple-attenuation algorithm is often called upon due to its unique and unmatched ability to attenuate internal multiples. It can predict internal multiples (accurately in time and approximately in amplitude) that are generated by any reflectors below the free surface without needing sub-surface information. While this algorithm is the most capable algorithm currently available for attenuating internal multiples, there are an increasing number of offshore and onshore circumstances where the problem of removing internal multiples is beyond the current ISS internal-multiples-attenuation algorithm's ability to address. For example, an open issue and specific problem is removing internal multiples which are proximal to or interfering with the primaries (Weglein et al., 2011). This invites us to pursue solutions that can address this type of challenge. Recent work by Herrera and Weglein (2013) and Zou and Weglein (2013) extend the current ISS internal-multiple-attenuation algorithm to the ISS first-order internal-multiple-elimination algorithm. Ma et al. (2011) and Liang et al. (2013) show the spurious predictions (events that do not exist in the data) that the current ISS internal-multiple-attenuation algorithm can produce when the input data are generated by three or more reflectors, and internal multiples in the input data are treated as subevents. That spurious event issue is only a problem for the ISS leading-order term (the term used to derive the current ISS internal-multiple-attenuation algorithm), specific higher-order terms from ISS will remove those spurious events. We develop the new higher-order ISS internal-multiple-attenuation algorithm and show examples of how it can effectively address that limitation (spurious predictions) of the current ISS internal-multiple-attenuation algorithm while at the same time retaining the current algorithm's recognized strengths.



## INTRODUCTION

The Inverse Scattering Series is a comprehensive seismic data-processing tool from which distinct task-specific subseries can be isolated to perform specific tasks (Weglein et al., 2003). For example, the current ISS leading-order internal-multiple-attenuation algorithm was first developed by Araujo et al. (1994) and Weglein et al. (1997) from the ISS internal-multiple-attenuation subseries. The strengths (always present independent of the circumstances and complexity of the geology and the play) of the ISS internal-multiple-attenuation algorithm are: (1) this algorithm does not need any sub-surface information for predicting the internal multiples, and (2) all first-order internal multiples generated by any reflectors below the free surface are predicted at once with accurate time and approximate amplitude. The tests on ISS internal-multiple-attenuation algorithm have shown promising results and unique value compared with other multiple-suppression methods (Fu et al., 2010; Hsu et al., 2010; Andre, 2011; Terenghi et al., 2011; Luo et al., 2011; Weglein et al., 2011; Kelamis et al., 2013). However, Weglein et al. (2011) point out limitations of the current ISS internal-multiple-attenuation algorithm: (1) this algorithm is always an attenuation algorithm, and (2) spurious predictions can occur only if there are three or more reflectors, and internal multiples in the input data are treated as subevents.

It should be mentioned that those two limitations will not always matter. For example, in the cases in which there are several strong internal-multiple generators, and primaries, internal multiples and spurious events are isolated from each other, the current ISS internal-multiple-attenuation algorithm, combined with the energy-minimization adaptive subtraction methods, will remove internal multiples and spurious events completely.

However, there are times when those two limitations do matter. For example, in some offshore (e.g., North Sea) and most on-shore (e.g., Middle East) plays with many internal multiple generators, internal multiples will be proximal to or interfere with primaries, the current ISS internal-multiple-attenuation algorithm plus energy-minimization adaptive subtraction methods will not remove internal multiples and spurious predictions. In these circumstances, a complete internal-multiple-elimination algorithm without spurious predictions is called upon.

In this paper, we will focus on addressing the second limitation (i.e., spurious predictions) and exemplifying that including the higher-order terms for addressing the spurious prediction will provide added values and better prediction results.

## AN OVERVIEW OF THE ISS LEADING-ORDER INTERNAL MULTIPLE ATTENUATION ALGORITHM

We refer the current ISS internal-multiple-attenuation algorithm as ISS leading-order internal-multiple-attenuation algorithm (leading-order means this algorithm predicts internal multiples with the exact time but approximate amplitude). This algorithm starts with the input data, $D(k_g, k_s, \omega)$, in 2D, which are the Fourier transform of the deghosted prestack data, and with the wavelet deconvolved and direct wave and free-surface multiples removed. The second term is the prediction of the first-order internal multiples. In a 2D earth, this prediction is (Weglein et al., 2003)

$$b_3(k_g,k_s,q_s+q_s)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}dk_1\int_{-\infty}^{\infty}dk_2 e^{-iq_1(z_g-z_s)}e^{iq_2(z_g-z_s)}\times\int_{-\infty}^{\infty}dz_1 b_1(k_g,k_1,z_1)e^{i(q_g+q_1)z_1}\times\int_{-\infty}^{z_1-\varepsilon}dz_2 b_1(k_1,k_2,z_2)e^{-i(q_1+q_2)z_2}\times\int_{z_2+\varepsilon}^{\infty}dz_3 b_1(k_2,k_s,z_3)e^{i(q_2+q_s)z_3}, \quad (1)$$

where $k_s$ and $k_g$ are the horizontal wavenumbers for the source and receiver coordinates, respectively; $q_g$ and $q_s$ are the vertical source and receiver wavenumbers defined by $q_i = sgn(\omega)\sqrt{\frac{\omega^2}{c_0^2}-k_i^2}$ for $i \in \{g,s\}$ ($\omega$ is the temporal freqency); $z_s$ and $z_g$ are source and receiver depths; and $z_j$ ($i \in \{1,2,3\}$) represents pseudo-depth by using a reference velocity migration. The quantity $b_1(k_g,k_s,z)$ corresponds to an uncollapsed migration (Weglein et al., 1997) of effective plane-wave incident data.

The data with their first-order internal multiple attenuated are

$$D(k_g,k_s,\omega)+D_3(k_g,k_s,\omega), \quad (2)$$

where $b_3(k_g,k_s,\omega) = -2iq_s D_3(k_g,k_s,\omega)$.

For a 1-D earth and a normal incident plane wave, equation 1 reduces to

$$b_3(k)=\int_{-\infty}^{\infty}dz_1 e^{ikz_1}b_1(z_1)\int_{-\infty}^{z_1-\varepsilon}dz_2 e^{-ikz_2}b_1(z_2)\times\int_{z_2+\varepsilon}^{\infty}dz_3 e^{ikz_3}b_1(z_3). \quad (3)$$

The deghosted data, $D(t)$, for an incident plane wave, satisfy $D(\omega) = b_1(\frac{2\omega}{c_0})$, $D(\omega)$ is the temporal Fourier transform of $D(t)$, $b_1(z) = \int_{-\infty}^{\infty} e^{ikz}b_1(k)dk$, and $k = \frac{2\omega}{c_0}$ is the vertical wavenumber.

Equation 2 then reduces to

$$D(t)+D_3(t), \quad (4)$$

where $D_3(t)$ is Inverse Fourier transform of $D_3(\omega)$, and $D_3(\omega) = b_3(\frac{2\omega}{c_0})$, where $k = \frac{2\omega}{c_0}$.

The idea behind using equation 1 or equation 3 to predict the first-order internal multiple is to treat primaries (events that experience only one upward reflection) in the data as subevents, and to combine different subevents that satisfying the " lower(A)-higher(B)-lower(C) " requirement in the pseudo-depth domain (see Figure 1).

We denote the three primary-subevents combination as "*PPP*", where $P$ stands for primary. Equation 1 or equation 3 can predict all first-order internal multiples without needing any subsurface information, and those predicted internal multiples will have an accurate time and an approximate amplitude. Its limitations (e.g., spurious predictions) and resolutions are pointed out in in Weglein et al. (2011). In the next section, we will briefly review the generation of those spurious predictions and the proposed algorithms to reduce them (Ma et al., 2011; Liang et al., 2013).

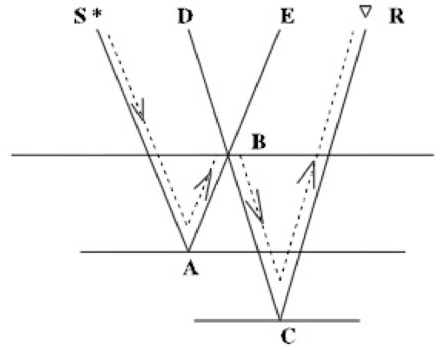


Figure 1: Combination of subevents for the first-order internal multiple (dashed line), $(SABE)_{time} + (DBCR)_{time} - (DBE)_{time} = (SABCR)_{time}$, figure adapted from Weglein et al. (2003)

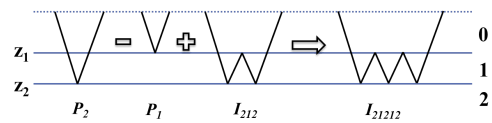


Figure 2a: In a two-reflector example, a "Primary – Primary – Internal multiple (*PPI*)" combination predicts a second-order internal multiple.

## THE HIGHER-ORDER MODIFICATION OF THE ISS INTERNAL-MULTIPLE LEADING-ORDER ALGORITHM

The work of Araujo et al. (1994) and Weglein et al. (1997) focuses on the analysis of the leading-order prediction of first-order internal multiples (i.e., equation 1) by treating primaries in the data as subevents (see Figure 1). However, data consist of both primaries and internal multiples. Hence, when the data are input into equation 1, the internal multiples are inevitably also treated as subevents. When both primaries and internal multiples are treated as subevents,

$$\begin{aligned} b_3 &= b_1 * b_1 * b_1 \\ &= (P+I)(P+I)(P+I) \\ &= PPP+PPI+PIP+IPP+PII+IPI+IIP+III, \quad (5) \end{aligned}$$

where $*$ stands for the nonlinear interaction between the data (see equation 1), and $P$ and $I$ stand for primaries and internal multiples. Notice that we use the above expression to categorize different possible subevent combinations.

When internal multiples are treated as subevents, Zhang and Shaw (2010) use a two-reflector model to show that a second-order internal multiple can be predicted (see Figure 2a); Ma et al. (2011) and Liang et al. (2013) use three-reflector and four-reflector examples to show that spurious events are generated, respectively (see Figures 2b and 2c).

It is worth noting that because of the "lower-higher-lower" requirement of the algorithm (see Figure 1), the spurious event

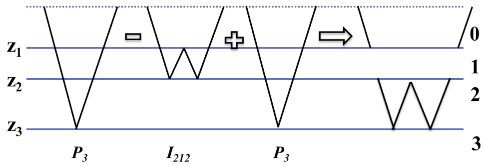


Figure 2b: In a three-reflector example, a "Primary – Internal multiple – Primary (*PIP*)" combination predicts a spurious event.

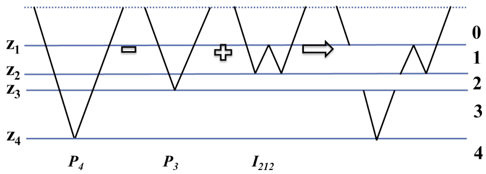


Figure 2c: In a four- reflector example, a "Primary – Primary – Internal multiple (*PPI*)" combination predicts another type of spurious event.

in Figure 2b (i.e., $P_3$–$I_{212}$–$P_3$), can be generated only when the arrival time of the third primary ($P_3$) is greater than that of the internal multiple ($I_{212}$). Otherwise, this spurious event would not be produced.

In Figure 2c, the condition for the prediction of spurious event (i.e., $P_4$–$P_3$–$I_{212}$) is that the arrival time of the third primary ($P_3$) is smaller than that of the internal multiple ($I_{212}$).

In order to eliminate the prediction of spurious events, we must remove the effects of internal multiples acting as subevents. The higher-order terms from ISS are isolated to address the two types of spurious events shown in 2b and 2c:

$$b_5^{PIP}(k) = \int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} b_3(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} b_1(z_3), \tag{6}$$

and

$$b_5^{PPI}(k) = 2\int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_3(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} b_1(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} b_1(z_3), \tag{7}$$

where $b_1(z)$ is the same as in equation 3, and $b_3(z) = \int_{-\infty}^{\infty} e^{ikz} b_3(k) dk$. The superscripts *PIP* and *PPI* in equations 6 and 7 indicate that higher-order terms, $b_5^{PIP}$ and $b_5^{PPI}$, are included to address the spurious prediction generated by Primary–Internal multiple–Primary and Primary–Primary-Internal multiple, respectively. The factor of 2 is used in equation 7 because an internal multiple can act as a subevent in either the innermost integral or the outermost integral.

By including the higher-order terms in equations 6 and 7, the proposed new algorithm in 1-D is

$$D(t) + D_3(t) + D_5^{PIP}(t) + D_5^{PPI}(t), \tag{8}$$

where $D(t)$ and $D_3(t)$ are the same as in equation 4, and $D_5^{PIP}(t)$ and $D_5^{PPI}(t)$ are Inverse Fourier transforms of $D_3(\omega), D_5^{PIP}(\omega)$ and $D_5^{PPI}(\omega)$, respectively, and $D_5^{PIP}(\omega) = b_5^{PIP}(k)$ and $D_5^{PPI}(\omega) = b_5^{PPI}(k)$ with $k = \frac{2\omega}{c_0}$.

It should be mentioned that, in the cases where there are only three reflectors, only $D_5^{PIP}$ is needed to address *PIP*-type spurious events because *PPI*-type spurious events arise only when there are four or more reflectors.

## 1-D NORMAL INCIDENT EXAMPLES WITH INTERFERING PRIMARIES AND INTERNAL MULTIPLES

In this section, we test both the $D_5^{PIP}$ and $D_5^{PPI}$ terms by using more realistic and practical synthetic data (generated by many reflectors with interfering primaries and internal multiples), compare the reference internal multiples to the prediction results with/without the inclusion of higher-order terms.

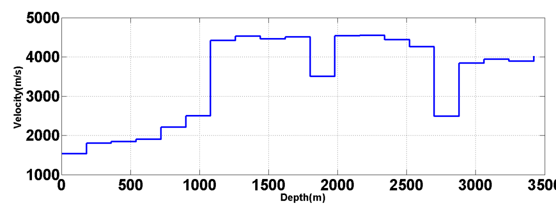


Figure 3: Velocity model used to generate synthetic data (courtesy of Saudi Arabian Oil Co.).

The first synthetic data are generated based on velocity model in Figure 3 by using reflectivity method with a ricker wavelet of peak frequency at 25 Hz .

The comparison is shown in Figure 4. When input contains only primaries, the leading-order algorithm predicts the first-order internal multiples very well (see Figure 4a). The prediction result shows degradation (because of the higher-order internal multiples and spurious events in the prediction) when the input contains both primaries and internal multiples (see Figure 4b). With the inclusion of higher-order terms, the prediction result improves (see Figure 4c).

In the second test, instead of comparing the prediction results with the reference internal multiples of **first-order**, we will

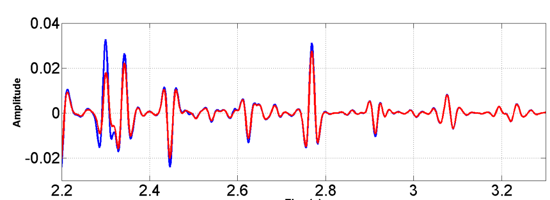


Figure 4a: Comparison between the reference first-order internal multiples (in blue) and leading-order prediction (in red) with primaries as input.

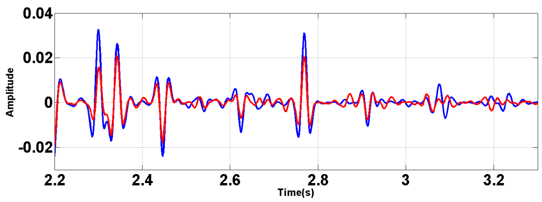


Figure 4b: Comparison between the reference first-order internal multiples (in blue) and leading-order prediction (in red) with primaries and internal multiples as input.

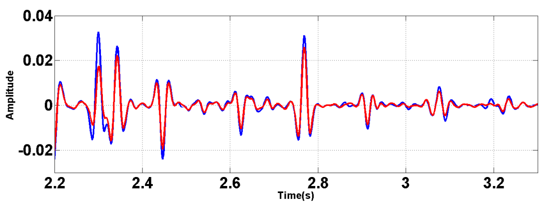


Figure 4c: Comparison between the reference first-order internal multiples (in blue) and leading-order plus higher-order prediction (in red) with primaries and internal multiples as input.

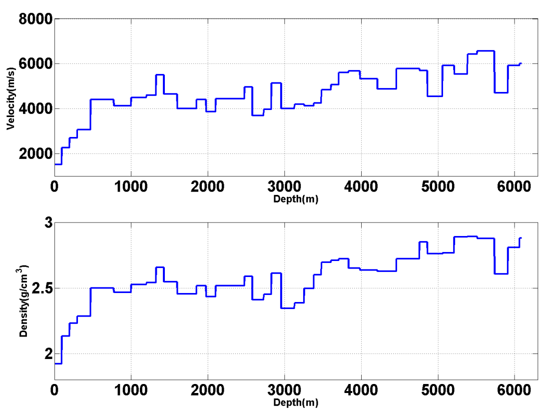


Figure 5: Velocity and density blocking from well-log data (courtesy of Kuwait Oil Company).

compare the prediction results with the reference internal multiples of **all orders**. The model (data courtesy of Kuwait Oil Company) is shown in Figure 5 with both velocity and density varying. We use reflectivity methods with a ricker wavelet of peak freqency at 25 Hz to generate the test data corresponding the model in Figure 5.

The comparison among the reference internal multiples (blue in Figure 6), leading-order prediction (red in Figure 6a), and leading-order plus higher-order prediction (red in Figure 6b) are shown in Figure 6 where arrows point to the significant improvements. Notice that the inputs for both predictions contain primaries and internal multiples. The results show that inclusion of higher-order terms improves the prediction results in the cases in which events are interfering with each other.

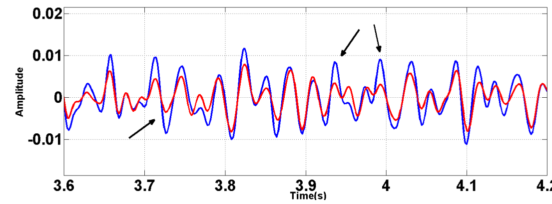


Figure 6a: Comparison between the reference internal multiples (in blue) and leading-order prediction (in red).

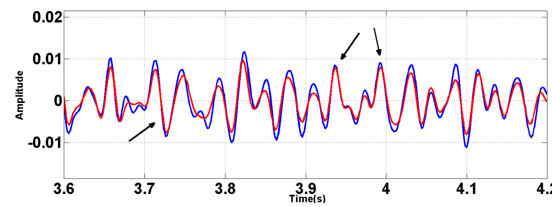


Figure 6b: Comparison between the reference internal multiples (in blue) and leading-order plus higher-order prediction (in red).

## CONCLUSIONS

In this paper, we exemplified a serious shortcoming (i.e., spurious predictions) of the current ISS leading-order internal-multiple-attenuation algorithm. We develop, test and analyze the resolution with a new higher-order ISS algorithm that anticipates and removes the spurious events. This higher-order ISS internal-multiple-attenuation algorithm retains the strengths of the current leading-order ISS internal-multiple-attenuation algorithm and addresses one of its limitations.

The synthetic tests on the realistic well-log based data sets in this paper show the significance and value of including the higher-order ISS terms to address the spurious predictions.

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