Predicting reference medium properties from invariances in Green's theorem reference wave prediction: towards an on-shore near surface medium and reference wave prediction

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SUMMARY

The Inverse Scattering Series (ISS) methods require prerequisites to reach their potential. Seismic data preprocessing for ISS methods includes identifying and removing the reference wave, estimating the source wavelet and radiation pattern, and source and receiver deghosting. For on-shore seismic exploration, these preprocessing steps still have many serious challenges. To study how to determine the reference velocity for land application, this paper uses the marine environment and a point source as a starting point, and shows that the invariance of the estimated source signature for different output points below the cable could be a criterion to find the correct reference velocity. In addition, for the case of a source signature and radiation pattern, the invariance of the source wavelet in one radiation angle could be the criterion for having the right reference velocity.

INTRODUCTION

The current petroleum industry trend to deep water and complex geology, where primary and multiple events may often experience interfering or proximal with each other. In this case, removal of the multiple events becomes a big challenge. Inverse Scattering Series (ISS) methods offer a direct way of removing free-surface multiples and attenuating internal multiples without requiring any subsurface information. However, these methods have prerequisites that need to be satisfied. The prerequisites include identifying and removing the reference wave, estimating the source wavelet and radiation pattern, and source and receiver deghosting. In order to deliver the high fidelity of ISS multiple predictions, effective preprocessing methods need to be developed and improved (Zhang (2007), Mayhan et al. (2011), Mayhan and Weglein (2013), Tang et al. (2013), Yang et al. (2013)).

As seismic exploration goes to more complex and difficult on-shore and offshore plays, there are more fundamental issues and challenges need to be resolved. Among these issues and challenges, removal of the reference wave on land is one pressing and important topic. Why do we need to remove the reference wave first? Scattering theory separates the real world into two parts: the reference medium, whose property is known, plus a perturbation. The wave that travels in the reference medium is called the reference wave, which does not experience the earth that we are interested in. So it is important to identify the reference wave and remove it before the following data processing steps, such as multiple removal and depth imaging. We need to identify it because it also contains the information of the source signature, which is essential information in the subsequent processing steps. ISS methods require that the reference medium agrees with the actual medium on and above the measurement surface (Weglein et al. (2003)). Green's theorem provides a good mathematical tool to achieve these prerequisites that are consistent with the ISS methods they are meant to serve.

For on-shore seismic application, the property of the medium near surface is often complicated and not easy to determine, e.g., because the conditions of rocks, soil or minerals in the near surface are not easy to define due to weathering. Strong ground roll could be generated that can obscure reflected seismic data. To remove the ground roll/reference wave, the physical properties of the near surface is needed. Our purpose in looking for a way to determine the velocity of near surface medium on land, is to provide a foundation for the study of on-shore seismic data preprocessing methods. It is part of the comprehensive Inverse Scattering Series multiple removal strategy.

In order to study the complex on-shore or ocean bottom near surface property, we propose to start from seeking criteria which can determine whether we have the correct reference medium information or not. The criteria could be the presence of some invariance that only the correct reference velocity would satisfy. We use a marine seismic application as a starting point to pursue this idea. First, consider an isotropic point source, which has an isotropic source wavelet in every radiation direction. Using Green's theorem, we can estimate the wavelet signature everywhere below the measurement surface. When using the correct reference velocity, the results for the wavelet should be invariant for all output points below the measurement surface. Thus, the value of reference velocity we use in the wavelet calculation that leads to an invariance of the estimated source wavelet is the correct reference velocity. Furthermore, instead of a single point source, in practice, source arrays which have angle radiation pattern are widely used in industry (Loveridge et al. (1984)). Then the invariance of the estimated wavelet will happen when estimating the wavelet at different points along one radiation angle. Similarly, only the correct reference velocity can lead to the invariance. Thus, the invariances of the source wavelet indicate that we have found the correct reference velocity.

This paper will discuss the criteria of predicting the reference medium properties from invariances in Green's theorem-based wavelet estimation, for both a point source and for source array cases. For a point source, the source wavelet estimated at any points beneath the measurement surface should stay the same, while for source array, estimated source wavelet results in one radiation angle should be invariant. These invariances could be criteria of having the correct reference velocity. Future study will extend this research from marine example to complex on shore elastic model.

THEORY

Green's theorem for wavelet estimation

The theory of wavelet estimation using Green's theorem was first described in Weglein and Secrest (1990). Assume that source $A(\omega)$ is placed at $\vec{r}_s$ and the receiver is at $\vec{r}$. The pressure wavefield P satisfies constant density acoustic wave equation in the frequency domain,

$$\left(\nabla^2+\frac{\omega^2}{c^2(\vec{r})}\right)P(\vec{r},\vec{r}_s,\omega)=A(\omega)\delta(\vec{r}-\vec{r}_s) \tag{1}$$

In scattering theory, we treat the actual medium as a combination of an unperturbed medium, called the reference medium, plus a perturbation. Introduce perturbation α defined by

$$\frac{1}{c^2(\vec{r})}=\frac{1}{c_0^2}[1-\alpha(\vec{r})],$$

where c_0 is the velocity in a homogeneous reference medium. Then Equation 1 becomes

$$\left(\nabla^2+\frac{\omega^2}{c_0^2}\right)P(\vec{r},\vec{r}_s,\omega)=\underbrace{\frac{\omega^2}{c_0^2}\alpha(\vec{r})P(\vec{r},\vec{r}_s,\omega)+A(\omega)\delta(\vec{r}-\vec{r}_s)}_{\rho}. \tag{2}$$

The right hand side of the equation can be viewed as the source of the wavefield P, which consists of two terms: the perturbation α, which generates scattered wave P_s, and the active source $A(\omega)$, the energy source that generates the wave, P. The corresponding Green's function satisfies,

$$\left(\nabla^2+\frac{\omega^2}{c_0^2}\right)G_0(\vec{r},\vec{r}',\omega)=\delta(\vec{r}-\vec{r}'). \tag{3}$$

Having a causal Green's function G_0^+, we can have wavefield P,

$$\begin{aligned} P(\vec{r},\omega) &= \int_\infty G_0^+(\vec{r},\vec{r}',\omega)\rho(\vec{r}',\omega)d\vec{r}' \\ &= \int_\infty G_0^+(\vec{r},\vec{r}',\omega)\frac{\omega^2}{c_0^2}\alpha(\vec{r}')P(\vec{r}',\omega)d\vec{r}' \\ &\quad +A(\omega)G_0^+(\vec{r},\vec{r}_s,\omega). \end{aligned} \tag{4}$$

The first term on the right hand side of Equation 4 is the source that generates the difference between the total wavefield P and the reference wavefield P_0, where $P_0=A(\omega)G_0$.

On the other hand, from Green's second identity, plugging P and G_0 in Equation 2 and Equation 3 in, we have,

$$\begin{aligned} &\int_V (P\nabla'^2G_0-G_0\nabla'^2P)d\vec{r}' \\ =& \int_V \Bigg(P(\vec{r}',\vec{r}_s,\omega)\left[-\frac{\omega^2}{c_0^2}G_0(\vec{r}',\vec{r},\omega)+\delta(\vec{r}-\vec{r}')\right] \\ & -G_0(\vec{r}',\vec{r},\omega)\left[-\frac{\omega^2}{c_0^2}P(\vec{r}',\vec{r}_s,\omega)+\rho(\vec{r}')\right]\Bigg)d\vec{r}' \\ =& \int_V P(\vec{r}',\vec{r}_s,\omega)\delta(\vec{r}-\vec{r}')d\vec{r}' \\ -& \int_V G_0(\vec{r}',\vec{r},\omega)\left[\frac{\omega^2}{c_0^2}\alpha(\vec{r}')P(\vec{r}',\vec{r}_s,\omega)+\delta(\vec{r}'-\vec{r}_s)A(\omega)\right]d\vec{r}' \\ =& \oint_S dS\hat{n}\cdot \\ & \left[P(\vec{r}',\vec{r}_s,\omega)\nabla'G_0(\vec{r}',\vec{r},\omega)-G_0(\vec{r}',\vec{r},\omega)\nabla'P(\vec{r}',\vec{r}_s,\omega)\right] \end{aligned} \tag{5}$$

When choosing the volume as the infinite space below the measurement surface, and $\vec{r}$ is chosen to be below the measurement surface (inside the volume V), as shown in Figure 1. Equation 5 becomes

$$\begin{aligned} P(\vec{r},\vec{r}_s,\omega) &= \int_V G_0(\vec{r},\vec{r}',\omega)\frac{\omega^2}{c_0^2}\alpha(\vec{r}')P(\vec{r}',\vec{r}_s,\omega)d\vec{r}' \\ &\quad +\oint_S\left[P\nabla'G_0-G_0\nabla'P\right]\cdot\hat{n}dS. \end{aligned} \tag{6}$$

Choosing G_0^+ in Equation 6, let's compare Equation 6 and Equation 4. When the support of perturbation $\alpha(\vec{r})$ is within the volume V, the integral of α over infinity equals integral over volume V. Thus, with $\vec{r}$ inside the volume, the support of α within the volume, both Equations 6 and 4 should give the same wavefield. Therefore,

$$\begin{aligned} & A(\omega)G_0^+(\vec{r},\vec{r}_s,\omega) \\ =& \oint_S dS\hat{n}\cdot \\ & \left[P(\vec{r}',\vec{r}_s,\omega)\nabla'G_0^+(\vec{r}',\vec{r},\omega)-G_0^+(\vec{r}',\vec{r},\omega)\nabla'P(\vec{r}',\vec{r}_s,\omega)\right]. \end{aligned} \tag{7}$$

So source signature $A(\omega)$ can be estimated by a surface integral and then divided by the Green's function. Using Sommerfeld's radiation condition for G_0^+, the wavefield contribution at $\vec{r}$ in V from the infinite far away boundary vanishes. Then,

$$\begin{aligned} A(\omega) &= \frac{1}{G_0^+(\vec{r},\vec{r}_s,\omega)}\cdot\int_{m.s.}dS\hat{n}\cdot \\ & \left[P(\vec{r}',\vec{r}_s,\omega)\nabla'G_0^+(\vec{r}',\vec{r},\omega)-G_0^+(\vec{r}',\vec{r},\omega)\nabla'P(\vec{r}',\vec{r}_s,\omega)\right]. \end{aligned} \tag{8}$$

From Equation 8, we can see that the wavelet $A(\omega)$ is independent of the observation point $\vec{r}$. The estimation result of source wavelet should stay the same at any observation point $\vec{r}$ below the measurement surface . This condition will only hold when using the correct reference velocity. Therefore, the invariance of the estimated wavelet can be a criterion of having the correct reference velocity. Later, we will present test result to support this conclusion.

Radiation pattern

In the previous section, we focused on solving the wavelet from a point source at $\delta(\vec{r}-\vec{r}_s)$. In a more general case, a extended source array that consists of several point source could be used in seismic exploration. In this case, the source displays a radiation pattern in different radiation angles. The radiation pattern from a single effective point source could be estimated by assuming that $A(\omega)$ is a function of the radiation angle θ (using far field approximation).

Assume that a general extended source $\rho(\vec{r})$ as Figure 2 shows. Wavefield at $\vec{r}$ generated from this source array can be calculated from the integral,

$$P_0(\vec{r},\omega)=\int G_0(\vec{r},\vec{r}',\omega)\rho(\vec{r}')d\vec{r}'. \tag{9}$$

In 3D propagation, Green's function in frequency domain can be written as

$$G_0(\vec{r},\vec{r}',\omega) = \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}. \tag{10}$$

In the far field, $|\vec{r}| >> |\vec{r}'|$, we have approximation,

$$\begin{aligned}
|\vec{r}-\vec{r}'| &= \sqrt{(\vec{r}-\vec{r}')^2} \\
&= \sqrt{r^2 - 2\vec{r}\cdot\vec{r}' + r'^2} \\
&= r[1 - \frac{2\vec{r}\cdot\vec{r}'}{r^2} + \frac{r'^2}{r^2}]^{1/2} \\
&= r(1 - \frac{\vec{r}\cdot\vec{r}'}{r^2} + \frac{r'^2}{2r^2} + ...) \\
&= r - \hat{n}\cdot\vec{r}' + O(\frac{1}{r}).
\end{aligned} \tag{11}$$

The above equation uses Taylor series $(1+x)^{1/2} = 1 + \frac{1}{2}x + O(x^2)$, and $\hat{n}$ is the unit vector in the direction of $\vec{r}$. And similarly,

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r} + \frac{\hat{n}\cdot\vec{r}'}{r^2} + ... = \frac{1}{r} + O(\frac{1}{r^2}). \tag{12}$$

Then in the far field, Equation 9 becomes

$$\begin{aligned}
P_0(\vec{r},\omega) &= \int \frac{e^{ik(r-\hat{n}\cdot\vec{r}')}}{r}\rho(\vec{r}')d\vec{r}' \\
&= \frac{e^{ikr}}{r}\int e^{-ik\hat{n}\cdot\vec{r}'}\rho(\vec{r}')d\vec{r}' \\
&= \frac{e^{ikr}}{r}\tilde{\rho}(k\hat{n}).
\end{aligned} \tag{13}$$

Therefore, in the far field if we process seismic data generated from the source array as if a point source, we can get the source wavelet

$$A(\omega,\theta) = \frac{P_0}{G_0} = \tilde{\rho}(k\hat{n}).$$

Since $\hat{n}$ is the direction from the source to the observation point, the estimated wavelet result will display variances in different radiation angle. While in one radiation angle, wavelet $A(\omega,\theta)$ will be the same. This could be a criterion of determining the correct reference velocity. If using a wrong reference velocity, this invariance at one radiation angle will not be satisfied.

POINT SOURCE

In this test, we use Cagnidard-de Hoop method to model over-under cable data. Then using Green's theorem of Equation 8, we estimate the wavelet, $A(\omega)$, at different points at a fixed depth below the cable. We predict the estimated wavelet results using different reference velocities:

(1) correct reference velocity $c_0 = 1500m/s$;
(2) wrong reference velocity $c_0 = 1490m/s$;
(3) further wrong reference velocity $c_0 = 1450m/s$.

The estimated reference wavefields P_0 are shown in Figure 3, and corresponding wavelet results in Figure 4. Figure 3 indicates that the wrong reference velocities also lead to errors in the prediction of P_0. The estimated source wavelet results show that when using the correct reference velocity, the wavelet displays invariance at different offset, while wrong velocities give different wavelet prediction at different output points.

Therefore, only the correct reference velocity can result in the invariance of estimated wavelet. When the velocities are further from the reference velocity, the errors of wavelet invariance also becomes larger. This conclusion will also help us in finding the correct reference velocity.

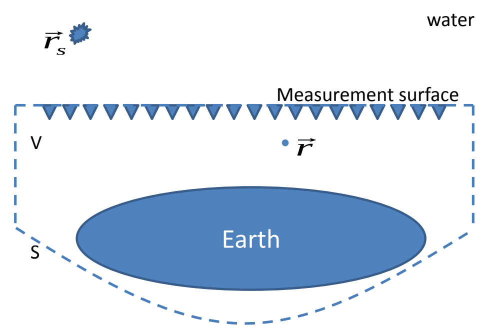

Figure 1: Volume chosen as half infinite space below the measurement surface.

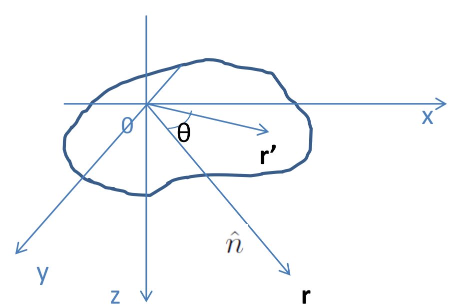

Figure 2: A general extended source.

SOURCE ARRAY

In this section, instead of a point source, data generated by a source array will be tested. The source array consists of 7 point sources separated at 3 m, as shown in Figure 5. First, we will estimate source wavelet along a horizontal cable, whose radiation angles are different. We predict source wavelet at depth 56 m, from offset 0 m to 606 m, whose radiation angles are from $0°$ to $85°$. The results in Figure 5 show the radiation pattern in different offset (radiation angle). Next, we estimate the wavelet $A(\omega,\theta)$ in one radiation angle. The estimated wavelet in angle $5.8°$ using different velocities is shown in Figure 7.

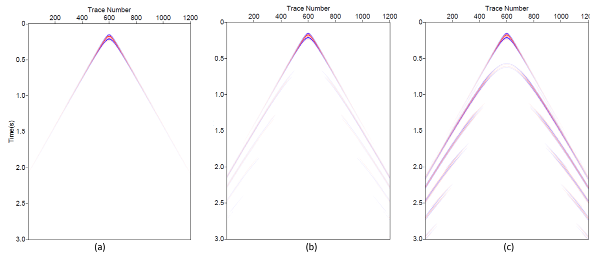

Figure 3: P_0 estimated using (a) correct $c_0 = 1500m/s$, (b) wrong $c_0 = 1490m/s$, (c) wrong $c_0 = 1450m/s$.

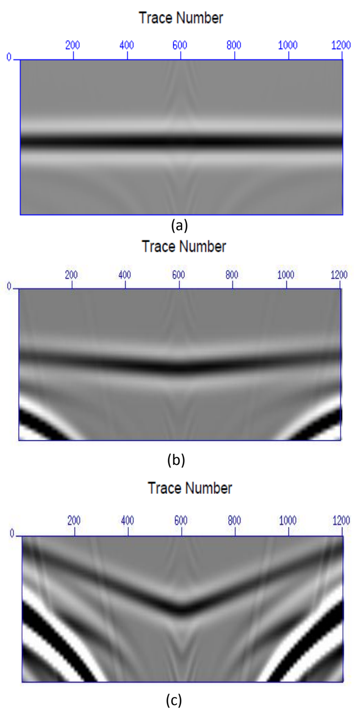

Figure 4: $A(t)$ estimated using (a) correct $c_0 = 1500m/s$, (b) wrong $c_0 = 1490m/s$, (c) wrong $c_0 = 1450m/s$

Similar to the conclusion above, we can see that only the correct velocity gives us the invariance of the source array wavelet in one angle, while the wrong reference velocity will lead to differences of the wavelet in one radiation angle.

CONCLUSIONS

We have shown that an output point invariance of the estimated wavelet using Green's theorem could be a criterion for determining the correct reference velocity. For a point source, the invariance occurs for the output point anywhere below the measurement surface, while for a source array, the invariance

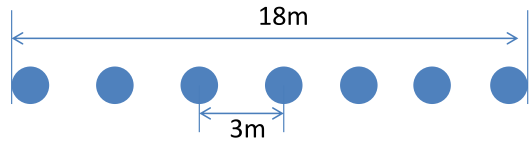

Figure 5: Source array

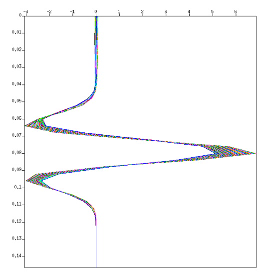

Figure 6: Radiation pattern of source array in Figure 5, estimated from offset 0m to 606m.

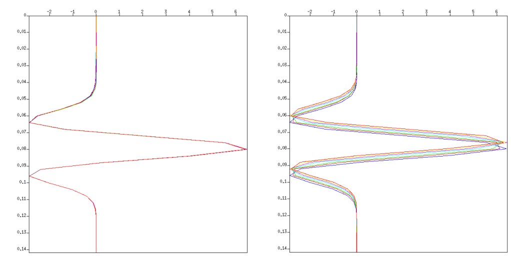

Figure 7: Wavelet estimated at depth 36m, 56m, 76m, 96m, 116m, 136m, 156m in the same radiation angle using (a) correct reference wave $c_0 = 1500m/s$ and (b) wrong reference velocity $c_0 = 1450m/s$.

is for output points along one radiation angle. Using marine seismic application as a starting point, this paper shows that invariances of Green's theorem-based wavelet estimation could be a criterion of determining the reference velocity. Using similar thinking, in the future study we will focus on solving the complex on-shore or ocean bottom near surface medium problems. For on-shore or ocean bottom problems, understanding of the near surface property could enable us to predict and remove the ground roll/reference wave on land, and thereby enhance the capability of subsequent multiple removal processing steps for the challenge of on-shore multiple attenuation.

ACKNOWLEDGEMENTS

We are grateful to all M-OSRP sponsors for encouragement and support in this research. We would like to thank Jinlong Yang and Chao Ma for reviewing this paper.

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