The internal-multiple elimination algorithm for all reflectors for 1D earth
Part I: strengths and limitations

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SUMMARY

The ISS(Inverse-Scattering-Series) internal-multiple attenuation algorithm(Araújo et al. (1994) and Weglein et al. (1997)) can predict the correct time and approximate amplitude for all first-order internal multiples without any information of the earth. This algorithm is effective and can attenuate internal multiples in many cases. However, in certain places, both offshore and onshore, the multiple is often proximal to or interfering with the primaries. Therefore, the task of removing internal multiples without damaging primaries becomes more challenging and subtle and currently beyond the collective capability of the petroleum industry. Weglein et al. (2003) proposed a three-pronged strategy for providing an effective response to this pressing and prioritized challenge. One part of the strategy is to develop an internal-multiple elimination algorithm that can predict both the correct amplitude and correct time for all internal multiples. The ISS internal-multiple elimination algorithm for all first-order internal multiples generated from all reflectors in a 1D earth is proposed in part I of this paper. The primaries in the reflection data that enters the algorithm provides that elimination capability, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. That is a limitation of this new algorithm. In part II of this two part paper, we show how the ISS anticipates that shortcoming. Higher order ISS terms when combined with this new algorithm will provide elimination ability without the current shortcoming. The basic algorithm is developed,evaluated and tested in part I. The next version with higher order ISS terms that rewrites the elimination algorithm without a downside is presented and tested in part II. Moreover, this elimination algorithm based on the ISS internal-multiple attenuation algorithm is derived by using reverse engineering to provide the difference between eliminate and attenuate for a 1D earth. This particular elimination algorithm is model type dependent since the reverse engineering method is model type dependent. The ISS internal-multiple attenuation algorithm is model type independent and in future work we will pursue the development of an eliminator for a multi-dimensional earth by identifying terms in the inverse scattering series that have that purpose.

INTRODUCTION

The inverse-scattering-series allows all seismic processing objectives, such as free-surface-multiple removal and internal-multiple removal to be achieved directly in terms of data, without any estimation of the earth's properties. For internal-multiple removal, the ISS internal-multiple attenuation algorithm can predict the correct time and approximate and well-understood amplitude for all first-order internal multiples generated from all reflectors without any subsurface information. If the events in data are isolated, the energy minimization adaptive subtraction can fix the gap between attenuation algorithm and elimination algorithm plus all factors that are outside the assumed physics of the subsurface and acquisition, et al. However, in certain places, events often interfere with each other in both on-shore and off-shore seismic data. In these cases, the criteria of energy minimization adaptive subtraction may fail and completely removing internal multiples becomes more challenging and beyond the current capability of the petroleum industry.

For dealing with this challenging problem, Weglein et al. (2003) proposed a three-pronged strategy including (1)Develop the ISS prerequisites for predicting the reference wave field and to produce de-ghosted data Mayhan and Weglein (2014). (2)Develop internal-multiple elimination algorithms from ISS. (3)Develop a replacement for the energy-minimization criteria for adaptive subtraction. For the second part of the strategy, that is, to upgrade the ISS internal-multiple attenuator to eliminator, the strengths and limitations of the ISS internal-multiple attenuator are noted and reviewed. The ISS internal-multiple attenuator always attenuates all first-order internal multiples from all reflectors at once, automatically and without subsurface information. That is a tremendous strength, and is a constant and holds independent of the circumstances and complexity of the geology and the play. The primaries in the reflection data that enters the algorithm provides that delivery, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, when they enter the ISS internal-multiple algorithm will alter the higher order internal multiples and thereby assist and cooperate with higher order ISS internal-multiple attenuation terms, to attenuate higher order internal multiples. However, there is a downside, a limitation. There are cases when internal multiples that enter the attenuator can predict spurious events. That is a well-understood shortcoming of the leading order term, when taken in isolation, but is not an issue for the entire ISS internal-multiple capability. It is anticipated by the ISS and higher order ISS internal multiple terms exist to precisely remove that issue of spurious event prediction, and taken together with the first order term, no longer experiences spurious event prediction. Ma et al. (2012) and Ma and Weglein (2014) provided those higher order terms and for spurious events removal. In a similar way, there are higher order ISS internal multiple terms that provide the elimination of internal multiples when taken together with the leading order attenuator term. There are early discussions in Ramírez (2007). And Wilberth Herrera and Weglein (2012) has derived an algorithm that can eliminate all first-order internal multiples generated at the shallowest reflector for 1D normal incidence. Part I of this paper proposes a general elimination algorithm for all first-order internal-multiples generated from all reflectors in a 1D earth. Similarly as the attenuator, The primaries in the reflection data that enters the algorithm provides that elimination capability, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. That is a limitation of current algorithm. In part II of this two part paper, we show how the ISS anticipate that shortcoming. Higher order ISS terms when combined with the current algorithm will provide elimination ability without the current shortcoming. The basic algorithm is developed and explained in part I. The newer version with higher order ISS terms that rewrites elimination algorithm without a downside is presented and tested in part II. Moreover, this elimination algorithm based on the ISS internal-multiple attenuation algorithm is derived by using reverse engineering method. It is model type dependent since the reverse engineering method is model type dependent. The ISS internal-multiple attenuation algorithm is model type independent.

ISS INTERNAL-MULTIPLE ATTENUATION ALGORITHM AND ATTENUATION FACTOR FOR 1D NORMAL INCIDENCE

First, we can have a review of the ISS internal-multiple attenuation algorithm before we introduce the internal-multiple elimination algorithm. The ISS internal-multiple attenuation algorithm is first given by Araújo et al. (1994) Weglein et al. (1997). The 1D normal incidence version of the algorithm is presented as follows:

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z''). \tag{1}$$

Where $b_1(z)$ which is closely related to the data is the water speed migration of the data due to a 1D normal incidence spike plane wave. ε_1 and ε_2 are two small positive number introduced to avoid self interaction. $b_3^{IM}(k)$ is the predicted internal multiples in the vertical wavenumber domain. This equation can predict the correct time and approximate amplitude of all first-order internal multiples.

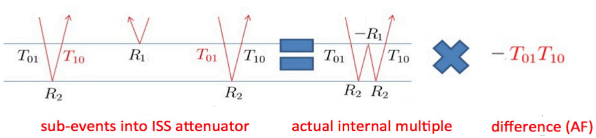

Figure 1: an example of the Attenuation Factor of a first-order internal multiple generated at the shallowest reflector, notice that all red terms are extra transmission coefficients

The procedure of predicting a first-order internal multiple generated at the shallowest reflector is shown in figure 1. The ISS internal-multiple attenuation algorithm uses three primaries in the data to predict a first-order internal multiple(Note that this algorithm is model type independent and it takes account all possible combinations of primaries that can predict internal multiples. These figures are just to show intuitively how it works). From the figure we can see, every sub event on the left hand side experiences several phenomena making its way

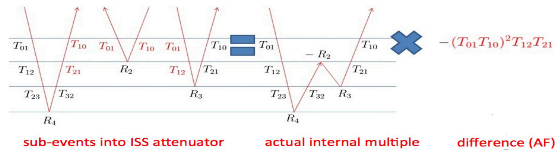

Figure 2: an example of the Attenuation Factor of a first-order internal multiple generated at the next shallowest reflector, notice that all red terms are extra transmission coefficients

down to the earth then back to the receiver. When compared with the internal multiple on the right hand side, the events on the left hand side have extra transmission coefficients as shown in red. Multiplying all those extra transmission coefficients, we get the attenuation factor $T_{01}T_{10}$ for this first-order internal multiple generated at the shallowest reflector. And all first-order internal multiples generated at the shallowest reflector have the same attenuation factor.

Figure 2 shows the procedure of predicting a first-order internal multiple generated at the next shallowest reflector. In this example, the attenuation factor is $(T_{01}T_{10})^2(T_{12}T_{21})$.

The attenuation factor, AF_j, in the prediction of internal multiples is given by the following:

$$AF_j = \begin{cases} T_{0,1}T_{1,0} \quad (for\ j=1) \\ \prod_{i=1}^{j-1}(T_{i-1,i}^2 T_{i,i-1}^2)T_{j,j-1}T_{j-1,j} \quad (for\ 1<j<J) \end{cases} \tag{2}$$

The attenuation factor AF_j can also be performed by using reflection coefficients:

$$AF_j = \begin{cases} 1-R_1^2 \quad (for\ j=1) \\ (1-R_1^2)^2(1-R_2^2)^2\cdots(1-R_{j-1}^2)^2(1-R_j^2) \quad (for\ 1<j<J) \end{cases} \tag{3}$$

The subscript j represents the generating reflector, and J is the total number of interfaces in the model. The interfaces are numbered starting with the shallowest location. The attenuation factor is directly related to the trajectory of the events, which forms the prediction of the internal multiple.

ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM FOR 1D NORMAL INCIDENCE

The discussion above demonstrates that all first-order internal multiples generated at the same reflector have the same attenuation factor. We can see the attenuation factor contains all transmission coefficients from the shallowest reflector down to the reflector generating the multiple. And from the examples(shown in figure 1 and 2) we can see the middle event contains all the information about those transmission coefficients .Therefore, our idea is to modify the middle term in the attenuation algorithm to remove the attenuation factor and make the attenuation algorithm an eliminator. That is from

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \tag{4}$$

to

$$b_E^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \tag{5}$$

By introducing a new function called g(z) in which the amplitude of each event corresponds to a reflection coefficient, we find a way to construct $F[b_1(z)]$ by using $b_1(z)$ and g(z). After that, we find an integral equation about $b_1(z)$ and g(z). The $F[b_1(z)]$ is discovered Zou and Weglein (2013):

$$F[b_1(z)] = \frac{b_1(z)}{[1-(\int_{z-\varepsilon}^{z+\varepsilon} dz' g(z'))^2][1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')]^2} \tag{6}$$

$$g(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')} \tag{7}$$

To derive the $F[b_1(z)]$ from $b_1(z)$, g(z) must first be solved in equation (7). Thereafter, g(z) is integrated into equation (6). Now we will show one way to solve these equations. By iterating $g(z)$ in (7), we can get more accurate approximation. Substitute more accurate approximations of $g(z)$ into $F[b_1(z)]$, we will achieve or obtain higher orders of approximation of the elimination algorithm which can predict correct amplitude of first-order internal multiples generated at deeper reflectors.

ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM FOR 1D PRESTACK

Now we get the algorithm in 1D normal incidence and it lights the way to find an algorithm in 1D pre-stack. Let us discussed an example in a 2D world with 1D earth. In this example, the reflection coefficients and transmission coefficients are both angle dependent. With discussions about this example Zou and Weglein (2014), we find that the attenuation factors consist of angle dependent transmission coefficients. Following early discussions and work in Ramírez (2007) and Wilberth Herrera and Weglein (2012), we discovered the elimination algorithm in 1D pre-stack.

Below shows the 1D pre-stack internal-multiple elimination algorithm for acoustic medium (Note that the ISS internal-multiple attenuation algorithm is model type independent). Due to the angle dependent reflection coefficients, we can no longer just integrate the data in k-z domain to get the reflection coefficients as we did in 1D normal incidence, we need to go to k-q domain where each (k, q) corresponds to one reflection coefficient. The differences between the 1D pre-stack and 1D normal incidence algorithms are (1) the 1D pre-stack algorithm has one more variable k, and (2) use the reflection coefficients in the k-q domain instead of direct integral in k-z domain.

$$b_E^{IM}(k,2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k,z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} F[b_1(k,z')] \times \int_{z'+\varepsilon_2}^{\infty} dz'' e^{2iqz''} b_1(k,z'')$$

$$F[b_1(k,z)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k,z')}{[1-\int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-iq'z'''}]^2} \times \frac{1}{1-|\int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k,z'') e^{iq'z''}|^2}$$

$$g(k,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k,z')}{1-\int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-iq'z'''}}$$

NUMERICAL TESTS FOR 1D PRESTACK ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM

We test the 1D pre-stack acoustic internal multiple elimination algorithm for a two-reflector model. Each layer has density $1.0 g/cm^3$, $1.2 g/cm^3$, $2.0 g/cm^3$ and velocity 1500m/s 3000m/s and 4500m/s respectively. Figure 3 shows the data and figure 4 and 5 show the attenuation and elimination algorithm predictions respectively. Figure 6 to Figure 13 show different traces in different offsets (the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue)). We can see the elimination algorithm keeps the correct time and can predict better amplitude.

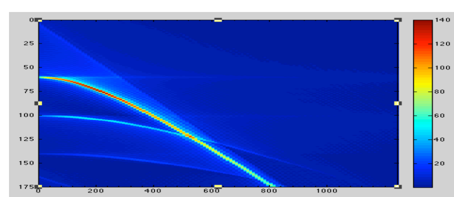

Figure 3: data

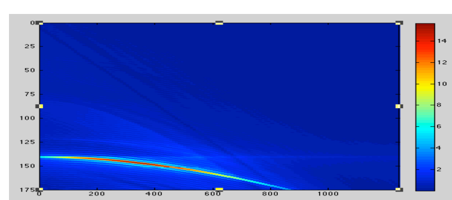

Figure 4: internal multiple attenuation prediction

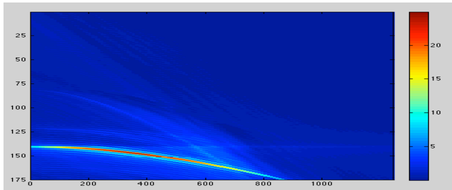

Figure 5: internal multiple elimination prediction

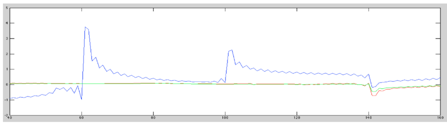

Figure 6: the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue) at offset = 0m

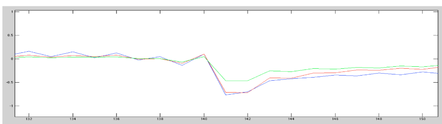

Figure 7: the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue) at offset = 0m. After removing the tails of primaries.

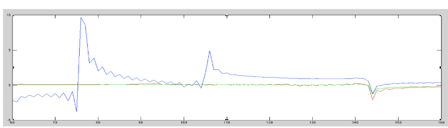

Figure 8: the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue) at offset = 200m

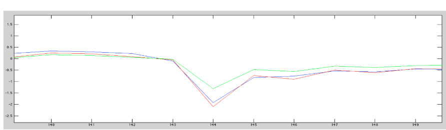

Figure 9: the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue) at offset = 200m. After removing the tails of primaries.

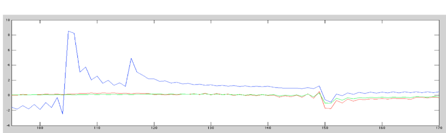

Figure 10: the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue) at offset = 400m

CONCLUSION

The pre-stack 1D ISS internal multiple elimination algorithm for all first-order internal multiples from all reflectors is pro-

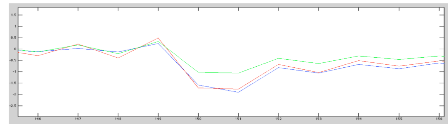

Figure 11: the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue) at offset = 400m. After removing the tails of primaries.

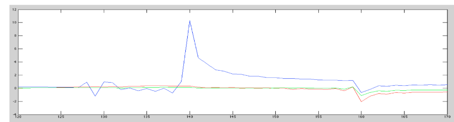

Figure 12: the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue) at offset = 600m

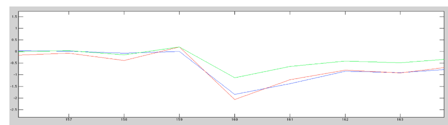

Figure 13: the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue) at offset = 600m. After removing the tails of primaries.

posed in part I of this paper. Numerical tests are carried out to evaluate this new algorithm and to determine the strengths and limitations. The results shows the elimination algorithm can predict better amplitude of the internal multiples. In discussing the elimination algorithm, the primaries in the reflection data that enters the algorithm provides that elimination capability, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. That is a limitation of this new algorithm. In part II of this two part paper, we show how the ISS anticipates that shortcoming. Higher order ISS terms when combined with the current algorithm will provide elimination ability without the current shortcoming. The basic algorithm is developed and explained in part I. The newer version with higher order ISS terms that rewrites the elimination algorithm without a downside is presented and tested in part II.This algorithm is a part of the three-pronged strategy which is especially relevant and provide value when primaries and internal multiples are proximal to and/or interfere with each other in both on-shore and off-shore data.

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