# The internal-multiple elimination algorithm for all reflectors for 1D earth Part II: addressing the limitations

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## SUMMARY

Driven by the demand for more capabilities in removing the internal multiples, the strengths and limitations of the ISS(Inverse-Scattering-Series) internal-multiple attenuation algorithm(Araújo et al. (1994) and Weglein et al. (1997)) are noted and reviewed. The ISS internal multiple attenuation algorithm has tremendous strength that it can predict the correct time and approximate amplitude for all first-order internal multiples without any information of the earth. As the first term in the internal-multiple elimination sub-series, the ISS internal-multiple attenuation algorithm has its own limitations, as noted in Weglein et al. (2003): in certain circumstances, it may generate spurious events Ma et al. (2012) and can not predict exact correct amplitude.That is a well-understood shortcoming of the leading order term, when taken in isolation, but is not an issue for the entire ISS internal multiple capability. In part I of this paper, a new elimination algorithm for all first-order internal multiples for one dimensional earth has been derived based on the ISS internal-multiple attenuation algorithm. This elimination algorithm based on the ISS internal-multiple attenuation algorithm is derived by using reverse engineering method. This elimination algorithm is model type dependent since the reverse engineering method is model type dependent while the ISS internal-multiple attenuation algorithm is model type independent. The primaries in the reflection data that enters this elimination algorithm provides that elimination capability, without requiring the primaries to be identified or in any way separated. The other events in the reflection data may alter the amplitude and need assist and cooperate with other ISS terms to completely eliminate the internal multiples. In part II of this paper, a modified strategy is proposed to address this limitation of the new elimination algorithm.


## INTRODUCTION

The ISS internal-multiple attenuation algorithm(Araújo et al. (1994) and Weglein et al. (1997)) can predict the correct time and approximate amplitude for all first-order internal multiples without any information of the earth. This algorithm is effective and can attenuate internal multiples in many cases. However, in certain places, both offshore and onshore, the multiple is often proximal to or interfering with the primaries. Therefore, the task of removing internal multiples without damaging primaries becomes more challenging and subtle and currently beyond the collective capability of the petroleum industry. Weglein et al. (2003) proposed a three-pronged strategy for providing an effective response to this pressing and prioritized challenge. One part of the strategy is to develop an internal-multiple elimination algorithm that can predict both the correct amplitude and correct time for all internal multiples. Part I of this paper proposes a general elimination algorithm for all first-order internal-multiples generated from all reflectors in a 1D earth. The primaries in the reflection data that enters the algorithm provides that elimination capability, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. That is a limitation of current algorithm. In part II of this two part paper, we show how the ISS anticipate that shortcoming. Higher order ISS terms when combined with the current algorithm will provide elimination ability without the current shortcoming. The basic algorithm is developed and explained in part I. The newer version with higher order ISS terms that rewrites elimination algorithm without a downside is presented and tested in part II. In this part II, we will first give a review of both the internal multiple attenuator and eliminator, then we will propose a modified strategy with higher order terms from the inverse scattering series for addressing the limitations of the eliminator and test the strategy in a layered medium.

## ISS INTERNAL-MULTIPLE ATTENUATION ALGORITHM AND ATTENUATION FACTOR FOR 1D NORMAL INCIDENCE

The ISS internal-multiple attenuation algorithm is first given by Araújo et al. (1994) and Weglein et al. (1997). The 1D normal incidence version of the algorithm is presented as follows:

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z''). \tag{1}$$

Where $b_1(z)$ which is closely related to the data is the water speed migration of the data due to a 1D normal incidence spike plane wave. $\varepsilon_1$ and $\varepsilon_2$ are two small positive number introduced to avoid self interaction. $b_3^{IM}(k)$ is the predicted internal multiples in vertical wavenumber domain. This equation can predict the correct time and approximate amplitude of all first-order internal multiples.

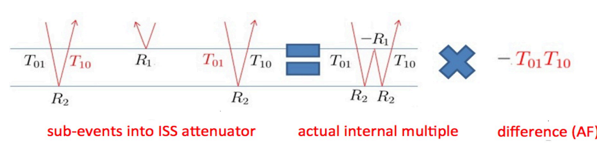


Figure 1: an example of the Attenuation Factor of a first-order internal multiple generated at the shallowest reflector, notice that all red terms are extra transmission coefficients

The procedure of predicting a first-order internal multiple generated at the shallowest reflector is shown in figure 1. The ISS internal-multiple attenuation algorithm uses three primaries in data to predict a first-order internal multiple. Multiplying all

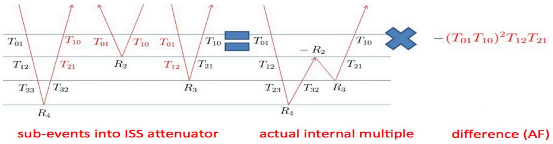


Figure 2: an example of the Attenuation Factor of a first-order internal multiple generated at the next shallowest reflector, notice that all red terms are extra transmission coefficients

those extra transmission coefficients, we get the attenuation factor $T_{01}T_{10}$ for this first-order internal multiple generated at the shallowest reflector. Figure 2 shows the procedure of predicting a first-order internal multiple generated at the next shallowest reflector. In this example, the attenuation factor is $(T_{01}T_{10})^2(T_{12}T_{21})$. The attenuation factor, $AF_j$, in the prediction of internal multiples is given by the following:

$$AF_j = \begin{cases} T_{0,1}T_{1,0} \quad (for\ j=1) \\ \prod_{i=1}^{j-1}(T_{i-1,i}^2 T_{i,i-1}^2)T_{j,j-1}T_{j-1,j} \quad (for\ 1<j<J) \end{cases} \tag{2}$$

The attenuation factor $AF_j$ can also be performed by using reflection coefficients:

$$AF_j = \begin{cases} 1-R_1^2 \quad (for\ j=1) \\ (1-R_1^2)^2(1-R_2^2)^2\cdots(1-R_j^2) \quad (for\ 1<j<J) \end{cases} \tag{3}$$

The subscript j represents the generating reflector, and J is the total number of interfaces in the model.

## ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM FOR 1D NORMAL INCIDENCE

The discussion above demonstrates that all first-order internal multiples generated at the same reflector have the same attenuation factor. We can see the attenuation factor contains all transmission coefficients from the shallowest reflector down to the reflector generating the multiple. Zou and Weglein (2013) proposed an elimination algorithm that can remove all the attenuation factors for all first-order internal multiples from all reflectors. The algorithm is shown as following:

$$F[b_1(z)] = \frac{b_1(z)}{[1-(\int_{z-\varepsilon}^{z+\varepsilon}dz'g(z'))^2][1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')]^2} \tag{4}$$

$$g(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')} \tag{5}$$

g(z) is a intermediate function to help construct the above close formula. To derive the $F[b_1(z)]$ from $b_1(z)$, g(z) must first be solved in equation (5). Thereafter, g(z) is integrated into equation (4). By iterating $g(z)$ in (5), we can get more accurate approximation. Substitute more accurate approximations of $g(z)$ into $F[b_1(z)]$, we will get higher orders of approximation of the elimination algorithm which can predict correct amplitude of first-order internal multiples generated at deeper reflectors.

**First Type of Equation Approximation for g(z)**

The simplest approximation for g(z) is presented as follows:

$$\begin{aligned} g(z) =& \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')} \\ \approx& \frac{b_1(z)}{1-0} \\ \approx& b_1(z) \end{aligned} \tag{6}$$

**Second Type of Equation Approximation for g(z)**

A more accurate approximation for g(z) is presented as follows::

$$\begin{aligned} g(z) =& \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')} \\ \approx& \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''b_1(z'')} \end{aligned} \tag{7}$$

**Higher order approximations**

By iterating $g(z)$ in (5), we can get more accurate approximation, as shown in figure 3. Substitute more accurate approximations of $g(z)$ into $F[b_1(z)]$, we will get better approximation of the elimination algorithm which can predict correct amplitude of first-order internal multiples generated at deeper reflectors.

| | |
|---|---|
| First type approximation for $g(z)$ | $g(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')}$ $\approx \frac{b_1(z)}{1-0}$ $\approx b_1(z)$ |
| Second type approximation for $g(z)$ | $g(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')}$ $\approx \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''b_1(z'')}$ |
| Third type approximation for $g(z)$ | $g(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')}$ $\approx \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''\frac{b_1(z'')}{1-\int_{-\infty}^{z''-\varepsilon}dz'''b_1(z''')\int_{z'''-\varepsilon}^{z'''+\varepsilon}dz^{(4)}b_1(z^{(4)})}}$ |
| • • • | • • • |

Figure 3: different approximations for g(z)

## ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM FOR 1D PRESTACK

In part I of this paper, a new algorithm dealing with the amplitude issue for all first-order internal multiples for one dimensional earth has been derived based on the ISS internal-multiple attenuation algorithm. The algorithm is shown as follows:

$$\begin{aligned} b_E^{IM}(k,2q) =& \int_{-\infty}^{\infty}dze^{2iqz}b_1(k,z)\int_{-\infty}^{z-\varepsilon_1}dz'e^{-2iqz'}F[b_1(k,z')] \\ &\times\int_{z'+\varepsilon_2}^{\infty}dz''e^{2iqz''}b_1(k,z'') \end{aligned}$$

$$F[b_1(k,z)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dz' dq'$$
$$\times \frac{e^{-iq'z}e^{iq'z'} b_1(k,z')}{[1-\int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-iq'z'''}]^2}$$
$$\times \frac{1}{1-|\int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k,z'') e^{iq'z''}|^2}$$

$$g(k,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dz' dq'$$
$$\times \frac{e^{-iq'z}e^{iq'z'} b_1(k,z')}{1-\int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-iq'z'''}}$$

## A MODIFIED STRATEGY OF USING $B_1 + B_3$ INSTEAD OF $B_1$ AS THE INPUT DATA FOR THE ELIMINATION ALGORITHM

The primaries in the reflection data that enters the elimination algorithm (both 1D normal incidence and 1D pre-stack) provides that elimination capability, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. That is a limitation of current algorithm. Now, we show the modified strategy and newer version of internal-multiple elimination algorithm. The limitation is due to internal multiples in the input data. Fortunately, we have a good approximations of the internal multiples ($b_3$) and if we use $b_1 + b_3$ instead of $b_1$ as the input data for the elimination algorithm, we will be able to significantly reduce the errors due to the multiples in the data. In figure 4, $b_1$, which is very close to data, contains primaries, first-order internal multiples and higher-order internal multiples. We use the attenuation algorithm to predict first-order internal multiples($b_3$) with correct time and approximate amplitude. Due to the multiples in the data, the attenuation algorithm also generates spurious events Ma et al. (2012) and makes prediction for higher-order multiples at the same time. However, the elimination algorithm assumes the data contains only primaries. Here is the strategy, since in $b_1 + b_3$ the first-order internal multiples are attenuated and it is a good approximation for data with only primaries. If we use $b_1 + b_3$ instead of $b_1$ for the elimination algorithm, the predicted spurious events and higher-order multiples due to first-order internal multiples in the data are also attenuated. All events in the red circle including other events are small compared with the first-order internal multiples and can be ignored.

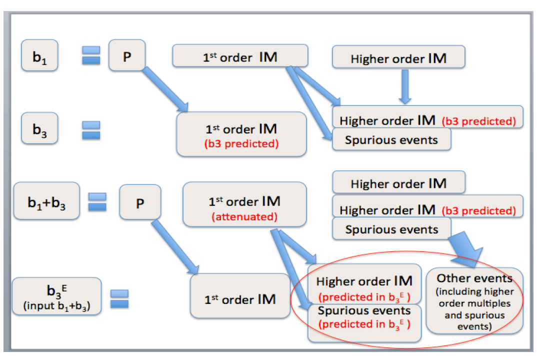


Figure 4: using $b_1 + b_3$ instead of $b_1$ as the input data for the elimination algorithm

## NUMERICAL TESTS ON A 34-REFLECTOR MODEL

In this section, we will test the modified strategy for a 34-reflector model under 1D normal incidence. And the modified strategy we proposed in this paper can be easily extended to the 1D pre-stack version. In figure 5, is a 34-reflector model. The input data is shown in figure 6. In this test we used a 40th approximation of the algorithm as shown in figure 7. We test the ISS internal-multiple attenuation algorithm, the elimination algorithm with input $b_1$ and input $b_1$+$b_3$ respectively. From the result we conclude that using $b_1$+$b_3$ as the input significantly reduced errors and makes better prediction for all first-order internal multiples generated from all reflectors.

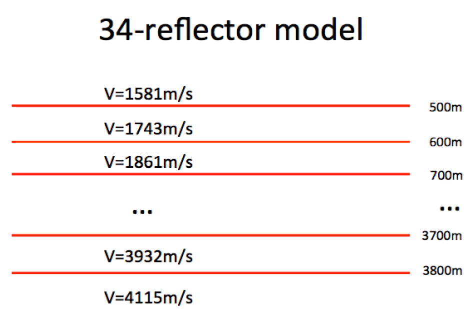


Figure 5: model

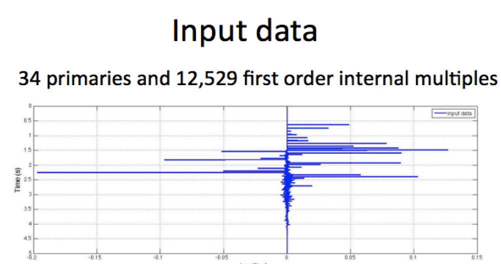


Figure 6: input data

Figure 8,9,10 show the prediction of different algorithms/strategies compared with the input data. Figure 11,12,13 shows a small time interval of figure 8,9,10 respectively.

$$b_E^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

$$F[b_1(z)] = \frac{b_1(z)}{[1-(\int_{z-\varepsilon}^{z+\varepsilon} dz' g(z'))^2][1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')]^2}$$

$$g(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')}$$

get g(z) by iteration

$$g_1(z) = b_1(z)$$

$$g_{n+1}(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g_n(z'')}$$

I used $g_{40}(z)$ in this test.

Figure 7: iteration to get $g_{40}(z)$

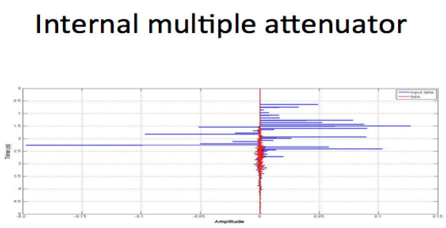


Figure 8: Internal multiple attenuator prediction(red) compared with the input data(blue)

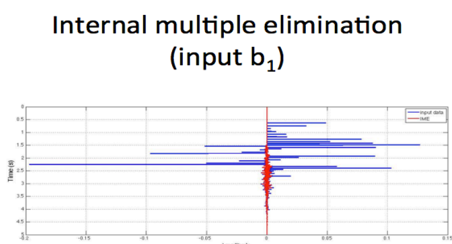


Figure 9: internal multiple elimination algorithm (with $b_1$ as the input data) prediction(red) compared with the input data(blue)

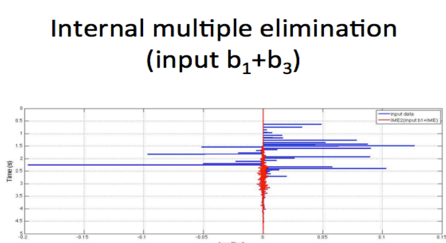


Figure 10: Internal multiple elimination algorithm (with $b_1 + b_3$ as the input data) prediction(red) compared with the input data(blue)

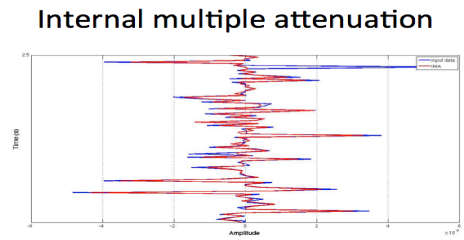


Figure 11: A small time interval of figure 8

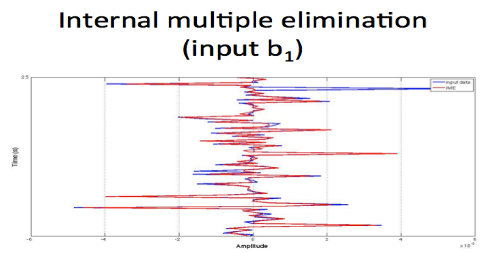


Figure 12: A small time interval of figure 9

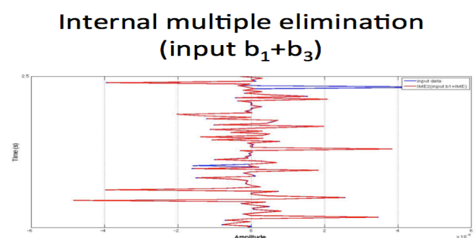


Figure 13: A small time interval of figure 10

## CONCLUSION

In part I of this paper, a new elimination algorithm for all first-order internal multiples for one dimensional earth has been derived based on the ISS internal-multiple attenuation algorithm. The primaries in the reflection data that enters this elimination algorithm provides that elimination capability, without requiring the primaries to be identified or in any way separated. The other events in the reflection data may alter the amplitude and need assist and cooperate with higher order ISS terms to completely eliminate the internal multiples. In this part II of this two part set of paper, a modified strategy is proposed to address this limitation of the current elimination algorithm. In the end we tested the strategy in a layered medium and the results are very encouraging.

## ACKNOWLEDGMENTS

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## REFERENCES

Araújo, F. V., A. B. Weglein, P. M. Carvalho, and R. H. Stolt, 1994, Inverse scattering series for multiple attenuation: An example with surface and internal multiples: SEG Technical Program Expanded Abstracts, 1039–1041.

Ma, C., H. Liang, and A. B. Weglein, 2012, Modifying the leading order iss attenuator of first-order internal multiples to accommodate primaries and internal multiples: fundamental concept andtheory, development, and examples exemplified when three reflectors generatethe data: Mission Oriented Seismic Research Program Annual Report, 133–147.

Weglein, A. B., F. V. Araújo, P. M. Carvalho, R. H. Stolt, K. H. Matson, R. T. Coates, D. Corrigan, D. J. Foster, S. A. Shaw, and H. Zhang, 2003, Inverse scattering series and seismic exploration: Inverse Problems, R27–R83.

Weglein, A. B., F. A. Gasparotto, P. M. Carvalho, and R. H. Stolt, 1997, An inverse-scattering series method for attenuating multiples in seismic reflection data: Geophysics, **62**, 1975–1989.

Zou, Y., and A. B. Weglein, 2013, Internal multiple removal: an amplitude correction equation for internal-multiple attenuator(1d normal incidence): Mission Oriented Seismic Research Program Annual Report, 167–188.