M-OSRP
Misson - Oriented Seismic Research Program
Solve The Right Problem

Annual Technical Review and Meeting

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May 27-30, 2014
Austin, TX

M-OSRP

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Seismic E&P challenges

- Methods make assumptions; when they are satisfied the methods are effective and when they are not satisfied the methods have difficulty and/or fail – challenges arise from that breakdown or failure.

Seismic E&P challenges

- Among assumptions:
  - acquisition
  - compute power
  - innate algorithmic assumptions/requirements

How to address challenges?

Two ways:

- Remove the assumption violation by satisfying the assumptions; or

- Avoid the assumption violation by deriving methods that do not make those assumptions.

Either of these is reasonable and indicated under different circumstances.

We recognize the value in each type of response and adopt distinctly different attitudes for different issues:

1. Help improve requirements

   e.g., recorded data collection/extrapolation.

   - we cannot (yet) develop a method that avoids this requirement for seismic data.
   - hence, we seek to help satisfy that requirement.

- M-OSRP takes ownership and responsibility for the entire processing chain.

- One way to understand the projects within the M-OSRP program is to consider the processing chain.

Innate algorithmic assumptions

- Many processing methods require subsurface information
- In complex and ill-defined areas that requirement can be difficult or impossible to satisfy
- The inability to satisfy that requirement can lead to algorithmic failure and dry hole drilling

A key goal

- A consistent and aligned linked set of processes where the success of earlier links always supports and facilitates later links along the chain.

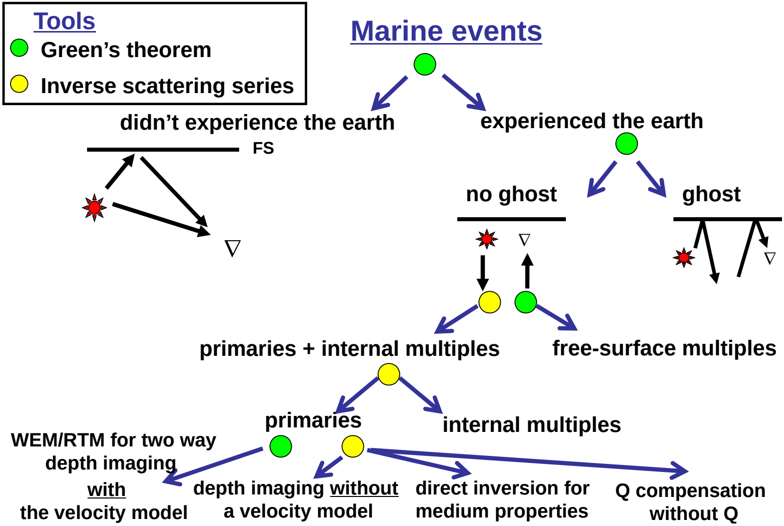

Figure 1: *Classification of marine events and how they are processed.*

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Tutorial on Green's theorem for source signature, deghosting, one-way migration, and RTM

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Green's theorem tutorial

To start with preprocessing, we present a tutorial on the different projects within M-OSRP that derive from Green's theorem

Green's theorem-derived methods

Wavefield separation: no subsurface information required

- Predicting the reference wave and the scattered wavefield (the reflection data) from the total wavefield
- Predicting the source signature and radiation pattern
- Source and receiver deghosting

Green's theorem-derived methods (continued)

Wavefield prediction for migration: subsurface information required

- Wave equation migration for one way waves
- Wave equation migration for two way waves (RTM)

Green's theorem and seismic processing (wave separation or wave prediction)

By way of illustration, consider an inhomogeneous acoustic medium

$$\left[\nabla^2 + \frac{\omega^2}{c^2(\vec{r})}\right] P(\vec{r}, \vec{r}_s, \omega) = A(\omega)\delta(\vec{r} - \vec{r}_s) \qquad (1)$$

Characterize the velocity field in terms of a reference, c_0, and a perturbation, $\alpha(\vec{r})$.

$$\frac{1}{c^2(\vec{r})} = \frac{1}{c_0^2}(1 - \alpha(\vec{r})) \qquad (2)$$

$$k = \frac{\omega}{c_0}$$

Green's theorem and seismic processing (wave separation or wave prediction)

$$\left[\nabla^2 + \frac{\omega^2}{c_0^2}\right] P(\vec{r}, \vec{r}_s, \omega) = k^2 \alpha(\vec{r}) P + A(\omega)\delta(\vec{r} - \vec{r}_s) \quad (3)$$

Define $\rho(\vec{r}, \omega) \equiv k^2 \alpha(\vec{r}) P + A(\omega)\delta(\vec{r} - \vec{r}_s)$ and equation 2 becomes

$$\left(\nabla^2 + \frac{\omega^2}{c_0^2}\right) P = \rho(\vec{r}, \omega) \quad (4)$$

Green's theorem tutorial (continued)

Introduce

$$(\nabla^2 + k^2) G_0 = \delta(\vec{r} - \vec{r}\,') \tag{5}$$

Equation 4 can be solved in terms of the solution of equation 5

$$\underset{Causal}{P(\vec{r}, \omega)} = \int_{\infty} d\vec{r}\,' \rho(\vec{r}\,', \omega) G_0^+(\vec{r}, \vec{r}\,', \omega) \tag{6}$$

$\vec{r}$ in ∞ (anywhere)

Green's theorem tutorial (continued)

Green's second identity

$$\int_V d\vec{r}\,'(P\nabla^2 G_0 - G_0\nabla^2 P) = \oint_S dS\,\hat{n}\cdot(P\nabla G_0 - G_0\nabla P) \tag{7}$$

Substituting $\nabla^2 P$ and $\nabla^2 G_0$ from equations 4 and 5 in equation 7

Green's theorem tutorial (continued)

$$\underset{\text{for } \vec{r} \text{ in } V}{P(\vec{r},\omega)} = \int_V d\vec{r}\,'\rho G_0 + \oint_S dS\,\hat{n}\cdot(P\nabla G_0 - G_0\nabla P) \quad (8)$$

Valid for any choice G_0 that satisfies equation 5.

- Different <u>choices</u> of solutions for G_0 will derive each of the Green's theorem applications we listed
- If we choose $G_0 = G_0^+$ then equation 8 becomes

$$\underset{\vec{r} \text{ in } V}{P} = \int_V d\vec{r}\,'\rho G_0^+ + \oint_S dS\,\hat{n}\cdot(P\nabla G_0^+ - G_0^+\nabla P) \quad (9)$$

Green's theorem tutorial (continued)

Equation 6 can be written

$$\underset{\textit{for any } \vec{r}}{P} = \int_V d\vec{r}\,'\rho G_0^+ + \int_{\infty - V} d\vec{r}\,'\rho G_0^+ \qquad (10)$$

Green's theorem tutorial (continued)

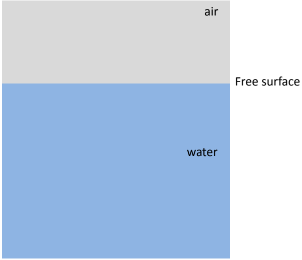
Comparing equations 9 and 10 we have
<u>for $\vec{r}$ in the volume</u>

$$\oint_S dS\,\hat{n}\cdot(P\nabla G_0^+ - G_0^+\nabla P) = \int_{\infty - V} d\vec{r}\,'\rho G_0^+ \qquad (11)$$

Therefore for $\vec{r}$ in the volume $\oint_S dS\,\hat{n}\cdot(P\nabla G_0^+ - G_0^+\nabla P)$ is the portion of the wavefield at a point $\vec{r}$ in the volume, V due to the sources outside V.

Green's theorem tutorial (continued)

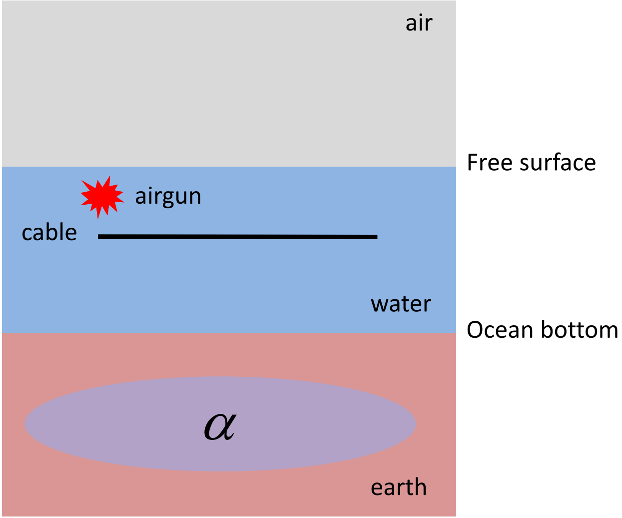
The reference medium is arbitrary. If we choose



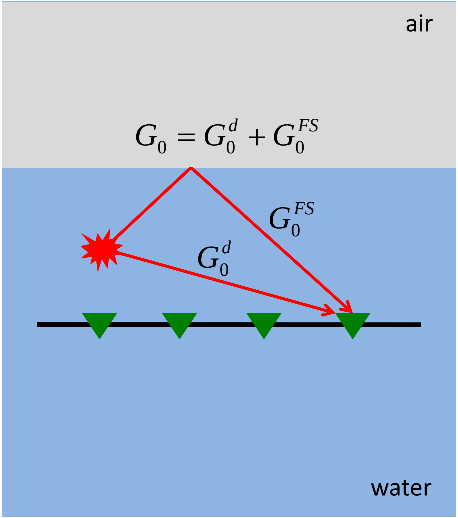
Reference medium

as the reference medium then 2 "sources," airguns and earth.

Green's theorem tutorial (continued)



Green's theorem tutorial (continued)

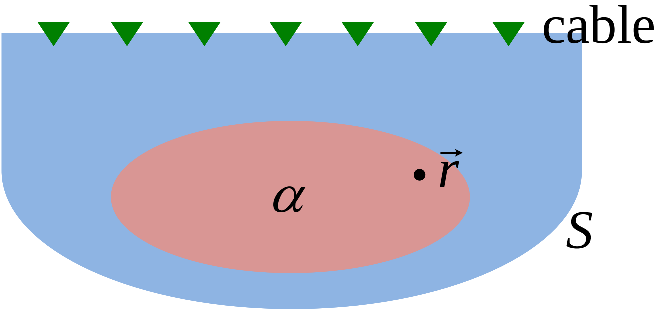


Free surface

The reference Green's function

Green's theorem tutorial (continued)

$\int dS\, \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P)$ when evaluated at $\vec{r}$ in a volume shown below provides the portion of P due to sources outside the volume, i.e., due only to sources above the cable



Green's theorem tutorial (continued)

$$\int_{\infty - V} d\vec{r}\,'\rho_{\text{airguns}}(\vec{r}\,', \omega) G_0^+ = P_0(\vec{r}, \omega)$$

$G_0 = G_0^d + G_0^{FS}$ the wavefield due to the airguns in <u>the reference</u> medium

$$P_0(\vec{r}, \omega) = \oint_S dS\, \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P)$$

The source signature and radiation pattern of the wave that leaves the source.

Green's theorem tutorial (continued)

If $P_0(\vec{r}, \omega) = A(\omega) G_0^+(\vec{r}, \vec{r}_s, \omega)$ then

$$A(\omega) = \frac{\oint_S dS\, \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P)}{G_0^+(\vec{r}, \vec{r}_s, \omega)}$$

(source signature estimation and radiation pattern)

Green's theorem tutorial (continued)

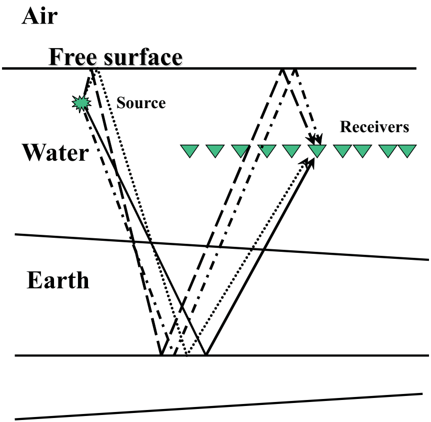
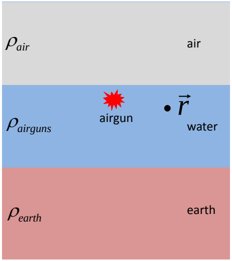


Figure 2: *Primary and its ghosts. Solid line: Primary; Dotted line: Source ghost of primary; Dashed-dot line: Receiver ghost of primary; Dashed line: Source-receiver ghost of primary (Zhang, 2007, Fig. 1.2).*

Green's theorem tutorial (continued)

For deghosting, choose a different reference medium — choose a homogeneous whole space of water. Now we have three sources.



$\vec{r}$ is lower than the airguns. $G_0 = \exp(ikR)/R = G_0^{d+}$.

Green's theorem tutorial (continued)

Then

$$\int_V d\vec{r}\,'\rho_{\mathrm{earth}} G_0^{d+} = \oint_S dS\,\hat{n}\cdot\left(P\nabla G_0^{+} - G_0^{+}\nabla P\right)$$

$\int \rho_{\mathrm{air}} G_0^{d+}$ is downgoing
$\int \rho_{\mathrm{airguns}} G_0^{d+}$ is downgoing
$\int \rho_{\mathrm{earth}} G_0^{d+}$ is <u>upgoing</u>

Green's theorem tutorial (continued)

Therefore

$$\int \rho_{\text{earth}} G_0^{d+} = \oint dS\, \hat{n} \cdot (P \nabla G_0^{+} - G_0^{+} \nabla P)$$

is the receiver deghosted data for $\vec{r}$ above the cable and at a depth below (deeper than) the sources
This requires P and dP/dn along the cable for a given shot record.

Green's theorem tutorial (continued)

If you don't have P' along the cable — then for an isotropic point source, and known wavelet, plus P along the cable you can predict P and P' (at a new cable above the actual one) from equation 8

$$P = A(\omega) G_0^{DD} + \int P \frac{\partial G_0^{DD}}{\partial z'} dx$$

$$\frac{dP}{dz} = A(\omega) \frac{\partial G_0^{DD}}{\partial z} + \int P \frac{\partial^2 G_0^{DD}}{\partial z \partial z'} dx$$

where $G_0^{DD} = 0$ at the free surface and at the cable.

Summary

Deghosting: Green's theorem with P and P' at one depth

$$
\begin{aligned}
P &= A \exp(ikz) + B \exp(-ikz) \\
P(0) &= A + B \\
P'(0) &= ik(A - B) \\
ikP(0) + P'(0) &= 2ikA \\
A &= \frac{P'(0) + ikP(0)}{2ik} \\
B &= \frac{P'(0) - ikP(0)}{2ik}
\end{aligned}
$$

stable

Summary (continued)

For measurements at two depths

$$\begin{aligned}
P(0) &= A + B \\
P(a) &= A\exp(ika) + B\exp(-ika) \\
P(0)\exp(-ika) - P(a) &= A\exp(-ika) - A\exp(ika) \\
A &= \frac{P(0)\exp(-ika) - P(a)}{\exp(-ika) - \exp(ika)} \\
B &= \frac{P(0)\exp(ika) - P(a)}{\exp(ika) - \exp(-ika)}
\end{aligned}$$

Summary (continued)

Sensitive near notches

If interest is away from notches — one-source experiments will be fine for source and receiver deghosting

If interest includes the notches — two-source experiment will provide more stability for source-side deghosting

Summary (continued)

- Depends on band width and depth of sources and receivers
- If your sources and receivers are at the ocean bottom, notches come up early and double sources would be indicated for source deghosting

Summary (continued)

$$P'_R(\vec{r}_g\,', \vec{r}_s, \omega) = \int_{\text{m.s.}} dS\, \vec{\hat{n}} \cdot \tag{12}$$
$$[P(\vec{r}, \vec{r}_s, \omega)\nabla G_0^+(\vec{r}, \vec{r}_g\,', \omega) - G_0^+(\vec{r}, \vec{r}_g\,', \omega)\nabla P(\vec{r}, \vec{r}_s, \omega)]$$
$$P'_{SR}(\vec{r}_g\,', \vec{r}_s\,', \omega) = \int_{\text{sources}} dS\, \vec{\hat{n}} \cdot \tag{13}$$
$$[P'_R(\vec{r}_g\,', \vec{r}, \omega)\nabla G_0^+(\vec{r}, \vec{r}_s\,', \omega) - G_0^+(\vec{r}, \vec{r}_s\,', \omega)\nabla P'_R(\vec{r}_g\,', \vec{r}, \omega)].$$

Over/under receivers
Over/under sources

Summary (continued)

Equation 12 over/under receivers
but only one source if away from notches,
using reciprocity and the free surface as another measurement

References I

Zhang, Jingfeng. *Wave theory based data preparation for inverse scattering multiple removal, depth imaging and parameter estimation: analysis and numerical tests of Green's theorem deghosting theory*. PhD thesis, University of Houston, 2007.