Seismic E&P challenges

- Methods make assumptions; when they are satisfied the methods are effective and when they are not satisfied the methods have difficulty and/or fail – challenges arise from that breakdown or failure.
Seismic E&P challenges

Among assumptions:

- acquisition
- compute power
- innate algorithmic assumptions/requirements
How to address challenges?

Two ways:

• Remove the assumption violation by satisfying the assumptions; or

• Avoid the assumption violation by deriving methods that do not make those assumptions.

Either of these is reasonable and indicated under different circumstances.
We recognize the value in each type of response and adopt distinctly different attitudes for different issues:

1. Help improve requirements  
   *e.g.*, recorded data collection/extrapolation.  
   - we cannot (yet) develop a method that avoids this requirement for seismic data.  
   - hence, we seek to help satisfy that requirement.
- M-OSRP takes ownership and responsibility for the entire processing chain.

- One way to understand the projects within the M-OSRP program is to consider the processing chain.
Innate algorithmic assumptions

- Many processing methods require subsurface information
- In complex and ill-defined areas that requirement can be difficult or impossible to satisfy
- The inability to satisfy that requirement can lead to algorithmic failure and dry hole drilling
A key goal

- A consistent and aligned linked set of processes where the success of earlier links always supports and facilitates later links along the chain.
Figure 1: Classification of marine events and how they are processed.
Tutorial on Green’s theorem for source signature, deghosting, one-way migration, and RTM

Arthur B. Weglein

May 27-30, 2014
Austin, TX
Green’s theorem tutorial

To start with preprocessing, we present a tutorial on the different projects within M-OSRP that derive from Green’s theorem.
Green’s theorem-derived methods

Wavefield separation: no subsurface information required

- Predicting the reference wave and the scattered wavefield (the reflection data) from the total wavefield
- Predicting the source signature and radiation pattern
- Source and receiver deghosting
Green’s theorem-derived methods (continued)

Wavefield prediction for migration: subsurface information required

- Wave equation migration for one way waves
- Wave equation migration for two way waves (RTM)
Green’s theorem and seismic processing (wave separation or wave prediction)

By way of illustration, consider an inhomogeneous acoustic medium

\[
\left[ \nabla^2 + \frac{\omega^2}{c^2(\vec{r})} \right] P(\vec{r}, \vec{r}_s, \omega) = A(\omega)\delta(\vec{r} - \vec{r}_s) \tag{1}
\]

Characterize the velocity field in terms of a reference, \( c_0 \), and a perturbation, \( \alpha(\vec{r}) \).

\[
\frac{1}{c^2(\vec{r})} = \frac{1}{c_0^2}(1 - \alpha(\vec{r})) \tag{2}
\]

\[
k = \frac{\omega}{c_0}
\]
Green’s theorem and seismic processing (wave separation or wave prediction)

\[
\left[ \nabla^2 + \frac{\omega^2}{c_0^2} \right] P(\vec{r}, \vec{r}_s, \omega) = k^2 \alpha(\vec{r}) P + A(\omega) \delta(\vec{r} - \vec{r}_s) \tag{3}
\]

Define \( \rho(\vec{r}, \omega) \equiv k^2 \alpha(\vec{r}) P + A(\omega) \delta(\vec{r} - \vec{r}_s) \) and equation 2 becomes

\[
\left( \nabla^2 + \frac{\omega^2}{c_0^2} \right) P = \rho(\vec{r}, \omega) \tag{4}
\]
Green’s theorem tutorial (continued)

Introduce

\[ (\nabla^2 + k^2)G_0 = \delta(\vec{r} - \vec{r}') \]  (5)

Equation 4 can be solved in terms of the solution of equation 5

\[ P(\vec{r}, \omega) = \int_{\infty} \, d\vec{r}' \rho(\vec{r}', \omega) G_0^+(\vec{r}, \vec{r}', \omega) \]  (6)

\( \vec{r} \) in \( \infty \) (anywhere)
Green’s theorem tutorial (continued)

Green’s second identity

\[ \int_V d\mathbf{r}' (P \nabla^2 G_0 - G_0 \nabla^2 P) = \int_S dS \mathbf{\hat{n}} \cdot (P \nabla G_0 - G_0 \nabla P) \]  
(7)

Substituting \( \nabla^2 P \) and \( \nabla^2 G_0 \) from equations 4 and 5 in equation 7
Green’s theorem tutorial (continued)

\[
P(\vec{r}, \omega) = \int_V d\vec{r}' \rho G_0 + \oint_S dS \hat{n} \cdot (P \nabla G_0 - G_0 \nabla P) \quad (8)
\]

Valid for any choice \( G_0 \) that satisfies equation 5.

- Different choices of solutions for \( G_0 \) will derive each of the Green’s theorem applications we listed.

- If we choose \( G_0 = G_0^+ \) then equation 8 becomes

\[
\bar{P} \text{ in } V = \int_V d\vec{r}' \rho G_0^+ + \oint_S dS \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P) \quad (9)
\]
Equation 6 can be written

\[ P = \int_V d\bar{r}' \rho G_0^+ + \int_{\infty - V} d\bar{r}' \rho G_0^+ \]  

(10)
Comparing equations 9 and 10 we have for $\vec{r}$ in the volume

$$\oint_{S} dS \hat{n} \cdot (P \nabla G_{0}^{+} - G_{0}^{+} \nabla P) = \int_{\infty - V} d\vec{r}' \rho G_{0}^{+}$$

(11)

Therefore for $\vec{r}$ in the volume $\int_{S} dS \hat{n} \cdot (P \nabla G_{0}^{+} - G_{0}^{+} \nabla P)$ is the portion of the wavefield at a point $\vec{r}$ in the volume, $V$ due to the sources outside $V$. 
Green’s theorem tutorial (continued)

The reference medium is arbitrary. If we choose as the reference medium then 2 “sources,” airguns and earth.
Green's theorem tutorial (continued)
Green’s theorem tutorial (continued)

The reference Green’s function

\[ G_0 = G_0^d + G_0^{FS} \]
Green’s theorem tutorial (continued)

\[ \int dS \, \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P) \] when evaluated at \( \vec{r} \) in a volume shown below provides the portion of \( P \) due to sources outside the volume, i.e., due only to sources above the cable.
Green’s theorem tutorial (continued)

\[ \int_{\infty-V} d\bar{r}' \rho_{\text{airguns}}(\bar{r}', \omega) G_0^+ = P_0(\bar{r}, \omega) \]

\[ G_0 = G_0^d + G_0^{FS} \] the wavefield due to the airguns in the reference medium

\[ P_0(\bar{r}, \omega) = \oint_S dS \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P) \]

The source signature and radiation pattern of the wave that leaves the source.
Green’s theorem tutorial (continued)

If $P_0(\vec{r}, \omega) = A(\omega) G_0^+(\vec{r}, \vec{r}_s, \omega)$ then

$$A(\omega) = \oint_S dS \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P) \over G_0^+(\vec{r}, \vec{r}_s, \omega)$$

(source signature estimation and radiation pattern)
The actual definitions of source wavelet, ghost/deghosting, free surface and internal multiple will be provided later in this chapter. The point of the list above is that seismic processing is a linked sequence or chain of steps, where the effectiveness of any given step not only depends upon how well its own assumptions are satisfied, but also how well all the earlier tasks in the chain have been achieved. Deghosting, as discussed in the next section, not only is required for conventional data processing, but also is a critical part of the new efforts to address outstanding and pressing seismic exploration and production challenges described in the next section.

In the following, I will provide a set of definitions of seismic terms relevant to the problem being addressed and the method to address that problem. Seismic data recorded by receivers are a collection of seismic events which include primaries, multiples, the direct wave and their ghosts. Weglein et al. (2003) provide the definitions of these events. In this

A seismic event is a temporally localized arrival of seismic energy.

Figure 2: Primary and its ghosts. Solid line: Primary; Dotted line: Source ghost of primary; Dashed-dot line: Receiver ghost of primary; Dashed line: Source-receiver ghost of primary (Zhang, 2007, Fig. 1.2).
Green’s theorem tutorial (continued)

For deghosting, choose a different reference medium — choose a homogeneous whole space of water. Now we have three sources.

\[ \vec{r} \text{ is lower than the airguns. } G_0 = \exp(ikR)/R = G_{0}^{d+}. \]
Green’s theorem tutorial (continued)

Then

$$\int_V d\mathbf{r}' \rho_{\text{earth}} G_0^{d+} = \int_S dS \, \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P)$$

- $\int \rho_{\text{air}} G_0^{d+}$ is downgoing
- $\int \rho_{\text{airguns}} G_0^{d+}$ is downgoing
- $\int \rho_{\text{earth}} G_0^{d+}$ is upgoing
Therefore

\[ \int \rho_{\text{earth}} G_0^{d+} = \int dS \hat{n} \cdot (P \nabla G_0^+ - G_0^+ \nabla P) \]

is the receiver deghosted data for \( \vec{r} \) above the cable and at a depth below (deeper than) the sources. This requires \( P \) and \( dP/dn \) along the cable for a given shot record.
Green’s theorem tutorial (continued)

If you don’t have $P’$ along the cable — then for an isotropic point source, and known wavelet, plus $P$ along the cable you can predict $P$ and $P’$ (at a new cable above the actual one) from equation 8

\[
P = A(\omega) G_{0}^{DD} + \int P \frac{\partial G_{0}^{DD}}{\partial z'} dx
\]

\[
dP \frac{dP}{dz} = A(\omega) \frac{\partial G_{0}^{DD}}{\partial z} + \int P \frac{\partial^2 G_{0}^{DD}}{\partial z \partial z'} dx
\]

where $G_{0}^{DD} = 0$ at the free surface and at the cable.
Deghosting: Green’s theorem with $P$ and $P'$ at one depth

\[ P = A \exp(ikz) + B \exp(-ikz) \]

\[ P(0) = A + B \]

\[ P'(0) = ik(A - B) \]

\[ ikP(0) + P'(0) = 2ikA \]

\[ A = \frac{P'(0) + ikP(0)}{2ik} \]

\[ B = \frac{P'(0) - ikP(0)}{2ik} \]
Summary (continued)

For measurements at two depths

\[ P(0) = A + B \]
\[ P(a) = A \exp(ika) + B \exp(-ika) \]
\[ P(0) \exp(-ika) - P(a) = A \exp(-ika) - A \exp(ika) \]

\[ A = \frac{P(0) \exp(-ika) - P(a)}{\exp(-ika) - \exp(ika)} \]
\[ B = \frac{P(0) \exp(ika) - P(a)}{\exp(ika) - \exp(-ika)} \]
Sensitive near notches
If interest is away from notches — one-source experiments will be fine for source and receiver deghosting
If interest includes the notches — two-source experiment will provide more stability for source-side deghosting
Summary (continued)

- Depends on band width and depth of sources and receivers
- If your sources and receivers are at the ocean bottom, notches come up early and double sources would be indicated for source deghosting
\[ P'_R(\vec{r}_g', \vec{r}_s, \omega) = \int_{\text{m.s.}} dS \, \hat{n} \cdot \] 

\[ \left[ P(\vec{r}, \vec{r}_s, \omega) \nabla G_0^+(\vec{r}, \vec{r}_g', \omega) - G_0^+(\vec{r}, \vec{r}_g', \omega) \nabla P(\vec{r}, \vec{r}_s, \omega) \right] \] 

\[ P'_{SR}(\vec{r}_g', \vec{r}_s', \omega) = \int_{\text{sources}} dS \, \hat{n} \cdot \] 

\[ \left[ P'_R(\vec{r}_g', \vec{r}, \omega) \nabla G_0^+(\vec{r}, \vec{r}_s', \omega) - G_0^+(\vec{r}, \vec{r}_s', \omega) \nabla P'_R(\vec{r}_g', \vec{r}, \omega) \right]. \]

Over/under receivers
Over/under sources

\[ P'_R(\vec{r}_g', \vec{r}_s, \omega) = \int_{\text{m.s.}} dS \, \hat{n} \cdot \] 

\[ \left[ P(\vec{r}, \vec{r}_s, \omega) \nabla G_0^+(\vec{r}, \vec{r}_g', \omega) - G_0^+(\vec{r}, \vec{r}_g', \omega) \nabla P(\vec{r}, \vec{r}_s, \omega) \right] \] 

\[ P'_{SR}(\vec{r}_g', \vec{r}_s', \omega) = \int_{\text{sources}} dS \, \hat{n} \cdot \] 

\[ \left[ P'_R(\vec{r}_g', \vec{r}, \omega) \nabla G_0^+(\vec{r}, \vec{r}_s', \omega) - G_0^+(\vec{r}, \vec{r}_s', \omega) \nabla P'_R(\vec{r}_g', \vec{r}, \omega) \right]. \]
Equation 12 over/under receivers but only one source if away from notches, using reciprocity and the free surface as another measurement