M-OSRP
Misson - Oriented Seismic Research Program
Solve The Right Problem

Wave theory RTM

Arthur B. Weglein

May 27-30, 2014
Austin, TX

Table of Contents I

Table of Contents

Green's theorem-derived methods

Wavefield separation: no subsurface information required

- Predicting the reference wave and the scattered wavefield (the reflection data) from the total wavefield
- Predicting the source signature and radiation pattern
- Source and receiver deghosting

Green's theorem-derived methods (continued)

Wavefield prediction for migration: subsurface information required

- One-way-wave pre-stack Stolt FK migration
- Two-way-wave wave-equation-migration RTM

Green's theorem and seismic processing (wave separation or wave prediction)

By way of illustration, consider an inhomogeneous acoustic medium

$$\left[\nabla^2 + \frac{\omega^2}{c^2(\vec{r})}\right] P(\vec{r}, \vec{r}_s, \omega) = A(\omega)\delta(\vec{r} - \vec{r}_s) \tag{1}$$

Characterize the velocity field in terms of a reference, c_0, and a perturbation, $\alpha(\vec{r})$.

$$\frac{1}{c^2(\vec{r})} = \frac{1}{c_0^2}(1 - \alpha(\vec{r})) \tag{2}$$

$$k = \frac{\omega}{c_0}$$

Green's theorem and seismic processing (wave separation or wave prediction)

$$\left[\nabla^2 + \frac{\omega^2}{c_0^2}\right] P(\vec{r}, \vec{r}_s, \omega) = k^2 \alpha(\vec{r}) P + A(\omega) \delta(\vec{r} - \vec{r}_s) \quad (3)$$

Define $\rho(\vec{r}, \omega) \equiv k^2 \alpha(\vec{r}) P + A(\omega) \delta(\vec{r} - \vec{r}_s)$ and equation 2 becomes

$$\left(\nabla^2 + \frac{\omega^2}{c_0^2}\right) P = \rho(\vec{r}, \omega) \quad (4)$$

Green's theorem tutorial (continued)

Introduce

$$(\nabla^2 + k^2) G_0 = \delta(\vec{r} - \vec{r}\,') \tag{5}$$

Equation 4 can be solved in terms of the solution of equation 5

$$\underset{Causal}{P(\vec{r}, \omega)} = \int_{\infty} d\vec{r}\,' \rho(\vec{r}\,', \omega) G_0^{+}(\vec{r}, \vec{r}\,', \omega) \tag{6}$$

$\vec{r}$ in ∞ (anywhere)

Green's theorem tutorial (continued)

Green's second identity

$$\int_V d\vec{r}\,'(P\nabla^2 G_0 - G_0\nabla^2 P) = \oint_S dS\,\hat{n}\cdot(P\nabla G_0 - G_0\nabla P) \tag{7}$$

Substituting $\nabla^2 P$ and $\nabla^2 G_0$ from equations 4 and 5 in equation 7

Green's theorem tutorial (continued)

$$\underset{\text{for } \vec{r} \text{ in } V}{P(\vec{r},\omega)} = \int_V d\vec{r}\,'\rho G_0 + \oint_S dS\,\hat{n}\cdot(P\nabla G_0 - G_0\nabla P) \quad (8)$$

Valid for any choice G_0 that satisfies equation 5.

- Different choices of solutions for G_0 will derive each of the Green's theorem applications we listed
- If we choose $G_0 = G_0^+$ then equation 8 becomes

$$\underset{\vec{r} \text{ in } V}{P} = \int_V d\vec{r}\,'\rho G_0^+ + \oint_S dS\,\hat{n}\cdot(P\nabla G_0^+ - G_0^+\nabla P) \quad (9)$$

Table of Contents

Models for migration

The earliest wave equation migration pioneers viewed the backpropagation region as an infinite hemispherical half space with known mechanical properties, whose upper plane surface corresponded to the measurement surface, as in, *e.g.*, Schneider (1978) and Stolt (1978). See Fig. 1.

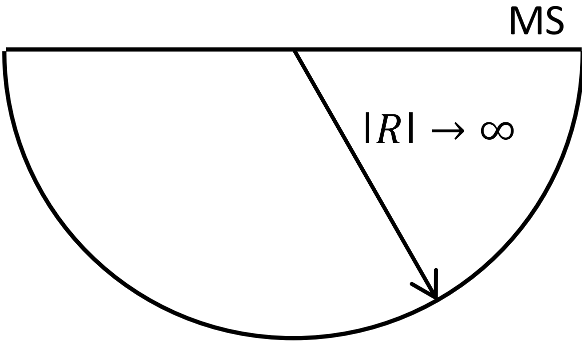


Figure 1: *The infinite hemispherical migration model. The measurement surface is denoted by MS.*

Models for migration

There are several problems with the infinite hemispherical migration model. That model assumes:
(1) that all subsurface properties beneath the measurement surface (MS) are known, and
(2) that an anticausal Green's function (*e.g.*, Schneider (1978)), with a Dirichlet boundary condition on the measurement surface, would allow measurements (MS) of the wave-field, P, on the upper plane surface of the hemisphere to determine the value of P within the hemispherical volume, V.

Models for migration

The first assumption leads to the contradiction that we have not allowed for anything that is unknown to be determined in our model, since everything within the closed and infinite hemisphere is assumed to be known. Within the infinite hemispherical model there is nothing and/or nowhere below the measurement surface where an unknown scattering point or reflection surface can serve to produce reflection data whose generating reflectors are initially unknown and being sought by the migration process.

Models for migration

The second assumption, in early infinite hemispherical wave equation migration, assumes that Green's theorem with wave-field measurements on the upper plane surface and using an anticausal Green's function satisfying a Dirichlet boundary condition can determine the wave-field within V. That conclusion assumes that the contribution from the lower hemispherical surface of S vanishes as the radius of the hemisphere goes to infinity. That is not the case.

Models for migration

The finite model for migration assumes that we know or can adequately estimate earth medium properties (velocity) down to the reflector we seek to image. The finite volume model assumes that beneath the sought after reflector the medium properties are and remain unknown. The "finite volume model" corresponds to the volume within which we assume the earth properties are known and within which we predict the wave-field from surface measurements.

Models for migration

We have moved away from the two issues of the infinite hemisphere model, *i.e.*, (1) the assumption we know the subsurface to all depths and (2) that the surface integral with an anticausal Green's function has no contribution to the field being predicted in the earth. The finite volume model takes away both assumptions. However, we are now dealing with a finite volume V, and with a surface S, consisting of upper surface S_U, lower surface S_L and walls, S_W (Fig. 2). We only have measurements on S_U.

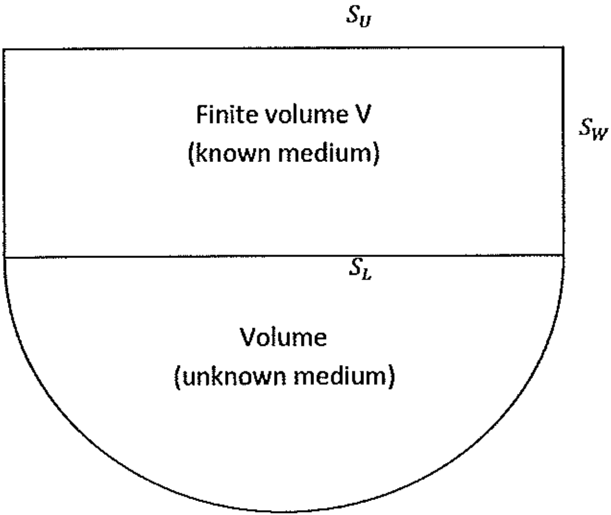


Figure 2: *A finite volume model*

Table of Contents

Stolt FK migration from Green's theorem

Consider a 1D up-going plane wave-field $P = Re^{-ikz}$ propagating upward through the 1D homogeneous volume without sources between $z = a$ and $z = b$ (Fig. 3). The wave P inside V can be predicted from

$$P(z,\omega) = \Big|_{z'=a}^{b} \Big\{ P(z',\omega)\ \frac{dG_0}{dz'}(z,z',\omega) - G_0(z,z',\omega)\frac{dP}{dz'}(z',\omega) \Big\}, \tag{10}$$

with the Green's function, G_0, that satisfies

$$\left(\frac{d^2}{dz'^{\,2}} + k^2\right)\ G_0(z,z',\omega)\ =\ \delta(z-z'), \tag{11}$$

for z and z' in V.

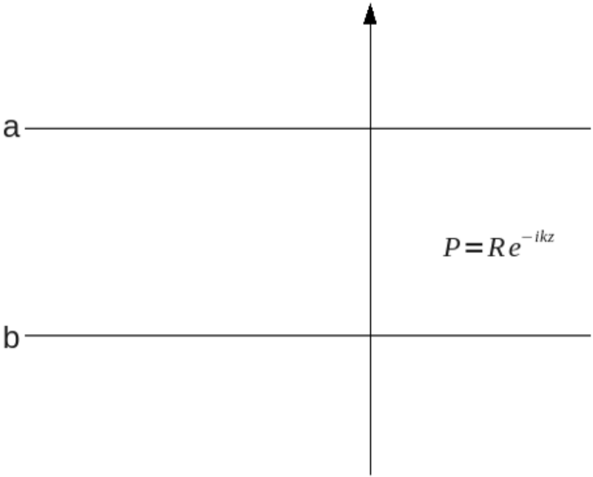


Figure 3: *1D up-going plane wave-field*

Stolt FK migration from Green's theorem

We can easily show that for an upgoing wave, $P = Re^{-ikz}$, that if one chooses $G_0 = G_0^+$ (causal, $e^{ik|z-z'|}/(2ik)$), the lower surface (i.e. $z' = b$) constructs P in V and the contribution from the upper surface vanishes. On the other hand, if we choose $G_0 = G_0^-$ (anticausal solution $e^{-ik|z-z'|}/(-2ik)$), then the upper surface $z = a$ constructs $P = Re^{-ikz}$ in V and there is no contribution from the lower surface $z' = b$.

Stolt FK migration from Green's theorem

This makes sense since information on the lower surface $z' = b$ will move with the upwave into the region between a and b, with a forward propagating causal Green's function, G_0^+. At the upper surface $z' = a$, the anticausal G_0^- will predict from an upgoing wave measured at $z' = a$, where the wave was previously and when it was moving up and deeper than $z' = a$.

Stolt FK migration from Green's theorem

Since in exploration seismology the reflection data is typically upgoing, once it is generated at the reflector, and we only have measurements at the upper surface $z' = a$, we choose an anticausal Green's function G_0^- in one-way wave back propagation in the finite volume model. If in addition we want to rid ourselves of the need for dP/dz' at $z' = a$ we can impose a Dirichlet boundary condition on G_0^-, to vanish at $z' = a$. The latter Green's function is labeled G_0^{-D}.

$$G_0^{-D} = -\frac{e^{-ik|z-z'|}}{2ik} - \left(-\frac{e^{-ik|z_I-z'|}}{2ik}\right) \qquad (12)$$

where z_I is the image of z through $z' = a$.

Stolt FK migration from Green's theorem

It is easy to see that $z_I = 2a - z$ and that

$$P(z) \;=\; -\frac{dG_0^{-D}}{dz'}(z,z',\omega)\Big|_{z'=a} P(a) \;=\; e^{-ik(z-a)}P(a) \tag{13}$$

in agreement with a simple Stolt FK phase shift for back propagating an up-field. Please note that $P(z,\omega) \;=\; -dG_0^{-D}/dz'(z,z',\omega)|_{z'=a}P(a,\omega)$ back propagates $P(z'=a,\omega)$, not G_0^{-D}. The latter thinking that G_0^{-D} back propagates data is a fundamental mistake/flaw in many seismic back propagation migration and inversion theories.

Table of Contents

The Green's function

For one-way wave propagation the double downward continued data, D is

$$D(\text{at depth}) = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} D\, dS_g\, dS_s \quad , \quad (14)$$

where D in the integrand $= D$(on measurement surface), $\partial G_0^{-D}/\partial z_s =$ anticausal Green's function with Dirichlet boundary condition on the measurement surface, $s =$ shot, and $g =$ receiver.

The Green's function

For two-way wave double downward continuation:

$$
\begin{aligned}
D(\text{at depth}) \;&=\; \int_{S_s} \left[\frac{\partial G_0^{DN}}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g \right. \\
&+\; \left. G_0^{DN} \frac{\partial}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g \right] dS_s \qquad (15)
\end{aligned}
$$

where D in the integrands $= D$(on measurement surface). G_0^{DN} is *neither* causal nor anticausal. G_0^{DN} is not an *anti*causal Green's function; it is not the inverse or adjoint of any physical propagating Green's function. It is the Green's function needed for RTM.

Details can be found in Weglein et al. (2011a,b).

This Report is an attachment in this year's Annual Report.

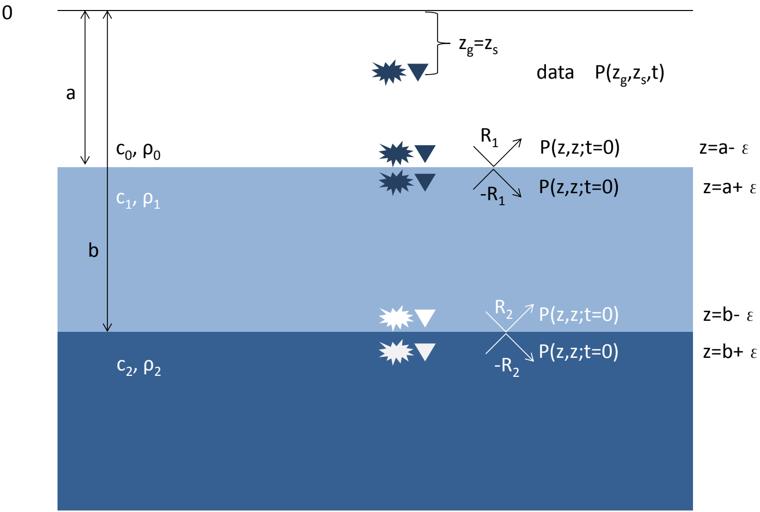
M-OSRP
Misson - Oriented Seismic Research Program
Solve The Right Problem

The first *wave theory* RTM, examples with a layered medium, predicting the source and receiver at depth and then imaging, providing the correct location and reflection amplitude at every depth location, and where the data includes primaries and all internal multiples

Fang Liu and Arthur B. Weglein

May 27-30, 2014
Austin, TX

Figure 4: *Imaging with primaries and internal multiples.*



M-OSRP

Sponsors

Anadarko Petroleum Corporation · CNPC BGP · bhpbilliton · bp · Chevron · CNPC · ConocoPhillips · devon · encana natural gas · ExxonMobil · HESS · IBM · ion · PEMEX · BR PETROBRAS · PGS · REPSOL · أرامكو السعودية Saudi Aramco · Spectrum · Statoil · TOTAL · WesternGeco

U.S. Federal Government Research Support

B.E.S.

NATIONAL SCIENCE FOUNDATION NSF C.M.G.

References I

Schneider, William A. "Integral formulation for migration in two and three dimensions." *Geophysics* 43 (February 1978): 49–76.

Stolt, R. H. "Migration by Fourier transform." *Geophysics* 43 (1978): 23–48.

Weglein, A. B., R. H. Stolt, and J. D. Mayhan. "Reverse-time migration and Green's theorem: Part I — The evolution of concepts, and setting the stage for the new RTM method." *Journal of Seismic Exploration* 20 (February 2011): 73–90.

References II

Weglein, A. B., R. H. Stolt, and J. D. Mayhan. "Reverse time migration and Green's theorem: Part II — A new and consistent theory that progresses and corrects current RTM concepts and methods." *Journal of Seismic Exploration* 20 (May 2011): 135–159.