

Multi-dimensional seismic imaging using the inverse scattering series

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Summary

The inverse scattering series (ISS) is a comprehensive multidimensional theory for processing and inverting seismic reflection data, that may be task-separated such that meaningful sub-problems of the seismic inverse problem may be accomplished individually, each without an accurate velocity model. We describe a task-separated subseries of the ISS geared towards accurate location in depth of reflectors, in particular the mechanisms of the series that act in multiple dimensions. We show that some 2D ISS imaging terms have analogs in previously developed 1D ISS imaging theory (e.g., Weglein et al., 2002; Shaw, 2005) and others do not; the former are used to create a 2D depth-only imaging prototype algorithm which is tested on synthetic salt-model data, and the latter are used to discuss ongoing research into reflector location activity within the series that acts only in the case of lateral variation and the presence of, e.g., diffraction energy in the data. Numerical tests are encouraging and show clear added value.

Introduction

The inverse scattering series (ISS) is a direct, non-linear inverse procedure for the reconstruction of an unknown spatial distribution of multidimensional medium parameters in terms of only measurements of a reflected wave field. The history of its investigation as a tool for the processing and inversion of seismic data, and the development of the task-separated treatment of the ISS, is detailed by Weglein et al. (2003). The ISS had been cast to individually carry out what are externally defined to be classical objectives of seismic data processing and inversion: (1) elimination of free surface multiples, (2) attenuation of internal multiples, (3) location in depth of rapid variations of medium parameters (imaging), and (4) determination of the parameter changes at those locations (inversion). Recent work on task (4) of the ISS can be found in, e.g., (Zhang and Weglein, 2005). The ISS expands the desired output as an infinite series in terms of only the data and a chosen (often very simple) reference Green's function, thus each of the above tasks is carried out without an accurate input velocity model.

Task (3) above, the problem of non-linear, ISS-based imaging (Weglein et al., 2002; Shaw et al., 2004) in a constant-density acoustic medium, has been earlier posed and described for the 1D pre-stack case by Shaw (2005). We point out that those cases are, however, 1D results due to a fully multidimensional theory. In this paper we extend the construction of direct non-linear imaging algo-

rithms to media with lateral and vertical variation (early results of this research are also discussed by Liu et al., 2005). The extension produces both terms that are clear analogs to those generated in the 1D case, and terms that are unique to the 2D environment. Those that have a 1D analog follow patterns recognizable from earlier research, and therefore map to potential 2D algorithms, applicable numerically, that work to correctly locate reflectors in depth while leaving errors due to lateral variation, and issues such as diffractions, largely intact. We demonstrate the added value of this type of algorithm on two synthetic models including a 2D salt model. Those that do not have 1D analogs are discussed as low-order terms in new, fully 2D subseries, which (although not yet sufficiently developed for inclusion in an algorithm) have activity locally wherever purely 2D effects (e.g., diffractions) are found in the data, and are expected to operate to correct the location error and artifacts remaining beyond the depth problem.

In this paper, we begin with a brief review of the ISS, followed by a discussion of the form of the linear inverse in the presence of a line source in a homogeneous reference medium; this linear inverse is the input to the non-linear terms of the ISS. The patterns of the imaging terms of the ISS are next described; several second-order terms deemed to be responsible for 2D reflector location tasks are presented and described, including those with and those without 1D analogs. The imaging prototype algorithm terms are then derived from the former in a $k_h = 0$ setting (k_h is the offset-conjugate wavenumber; see Clayton and Stolt (1981) for more detailed definition), additionally incorporating terms which will become pronounced for large contrast velocity models. The result is a depth-only 2D subseries, that performs 1D-like corrections at each lateral correction. This prototype algorithm, with alternate forms suitable for varying degrees of model contrast, is then numerically tested on a salt model.

Background

In operator form, the differential equations describing wave propagation in an actual and a reference medium can be written as

$$LG = -I \quad L_0 G_0 = -I \quad (1)$$

where L , L_0 and G , G_0 are the actual and reference differential and Green's operators, respectively, for a single temporal frequency (ω) and I is the identity operator. The perturbation V is defined as $V = L_0 - L$. The Lippmann-Schwinger equation, $G = G_0 + G_0 V G$, may

be expanded to form the forward scattering series:

$$G - G_0 = G_0 V G_0 + G_0 V G_0 V G_0 + \cdots \quad (2)$$

As detailed by Weglein et al. (2003), the representation of $V = V_1 + V_2 + V_3 + \cdots$ in equation 2 as an infinite series in orders of the measured scattered wave field $G - G_0$, gives rise to the ISS when like orders are equated:

$$\begin{aligned} D &= G - G_0 = G_0 V_1 G_0 \\ 0 &= -G_0 V_2 G_0 - G_0 V_1 G_0 V_1 G_0 \\ 0 &= -G_0 V_3 G_0 - G_0 V_1 G_0 V_2 G_0 - G_0 V_2 G_0 V_1 G_0 \\ &\quad - G_0 V_1 G_0 V_1 G_0 V_1 G_0, \end{aligned} \quad (3)$$

etc. For a constant density acoustic reference medium characterized by wavespeed c_0 , the relationship between the perturbation and the velocity is: $c_0^2 V / \omega^2 = \alpha(z)$, where $\alpha(z) = 1 - c_0^2 / c^2(z)$. Through the use of a variety of changes of integration variable and instances of integration by parts, Shaw (2005) identifies a portion of the ISS sum $\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \cdots$ that acts *only* to alter the locations of the discontinuities of the linear inverse α_1 from the wrong depth to the correct depth. First we consider the leading-order imaging subseries and its closed-form (c.f. Shaw et al., 2004):

$$\begin{aligned} \alpha_{IM}(z) &= \sum_{n=0}^{\infty} \frac{(-1/2)^n}{n!} d^n \alpha_1 / dz^n \left(\int_0^z \alpha_1(z') dz' \right)^n \\ &= \alpha_1 \left(z - \frac{1}{2} \int_0^z \alpha_1(z') dz' \right) \end{aligned} \quad (4)$$

The terms in the series above have two characteristics: they involve (1) derivatives of the linear inverse with respect to the coordinate in which the reflector location is being corrected, and (2) they are weighted by a depth integral of the same linear inverse. We proceed with a study of related forms in the more complex 2D case.

Equations for multidimensional imaging

Equations 3 can be solved for 2D constant density acoustic media, in which the single perturbation parameter,

$$\alpha(x, z) = 1 - c_0^2 / c^2(x, z), \quad (5)$$

is the essential quantity. In the ISS representation our objective is solved for as an infinite series, $\alpha(x, z) = \alpha_1(x, z) + \alpha_2(x, z) + \alpha_3(x, z) + \cdots$. The first term (the linear inverse) is expressible in terms of the data via the solution of the first equation in (3) (e.g., Clayton and Stolt, 1981):

$$\tilde{\alpha}_1(k_m, k_z) = -\frac{4k_z^2}{k_z^2 + k_m^2} \tilde{D} \left(k_m, \frac{c_0 k_z}{2} \sqrt{1 + \frac{k_m^2}{k_z^2}} \right), \quad (6)$$

in the midpoint conjugate (k_m) and depth conjugate (k_z) domains with the restriction $k_h = 0$; the data quantity

is computed from wave field information on the measurement surface:

$$\begin{aligned} \tilde{D}(k_m, \omega) &= \int_{-\infty}^{\infty} dx_m \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx_h \\ &\quad \times e^{i(\omega t - k_m x_m)} D \left(x_m + \frac{x_h}{2}, x_m - \frac{x_h}{2}, t \right) \end{aligned} \quad (7)$$

where the data in the integrand are considered in the source and receiver coordinates: $D(x_g, x_s, t)$. With this computation of the linear inverse, in lateral and depth coordinates, we next turn to the non-linear terms of the ISS, and express them as operations on $\alpha_1(x, z)$, applying an integration-by-parts strategy similar to that resulting in 1D forms (e.g., Weglein et al., 2002) to extract terms with the imaging-like aspect visible in equation (4). Solving the second equation in (3) for $\alpha_2(x, z)$, and manipulating the results accordingly, produces, amongst other terms (Liu et al., 2005), two which we label (a) and (b):

$$\alpha_2^{(a)}(x, z) = -\frac{1}{2} \frac{\partial \alpha_1(x, z)}{\partial z} \int_0^z \alpha_1(x, z') dz', \quad \text{and} \quad (8)$$

$$\alpha_2^{(b)}(x, z) = -\frac{1}{2} \frac{\partial \alpha_1(x, z)}{\partial x} \int_0^z \int_0^{z'} \frac{\partial \alpha_1(x, z'')}{\partial x} dz'' dz'. \quad (9)$$

Term (a) has a direct 1D analogy (c.f. the discussion of eqn. 4), and term (b) has no such analogy, but both exhibit the salient characteristics of ISS non-linear imaging. Equation (8) involves a first derivative of α_1 with respect to depth weighted by the integral of α_1 down to that depth. The term in equation (9), meanwhile, has the expected hallmarks of a lateral corrector at second order, involving a first derivative with respect to the lateral coordinate, weighted by the depth integral of the rate of change of α_1 . Notice that this term will vanish if the linear inverse does not vary laterally. We surmise that this term is the first in an infinite series correcting the lateral error in the linear inverse.

The above analysis leads to two main conclusions. First, the presence, in the multi-D case, of an exact reproduction of the 1D depth imaging engine, as terms that are zeroth order in $\partial \alpha_1 / \partial x$ (and the tendency of the imaging terms of the ISS to behave like nested, or cascaded Taylors series), suggests that we consider the vertical and lateral imaging problem as being akin to a series expansion about the purely vertical imaging problem. Lateral corrector terms that are of low order should be effective when applied to problems involving slow lateral variability; rapid lateral variations will evidently require terms of higher-order in $\partial \alpha_1 / \partial x$. Second, this re-appearance of the same patterns as those found in the 1D case allow for the same summations to closed-form that exists in 1D scenarios. Hence, a single expression that is the *zeroth order lateral corrector* and the *leading order depth corrector* for

the 2D case may be derived (c.f. equation (4)):

$$\alpha_{LOIS}(x, z) = \alpha_1 \left(x, z - \frac{1}{2} \int_0^z \alpha_1(x, z') dz' \right). \quad (10)$$

This is a depth corrector that involves the same engine as in the 1D case, but with a different quantity under scrutiny at each x . We refer to the quantity above as “leading-order imaging subseries” (LOIS) to conform with descriptions of the 1D imaging algorithm. *Leading order* refers to the fact that the subseries coefficients are approximated as the integral of the first power of α_1 only (Shaw, 2005). Similarly, then, to the imaging mechanism of Shaw’s analysis, equation (10) can achieve accurate location of reflectors in depth for media of low- to moderate-velocity contrast.

We have had occasion to study idealized Earth models whose contrast levels have been of a size too large for the leading order mechanisms alone to be used; this has led to some effort at incorporation of a greater complement of the reflector location terms within the ISS, as part of a new (but still partial) “higher-order” imaging without inversion formula. Innanen (2005) describes a capture of these terms within a 1D milieu that couples the imaging and inversion problem. Bringing that location capability to the 2D task-separated algorithm, a non-trivial development outside the scope of this paper, leads to the algorithm form:

$$\alpha_{HOIS}(x, z) = \alpha_1 \left(x, z - \frac{1}{2} \int_0^z \frac{\alpha_1(x, z') dz'}{1 - 0.25\alpha_1(x, z')} \right). \quad (11)$$

Either of eqns. (10) or (11), with the linear inverse of eqn. (6) as input, may be computed numerically to explore the capability of this partial ISS imaging algorithm.

Numerical examples

We present an example of the 2D imaging algorithms of the previous sections, i.e., in eqns (6) and (11). Fig 1 illustrates the salt model. The data are created using a fourth-order finite difference scheme, with a temporal sampling rate of 2ms and a lateral spatial sampling rate of 5m. The source signature is the first derivative of a Gaussian (peak frequency of 28Hz). The resultant data are used as described above to compute the linear inverse associated with a homogeneous reference medium of wavespeed $c_0 = 1500m/s$. First, the linear term α_1 is calculated from data according to equation (6), then the imaging algorithm is calculated via equation (11), and the first derivative in depth of the result is displayed in Figure 3.

In the salt model example, the depth-only 2D imaging algorithm can be seen to produce a target reflection location close to that of the actual, correct target. The correction of the depth issues of the location problem is visible, as is the absence of correction on the purely lateral/2D issues

(e.g., the remnant diffraction energy visible on the flanks of the salt body). Emphasizing that in generating this example the algorithm was accorded no velocity information before or during the calculation, we regard this as a very encouraging result.

Conclusions

We present an extension of the velocity-independent imaging methods of the ISS to accommodate media that vary laterally as well as in depth. Those prototype methods, which in 2D have some terms with 1D analogs and some without, call for specific and reasonably straightforward non-linear data activity, with the former of which we demonstrate very encouraging numerical examples on 2D synthetic data generated over a salt model. Ongoing research is geared towards finding and grouping terms that are more specifically present for lateral/2D processing issues, and studying the structure of the algorithm in a framework generalizing beyond $k_h = 0$.

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References

- Innanen, K. A., Two non-linear forward and inverse approximations for wave fields in the presence of sustained medium perturbations. *Proceedings of the 75th Annual Meeting of the Society of Exploration Geophysicists, Houston, TX*, (2005).
- Liu, F., Weglein, A. B., Innanen, K. A. and Nita, B. G., Extension of the non-linear imaging capability of the inverse scattering series to multidimensional media: strategies and numerical results. *Proc. 9th Int. Cong. SBGF and 6th Latin American Geop. Conf., Salvador de Bahia, Brazil*, (2005).
- Shaw, S. A., Weglein, A. B., Foster, D. J., Matson, K. H., and Keys, R. G., Isolation of a leading order depth imaging series and analysis of its convergence properties for a 1D acoustic medium. *Journal of Seismic Exploration* 13, (2004): 99-120.
- Shaw, S. A., An inverse scattering series algorithm for depth imaging of reflection data from a layered acoustic medium. Ph.D. Thesis, University of Houston, (2005).
- Weglein, A. B., Foster, D. J., Matson, K. H., Shaw, S. A., Carvalho, P. M., and Corrigan, D., Predicting the

correct spatial location of reflectors without knowing or determining the precise medium and wave velocity: initial concept, algorithm and analytic and numerical example. *Journal of Seismic Exploration* 10, (2002).

Weglein, A. B., Araujo, F. A., Carvalho, P. M., Stolt, R. H., Matson, K. H., Coates, R., Corrigan, D., Foster, D. J., Shaw, S. A., and Zhang, H. Topical Review: Inverse-scattering Series and Seismic Exploration. *Inverse Problems* 19, (2003):R27–R83.

Zhang, H., and Weglein, A. B. The inverse scattering series for tasks associated with primaries: depth imaging and direct non-linear imaging of 1D variable velocity and density acoustic media *Proceedings of the 75th Annual Meeting of the Society of Exploration Geophysicists, Houston, TX*, (2005).

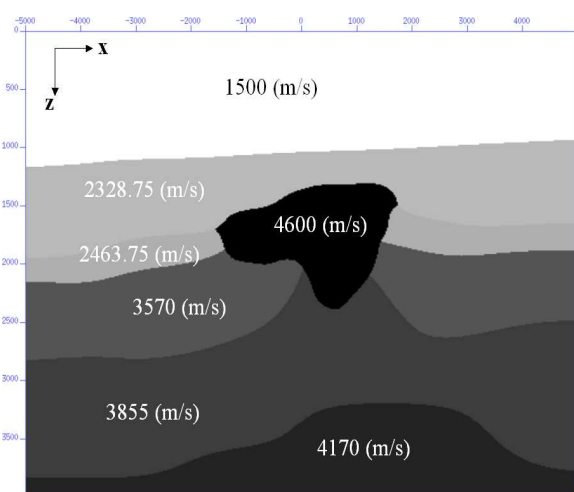


Fig. 1: The geometry and velocity of each layer in the salt model.

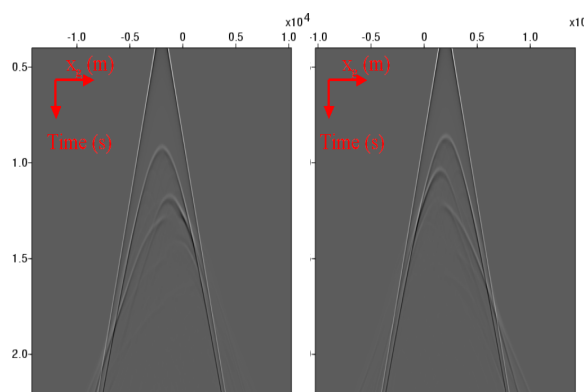


Fig. 2: Left: shot gather with $x_s = -2000(m)$. Right: shot gather with $x_s = 2000(m)$. In both shot gathers, conflicting hyperbolas are present which will cause ambiguities in velocity analysis.

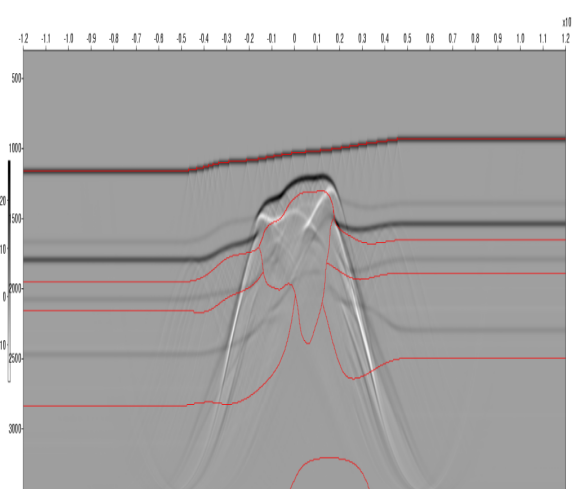


Fig. 3: Salt model linear imaging results, having taken the partial derivative of the linear term over depth ($\frac{\partial \alpha_1}{\partial z}$) (the purpose of this partial derivative is to make the reflectors clearer in the section). In the linear image, only the water bottom was correctly imaged by the whole-space water velocity, and the bottom reflector was more than 1km away from its correct location.

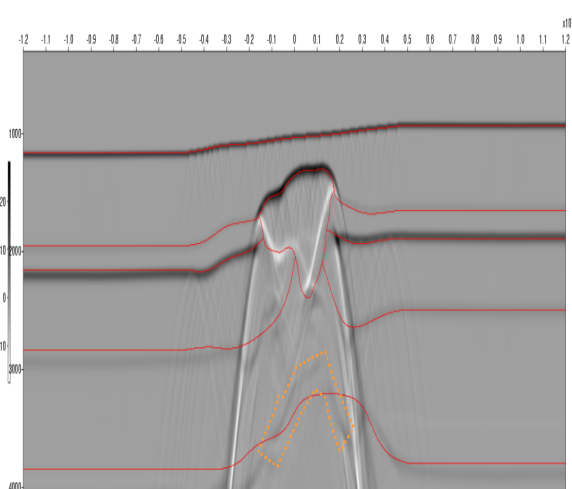


Fig. 4: Large contrast imaging subseries prototype algorithm output given the input in Figure 3. The actual locations of the reflectors are indicated. The second reflector (including the top salt) was correctly imaged. The third reflector (including the salt bottom) was imaged very close to their actual locations. Outside the salt flank, the locations of the fourth reflector was moved much closer to its actual location. A dashed polygon (in orange) is overlaid around the portion of the fifth reflector below the salt.

EDITED REFERENCES

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REFERENCES

- Innanen, K. A., 2005, Two non-linear forward and inverse approximations for wavefields in the presence of sustained medium perturbations: 75th Annual International Meeting, SEG, Expanded Abstracts, 2362–2365.
- Liu, F., A. B. Weglein, K. A. Innanen, and B. G. Nita, 2005, Extension of the non-linear imaging capability of the inverse scattering series to multidimensional media: strategies and numerical results: Proceedings of the 9th International Conference, SBGf.
- Shaw, S. A., 2005, An inverse scattering series algorithm for depth imaging of reaction data from a layered acoustic medium: Ph.D. thesis, University of Houston.
- Shaw, S. A., A. B. Weglein, D. J. Foster, K. H. Matson, and R. G. Keys, 2004, Isolation of a leading order depth imaging series and analysis of its convergence properties for a 1D acoustic medium: *Journal of Seismic Exploration*, **13**, 99–120.
- Weglein, A. B., F. A. Araujo, P. M. Carvalho, R. H. Stolt, K. H. Matson, R. Coates, D. Corrigan, D. J. Foster, S. A. Shaw, and H. Zhang, 2003, Topical review: Inverse-scattering series and seismic exploration: *Inverse Problems*, **19**, R27–R83.
- Weglein, A. B., D. J. Foster, K. H. Matson, S. A. Shaw, P. M. Carvalho, and D. Corrigan, 2002, Predicting the correct spatial location of reflectors without knowing or determining the precise medium and wave velocity: Initial concept, algorithm, and analytic and numerical example: *Journal of Seismic Exploration*, **10**.
- Zhang, H., and A. B. Weglein, 2005, The inverse scattering series for tasks associated with primaries: Depth imaging and direct non-linear imaging of 1D variable velocity and density acoustic media: 75th Annual International Meeting, SEG, Expanded Abstracts, 1705–1708.