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A direct inverse method for subsurface properties: the conceptual and practical benefit and added-value in comparison with all current indirect methods, for example, AVO and FWI

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GEO-v1

Running head: **A direct inverse solution**

ABSTRACT

We begin with a context and perspective that facilitates the logic flow, the message and advances in this paper to be accessible, comprehended and understood. Towards that end we (1) provide a brief overview that defines modeling and direct and indirect inversion, (2) describe the inverse scattering series (ISS), the only direct inversion method for a multidimensional subsurface; (3) provide a clear understanding of why each term in the ISS can be computed directly in terms of data, and without subsurface information, (4) show how [item (2)] leads to distinct isolated task ISS subseries that can achieve every seismic processing task, directly and without subsurface information. We then focus on the seismic processing task of parameter identification and provide the direct inverse solution for elastic isotropic

mechanical properties. Among the key objectives of this paper is to clearly and convincingly demonstrate, in detail, why solving a forward problem in an inverse sense is not equivalent to solving an inverse problem directly. AVO and FWI are solving a forward problem in an inverse sense. We provide a direct inverse solution (from ISS) for the goals and objectives of AVO and FWI. We describe the precise difference between this ISS direct parameter estimation method and all current conventional approaches for those same objectives. A new set of significant conceptual and practical differences, and the added value, benefit and consequences of the direct methodology in comparison with all current indirect methods are clearly defined, illustrated and examined. A key and unique advantage of direct inversion is being able to: (1) know you have actually solved the problem of interest and equally if not more important (2) being able to distinguish between a problem of interest and the problem that we need to be interested in solving.

INTRODUCTION

The objective of seismic processing in seismic exploration is to use recorded reflection data to extract useful subsurface information that is relevant to the location and production of hydrocarbons. There is typically a coupled chain of intermediate steps and processing that takes place towards that objective, and we refer to those intermediate steps, stages and tasks as objectives "associated with inversion" and the ultimate subsurface information extraction goal and objective. All seismic processing methods used to extract subsurface information make assumptions and have prerequisites and conditions that need to be satisfied.

A seismic method will be effective when those assumptions/conditions/requirements are satisfied. When those assumptions are not satisfied the method can have difficulty and/or fail. That failure can and will contribute to dry-hole drilling or drilling suboptimal appraisal and development wells.

Challenges in seismic processing and seismic exploration and production derive from the violation of assumptions/requirements behind seismic processing methods. Advances in seismic processing effectiveness are measured in terms of whether the new capability results in/contributes to more successful plays and positive, well-informed and successful drilling decisions.

The purpose of seismic research is to identify and address seismic challenges and to thereby add options to the seismic processing toolbox. These new options can be called upon when indicated, appropriate and necessary to increase the drill success rate. As we will point out below "identify the problem" is the first, the essential and sometimes the most difficult (and often ignored and/or underappreciated) aspect of seismic research.

Clearly identifying and delineating the actual problems behind seismic processing challenges is essential for a serious, substantive and sustainable commitment to delivering a response to actual priorities and pressing challenges. This paper provides a new insight, guide, contribution and advance for the first and critical step of problem identification. The methods used to achieve seismic processing objectives can be classified as modeling and inversion.

MODELING AND INVERSION

(Please see figure 1.) Modeling, as a seismic processing tool, starts with a prescribed energy mechanism/description and a model type (e.g., acoustic, elastic, anisotropic, anelastic, . . .) and then properties are defined within the model type for a given medium (e.g., velocities, density, reflector location, . . .). The modeling procedure then synthetically/numerically generates the seismic wave field that the energy source produces at all points inside and outside the medium.

Inversion also starts with an assumed known and prescribed energy source, and in addition has measurements of the resulting wavefield outside the medium being interrogated. The objective of seismic inversion is to use the latter source description and wavefield measurement information to make inferences about the subsurface medium that are relevant to the location and production of hydrocarbons.

DIRECT AND INDIRECT INVERSION

Inversion methods can be classified as direct or indirect. A direct inversion method can solve an inverse problem (as its name suggests) directly. On the other hand, an indirect inversion method seeks to solve an inverse problem circuitously through indirect or assumed aligned necessary (but typically not sufficient) — conditions, and properties, and often mistakenly considered and treated though it was equivalent to a direct method and solution. Among indicators, identifiers and examples of "indirect" inverse solutions (Weglein, 2015a) are: (1) model matching, (2) objective/cost functions, (3) local and global search algorithms, (4) iterative linear inversion, (5) methods corresponding to necessary but not sufficient conditions, e.g., common image gather flatness as an indirect migration velocity analysis method and (6) solving a forward problem in an inverse sense, e.g., AVO and FWI, that we will show are not equivalent to a direct inverse solution for those same objectives.

As a simple illustration, [see figure 2] a quadratic equation

$$ax^2 + bx + c = 0 \tag{1}$$

can be solved through a direct method as

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{2}$$

or it can be solved by an indirect method searching for x such that, e.g., some functional of

$$(ax^2 + bx + c)^2 \text{ is a minimum.} \tag{3}$$

There is an earlier paper on this specific topic and discussion (Weglein, 2013) that contains a useful background and communication, as well as an extensive list of papers on AVO and FWI. .

THE IMPORTANT QUADRATIC EQUATION EXAMPLE

The direct quadratic formula and solution equation (2) explicitly and immediately outputs the exact roots, independent of whether they are real and distinct, a real double root, imaginary and/or complex roots. This is a very simple and very insightful example. How would a search algorithm know after a double root is found that it is the only root and to not keep looking and searching forever for a second nonexistent root? How would a search algorithm know to search for only real or for real and complex roots. How would a search algorithm accurately locate an irrational root like $\sqrt{3} \cong 1.732\ldots$ as $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ would directly and precisely and immediately produce. Indirect methods like model matching and seeking and searching and determining roots as in equation (3) is ad hoc, and does not rely on a firm framework and foundation and never provides the confidence that we are actually solving the problem of interest.

WHAT'S THE POINT? AND WHAT'S THE PRACTICAL BIG DEAL ABOUT A DIRECT SOLUTION?

How can this example and discussion of the quadratic equation possibly be relevant to exploration seismology? Please imagine for a moment that equation (1) $ax^2 + bx + c = 0$ was an equation whose inverse and solution for x given by equation (2) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ had seismic exploration prediction consequence. And furthermore suppose that this direct solution for x did not lead to successful and/or improved drilling decisions. If that was the case, we could not blame or question the method of solution of equation (1), since equation (2) is direct and unquestionably solving equation (1). If equation (2) was not producing useful and beneficial results we know that our starting equation (1) is the issue, and we have identified the problem. With equation (3), an indirect method, negative exploration consequences could be due to either the choice within the plethora of indirect approaches and/or the equation you are seeking to invert.

That lack of clarity and definitiveness within indirect methods obfuscates the underlying issue and makes identification of the problem (and what's behind a seismic challenge) considerably more difficult and illusive. Indirect methods with search engines such as equation (3), lead to "workshops" for solving equation (1) and grasping at mega HPC straws (and capital expenditure investment for buildings full of HPC) that are required to search, seek and find. The more HPC we invest in, and is required, the more we are committed and therefore convinced of the unquestioned validity of the starting point and indirect approach.

Therefore, beyond the benefit of a direct method like equation (2) providing assurance that we are actually solving the problem of interest, equation (1), there is the unique problem location and identification benefit of a direct inverse when a seismic method produces unsatisfactory E&P results.

To bring this closer to seismic experience, if, for example, if you are not satisfied (in terms of improved drill location and success rate) with a direct inverse of the elastic isotropic equation for amplitude analysis, you know you need to go to a different starting point, perhaps a more complete and realistic wave propagation equation, since you can exclude the direct solution method as the problem and issue.

HOW TO DISTINGUISH BETWEEN THE "PROBLEM OF INTEREST" AND THE PROBLEM WE NEED TO BE INTERESTED IN

Direct inverse methods provide value for both knowing that you have actually solved the problem of interest, and, in addition, there is the additional value of knowing whether our starting point, the "problem of interest", is the problem we need to be interested in.

With that introduction we move onto the direct inverse solution for a multidimensional subsurface. Scattering theory is the starting point for that discussion, providing a direct inversion method for all seismic processing objectives realized by the isolated task subseries of the inverse scattering series (ISS) (Weglein et al., 2003). Each term in the inverse scattering series (and the distinct and specific collection of terms that achieve different specific inversion associated tasks) is computable (1) directly and (2) in terms of recorded reflection data and without any subsurface information known, estimated or determined before, during or after the task is performed and the specific processing objective is achieved.

For certain distinct tasks, and subseries, e.g. free surface multiple elimination and internal multiple attenuation, the algorithms not only do not require subsurface information they are independent of earth model type (Weglein et al., 2003). That is, the distinct ISS free surface and internal multiple algorithms are unchanged, without a single line of code having the slightest change for acoustic, elastic, anisotropic and anelastic earth models (Weglein et al., 2003; Wu and Weglein, 2014). For those who subscribe to indirect inversion methods as the "be all and end all", and, e.g., model matching approaches, it would be a useful exercise for them to consider how they would formulate a model-type independent model matching scheme for free surface and internal multiple removal.

For the topic and focus of this paper, the task of parameter estimation, there is an obvious need to specify model type and what parameters are to be determined. It is for that parameter estimation/medium property objective, and that specific ISS subseries, that the difference between "the problem of interest" and the problem that we need to be interested in, is both relevant, central and significant. Only direct inversion methods for earth mechanical properties provides that essential starting point information, clarity and distinction.

A direct inverse solution for parameter estimation can be derived from an operator identity that relates the change in a medium's properties and the commensurate change in the wavefield. That operator identity is valid and can accommodate any seismic model-type, for example, acoustic, elastic, anisotropic, heterogeneous, and inelastic earth models. That operator identity can be the starting point and basis of both: (1) perturbative scattering theory modeling methods and (2) a firm and solid math-physics foundation and framework for direct inverse methods.

THEORY

Let's consider an energy source that generates a wave in a medium with prescribed properties. Now let's consider a change in the medium and the wave that results from the same energy source. Scattering theory is a perturbation theory that relates a change (or perturbation) in a medium to a corresponding change (or perturbation) in the associated wavefield. When the medium changes the resulting wavefield changes. The direct inverse solution (Weglein et al., 2003; Zhang, 2006) for determining earth mechanical properties is derived from the operator identity that relates the change in a medium's properties and the commensurate change in the wavefield both within and exterior to the medium. Let L_0, L, G_0, and G be the differential operators and Green's functions for the reference and actual media, respectively, that satisfy:

$$L_0 G_0 = \delta \quad \text{and} \quad LG = \delta,$$

where δ is a Dirac δ-function. We define the perturbation operator, V, and the scattered wavefield, ψ_s, as follows:

$$V \equiv L_0 - L \quad \text{and} \quad \psi_s \equiv G - G_0.$$

The operator identity

The relationship (called the Lippmann-Schwinger or scattering theory equation)

$$G = G_0 + G_0 V G \tag{4}$$

is an operator identity that follows from

$$L^{-1} = L_0^{-1} + L_0^{-1}(L_0 - L)L^{-1},$$

and the definitions of L_0, L, and V.

Direct forward series and direct inverse series

The operator identity equation (4) [for a fixed source function] is the exact relationship between changes in a medium and changes in the wavefield; it is a relationship and not a solution. However the operator identity equation (4) can be solved for G as

$$G = (1 - G_0 V)^{-1} G_0, \tag{5}$$

or

$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots . \tag{6}$$

For forward modeling the wavefield, G, in equation 6 for a medium described by L is given in terms of the two parts of L, L_0 and V where L_0 enters as G_0 and V enters as V itself. Equation 6 communicates that modeling using scattering theory requires a complete and detailed knowledge of the earth model type and medium properties within the model type. Equation 6 communicates that any change in medium properties, V will lead to a change in the wavefield, $G - G_0$, that is always non-linearly related to the medium property change,

V. Equation 6 is called the Born or Neumann series in scattering theory literature (see e.g., Taylor, 1972). Equation (6) has the form of a generalized geometric series

$$G - G_0 = S = ar + ar^2 + \cdots = \frac{ar}{1-r} \text{ for } |r| < 1, \tag{7}$$

where we identify $a = G_0$ and $r = VG_0$ in equation (6), and

$$S = S_1 + S_2 + S_3 + \cdots , \tag{8}$$

where the portion of S that is linear, quadratic, ... in r is:

$$S_1 = ar,$$
$$S_2 = ar^2,$$
$$\vdots$$

and the sum is

$$S = \frac{ar}{1-r}. \tag{9}$$

Solving equation (9) for r, in terms of S/a produces the inverse geometric series,

$$\begin{aligned} r = \frac{S/a}{1+S/a} &= S/a - (S/a)^2 + (S/a)^3 + \cdots \\ &= r_1 + r_2 + r_3 + \cdots , \text{ when } |S/a < 1|. \end{aligned} \tag{10}$$

where r_i is the portion of r that is i-th order in S/a. When S is a geometric power series in r, then r is a geometric power series in S. The former is the forward series and the latter is the inverse series. That is exactly what the inverse series represents, the inverse geometric series of the forward series equation (6). This is the simplest prototype of an inverse series for r, i.e., the inverse of the forward geometric series for S. For the seismic inverse problem, we associate S with the measured data (see e.g. Weglein et al., 2003)

$$S = (G - G_0)_{ms} = \text{Data},$$

and the forward and inverse series follow from treating the forward solution as S in terms of V, and the inverse solution as V in terms of S (measured values of $G - G_0$). The inverse series is the analog of equation (10) where $r_1,\ r_2,\ \cdots$ is simply replaced with $V_1,\ V_2,\ \cdots$.

$$V = V_1 + V_2 + V_3 + \cdots\ , \tag{11}$$

where V_n is the portion of V that is n^{th} order in measured data, D. Equation (6) is the forward series; and equation (11) is the inverse series. The identity (equation 4) provides a geometric forward series, a very special case of a Taylor series. A Taylor series of a function, $S(r)$,

$$\begin{aligned} S(r) &= S(0) + S'(0)r + \frac{S''(0)r^2}{2} + \cdots \\ s(r) = S(r) - S(0) &= S'(0)r + \frac{S''(0)r^2}{2} + \cdots \end{aligned} \tag{12}$$

whereas the geometric series is

$$S(r) - \underbrace{S(0)}_{a} = ar + ar^2 + \cdots\ . \tag{13}$$

The Taylor series equation (12) would reduce to the special case of a geometric series equation (13) if

$$S(0) = S'(0) = \frac{S''(0)}{2} = \cdots = a$$

The geometric series equation (13) has an inverse series whereas the Taylor series geometric series equation (12) does not. In general, a Taylor series doesn't have an inverse series. That's the reason that inversionists committed to a Taylor series starting point adopt the indirect linear updating approach, where a linear approximate Taylor series is inverted. They attempt through updating to make the linear form an ever more accurate approximate — and it remains entirely and firmly indirect and ad hoc.

Please see figures 3-4. The relationship (6) provides a Geometric forward series that honors equation (4) rather than a truncated Taylor series.

All conventional current mainstream parameter estimation inversion, including iterative linear inversion, AVO and FWI, are based on a Taylor series concept, that doesn't honor and remain consistent with the identity equation (6).

We will show that in general solving a forward problem in an inverse sense is not the same as solving an inverse problem directly. The exception is when the exact direct inverse is linear, as e.g. in the theory of migration. For wave equation migration, given a velocity model the migration and structure map output is a linear function of recorded reflection data.

If we assume $S = ar$ (that is, that there is an exact linear forward relationship between S and r) then $r = S/a$ is solving the inverse problem directly. In that case, solving the forward problem in an inverse sense is the direct inverse solution.

However, if the forward exact relationship is non-linear, for example

$$S_n = ar + ar^2 + \cdots + ar^n$$

$$S_n - ar - ar^2 - \cdots - ar^n = 0 \tag{14}$$

and solving the forward problem (14) in an inverse sense for r will have n roots, $r_1, r_2, \ldots, r_n$

As $n \to \infty$, number of roots $\to \infty$

However, from the direct nonlinear forward problem $S = \frac{ar}{1-r}$, we found the direct inverse solution $r = \frac{S}{a+S}$ (one real root).

This discussion above provides a vivid and compelling illustration of how solving a forward problem in an inverse sense is not the same as solving the inverse problem directly. The difference between solving a forward problem in an inverse sense (for example using equation (6) to solve for V) and solving an inverse problem (for example, equations 15-17) are much more serious, substantive and practically significant when examining the direct and indirect inversion of the elastic heterogeneous wave equation for earth mechanical properties.

In terms of the expansion of V in equation (11), and G_0, G, $D = (G - G_0)_{ms}$, the inverse scattering series (Weglein et al., 2003) can be obtained as

$$
\begin{aligned}
G_0 V_1 G_0 =& D, && (15)\\
G_0 V_2 G_0 =& -G_0 V_1 G_0 V_1 G_0, && (16)\\
G_0 V_3 G_0 =& -G_0 V_1 G_0 V_1 G_0 V_1 G_0 \\
& - G_0 V_1 G_0 V_2 G_0 - G_0 V_2 G_0 V_1 G_0, && (17)\\
& \vdots
\end{aligned}
$$

To illustrate how to solve equations (15), (15), (15), ...for V_1, V_2, V_3, ...consider the marine case with L_0 corresponding to a homogeneous reference medium of water. G_0 is the Green's function for propagation in water. D is the data measured for example, with towed streamers, G being the total field the hydrophone receiver records on the measurement surface, and G_0 the field the reference wave would record at the receiver. V then represents the difference between earth properties L and water properties L_0. The solution for V is found using

$$V = V_1 + V_2 + V_3 + \cdots, \tag{18}$$

where V_i is the portion of V that is ith order in the data, D. Substituting equation (18) into the forward series equation (6), then evaluating equation (6) on the measurement surface and setting terms that are equal order in the data equal we find equations (15), (16), (17), ... Solving equation (15) for V_1 involves the data D and G_0 (water speed propagator) and solving for V_1 is analytic, and corresponds to a prestack water speed Stolt FK migration of the data D.

Hence solving for V_1 involves an analytic water speed FK migration of the data D. Solving for V_2 from equation (16) involves the same water speed analytic Stolt FK migration of $-G_0V_1G_0V_1G_0$, a quantity that depends on V_1 and G_0, where V_1 depends on data and water speed, and G_0 is the water speed Green's function. Each term in the series produces V_n as an analytic Stolt FK migration of a new "effective data", where the effective data, the right hand side of equations (15)-(17), are multiplicative combinations of factors that only depend on the data, D and G_0. Hence, every term in the ISS is directly computed in terms of data and water speed. That's the direct non-linear inverse solution.

The different specific combinations of terms (isolated task subseries) that achieve different inversion associated tasks:

Free surface multiple removal

internal multiple removal

depth imaging

non-linear direct amplitude analysis for earth mechanical properties

Q compensation without Q

are each direct and only involve the data, D and water speed properties through G_0. The inverse step is analytic and unchanged. There is one equation, equation (15), that exactly produces V_1, and V_1 is the exact portion of V that is linear in the measured data, D. The inverse operation to determine V_1, V_2, $V_3, \ldots$ is analytic, and never is updated with a bandlimited data, D. The band limited nature of D never enters an updating process as in iterative linear inversion, non-linear AVO and FWI.

The inverse scattering series provides a direct method for obtaining the subsurface properties contained within L, by inverting the series order-by-order to solve for the perturbation operator V, using only the measured data D and a reference Green's function G_0, for any assumed earth model type. We can imagine that a set of tasks need to be achieved to determine the subsurface properties, V, from recorded seismic data, D. These tasks are achieved within equations (15), (16), (17), ... The tasks that are within a direct inverse solution are: (1) free-surface multiple removal, (2) internal multiple removal, (3) depth imaging, (4) Q compensation without Q, and (5) non-linear parameter estimation. Each of these five tasks has its own task-specific subseries from the ISS for $V_1, V_2, \cdots$, and each of those tasks is achievable directly and without subsurface information (see, e.g., Weglein et al., 2003). Equations (15)-(17) provide V in terms of $V_1, V_2, \cdots$, and each of the V_i is computable directly in terms of D and G_0. In the next section, we review the details of equations (15)-(17) for a 2D heterogeneous elastic medium.

The operator identity for a 2D heterogeneous elastic medium

We describe the forward and direct inverse method for a 2D elastic heterogeneous earth (see Zhang, 2006).

The 2D elastic wave equation for a heterogeneous isotropic medium (Zhang, 2006) is

$$L\vec{u} = \begin{pmatrix} f_x \\ f_z \end{pmatrix} \quad \text{and} \quad \hat{L}\begin{pmatrix} \phi^P \\ \phi^S \end{pmatrix} = \begin{pmatrix} F^P \\ F^S \end{pmatrix}, \tag{19}$$

where $\vec{u}$, f_x, and f_z are the displacement and forces in displacement coordinates and ϕ_P, ϕ_S and F^P, F^S are the P and S waves and the force components in P and S coordinates, respectively. The operators L and L_0 in the actual and reference elastic media are

$$L = \left[\rho\omega^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \partial_x\gamma\partial_x + \partial_z\mu\partial_z & \partial_x(\gamma - 2\mu)\partial_z + \partial_z\mu\partial_x \\ \partial_z(\gamma - 2\mu)\partial_x + \partial_x\mu\partial_z & \partial_z\gamma\partial_z + \partial_x\mu\partial_x \end{pmatrix}\right],$$

$$L_0 = \left[\rho\omega^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \gamma_0\partial_x^2 + \mu_0\partial_z^2 & (\gamma_0 - \mu_0)\partial_x\partial_z \\ (\gamma_0 - \mu_0)\partial_x\partial_z & \mu_0\partial_x^2 + \gamma_0\partial_z^2 \end{pmatrix}\right],$$

and the perturbation V is

$$V \equiv L_0 - L = \begin{bmatrix} a_\rho\omega^2 + \alpha_0^2\partial_x a_\gamma\partial_x + \beta_0^2\partial_z a_\mu\partial_z & \partial_x(\alpha_0^2 a_\gamma - 2\beta_0^2 a_\mu)\partial_z + \beta_0^2\partial_z a_\mu\partial_x \\ \partial_z(\alpha_0^2 a_\gamma - 2\beta_0^2 a_\mu)\partial_x + \beta_0^2\partial_x a_\mu\partial_z & a_\rho\omega^2 + \alpha_0^2\partial_z a_\gamma\partial_z + \beta_0^2\partial_x a_\mu\partial_x \end{bmatrix},$$

where the quantities $a_\rho \equiv \rho/\rho_0 - 1$, $a_\gamma \equiv \gamma/\gamma_0 - 1$, and $a_\mu \equiv \mu/\mu_0 - 1$ are defined in terms of the bulk modulus, shear modulus and density (γ_0, μ_0, ρ_0, γ, μ, ρ) in the reference and actual media, respectively.

The forward problem is found from the identity equation (6) and the elastic wave equation (19) in PS coordinates as

$$\begin{aligned} &\hat{G} - \hat{G}_0 = \hat{G}_0\hat{V}\hat{G}_0 + \hat{G}_0\hat{V}\hat{G}_0\hat{V}\hat{G}_0 + \cdots, \\ &\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix} = \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \\ &+ \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} + \cdots, \end{aligned} \tag{20}$$

and the inverse solution, equations (15)-(17), for the elastic equation (19) is

$$\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix} = \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix},$$

$$\begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_2^{PP} & \hat{V}_2^{PS} \\ \hat{V}_2^{SP} & \hat{V}_2^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}$$

$$= - \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}, \tag{21}$$

$$\vdots$$

where $\hat{V}^{PP} = \hat{V}_1^{PP} + \hat{V}_2^{PP} + \hat{V}_3^{PP} + \cdots$ and any one of the four matrix elements of V requires the four components of the data

$$\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix}.$$

The 3D heterogeneous isotropic elastic generalization of the above 2D forward and direct inverse elastic isotropic method begins with the linear 3D form found in Stolt and Weglein (2012) page 159.

In summary, from equation (20), $\hat{D}^{PP}$ can be determined in terms of the four elements of V. The four components $\hat{V}^{PP}$, $\hat{V}^{PS}$, $\hat{V}^{SP}$, and $\hat{V}^{SS}$ require the four components of D. That's what the general relationship $G = G_0 + G_0VG$ requires, i.e., a direct non-linear inverse solution is a solution order-by-order in the four matrix elements of D (in 2D). The generalization of the forward series equation (20) and the inverse series equation (21) for a direct inversion of an elastic isotropic heterogeneous medium in 3D involves the 3×3 data, D, and V matrices in terms of P, SH and SV data and start with the linear $G_0V_1G_0 = D$

on page 179 of Stolt and Weglein (2012).

Direct inverse and indirect inverse

The direct inverse solution described above is not iterative linear inversion. Iterative linear inversion starts with equation (15). In that approach, we solve for V_1 and change the reference medium iteratively. The new differential operator L_0' and the new reference medium G_0' satisfy

$$L_0' = L_0 - V_1 \quad \text{and} \quad L_0' G_0' = \delta. \tag{22}$$

In the indirect iterative linear approach equation (15) with different reference background, L_0' and G_0' we return to

$$G_0' V_1' G_0' = D' = (G - G_0')_{ms}, \tag{23}$$

where V_1' is the portion of V linear in data $(G - G_0')_{ms}$. We can continually update L_0' and G_0', and hope to solve for the perturbation operator V. In contrast, the direct inverse solution equations (11) and (21) calls for a single unchanged reference medium, for computing $V_1, V_2, \ldots$. For a homogeneous reference medium, $V_1, V_2, \ldots$ are each obtained by a single unchanged analytic inverse. The inverse to find V_1 from data, is the same exact unchanged analytic inverse operation to find $V_2, V_3, \ldots$, from equations (15),(16),....

For ISS direct inversion, there are no numerical inverses, no generalized inverses, no inverses of matrices that are computed from and contain noisy band-limited data. The latter terribly troublesome, difficult and serious practical problem doesn't exist or occur with direct ISS methods but is a central and intrinsic characteristic and pitfall of indirect methods, model matching, updating, iterative linear inverse approaches (e.g. AVO and FWI).

Are there any circumstances where the indirect iterative linear inversion and the direct ISS parameter estimation would be equivalent?

Are there any circumstances (or circumstance) where the ISS direct inversion would be equivalent to and correspond to the indirect iterative linear approach? If we consider the simplest acoustic single reflector model. Consider a normal incident plane wave reflection data experiment and ideal full band-width perfect data, with the upper half space with velocity c_0 and lower half space with velocity c_1 and ask to use these two methods to use the reflected event to determine the velocity of the lower half space, c_1. Yang and Weglein (2015) examined and analyzed this problem and compared the results of the direct ISS method and the indirect iterative linear inversion. They showed that the direct ISS inversion to estimate c_1 converged to c_1 under all circumstances and all values of c_0 and c_1. In contrast, the indirect linear iterative inversion had a limited range of values of c_0 and c_1 where it converged to c_1, and in that range it converged much slower than the direct ISS parameter estimation for c_1. The iterative linear inverse simply shut down and failed when the reflection coefficient, R was greater than 1/4. The direct ISS parameter estimation method converged to c_1 for any value of the reflection coefficient R. Hence, under the simplest possible circumstance, and providing the iterative linear method with an analytic Frechet derivative, as a courtesy from and gift delivered to the linear iterative from the ISS direct inversion method, the ranges of usefulness and validity and the effectiveness were never equivalent or comparable. With band-limited data and more complex earth models (e.g., elastic multiparameter) this gap in the range of validity, usefulness and effectiveness will necessarily widen. The indirect iterative linear inversion and the direct ISS parameter estimation method are never equivalent, and there are absolutely no simple or complicated circumstances where they are equally effective.

Direct ISS parameter Inversion: A time lapse application

The direct inverse ISS elastic parameter estimation method [equation 21] was successfully applied (Zhang et al., 2006) in a time lapse sense to discriminate between pressure and fluid saturation changes. Traditional time-lapse linear estimation methods were unable to predict and match that direct inversion ISS discrimination.

The difference between iterative linear and the direct inverse of equation (21) is much more substantive and serious than merely a different way to solve $G_0V_1G_0 = D$ (equation 15), for V_1. If equation (15) is someone's entire basic theory, you can mistakenly think that $\hat{D}^{PP} = \hat{G}_0^P\hat{V}_1^{PP}\hat{G}_0^P$ is sufficient to update $\hat{D}^{PP} = \hat{G}_0'^P\hat{V}_1'^{PP}\hat{G}_0'^P$. This step loses contact with and violates the basic operator identity $G = G_0 + G_0VG$ for the elastic wave equation. That's as serious as considering problems involving a right triangle and violating the Pythagorean theorem within your method. That is, iteratively updating PP data with an elastic model violates the basic relationship between changes in a medium, V and changes in the wavefield, $G - G_0$, for the simplest elastic earth model.

This direct inverse method provides a platform for amplitude analysis, and the goals and objectives of AVO and FWI. A direct method communicates when an amplitude analysis method should work, in principle. Iteratively inverting multi-component data has the correct data but doesn't correspond to a direct inverse algorithm. To honor $G = G_0 + G_0VG$, you need both the data and the algorithm that direct inverse prescribes. Not recognizing the message that an operator identity and the elastic wave equation unequivocally communicate is a fundamental and significant contribution to the gap in effectiveness in current

AVO and FWI methods and application (equation 21). This analysis generalizes to 3D with P, S_h, and S_v data.

The role of direct and indirect methods

There's a role for direct and indirect methods in practical real world applications. Indirect methods are to be called upon for recognizing that the world is more complicated than the physics that we assume in our models and methods. For the part of the world that you are capturing in your model (and methods) nothing compares to direct methods for clarity and effectiveness. An optimal indirect method would seek to satisfy a cost function that derives from a property of the direct method. In that way the indirect and direct method would be aligned, consistent and cooperative for accommodating the part of the world described by your physical model and the part that is outside.

Model matching primaries and multiples

Indirect iterative linear inversion model matching is a search methodology, and ad-hoc, and without a firm and solid foundation and theoretical and conceptual framework. We can imagine and understand that model matching primaries and multiples, rather than only primaries, could improve and upgrade the matching criteria. However, model matching primaries and multiples remains ad hoc and always on much shakier footing than direct inversion for the same inversion goals and objectives. The practical value of identifying the problem or issue is never well defined in an ad-hoc method, and to address challenges one must begin by the clearest identification and delineation of the problem. Nothing comes close to direct methods for that usefulness and clarity.

For all multidimensional seismic applications, the direct inverse solution provided by the operator identity equation (4) is in the form of a series equations (15)-(15), referred to as the inverse scattering series (Weglein et al., 2003). It can achieve all processing objectives within a single framework and a single set of equations (15)-(15) without requiring any subsurface information. There are distinct isolated-task inverse scattering subseries derived from the ISS, which can perform free-surface multiple removal (Carvalho et al., 1992; Weglein et al., 1997), internal multiple removal (Araújo et al., 1994; Weglein et al., 2003), depth imaging (e.g. Shaw, 2005; Liu, 2006; Weglein et al., 2012), parameter estimation (Zhang, 2006; Liang, 2013; Li, 2011; Yang and Weglein, 2015), and Q compensation without Q (Innanen and Weglein, 2007; Innanen and Lira, 2010; Lira, 2009), and each achieves its objective directly and without subsurface information. The direct inverse solution (e.g., Weglein et al., 2003, 2009) provides a framework and firm math-physics foundation that unambiguously defines both the data requirements and the distinct algorithms to perform each and every associated task within the inverse problem, directly and without subsurface information. There are many other issues that contribute to the gap between a direct parameter estimation inversion solution from the ISS and e.g., conventional and industry standard amplitude-versus-offset (AVO) and full-waveform-inversion (FWI). However, starting with and employing a framework that provides confidence of the data and methods that are actually solving the problem of interest is a significant, fundamental, and practical contribution and starting point towards identifying all differences and issues (Weglein, 2015b).

Having an ad hoc method as the starting point places a cloud over issue identification when less than satisfactory results arise with field data. Is it the questionable algorithm itself or is it the noisy bandlimited data? In addition, we saw that direct inversion parameter estimation has a significantly less dependence on low frequency data components than all the indirect methods like nonlinear AVO and FWI. Only a direct solution can provide algorithmic clarity, confidence and effectiveness. The current industry standard AVO and FWI, using variants of model-matching and iterative linear inverse, are indirect methods and procedures, and iteratively linearly updating P data or multi-component data (with or without multiples) does not correspond to, and will not produce, a direct solution.

All direct methods for structural determination and amplitude analysis only require primaries

All direct methods for structural determination and amplitude analysis require only primaries. In Weglein (2016) and in Weglein et al. (2003) it is shown, in the former, that with a direct structure and amplitude analysis method that requires a discontinuous velocity model above the target, that free surface and internal multiples play absolutely no role, with the same image and inversion results with or without the multiples. In the latter (Weglein et al., 2003), the ISS (without subsurface information) removes free surface and internal multiples prior to the distinct subseries that input primaries and perform depth imaging and amplitude analysis, respectively, each directly and without subsurface information and only using primaries.

All direct inversion methods, both those with and those without subsurface/velsocity information, require only primaries for complete structural determination and amplitude analysis. Indirect methods are ad hoc without a clear or firm math physics foundation and framework, and they start without knowing whether “the solution” is in fact the solution. A fuller data set being matched with model and field data each with primaries and multiples could at times improve upon matching only primaries, but the entire approach is indirect and ad hoc with or without multiples.

Hence, model matching primaries and multiples remains indirect and ad hoc and without a solid foundation, and hence, clueless in terms of why problems arise since there is no theory to rely on, and to have confidence in that a solution is forthcoming under any circumstances.

All methods have assumptions and requirements. Some methods have a firm and clear math-physics foundation and framework that provides an assurance that at least in principle a solution is forthcoming. The problem (from my point of view) is the constant overstating and marketing of the indirect FWI method, that continued unabated and was even amplified at the recent 2016 SEG in Dallas. This ad hoc indirect model matching (so called FWI) of primaries and multiples was described by a leading FWI pioneer "as the ultimate and most capable method that can be imagined and will ever be devised for extracting information about the subsurface".

It's a method, and although ad hoc it can have some value, and if it can survive the eventual backlash of the groupthink and the outrageous and injurious (to the method itself) from overselling and overmarketing it will hopefully find a place (and a measured and limited and appropriate role) in the seismic toolbox. But it isn't a direct solution (which, of course would, itself, not be a panacea or final word) and in fact FWI is in reality as far as one can imagine from what the advocates are proclaiming and are too often communicating — no method, direct or indirect could ever match that overstatement and hype.

If we seek the parameters of an elastic heterogeneous isotropic subsurface, then the differential operator in the operator identity is the differential operator that occurs in the elastic, heterogeneous isotropic wave equation. From forty years of AVO and amplitude analysis application in the petroleum industry, the elastic isotropic model is the base-line minimally realistic and acceptable earth model-type for amplitude analysis, for example, for AVO and FWI. Then taking the operator identity (called the Lippmann-Schwinger or scattering theory equation) for the elastic wave equation, we can obtain a direct inverse solution for the changes in elastic properties and density. The direct inverse solution specifies both the data required and the algorithm to achieve a direct parameter estimation solution. In this paper we explain how this methodology differs from all current AVO and FWI methods, that are in fact forms of model matching (often, and in addition, with the wrong/innately inadequate/inappropriate model type and/or less than necessary (or too much (primaries and multiples) and unnecessary data) and are not direct solutions. Multicomponent data consisting of only primaries are needed for a direct inverse solution for subsurface properties. This paper focuses on one specific task, parameter estimation, within the overall and broader set of inversion objectives and tasks. Furthermore, the impact of band-limited data and noise, are discussed and compared for the direct ISS parameter estimation and indirect (AVO and FWI) inversion methods.

In this paper, we focused on analyzing and examining the direct inverse solution that the ISS inversion subseries provides for parameter estimation. The distinct issues of: (1) data requirements, (2) model-type, and (3) inversion algorithm for the direct inverse are all important (Weglein, 2015b). For an elastic heterogeneous medium, we show that the direct inverse requires multi-component/PS (P-component and S-component) data and prescribes how that data are utilized for a direct parameter estimation solution (Zhang and Weglein, 2006).

CONCLUSIONS

In this paper, we describe, illustrate and analyze the considerable conceptual and substantive and substantial practical benefit and added-value that a direct parameter inversion from the inverse scattering series provides in comparison with all current indirect inverse methods (e.g., AVO and FWI) for the amplitude analysis goals and objectives. A direct method provides “a solution” that: (1) we can have confidence is a solution to the defined problem of interest and (2) if the method doesn’t produce predictions that improve drilling decisions, then we know that the issue is the problem of interest and that is not the problem we need to be interested in. On the other hand, indirect methods like AVO and FWI, have a plethora of approaches and paths, and when less than satisfactory results occur we don’t know whether the issue is the chosen problem of interest or the choice of indirect solution.

The ability to clearly and unambiguously define the origin and root cause of seismic breakdown and challenges is an essential and critically important step in designing and executing a strategy to provide new and more capable methods to the seismic processing toolbox.

Only direct inversion methods can provide that clarity and definitiveness. They are also unique in providing the confidence that the problem of interest is actually being solved. For ISS parameter estimation while the recorded data is of course band limited, the band-limited data is never used to compute the updated inverse operator for the next linear step, since the inverse operator is fixed and analytic for every term in the inverse scattering series. That’s one of several important and substantive differences pointed out in this paper between the direct inverse ISS parameter estimation method and all indirect inversion methods, e.g., AVO and FWI.

Direct and indirect methods both can play an important role and function in seismic processing: where the former accommodates and addresses the assumed physics within the system and the latter provides a channel for real world phenomena beyond the assumed physics.

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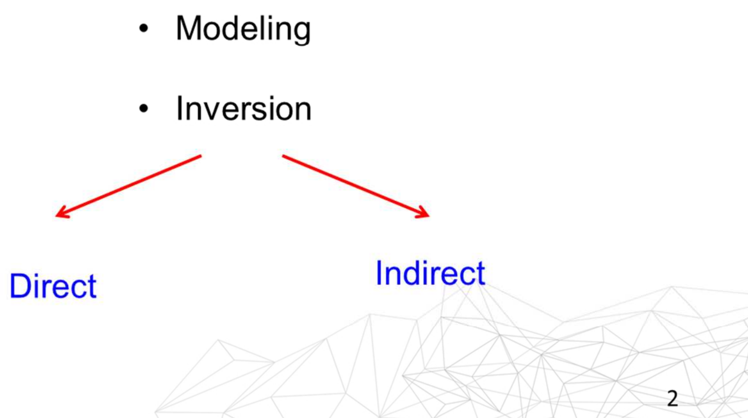
Figure 3: Slide 2.

Figure 4: Slide 3.

Figure 3: Slide 9.

Figure 4: Slide 14.


• Modeling
• Inversion
Direct
Indirect
2

Direct

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Indirect

Search for x such that

$$(ax^2 + bx + c)^2$$ is a minimum

3

DIRECT FORWARD AND DIRECT INVERSE

$$S = S_1 + S_2 + S_3 + \cdots$$

$$S = \frac{ar}{1-r}$$

Solve for r

$$r = \frac{S/a}{1 + S/a} = S/a - \left(S/a\right)^2 + \left(S/a\right)^3 + \cdots$$
$$= r_1 + r_2 + r_3 + \cdots$$

9

DIRECT FORWARD AND DIRECT INVERSE

$S = (G - G_0)_{ms} = Data$

Forward *S* in terms of *V*, inverse *V* in terms of *S*

$$V = V_1 + V_2 + \cdots \qquad (3)$$

where V_n is the portion of *V*, n-th order in the data

(2) $G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots$ is the forward series;

(3) $V = V_1 + V_2 + \cdots$ is the inverse series.

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