Short note: An alternative adaptive subtraction criteria (to energy minimization) for free surface multiple removal

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An idea: The idea starts by considering the following perturbation forms in 2D.

$$\text{1 parameter acoustic} \quad V_p = \omega^2 \left(\frac{1}{K_0} a_1(\vec{k} - \vec{p}) \right) = V_p(\vec{k}, \vec{p}, \omega)$$

$$\text{2 parameter acoustic} \quad V_{pp} = \omega^2 \left(\frac{1}{K_0} a_1(\vec{k} - \vec{p}) + \frac{1}{\omega^2} \frac{\vec{k} \cdot \vec{p}}{\rho_0} a_2(\vec{k} - \vec{p}) \right)$$

$$\text{elastic} \quad V_{pp} = \omega^2 \left(\frac{1}{K_0} a_1(\vec{k} - \vec{p}) + \frac{1}{\omega^2} \frac{\vec{k} \cdot \vec{p}}{\rho_0} a_2(\vec{k} - \vec{p}) - 2\frac{\beta_0^2}{\omega^4} |\vec{k} \times \vec{p}|^2 a_3(\vec{k} - \vec{p}) \right)$$

where $\vec{k}$ and $\vec{p}$ are arbitrary 2D vectors, K_0, ρ_0, and β_0 are the bulk modulus, density, and shear velocity of the reference medium, a_1 is the relative change in the bulk modulus, a_2 is the relative change in density, and a_3 is the relative change in shear modulus. On the measurement surface, $\vec{k} = \vec{k}_g$, $\vec{p} = \vec{k}_s$, and $|\vec{k}_g| = |\vec{k}_s| = \omega/c_0$. The input to the inverse scattering series free surface multiple algorithm is $V_1(\vec{k}_g, \vec{k}_s, \omega)$, which for the three forms listed above has an overall ω^2/c^2 factor.

$$D^{WOFS} = \underbrace{G_0^d V_1 G_0^d}_{\omega^2} + \underbrace{G_0^d V_1 G_0^{FS} V_1 G_0^d}_{\omega^4} + \underbrace{G_0^d V_1 G_0^{FS} V_1 G_0^{FS} V_1 G_0^d}_{\omega^6} + \ldots$$

where D^{WOFS} is deghosted data without free surface multiples and the first term on the right hand side is D^{WFS}, deghosted data with free surface multiples.

The new adaptive criteria is

$$\lim_{\omega \to 0} \left[\frac{D^{WFS} - D^{WOFS}}{\omega^2} \right] = 0$$

or minimize $(D^{WFS} - D^{WOFS})/\omega^2$ with respect to a "wavelet" factor.