Multiples: signal or noise?

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Summary

"The exclusive view" of seismic reflection data considers primaries as signal and multiples as noise. At the 2013 SEG "Recent Advances and the Road Ahead", a presentation entitled "Multiple Attenuation: Recent Advances and the Road Ahead (2013)" (also see Weglein, 2014) described the state of seismic multiple removal in terms of: (1) current industry capability, and (2) what are the significant and substantive open issues and challenges today. We described a three-pronged strategy that has the potential to close the current gap, for complex and complicated offshore and conventional and unconventional onshore plays. That entire activity and viewpoint is "the exclusive view" of seismic reflection data, where primaries are signal and multiples are a form of coherent noise that needs to be removed.

There is an alternative view, "the inclusive view" of processing seismic reflection data, where primaries and multiples are treated as signal. In that view both are considered useful, taken separately and/or taken together. "The inclusive view" seeks to provide added value over and beyond just using primaries as signal for seismic imaging. The inclusive view of utilizing both primaries and multiples, separately, or together, to enhance imaging has recently become a topic of increased discussion and interest. One purpose of this article is to examine this inclusive view and activity.

Seismic imaging

Since those pursuing the inclusive view are seeking added value in seismic imaging, we begin our discussion with a brief history of seismic imaging. That will allow us to define terms and place these recent "inclusive" efforts in perspective, and to assist in their examination and evaluation.

Let's begin by discussing the various concepts, objectives, and levels of ambition for seismic imaging. Migration has two ingredients: (1) a wave propagation component and (2) an imaging principle or concept. Jon Claerbout (Claerbout, 1971; Riley and Claerbout, 1976) was the initial and key wave-equation-migration imaging-concept pioneer and algorithm developer, and together with Stolt (1978) and Lowenthal et al. (1985), they introduced imaging conditions for locating reflectors at depth from surface-recorded data.

Imaging conditions

The three key imaging conditions that were introduced are:
(1) time and space coincidence of up and downgoing waves,
(2) the exploding-reflector model, and
(3) predicting a source and receiver experiment at a coincident-source-and-receiver subsurface point, and asking for time equals zero (the definition of Wave-Equation Migration (WEM)).

For a normal-incident spike plane wave on a horizontal reflector, these three imaging concepts are totally equivalent. However, a key point to make clear for this paper, is that for a non-zero-offset surface seismic-data experiment they are no longer equivalent, for either a one-dimensional or a multi-dimensional subsurface. For the purposes of determining quantitative information on the physical meaning of the image, the clear choice is predicting a source and receiver experiment at depth. Wave-equation migration (WEM) is defined as using the third imaging condition, (3), the predicted source and receiver experiment at depth at time equals zero. In anything beyond 1D normal-incidence or zero-offset data, the other two imaging concepts (for example, time coincidence of up and downgoing waves) turn out to be asymptotic ray travel-time-curve "Kirchhoff" algorithms with a trajectory of image candidates, that are summed, looking for constructive addition for structural determination. Lost is the definitive "yes" or "no" to a point being an image provided by a source and receiver experiment at a coincident subsurface point. Stolt and his colleagues (Clayton and Stolt, 1981; Stolt and Weglein, 1985; Stolt and Benson, 1986) extended the experiment-at-depth concept to allow a separated source and receiver at time equals zero, to not only provide a definitive "yes" or "no" to any given subsurface point being a reflector, but, in addition, provide the angle-dependent reflection coefficient. The other imaging concepts cannot provide that imaging definitiveness nor the quantitative angle-dependent reflection-coefficient information at the image point. In addition, and in general all pre-stack versions, variants, and extensions of the first two imaging conditions listed above, whether for one-way waves or two-way waves, or for data consisting of primaries, or primaries and multiples, are always asymptotic or ray approximates of the third imaging condition. Asymptotic migration, resulting from adopting imaging conditions (1) or (2), will impose asymptotic forms of wave propagation that relate to ray theory and do not satisfy the ubiquitous space-filling propagation and illumination of wave theory and wave-theory migration.

The properties and benefits of Wave-Equation Migration (WEM) in comparison to asymptotic "Kirchhoff-like" migration are:

(1) Definitiveness of a subsurface point corresponding to (or not corresponding to) structure from a predicted source and receiver experiment at that point;
(2) Quantitative angle-dependent reflection coefficient information at the imaged point; and
(3) Ubiquitous volume-filling wave propagation, coverage and illumination, compared to the limited propagation and illumination of ray theory.

RTM

When two-way migration was introduced by Whitmore, McMechan and their colleagues (Whitmore, 1983; McMechan, 1983), it was formulated and carried out first in post-stack and then in the pre-stack domain by running the data back in time (hence reversed time migration, or RTM) and the source field forward in time and then cross-correlating the two fields at zero lag. The post-stack and pre-stack versions were basically the earlier exploding-reflector model and the time coincidence of up and down-going wave-imaging concepts, respectively. That formulation of asymptotic RTM has become so widespread that it has been adopted even for one-way migration, where too often the very meaning of migration has come to be defined as:

$$\mathbf{I}(\mathbf{x}) = \sum_{\mathbf{x_s}} \sum_{\omega} \frac{\mathbf{S}'(\mathbf{x}, \mathbf{x_s}; \omega)\mathbf{R}(\mathbf{x}, \mathbf{x_s}; \omega)}{\mathbf{S}'(\mathbf{x}, \mathbf{x_s}; \omega)\mathbf{S}(\mathbf{x}, \mathbf{x_s}; \omega) + \varepsilon^2}, \qquad (1)$$

where R is the back-propagated reflection data, S is the forward-propagated source wavefield, the zero-lag cross-correlation is indicated by the sum over angular frequency, ω, and the sum over sources adds candidate-image traveltime trajectories. S' is the complex conjugate of S, and ε is a stabilization parameter.

The conventional RTM method represented by equation (1), consists of back propagating the receiver field and forward propagating the source field, where each is carried out using the wave equation. However, the cross-correlation at zero lag is the grown-up version of imaging condition (1) and the imaging condition (1) is the place that the method entered the land of asymptotics and "Kirchhoff" ray theory.

All current RTM methods (for primaries and multiples) use variants and extensions or higher-order terms based on equation (1), are asymptotic ray-based migration, and hence do not correspond to wave-equation migration.

That might come as a surprise to the very large number of researchers and those who apply equation (1) in oil and service companies, that with all the wave-equation computer effort and expense to implement and utilize equation (1) that it doesn't correspond to wave-equation migration. The use of equation (1) is ubiquitous, but the imaging method it employs and represents and the RTM migration itself is ray-theoretic and is therefore not ubiquitous in its subsurface coverage and illumination.

Wave-equation migration (WEM) for two-way waves, for diving waves, or for migrating primaries and multiples

Neither the post- nor pre-stack current versions of RTM (captured in equation (1)) corresponded to predicting a source and receiver experiment at depth and hence neither is WEM RTM. We suspect that many researchers that begin with migration forms such as equation (1) today, have no idea that they are starting with and remain in asymptotic rather than wave-equation migration concepts and algorithms. Weglein and his colleagues (Weglein et al., 2011a,b; Liu and Weglein, 2013) provided for two-way wave propagation the first predicted source and receiver experiment at depth and wave-equation migration, *i.e.*, WEM RTM. Green's theorem provides a solid basis and firm foundation for predicting a source and receiver experiment at depth from the wavefield on an upper surface of a volume. That's how wave-equation migration RTM is formulated for either: (1) turning-wave primaries, and (2) for reflection data consisting of both primaries and multiples. The benefits and added value of WEM RTM compared to all current and conventional RTM methods (equation (1)) are the same benefits as between wave-equation migration and asymptotic or Kirchhoff forms for one-way waves for one-way-wave migration: (1) definitiveness on whether a point in the subsurface corresponds to structure, (2) the angle-dependent reflection coefficient at the image point, and (3) the subsurface coverage, and illumination of waves versus rays. Equation (2) describes WEM migration for one-way waves, where D inside the integral is the surface data, and G_0^{-D} is the anti-causal Green's function that vanishes on the measurement surface. Equation (3) is WEM RTM where D in the integral is the surface data, and G_0^{DN} is the Green's function that along with its normal derivative vanishes on the lower surface and the walls of the volume.

$$D = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} D \, dS_g \, dS_s$$
$$\text{(Green, 1-way waves)} \qquad (2)$$

$$D = \int_{S_s} \left[\frac{\partial G_0^{DN}}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g + G_0^{DN} \frac{\partial}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g \right] dS_s$$
$$\text{(Green, 2-way waves)} \qquad (3)$$

Equation 2 is Stolt prestack one-way wave-equation migration, and equation 3 is wave-equation-migration RTM.

These new wave-equation-migration RTM methods (equation 3) provide for two-way wave propagation what earlier wave-equation migration methods (*e.g.*, Stolt, 1978) provided for one-way propagation (Weglein et al., 2011a; Stolt and Weglein, 2012).

Wave-Equation Migration Imaging for data consisting of primaries and multiples

In Figure 1, we illustrate (from Liu and Weglein, 2013) the result from applying equation (3) for WEM RTM to data that consists of all primaries and internal multiples, from a one-dimensional layered medium. The predicted coincident source and receiver experiment at time equals zero is shown at different locations in the subsurface, predicting the correct location of structure. In addition, the correct reflection coefficient is provided on each side of each reflector, by the experiment being predicted for a source and a receiver slightly above or slightly below each reflector, respectively. Hence, to migrate with primaries and multiples, you are required to simply follow what George Green prescribed in 1828 (Green, 1828) for a closed surface adjusted by Weglein et al. (2011b) for surface reflection data with an accurately known discontinuous medium in the volume, and a Green's function that corresponds to properties in that volume and vanishes along with its normal derivative on all surfaces except the upper surface. There is no "crosstalk", no need for "secondary distributed sources" caused by data, no higher-order scattering theory allusions and incantations, or other *ad hoc* or unclear and/or unnecessary constructs, including unnecessarily separating primaries and multiples. It's all in equation (3). Equation (3) is the wave-equation migration formula that predicts a source and receiver in a volume with two-way wave propagation, and combined with an imaging condition predicts both structure and the angle-dependent reflection amplitude. That's the wave-equation migration method for any two-way wave propagation in the volume.

Hence, "the inclusive view" is not in any way new, or requiring new theory, in fact it was historically the first, the original (*e.g.*, Green's theorem (1828)) of predicting a total wavefield inside a volume (*e.g.*, inside the earth) from total-wavefield surface measurements on the closed surface surrounding the volume.

Inclusive use of primaries and multiples to improve image illumination

Recent efforts at the inclusive use of primaries and multiples are aimed at improved image illumination. Illumination seeks to improve the clarity and resolution of the located image at depth.

The first step would seem to require a firm theory for correct depth imaging with primaries and multiples.

Having a better resolved and clearer but mislocated image is of little or no value.

Unfortunately, the methods currently put forth and pursued to realize "the inclusive view" for illumination do not hark back and begin their thinking and development with the solid foundation for wavefield prediction provided by Green (1828). Furthermore, the recent and current "inclusive view" activity very often has had shaky underpinnings, at best, and a lack of any clear and firm foundation and framework, with *ad hoc* constructs offered with confidence and conviction.

Those proposing to use primaries and multiples to enhance imaging have mainly confined their interest to improving the "illumination" for a structure map. Jon Claerbout famously and accurately observed, many years ago, that illumination is not an issue, in principle, for wave theory and wave-theory migration (WEM). Illumination is a fundamental and intrinsic issue for rays and all asymptotic (*e.g.*, Kirchhoff) migration methods and asymptotic RTM (equation (1)). Waves go everywhere and are space-filling. Rays don't. Where rays don't go, we have an intrinsic asymptotic-method-produced illumination issue. The conventional industry-applied RTM methods, represented by equation (1), are all asymptotic migration methods. Current industry RTM methods certainly use the wave equation in running the data backwards and the source forward and cross correlating at zero lag. However, using the wave equation is not the same as being a wave-equation migration. Wave-equation migration predicts a source and receiver experiment at depth, and all current RTM methods do not meet that requirement and are not wave-equation migration. Hence, all the currently employed RTM methods (equation (1)) are, in principle, and on their own, contributing to an intrinsic illumination issue and challenge. Furthermore, even with 100% perfect "illumination", asymptotic imaging provides a challenged image in terms of its ability to provide a reflection amplitude as a function of angle at the image point.

However, for those committed to asymptotic RTM and seeking to achieve improved "illumination" in order to better delineate structure by utilizing/including free-surface multiples using variants of equation (1), we recognize a certain added value, in particular, for relatively shallow targets (Berkhout and Verschuur, 1994, 1997; Whitmore et al., 2010, 2011b,a; Lu et al., 2011, 2013a,b; Lu and Whitmore, 2013; Ong et al., 2013). However, the latter methods also produce false events in the data (due to crosstalk) at deeper locations, and that issue can represent a serious downside. For example, imagine if such a generated false event interferes with a target primary. There doesn't seem to be a way, at the moment, to address that downside and to remove these false events. The basic reason those cross-talk-generated false images cannot be removed is there is no clear and firm wave-theory-based derivation of the method to begin with. Hence, we cannot go back and fix or avoid assumptions being made, that lead to injurious artifacts, since we don't have a starting point with a theory without those assumptions. Those crosstalk problems and artifacts occur whether the primaries and multiples are separated and utilized separately and then combined, or they are taken together at once (Wang et al., 2013).

Why did we want to remove multiples to begin with? Are those reasons any less valid today?

In general, it is important to remember why, in exploration seismology, we haven't used primaries and multiples for depth imaging and inversion. Have we overcome the fundamental reason for separating them and processing primaries? The answer is "no". Primaries are much more accepting of an approximate, smooth velocity for imaging. We very often cannot provide an adequate smooth velocity for imaging primaries, even when multiples have been effectively removed. Providing an adequate smooth velocity for imaging diving waves (with state-of-the-art RTM) going down and under salt remains a tough and daunting problem. For primaries and multiples in your data, as in Figure 1, wave-equation migration will require an accurate, discontinuous migration velocity with reflectors in the overburden for predicting a source and receiver experiment at depth. Determining an accurate discontinuous velocity model is not a realistic assumption, not now, and not for anytime in the foreseeable future.

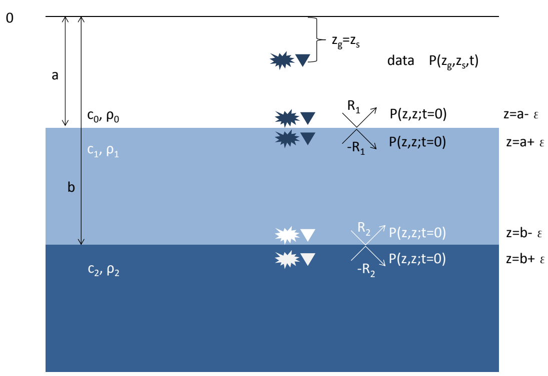

Fig. 1: *Imaging with primaries and internal multiples. A double Green's theorem is utilized with the data, and a Green's function that along with its normal derivative vanishes on the lower surface (and on the walls, in multi-D). That is what wave-equation migration means for waves that are two-way propagating in the medium.*

A migration velocity can be continuous or discontinuous. A smooth/continuous velocity model will output a false image for every free surface or internal multiple event. An accurate discontinuous velocity model will accurately WEM image data with primaries and all multiples, without false events (*e.g.*, Figure 1). Accurate discontinuous velocity models are not achievable in practice. Hence, in practice, multiples need to be removed, and are noise. The inclusive view for enhanced image illumination typically calls on a first step where multiples are separated from primaries, that is, on a highly effective result within the exclusive view. The serious and daunting open issues in predicting multiples today are too often ignored by those pursuing the inclusive view for seismic illumination.

Wave-equation migration imaging with primaries and free surface and/or internal multiples requires an accurate, discontinuous velocity model (to achieve any imaging benefit and objective). Among those considering internal multiples to enhance illumination are: Berkhout and Verschuur (1994, 1997, 2012), Soni et al. (2012), Davydenko and Verschuur (2013a,b), Fleury and Snieder (2011, 2012), and Wang et al. (2013, 2014).

Conclusion: Multiples contain information. Does containing information qualify multiples as signal?

Yes, multiples contain information, but that's not the point. The point is they contain too much information—containing information doesn't classify an event as signal; being able to reliably extract information from an event defines an event as signal. Multiples were and remain noise. Interest in illumination needs to start by selecting wave-equation migration and avoiding asymptotic migration; that selection of migration algorithms needs to come before considering placing different and additional events (*e.g.*, multiples), repeatedly and iteratively, into various forms of illumination-challenged migration (equation (1)).

The reason we separate primaries from multiples in exploration seismology is not due to lack of theory. The basic theory is almost 200 years old. It is due to the inability, in practice, to provide an adequate discontinuous velocity model necessary for the inclusive holistic and "all hold hands" whole-earth view. We need to be cognizant of that reality and to stay focused on delivering the next level of multiple-removal capability without requiring subsurface information. In general, we advocate a path that could require more data collected rather than detailed and accurate discontinuous subsurface information. The former is realizable and in general the latter is not.

The evolution and development of ever more effective multiple removal methods (stacking, FK and Radon filtering, Feedback and then ISS) represent an overall reduced/eliminated dependence on subsurface information. Methods that require more detailed subsurface information to be effective are pointed in exactly the wrong direction: technically and historically.

The recent interest in using multiples to enhance illumination has shown some promise for shallow reflectors, and needs to be placed on a firm footing, and encouraged and pursued. However, it cannot be used as a distraction from the main, central and overriding high priority objective to fill the gap between the pressing challenges in removing multiples and our current collective industry capability.

Below please find a link with references/documents that relate to this communication. http://mosrp.uh.edu/events/event-news/seg-annual-meeting-2013-2014

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