Inverse scattering internal multiple elimination: Leading-order and higher-order closed forms

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SUMMARY

Internal multiples are multiply reflected events in the measured wavefield that have experienced all of their downward reflections below the free surface. The order of an internal multiple is defined to be the number of downward reflections it experiences, without reference to the location of the downward reflection. The objective of internal multiple elimination using only recorded data and information about the reference medium is achievable directly through the inverse scattering task specific subseries formalism. The first term in the inverse scattering subseries for first-order internal multiple elimination is an attenuator, which predicts the correct traveltime and an amplitude always less than the true internal multiples' amplitude. The leading and higher-order terms in the elimination series correct the amplitude predicted by the attenuator moving the algorithm towards an eliminator. Leading-order as an eliminator means it eliminates a class of internal multiples and further attenuates the rest. Adding the leading-order terms in a closed form provides an algorithm that eliminates all internal multiples generated at the shallowest reflector. The generating reflector is the location where the downward reflection of a given first-order internal multiple took place. The higher-order subseries and its closed form correct the attenuation due to information on the overburden of deeper generating reflectors. A prestack form of the algorithm, which can be extended to a multidimensional form, is given for the leading-order subseries and its closed form.

INTRODUCTION

Seismic exploration is an inverse problem. The seismic data are inverted for the properties of the medium that created them. In exploration seismology, the medium properties correspond to the characteristics of the Earth's subsurface, and include the spatial location of the reflectors as well as the density and elastic properties of the layers between reflectors.

Events in recorded seismic data can be classified by the number of reflections they have experienced. Primaries are seismic events that have experienced one reflection, most often upward; whereas, multiples are seismic recorded events that have experienced more than one reflection. Multiples are further classified by the spatial location of the downward reflections within its history. A multiple that has at least one downward reflection at the free surface is a free surface multiple. A multiple that has all of its downward reflections below the free surface is an internal multiple. Source and/or receiver ghost events and direct waves are assumed to be removed before these definitions and classifications are applied.

In addition to the recorded data with the necessary wavefield components, some standard processing algorithms require source wavelet deconvolution, deghosting, seismic data reconstruction (interpolation and extrapolation), regularization and/or redatuming. Seismic data processing is usually accomplished in a sequence of steps, e.g., removal of multiples, depth imaging or migration, and inversion for changes in Earth properties. The standard practice is to perform these steps in a specific order because each step is a pre-processing condition for the next procedure. The removal of multiples is a longstanding problem, of considerable moment and interest, with outstanding theoretical and practical issues yet to be understood and addressed. The practical challenges to the removal of multiples usually intensify with deep water and highly complex, 2D or 3D, rapidly varying heterogeneous media, where medium properties and the reflectors that generate the multiples are difficult to adequately predict.

The inverse scattering series promises to directly address all seismic processing objectives with distinct algorithms that input only recorded data, and does not require in principle or practice any subsurface information whatsoever. Before the work reported here, the captured potential within the inverse scattering series was confined to the removal of free surface multiples and the reduction, but not removal, of internal multiples. This paper provides an additional capture of ability within the inverse series to move further towards matching its promise. This paper also provides algorithms that will further attenuate all internal multiples, and will eliminate a specific subset of internal multiples.

There are cases of high exploration priority where this further capture can have an impact on interpretation and subsequent drilling decisions. There are circumstances when internal multiple identification or attenuation are sufficient, and other situations when a residual left from internal multiple attenuation is a challenge and impediment to effective and reliable prediction. Among the latter circumstances are converted-wave internal multiples on towed-streamer data, and subtle plays in the subsalt where dim target primaries can interfere with weak proximal internal multiples.

Internal multiple elimination places greater demands on preprocessing steps such as wavelet estimation. However, methods based on inverse scattering *never* move from not needing to needing subsurface information when progressing from attenuating to eliminating internal multiples. The last comment further separates the inverse scattering multiple removal capability from the feedback loop internal multiple concept.

INTERNAL MULTIPLE ELIMINATION

The third term in the inverse scattering series: $(G_0V_1G_0V_1G_0V_1G_0)$ contains the leading-order contribution for the removal series of first-order internal multiples (Weglein et al., 2003). This leading-order term is the internal multiple attenuator. Assuming that the actual medium varies only in depth, the $1D$ Earth and normal incidence wave version (Araújo, 1994; Weglein et al., 1997) of the first-order internal multiple attenuator is

$$b_1(k) = D(\omega), \tag{1}$$

$$b_3^{IM_1}(k) = \int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_1(z_1) \times \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} b_1(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} b_1(z_3), \tag{2}$$

where $k = 2\frac{\omega}{c_0}$ is the vertical wavenumber, $D(\omega)$ is the temporal Fourier transforms of the measured scattered field (data), ε is a small positive parameter chosen to insure that the relations $z_1 > z_2$ and $z_3 > z_2$ are satisfied, the pseudodepths z_1 and z_2 are defined with the reference velocity c_0 to be $z_i = \frac{c_0 t_i}{2}$, and the superscript IM_1 refers to the $1st$ order internal multiple elimination series.

The attenuation algorithm prediction is performed by a nonlinear combination of three sets of data. This nonlinear combination predicts the traveltime of the true internal multiple in the data. The amplitude prediction is an estimate of the true internal multiple's amplitude. The estimate is always less than the actual amplitude given by an attenuation factor (AF) of (Ramírez and Weglein, 2005),

$$(AF)_j = \begin{cases} T_{01}T_{10} & \text{for } j=1 \\ \Pi_{i=1}^{j-1}\left(T_{i\ i-1}^2 T_{i-1\ i}^2\right) T_{j\ j-1}T_{j-1\ j} & \text{for } 1<j<J \end{cases} \tag{3}$$

where j represents the interface where the downward reflection took place, $T_{j-1\ j}$ and $T_{j\ j-1}$ are the transmission coefficients going down and up through the interface j, respectively, and J is the total number of interfaces in the model. The interfaces are numbered with integers, starting with the shallowest location. In a single-layer medium, the first-order internal multiple has an amplitude of $-T_{01}R_2R_1R_2T_{10}$ and $b_3^{IM_1}$ predicts a first-order internal multiple with an amplitude of $T_{01}T_{10}R_2R_1R_2T_{10}T_{01}$. In agreement with equation 3, the attenuation factor of the predicted internal multiple is $T_{01}T_{10}$. The attenuation factor is affected by the history of the event down to and including only the depth of the shallowest reflection, independent of the place where the two upward reflections occurred.

The terms in the elimination series use the data to predict multiples and remove them from the data itself. This removal is highly accurate when the prerequisites of the algorithm are satisfied (wavelet deconvolution, deghosting and free surface multiple elimination). The first-order internal multiple elimination series starts with $b_3^{IM_1}$. Because $b_3^{IM_1}$ has estimated the internal multiple amplitude attenuated by a factor of $(AF)_j$, the purpose of the higher-order terms in the elimination series is to remove the effect of this factor. The higher-order terms improve the effectiveness of the attenuator, towards the objective of completely subtracting the amplitude of multiples within the data. To achieve an elimination method, the inverse scattering subseries for internal multiples elimination should be able to predict the true amplitude for these events by correcting the attenuation factor in equation 3.

In the attenuator's prediction, the factor that multiplies the internal multiples generated at the first reflector*, $(IM)_{j=1}$, is $T_{01}T_{10}$. This attenuation factor corresponds to the first term in the Taylor expansion of $(T_{01}T_{10})/(T_{01}T_{10}) = 1$,

$$\begin{aligned} T_{01}T_{10}\left(\frac{1}{T_{01}T_{10}}\right) &= T_{01}T_{10}\frac{1}{(1-R_1^2)} \\ &= T_{01}T_{10}\left(1+R_1^2+R_1^4+R_1^6+R_1^8\cdots\right) \quad . \end{aligned} \tag{4}$$

In the attenuator's prediction, the factor $(T_{01}T_{10})^2T_{12}T_{21}$ that multiplies the internal multiples generated at the second reflector, $(IM)_{j=2}$, corresponds to the first term in the more complicated geometric series for:

$$\begin{aligned} \frac{(T_{01}T_{10})^2T_{12}T_{21}}{(T_{01}T_{10})^2T_{12}T_{21}} &= (T_{01}T_{10})^2T_{12}T_{21}\frac{1}{(1-R_1^2)^2(1-R_2^2)}, \\ &= (T_{01}T_{10})^2T_{12}T_{21}\left(1+2R_1^2+R_2^2+3R_1^4+2R_2^2R_1^2+\cdots\right). \end{aligned} \tag{5}$$

Each one of the terms in these Taylor expansions, equations 4 and 5, can be and are calculated by higher-order terms in the inverse scattering internal multiple elimination series. Identifying and adding these higher-order terms builds a sum of amplitude corrections that improves the subtraction of internal multiples from the data. The higher-order amplitude corrections are given by algorithms, found in the internal multiple elimination series $b_3^{IM_1}+b_5^{IM_1}+b_7^{IM_1}+\cdots$ (Ramírez and Weglein, 2005), that only required measured values of the scattered field and the reference Green's function.

The second term in the elimination series, ${b_5}^{IM_1}$, resides within the fifth term in the inverse series. It is the first step to move the attenuation algorithm towards an elimination of first-order internal multiples, and it is given by

$$\begin{aligned} b_5^{IM_1}(k) = &\int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \\ &\times \int_{-\infty}^{z-\varepsilon} dz' e^{-ikz'}\left[b_1(z')^3 + 2\,b_1(z')\int_{-\infty}^{z'-\varepsilon} dz'''\,b_1(z''')^2\right] \\ &\times \int_{z'+\varepsilon}^{\infty} dz'' e^{ikz''} b_1(z''). \end{aligned} \tag{6}$$

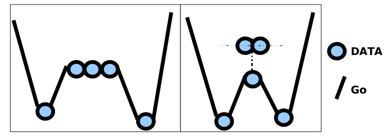

Figure 1: Diagrams for $b_{51}^{IM_1}$ (left) and $b_{52}^{IM_1}$ (right).

The second term in the first-order internal multiple elimination series can be separated in two parts, and represented with the diagrams in Figure(1),

$$\begin{aligned} b_{51}^{IM_1}(k) &= \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \\ &\times \int_{-\infty}^{z-\varepsilon} dz' e^{-ikz'} b_1(z')^3 \int_{z'+\varepsilon}^{\infty} dz'' e^{ikz''} b_1(z''), \end{aligned} \tag{7}$$

$$\begin{aligned} b_{52}^{IM_1}(k) &= \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \\ &\times \int_{-\infty}^{z-\varepsilon} dz' e^{-ikz'}\, 2\, b_1(z') \int_{-\infty}^{z'-\varepsilon} dz'''\, b_1(z''')^2 \int_{z'+\varepsilon}^{\infty} dz'' e^{ikz''} b_1(z''). \end{aligned} \tag{8}$$

The diagram located on the left side of Figure1 corresponds to equation 7 and it belongs to a series that eliminates all first-order internal multiples that were downward reflected at the shallowest reflector. This term combines nonlinearly five sets of data to give amplitude information and the correct traveltime of the internal multiples. The three hits in the diagram indicate triple self interaction of data at the generating reflector. Hence, the extra amplitude information given by the self-interacting data corresponds to powers of the reflection coefficient of each generating reflector, which is in agreement with the analysis in equations 4 and 5. The analysis of the properties of this term, using its diagram representation and numerical examples, showed that it is the main contribution of $b_5^{IM_1}$ to the elimination of internal multiples (Ramírez Pérez, 2007). Its mathematical representation resembles the one of the attenuator, which is the leading-order term of the series by itself. We can find the leading-order terms by examining each term in the internal multiple elimination series and selecting the ones that only have data self-interactions at the generating reflector. The sum of the leading-order terms in the series is

$$\begin{aligned} b_{LO}^{IM_1} &= \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \\ &\times \int_{-\infty}^{z-\varepsilon} dz' e^{-ikz'}\left(\frac{1}{1-b_1(z')^2}\right) b_1(z') \int_{z'+\varepsilon}^{\infty} dz'' e^{ikz''} b_1(z''). \end{aligned} \tag{9}$$

This equation is the infinite sum of the leading-order terms in the inverse scattering internal multiple elimination series. The leading-order eliminator, $b_{LO}^{IM_1}$, eliminates all first-order internal multiples generated at the shallowest reflector without requiring a-priori information, nor a velocity model. It is all done in terms of the effective data, b_1, and the reference velocity contained in $k=\frac{2\omega}{c_0}$. Furthermore, the leading-order eliminator helps to better attenuate all the internal multiples generated at deeper reflectors.

*We define the generating reflector of a first-order internal multiple as the reflector where the downward reflection took place.

The diagram located on the right of Figure 1 represents equation 8, containing $I = 2b_1(z')\int_{-\infty}^{z'-\varepsilon} dz'''\ b_1(z''')^2$ in the middle integral. The term I, represented by the middle part of the diagram, has two self-interacting data within the overburden of the generating reflector. This double self interaction provides the series with second order corrections for any interface above the generating reflector, and it only acts on internal multiples downward reflected at interfaces below the shallowest reflector. The internal multiples generated at the shallowest reflector are completely eliminated with the leading-order closed form term in equation 9. The double self-interacting diagram further attenuates all first-order internal multiples generated at deeper reflectors[†]. The main part of these second subseries can be summed in a higher-order closed form term,

$$b_{HO}^{IM_1} = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z)$$

$$\int_{-\infty}^{z-\varepsilon} dz' e^{-ikz'} \frac{2J(z')\int_{-\infty}^{z'-\varepsilon} dz''' J(z''') b_1(z''')}{1-\int_{-\infty}^{z'} dz''' J(z''') b_1(z''')} \int_{z'+\varepsilon}^{\infty} dz'' e^{ikz''} b_1(z''), \tag{10}$$

$$\text{where } J(z) = \frac{b_1(z)}{1-b_1(z)^2}. \tag{11}$$

The higher-order eliminator assumes that the action of the leading-order eliminator has taken effect prior to its calculation. Because the leading-order closed form eliminates all multiples generated at the first reflector, the only problem task left, in terms of internal multiples, is to finish correcting the amplitude of the deeper internal multiples and eliminate them. This is the task performed by the higher-order eliminator, $b_{HO}^{IM_1}$.

Equation 10 is the infinite sum of the main terms in the higher-order subseries of the internal multiple elimination series. The higher-order eliminator includes diagrams that have extra data self-interactions above the generating reflector. The reason it is not including all the higher-order terms is because, these terms in the inverse series for internal multiple elimination have different integer weights, which means that a specific higher-order diagram is required to act more than once in the removal process. From the form of equation 10, the closed form only contains a weighting factor of 2 (please refer to the middle integral) in agreement to the weighting factor needed by equation 8. The first term included in the higher-order closed form corresponds to equation 8.

An elimination algorithm for internal multiples based on inverse scattering series has the potential of removing difficult internal multiples, leaving all primaries unaffected. Although the internal multiple amplitudes are reduced by the attenuator, $b_3^{IM_1}$, and substantially reduced (and a subset is eliminated) by the leading-order closed form, $b_{LO}^{IM_1}$, there is, in some cases, an observable residual that can be further attenuated with the action of the higher-order closed form, $b_{HO}^{IM_1}$. The higher-order closed form term of the internal multiple elimination series complements the elimination of the amplitude of the remaining internal multiples by adding nonlinear contributions in terms of data and a reference Green's function. The combination of the leading-order closed form with the higher-order closed form term gives an improved algorithm for the removal of internal multiples.

2D EXTENSION OF THE ALGORITHM

In the theory presented in the previous section, no assumptions about the Earth below the receivers are made, this characteristic makes it ideal for addressing one of the current challenges in exploration seismology: removing multiples, locating and identifying targets in highly complex medium, when the velocity model is unobtainable. Hence, the extension to a multidimensional Earth model is a necessary step.

The attenuation algorithm for a $2D$ Earth model, presented in Araújo (1994); Weglein et al. (1997) and Weglein et al. (2003), is

$$b_1(k_g,k_s,q_g+q_s) = -2iq_s D(k_g,k_s,\omega), \tag{12}$$

$$\begin{aligned} b_3^{IM_1}(k_g,k_s,q_g+q_s) = {} & \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)}\int_{-\infty}^{\infty} dk_2 e^{iq_2(\varepsilon_g-\varepsilon_s)} \\ & \times \int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\ & \times \int_{-\infty}^{z_1-\varepsilon_2} dz_2 e^{i(-q_1-q_2)z_2} b_1(k_1,-k_2,z_2) \\ & \times \int_{z_2+\varepsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3), \end{aligned} \tag{13}$$

where ω represents the temporal frequency, c_0 is the acoustic velocity of water; k_g and k_s are the horizontal wavenumbers corresponding to receiver and source coordinates: x_g and x_s, respectively; the $2D$ wave vectors: $\mathbf{k}_g = (k_g,-q_g)$ and $\mathbf{k}_s = (k_s,q_s)$ are constrained by $|\mathbf{k}_g| = |\mathbf{k}_s| = \frac{\omega}{c_0}$; the vertical wavenumbers are $q_g = sgn(\omega)\sqrt{(\frac{\omega}{c_0})^2 - {k_g}^2}$ and $q_s = sgn(\omega)\sqrt{(\frac{\omega}{c_0})^2 - {k_s}^2}$, and ε_i is a small positive parameter chosen to insure that the relations $z_1 > z_2$ and $z_3 > z_2$ are satisfied.

In equations 12 and 13, the effective data $b_1(k_g,k_s,q_g+q_s)$ is defined as a source obliquity factor times the $2D$ measured values of the scattered field, D. The variable z is the Fourier conjugate to the sum of the vertical wave numbers, $k_z = -(q_g+q_s)$. The attenuation of multiples is performed by adding the attenuator, $b_3^{IM_1}$, to the effective data, b_1.

As we showed in $1D$, the second term in the first-order internal multiple elimination series can be separated in two equations. The 2D form of the first equation is

$$\begin{aligned} b_{51}^{IM_1}(k_g,k_s,q_g+q_s) = {} & \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)}\int_{-\infty}^{\infty} dk_2 e^{iq_2(\varepsilon_g-\varepsilon_s)} \\ & \times \int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\ & \times \int_{-\infty}^{z_1-\varepsilon_2} dz_2 e^{i(-q_1-q_2)z_2} \left[b_1(k_1,-k_2,z_2)\right]^3 \\ & \times \int_{z_2+\varepsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3), \end{aligned} \tag{14}$$

which have the same diagrammatic representation as shown in Figure 1. Studying the higher-order terms in the inverse scattering internal multiple elimination series in a multidimensional model type independent form, we find that the form of the terms with self-interacting data at the generating reflector conserves the properties and characteristics found in the simple $1D$ case. Analogous to the $1D$ case, the first term in the leading-order elimination series is the attenuator, equation 17, and the second term is given by equation 14. The next terms in the leading-order series have the form:

$$\begin{aligned} b_{51}^{IM_1}(k_g,k_s,q_g+q_s) = {} & \sum_{N=0}^{\infty}\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)}\int_{-\infty}^{\infty} dk_2 e^{iq_2(\varepsilon_g-\varepsilon_s)} \\ & \times \int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\ & \times \int_{-\infty}^{z_1-\varepsilon_2} dz_2 e^{i(-q_1-q_2)z_2} \left[b_1(k_1,-k_2,z_2)\right]^{2N+1} \\ & \times \int_{z_2+\varepsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3), \end{aligned} \tag{15}$$

We can add the leading-order terms in the multidimensional case to a

[†] Where deeper refers to all reflectors located below the shallowest one.

closed form, which is given by,

$$b_1(k_g,k_s,q_g+q_s) = -2iq_s D(k_g,k_s,\omega), \tag{16}$$

$$\begin{aligned} b_3(k_g,k_s,q_g+q_s) = &\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)} \int_{-\infty}^{\infty} dk_2 e^{iq_2(\varepsilon_g-\varepsilon_s)} \\ &\times \int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\ &\times \int_{-\infty}^{z_1-\varepsilon_2} dz_2 e^{i(-q_1-q_2)z_2} \frac{b_1(k_1,-k_2,z_2)}{1-b_1(k_1,-k_2,z_2)^2} \\ &\times \int_{z_2+\varepsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3), \end{aligned} \tag{17}$$

This is a $2D$ model type independent leading-order elimination algorithm for internal multiples. The leading-order eliminator is a data-driven algorithm written in terms of effective data b_1 (see equation 16). The leading-order closed form, $b_{LO}^{IM_1}$, gives the main contribution towards eliminating internal multiples. It completely removes all first-order internal multiples generated at the first reflector and improves the attenuation of the remaining multiples. 'Leading-order' as an eliminator means it eliminates a class of internal multiples and further attenuates the rest. In a $2D$ medium, the multiples that have no cumulative transmission error (the ones with downward reflection at the shallowest reflector) are eliminated by the algorithm in equation 17, $b_1 + b_{LO}^{IM_1}$. The higher-order closed form is being examined for a $2D$ extension. It is not always possible to generalize a $1D$ closed form to $2D$; an algorithm in $2D$ has more variables and different dependencies than the same algorithm in $1D$. However, we are studying the $2D$ expressions for the higher-order terms in the elimination series. For a multidimensional world, the leading-order eliminator provides the removal of all first-order internal multiples generated at the first reflector and effectively attenuates the rest of the multiples.

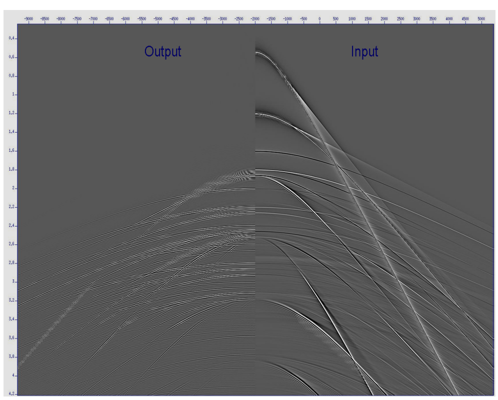

Figure 2: Left: Predicted internal multiples. Right: Data with primaries and internal multiples. The data and example were generated during an internship at ConocoPhillips, 2006.

A $1.5D$ numerical example of the internal multiple prediction with b_{LO}^{IM} in a half space of water and a horizontally layered elastic medium representing the Earth, is shown in figure 2. The finite difference synthetic data, on the right of this figure, contains primaries and internal multiples due to an elastic halfspace. The traces on the left show the predicted internal multiples. Notice that all multiples were predicted with their correct traveltime. The data were deconvolved with a statistical estimate of the wavelet. The wavelet used to model the data was not used in the prediction; hence, the predicted multiples have a different wavelet. The fact that the internal multiple elimination algorithm with an acoustic background, predicts internal multiples propagated in an elastic Earth is a remarkable effect of the model-type independence of the algorithm. The eliminator algorithm predicts all internal multiples in a model-type independent fashion. However, in situations when the background model-type does not correspond to the actual model-type where the internal multiples were created, the leading-order eliminator will not completely eliminate all the internal multiples generated at the first reflector. For example, in a towed-streamer marine acquisition, where the background is acoustic (water), the leading-order eliminator will attenuate all internal multiples better than the attenuator, and it will only eliminate those multiples that have a complete propagation path as pressure waves. The converted-wave internal multiples will be predicted with their correct arrival time and an attenuated amplitude. It is worth noting that the elimination algorithm is not at all more expensive than the attenuator. However, the sensitivity of the inverse scattering leading-order eliminator for input wavelet is expected to be higher. In particular, an accurate estimation of the source wavelet will be needed to perform the division in the innermost integral. It will also allow convergence of the leading-order closed form.

CONCLUSIONS

In many circumstances the first-order term in the inverse scattering internal multiple series, known as the attenuator, provides an effective solution. It predicts the correct arrival time and attenuates the amplitude of the internal multiples in the data. However, there are situations for towed-streamer pressure measurements where either the residual can be far from small (*e.g.*, converted-wave internal multiples) or where a small residual interferes with a target primary, and the latter is itself small. In these cases, the attenuation is not enough and we need to seek for algorithms that provide an elimination of these events in the data.

This work shows progress in the identification, analysis and mathematical manipulation of higher-order terms in the series for internal multiple elimination, where the first term is an attenuator. Two closed forms were obtained by adding subsets of the infinite series for internal multiple elimination: the leading-order and the higher-order closed forms. Leading-order as an eliminator means it eliminates a class of internal multiples and further attenuates the rest. The higher order algorithm provides a better estimate of the amplitudes, and represents an improvement towards the elimination of internal multiples. In this theory, no assumptions about the Earth below the receivers are made.

The internal multiple elimination algorithms do not require any knowledge of the subsurface properties, neither the distinction between internal multiples, nor the knowledge of the location where the downward reflection took place. The internal multiple algorithms are non-linear data-driven algorithms that only require a reference Green's function and the measured data. The extension to a multidimensional Earth was achieved for the leading-order algorithm. The leading order eliminator effectively attenuates all orders of internal multiples in the data, generated at any reflector below the measurement surface. The leading-order eliminator provides the removal of all first-order internal multiples generated at the shallowest reflector when the background model-type (*i.e.*, acoustic or elastic) agrees with the actual model-type. In situations when the background model-type is different to the actual it represents an improvement upon current internal multiple attenuation technology.

The extension to a multidimensional Earth of the higher-order terms, as well as extensions of definitions, is our current subject of study.

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EDITED REFERENCES

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