M-OSRP
Mission - Oriented Seismic Research Program

New Concepts and Methods to Enhance Time Lapse Capability for Reservoir Monitoring

ADNOC/UH Meeting
Thursday, May 1st, 2014

Arthur B. Weglein

M-OSRP/Physics/UH

This project relates to ISS capability for time lapse seismic , where the assumption going in is that multiples have been effectively removed. ISS applied in a time lapse test at ConocoPhillips was able to distinguish a fluid change from a pressure change, where other methods, that were tested, could not. This is a new fundamental research result in an embryonic stage of development.

Objectives of Seismic Exploration

- **Where is the location of the medium changes?**

  (Imaging or Migration)

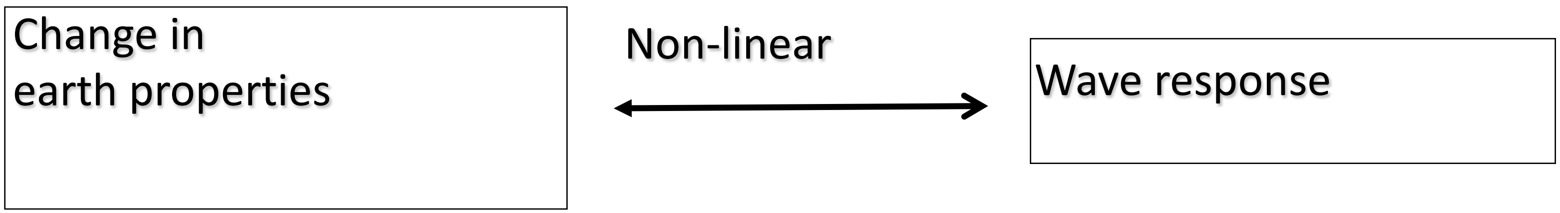
- **What are the medium changes? (Inversion or Target identification)**

Direct non-linear inversion of multi-parameter 1D elastic media using the inverse scattering series

Haiyan Zhang and Arthur B. Weglein

M-OSRP Annual Meeting
University of Houston
May 10 –12, 2006

Background and motivation



- **Current Inversion methods:**
  - **Linear inversion or Born approximation**
  - **Model matching**

- **Direct non-linear inversion method**
  - ***Towards fundamentally new comprehensive and realistic target identification.***

Derivation of the inverse series

In actual medium: $$LG = \delta$$

In reference medium: $$L_0 G_0 = \delta$$

Perturbation: $$V = L_0 - L$$

L-S equation: $$G = G_0 + G_0 V G$$

Forward scattering Series:

$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots$$

Inverse scattering series:

$$V = V_1 + V_2 + \cdots$$

$$D = (G - G_0)_{ms} = \left[G_0 V G_0 + G_0 V G_0 V G_0 + \cdots \right]_{ms}$$

Inverse Scattering Series

$$D = [G_0 V_1 G_0]_{ms}$$

Linear

$$0 = [G_0 V_2 G_0]_{ms} + [G_0 V_1 G_0 V_1 G_0]_{ms}$$

Non-linear

$$0 = [G_0 V_3 G_0]_{ms} + [G_0 V_2 G_0 V_1 G_0]_{ms} + [G_0 V_1 G_0 V_2 G_0]_{ms} + [G_0 V_1 G_0 V_1 G_0 V_1 G_0]_{ms}$$

$$\vdots \qquad \vdots$$

Elastic inversion: linear

$$\hat{D} = \hat{G}_0 \hat{V}_1 \hat{G}_0$$

$$\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix} = \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}$$

$$\hat{D}^{PP} = \hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P; \quad \hat{D}^{PS} = \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S;$$

$$\hat{D}^{SP} = \hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P; \quad \hat{D}^{SS} = \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S$$

Elastic inversion: linear

Then, in $(k_s, z_s; k_g, z_g; \omega)$ domain, we get $(z_s = z_g = 0)$

$$\widetilde{D}^{PP}(k_g,\nu_g) = -\frac{1}{4}\left(1-\frac{k_g^2}{\nu_g^2}\right)\widetilde{a}_{\rho}^{(1)}(-2\nu_g) - \frac{1}{4}\left(1+\frac{k_g^2}{\nu_g^2}\right)\widetilde{a}_{\gamma}^{(1)}(-2\nu_g) + \frac{2\beta_0^2}{\alpha_0^2}\cdot\frac{k_g^2}{\left(k_g^2+\nu_g^2\right)}\widetilde{a}_{\mu}^{(1)}(-2\nu_g).$$

$$\widetilde{D}^{PS}(\nu_g,\eta_g) = -\frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)\widetilde{a}_{\rho}^{(1)}(-\nu_g-\eta_g) - \frac{\beta_0^2}{2\omega^2}\cdot k_g\left(\nu_g+\eta_g\right)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)\widetilde{a}_{\mu}^{(1)}(-\nu_g-\eta_g)$$

$$\widetilde{D}^{SP}(\nu_g,\eta_g) = \frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)\widetilde{a}_{\rho}^{(1)}(-\nu_g-\eta_g) + \frac{\beta_0^2}{2\omega^2}\cdot k_g\left(\nu_g+\eta_g\right)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)\widetilde{a}_{\mu}^{(1)}(-\nu_g-\eta_g)$$

$$\widetilde{D}^{SS}\left(k_g,\eta_g\right) = -\frac{1}{4}\left(1-\frac{k_g^2}{\eta_g^2}\right)\widetilde{a}_{\rho}^{(1)}(-2\eta_g) - \left[\frac{k_g^2+\eta_g^2}{4\eta_g^2}-\frac{2k_g^2}{k_g^2+\eta_g^2}\right]\widetilde{a}_{\mu}^{(1)}(-2\eta_g).$$

Elastic inversion: non-linear

$$\hat{G}_0 \hat{V}_2 \hat{G}_0 = -\hat{G}_0 \hat{V}_1 \hat{G}_0 \hat{V}_1 \hat{G}_0$$

$$\begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_2^{PP} & \hat{V}_2^{PS} \\ \hat{V}_2^{SP} & \hat{V}_2^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} = -\begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}$$

$$G_0^P \hat{V}_2^{PP} G_0^P = -\hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P - \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P$$

$$G_0^P \hat{V}_2^{PS} G_0^S = -\hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S - \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S$$

$$G_0^S \hat{V}_2^{SP} G_0^P = -\hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P - \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P$$

$$G_0^S \hat{V}_2^{SS} G_0^S = -\hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S - \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S$$

Elastic inversion: linear

Then, in $\left(k_s, z_s; k_g, z_g; \omega\right)$ domain, we get $\left(z_s = z_g = 0\right)$

$$\widetilde{D}^{PP}(k_g,\nu_g) = -\frac{1}{4}\left(1-\frac{k_g^2}{\nu_g^2}\right)\widetilde{a}_\rho^{(1)}(-2\nu_g) - \frac{1}{4}\left(1+\frac{k_g^2}{\nu_g^2}\right)\widetilde{a}_\gamma^{(1)}(-2\nu_g) + \frac{2\beta_0^2}{\alpha_0^2}\cdot\frac{k_g^2}{\left(k_g^2+\nu_g^2\right)}\widetilde{a}_\mu^{(1)}(-2\nu_g).$$

$$\widetilde{D}^{PS}(\nu_g,\eta_g) = -\frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)\widetilde{a}_\rho^{(1)}(-\nu_g-\eta_g) - \frac{\beta_0^2}{2\omega^2}\cdot k_g\left(\nu_g+\eta_g\right)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)\widetilde{a}_\mu^{(1)}(-\nu_g-\eta_g)$$

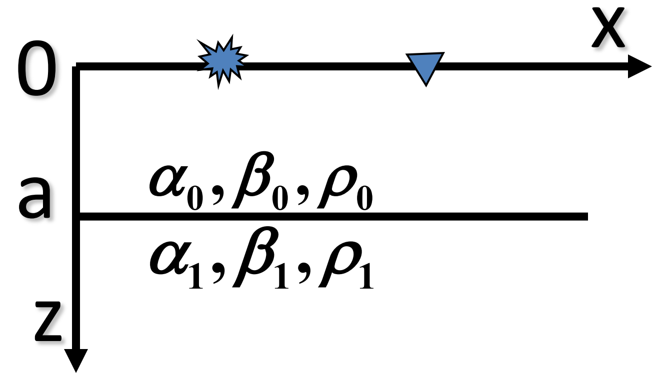
$$\widetilde{D}^{SP}(\nu_g,\eta_g) = \frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)\widetilde{a}_\rho^{(1)}(-\nu_g-\eta_g) + \frac{\beta_0^2}{2\omega^2}\cdot k_g\left(\nu_g+\eta_g\right)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)\widetilde{a}_\mu^{(1)}(-\nu_g-\eta_g)$$

$$\widetilde{D}^{SS}\left(k_g,\eta_g\right) = -\frac{1}{4}\left(1-\frac{k_g^2}{\eta_g^2}\right)\widetilde{a}_\rho^{(1)}(-2\eta_g) - \left[\frac{k_g^2+\eta_g^2}{4\eta_g^2} - \frac{2k_g^2}{k_g^2+\eta_g^2}\right]\widetilde{a}_\mu^{(1)}(-2\eta_g).$$

Numerical tests: PP data only

One interface,

$$z_s = z_g = 0$$



Model 1: shale (0.2 porosity) over oil sand (0.1 porosity)

$$\alpha_0 = 2627\, m/s, \alpha_1 = 4423\, m/s, \beta_0 = 1245\, m/s, \beta_1 = 2939\, m/s, \rho_0 = 2.32\, g/cm^3, \rho_1 = 2.46\, g/cm^3$$

Model 2: shale (0.2 porosity) over oil sand (0.2 porosity)

$$\alpha_0 = 2627\, m/s, \alpha_1 = 3251\, m/s, \beta_0 = 1245\, m/s, \beta_1 = 2138\, m/s, \rho_0 = 2.32\, g/cm^3, \rho_1 = 2.27\, g/cm^3$$

Model 3: shale (0.2 porosity) over oil sand (0.3 porosity)

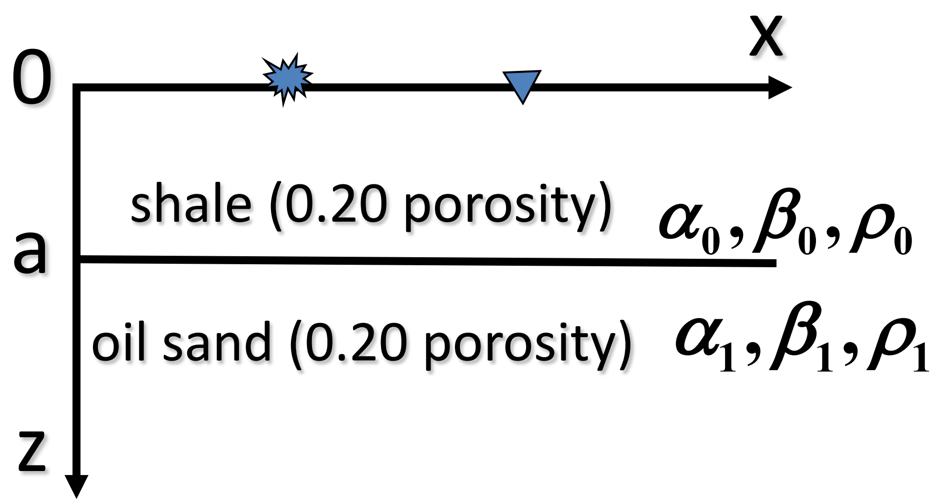
$$\alpha_0 = 2627\, m/s, \alpha_1 = 2330\, m/s, \beta_0 = 1245\, m/s, \beta_1 = 1488\, m/s, \rho_0 = 2.32\, g/cm^3, \rho_1 = 2.08\, g/cm^3$$

Model 4: oil sand (0.2 porosity) over wet sand (0.2 porosity)

$$\alpha_0 = 3251\, m/s, \alpha_1 = 3507\, m/s, \beta_0 = 2138\, m/s, \beta_1 = 2116\, m/s, \rho_0 = 2.27\, g/cm^3, \rho_1 = 2.32\, g/cm^3$$

Numerical tests: PP data only

Model 2



$$\alpha_0 = 2627\, m/s, \alpha_1 = 3251\, m/s,$$

$$\beta_0 = 1245\, m/s, \beta_1 = 2138\, m/s,$$

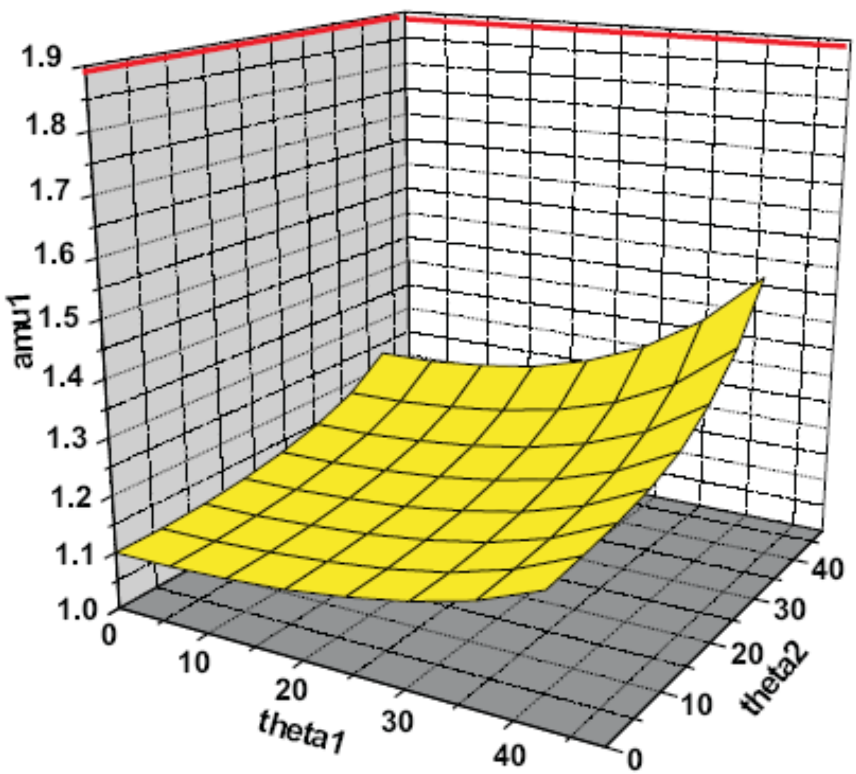
$$\rho_0 = 2.32\, g/cm^3, \rho_1 = 2.27\, g/cm^3$$

Inversion of Shear modulus

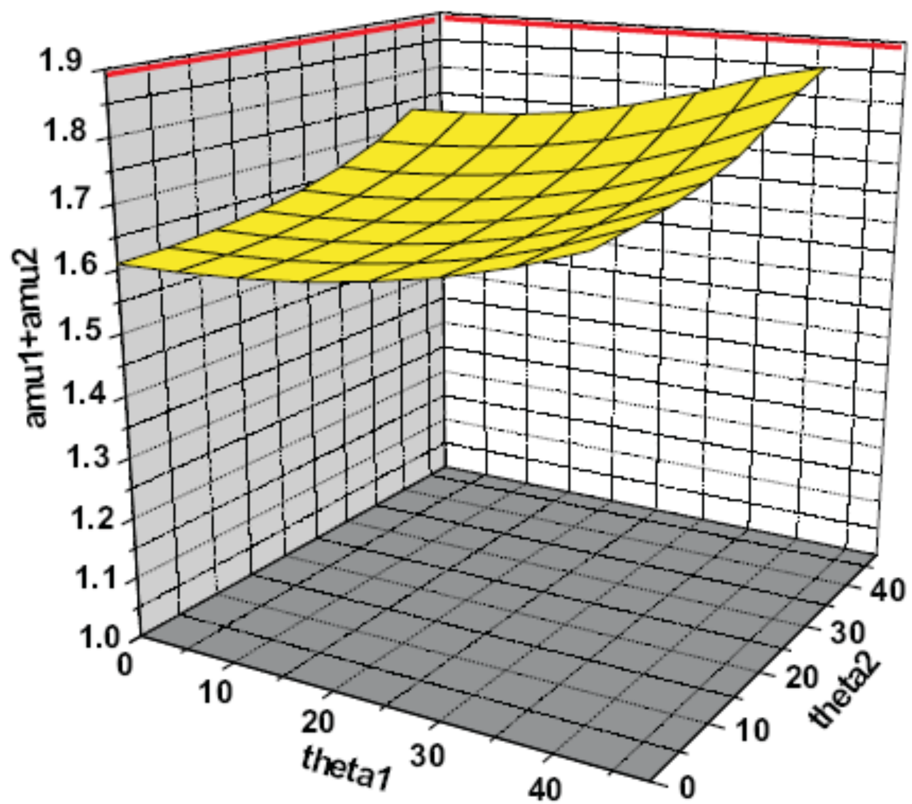
$$a_\mu = \Delta\mu/\mu_0 \quad \textbf{Exact value: 1.89}$$

$$\textbf{For very small angle}: a_\mu^{(1)} = 1.1,\ a_\mu^{(1)} + a_\mu^{(2)} = 1.61$$

Linear



Linear+nonlinear



Non-linear elastic inversion: all four components of data

Get the linear solution for $a_\rho^{(1)}, a_\gamma^{(1)}, a_\mu^{(1)}$ in terms of $\hat{D}^{PP}, \hat{D}^{PS}, \hat{D}^{SP}$ and $\hat{D}^{SS}$.

$$\begin{pmatrix} a_\rho^{(1)} \\ a_\gamma^{(1)} \\ a_\mu^{(1)} \end{pmatrix} = (O^T O)^{-1} O^T \begin{pmatrix} \hat{D}^{PP} \\ \hat{D}^{PS} \\ \hat{D}^{SP} \\ \hat{D}^{SS} \end{pmatrix},$$

where the matrix O is

$$\begin{pmatrix}
-\frac{1}{4}\left(1-\frac{k_g^{PP^2}}{\nu_g^{PP^2}}\right) & -\frac{1}{4}\left(1+\frac{k_g^{PP^2}}{\nu_g^{PP^2}}\right) & \frac{2\beta_0^2 k_g^{PP^2}}{\alpha_0^2\left(\nu_g^{PP^2}+k_g^{PP^2}\right)} \\
-\frac{1}{4}\left(\frac{k_g^{PS}}{\nu_g^{PS}}+\frac{k_g^{PS}}{\eta_g^{PS}}\right) & 0 & -\frac{\beta_0^2}{2\omega^2}k_g^{PS}\left(\nu_g^{PS}+\eta_g^{PS}\right)\left(1-\frac{k_g^{PS^2}}{\nu_g^{PS}\eta_g^{PS}}\right) \\
\frac{1}{4}\left(\frac{k_g^{SP}}{\nu_g^{SP}}+\frac{k_g^{SP}}{\eta_g^{SP}}\right) & 0 & \frac{\beta_0^2}{2\omega^2}k_g^{SP}\left(\nu_g^{SP}+\eta_g^{SP}\right)\left(1-\frac{k_g^{SP^2}}{\nu_g^{SP}\eta_g^{SP}}\right) \\
-\frac{1}{4}\left(1-\frac{k_g^{SS^2}}{\eta_g^{SS^2}}\right) & 0 & -\left[\frac{k_g^{SS^2}+\eta_g^{SS^2}}{4\eta_g^{SS^2}}-\frac{2k_g^{SS^2}}{k_g^{SS^2}+\eta_g^{SS^2}}\right]
\end{pmatrix},$$

Elastic inversion: linear

Then, in $(k_s, z_s; k_g, z_g; \omega)$ domain, we get $(z_s = z_g = 0)$

$$\widetilde{D}^{PP}(k_g,\nu_g) = -\frac{1}{4}\left(1-\frac{k_g^2}{\nu_g^2}\right)\widetilde{a}_\rho^{(1)}(-2\nu_g) - \frac{1}{4}\left(1+\frac{k_g^2}{\nu_g^2}\right)\widetilde{a}_\gamma^{(1)}(-2\nu_g) + \frac{2\beta_0^2}{\alpha_0^2}\cdot\frac{k_g^2}{\left(k_g^2+\nu_g^2\right)}\widetilde{a}_\mu^{(1)}(-2\nu_g).$$

$$\widetilde{D}^{PS}(\nu_g,\eta_g) = -\frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)\widetilde{a}_\rho^{(1)}(-\nu_g-\eta_g) - \frac{\beta_0^2}{2\omega^2}\cdot k_g\left(\nu_g+\eta_g\right)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)\widetilde{a}_\mu^{(1)}(-\nu_g-\eta_g)$$

$$\widetilde{D}^{SP}(\nu_g,\eta_g) = \frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)\widetilde{a}_\rho^{(1)}(-\nu_g-\eta_g) + \frac{\beta_0^2}{2\omega^2}\cdot k_g\left(\nu_g+\eta_g\right)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)\widetilde{a}_\mu^{(1)}(-\nu_g-\eta_g)$$

$$\widetilde{D}^{SS}\left(k_g,\eta_g\right) = -\frac{1}{4}\left(1-\frac{k_g^2}{\eta_g^2}\right)\widetilde{a}_\rho^{(1)}(-2\eta_g) - \left[\frac{k_g^2+\eta_g^2}{4\eta_g^2}-\frac{2k_g^2}{k_g^2+\eta_g^2}\right]\widetilde{a}_\mu^{(1)}(-2\eta_g).$$

Non-linear elastic inversion: all four components of data

Based on this idea, the solution for the first equation

$$G_0^P \hat{V}_2^{PP} G_0^P = -\hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P - \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P$$

is

$$\begin{aligned}
&\left(1-\tan^2\theta\right)a_\rho^{(2)}(z)+\left(1+\tan^2\theta\right)a_\gamma^{(2)}(z)-8\sin^2\theta b^2 a_\mu^{(2)}(z)=\\
&-\frac{1}{2}\left(\tan^4\theta-1\right)a_\gamma^{(1)}(z)a_\gamma^{(1)}(z)+\frac{\tan^2\theta}{\cos^2\theta}a_\gamma^{(1)}(z)a_\rho^{(1)}(z)+\frac{1}{2}\left[\left(1-\tan^4\theta\right)-\frac{2}{C+1}\left(\frac{1}{C}\right)\left(\frac{\alpha_0^2}{\beta_0^2}-1\right)\frac{\tan^2\theta}{\cos^2\theta}\right]a_\rho^{(1)}(z)a_\rho^{(1)}(z)\\
&-4\left[\tan^2\theta-\frac{2}{C+1}\left(\frac{1}{2C}\right)\left(\frac{\alpha_0^2}{\beta_0^2}-1\right)\tan^4\theta\right]b^2 a_\rho^{(1)}(z)a_\mu^{(1)}(z)+4\sin^2\theta\left(\tan^2\theta-\frac{\alpha_0^2}{\beta_0^2}\right)b^4 a_\mu^{(1)}(z)a_\mu^{(1)}(z)\\
&-\frac{2}{C+1}\frac{2}{C}\left(\frac{\alpha_0^2}{\beta_0^2}-1\right)\left(\tan^2\theta-\frac{\alpha_0^2}{\beta_0^2}\right)\tan^2\theta b^4 a_\mu^{(1)}(z)a_\mu^{(1)}(z)\\
&-\frac{1}{2}\left(\frac{1}{\cos^4\theta}\right)a_\gamma^{(1)'}\int_0^z dz'\left(a_\gamma^{(1)}-a_\rho^{(1)}\right)(z')-\frac{1}{2}\left(1-\tan^4\theta\right)a_\rho^{(1)'}\int_0^z dz'\left(a_\gamma^{(1)}-a_\rho^{(1)}\right)(z')+4b^2\tan^2\theta a_\mu^{(1)'}\int_0^z dz'\left(a_\gamma^{(1)}-a_\rho^{(1)}\right)(z')\\
&+\frac{2}{C+1}\frac{1}{C}\left(\frac{\alpha_0^2}{\beta_0^2}-1\right)\tan^2\theta\left(\tan^2\theta-C\right)b^2\int_0^z dz' a_\mu^{(1)'}\left(\frac{(C-1)z'+2z}{(C+1)}\right)a_\rho^{(1)}(z')H(z-z')\\
&-\frac{2}{C+1}\frac{2}{C}\left(\frac{\alpha_0^2}{\beta_0^2}-1\right)\tan^2\theta\left(\tan^2\theta-\frac{\alpha_0^2}{\beta_0^2}\right)b^4\int_0^z dz' a_\mu^{(1)'}\left(\frac{(C-1)z'+2z}{(C+1)}\right)a_\mu^{(1)}(z')H(z-z')\\
&+\frac{2}{C+1}\frac{1}{C}\left(\frac{\alpha_0^2}{\beta_0^2}-1\right)\tan^2\theta\left(\tan^2\theta+C\right)b^2\int_0^z dz' a_\mu^{(1)}(z')a_\rho^{(1)'}\left(\frac{(C-1)z'+2z}{(C+1)}\right)H(z-z')\\
&-\frac{2}{C+1}\frac{1}{2C}\left(\frac{\alpha_0^2}{\beta_0^2}-1\right)\tan^2\theta\left(\tan^2\theta+1\right)\int_0^z dz' a_\rho^{(1)}(z')a_\rho^{(1)'}\left(\frac{(C-1)z'+2z}{(C+1)}\right)H(z-z')
\end{aligned}$$

$$b=\beta_0/\alpha_0$$
$$C=\eta_g/\nu_g$$

Non-linear elastic inversion: all four components of data

The solution for the second equation

$$G_0^P \hat{V}_2^{PS} G_0^S = -\hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S - \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S$$

$b = \beta_0/\alpha_0$

$C = \eta_g/\nu_g$

is

$$-\frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)a_\rho^{(2)}(z)-\frac{\beta_0^2}{2\omega^2}k_g\left(\nu_g+\eta_g\right)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)a_\mu^{(2)}(z)$$

$$=-\left[\left(\frac{1}{2}+\frac{1}{C+1}\right)\frac{1}{\eta_g\nu_g^2}\left(\frac{\beta_0^4}{\alpha_0^4}Ck_g^3-3\frac{\beta_0^2}{\alpha_0^2}Ck_g^5\frac{\beta_0^2}{\omega^2}-k_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}+2k_g^5\nu_g^2\frac{\beta_0^4}{\omega^4}+2Ck_g^7\frac{\beta_0^4}{\omega^4}\right)\right.$$

$$+\left(\frac{1}{2}-\frac{1}{C+1}\right)\frac{1}{\eta_g\nu_g^2}\left(\frac{\beta_0^2}{\alpha_0^2}Ck_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}+2\frac{\beta_0^2}{\alpha_0^2}k_g^5\frac{\beta_0^2}{\omega^2}-2Ck_g^5\nu_g^2\frac{\beta_0^4}{\omega^4}-\frac{\beta_0^2}{\alpha_0^2}k_g^3+k_g^5\frac{\beta_0^2}{\omega^2}-2k_g^7\frac{\beta_0^4}{\omega^4}\right)$$

$$+\left(\frac{1}{2C}+\frac{1}{C+1}\right)\frac{1}{4\eta_g^2\nu_g}\left(6k_g^3-12k_g^5\frac{\beta_0^2}{\omega^2}-k_g\frac{\omega^2}{\beta_0^2}+8k_g^7\frac{\beta_0^4}{\omega^4}+8C^3\nu_g^2k_g^5\frac{\beta_0^4}{\omega^4}-4\frac{\beta_0^2}{\alpha_0^2}C^3\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}\right)$$

$$-\left(\frac{1}{2C}-\frac{1}{C+1}\right)\frac{1}{4\eta_g\nu_g^2}\left(4\frac{\beta_0^2}{\alpha_0^2}k_g^3-8k_g^5\frac{\beta_0^2}{\omega^2}-k_g\frac{\omega^2}{\alpha_0^2}+2k_g^3-4C\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}+8C\nu_g^2k_g^5\frac{\beta_0^4}{\omega^4}-4\frac{\beta_0^2}{\alpha_0^2}k_g^5\frac{\beta_0^2}{\omega^2}\right.$$

$$\left.\left.+8k_g^7\frac{\beta_0^4}{\omega^4}\right)-\frac{\beta_0^2}{\alpha_0^2}\frac{k_g^3}{\nu_g}\frac{\beta_0^2}{\omega^2}+\frac{k_g}{2\eta_g}\left(2k_g^2\frac{\beta_0^2}{\omega^2}-1\right)\right]a_\mu^{(1)}(z)a_\mu^{(1)}(z)$$

Non-linear elastic inversion: all four components of data

Continued

$$
\begin{aligned}
&-\left[\left(\frac{1}{2}+\frac{1}{C+1}\right)\frac{k_g}{8\eta_g\nu_g^2}\left(Ck_g^2+\nu_g^2\right)-\left(\frac{1}{2}-\frac{1}{C+1}\right)\frac{k_g}{8\eta_g\nu_g^2}\left(k_g^2+C\nu_g^2\right)\right.\\
&\left.+\left(\frac{1}{2C}+\frac{1}{C+1}\right)\frac{k_g}{8\eta_g^2\nu_g}\left(C^3\nu_g^2+k_g^2\right)-\left(\frac{1}{2C}-\frac{1}{C+1}\right)\frac{k_g}{8\eta_g\nu_g^2}\left(k_g^2+C\nu_g^2\right)\right]a_\rho^{(1)}(z)a_\rho^{(1)}(z)\\
&-\left[\left(\frac{1}{2}+\frac{1}{C+1}\right)\frac{\beta_0^2}{\alpha_0^2}\frac{1}{4\nu_g^3}k_g\left(k_g^2-\nu_g^2\right)+\left(\frac{1}{2}-\frac{1}{C+1}\right)\frac{1}{4\eta_g\nu_g^2}\left(k_g\frac{\omega^2}{\alpha_0^2}-2\frac{\beta_0^2}{\alpha_0^2}k_g^3\right)+\frac{\beta_0^2}{\alpha_0^2}\frac{k_g}{2\nu_g}\right]\\
&\times a_\mu^{(1)}(z)a_\gamma^{(1)}(z)\\
&+\left[\left(\frac{1}{2}+\frac{1}{C+1}\right)\frac{k_g\left(k_g^2+\nu_g^2\right)}{8\nu_g^3}-\left(\frac{1}{2}-\frac{1}{C+1}\right)\frac{k_g\left(k_g^2+\nu_g^2\right)}{8\eta_g\nu_g^2}\right]a_\rho^{(1)}(z)a_\gamma^{(1)}(z)\\
&-\left[\left(\frac{1}{2}+\frac{1}{C+1}\right)\frac{1}{4\eta_g\nu_g^2}\left(3\frac{\beta_0^2}{\alpha_0^2}Ck_g^3+\nu_g^2k_g-4Ck_g^5\frac{\beta_0^2}{\omega^2}-4k_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}\right)\right.\\
&-\left(\frac{1}{2}-\frac{1}{C+1}\right)\frac{1}{4\eta_g\nu_g^2}\left(\frac{\beta_0^2}{\alpha_0^2}C\nu_g^2k_g+k_g^3-4k_g^5\frac{\beta_0^2}{\omega^2}-4Ck_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}+2\frac{\beta_0^2}{\alpha_0^2}k_g^3\right)\\
&+\left(\frac{1}{2C}+\frac{1}{C+1}\right)\frac{1}{4\eta_g^2\nu_g}\left(k_g^3-2k_g^5\frac{\beta_0^2}{\omega^2}+\frac{\beta_0^2}{\alpha_0^2}C^3\nu_g^2k_g-4C^3\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}-\frac{1}{2}k_g\frac{\omega^2}{\beta_0^2}+2C^2\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}\right)
\end{aligned}
$$

Non-linear elastic inversion: all four components of data

Continued

$$-\bigg(\frac{1}{2C}-\frac{1}{C+1}\bigg)\frac{1}{4\eta_g\nu_g^2}\bigg(C\nu_g^2k_g-2C\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}+\frac{\beta_0^2}{\alpha_0^2}k_g^3-2k_g^5\frac{\beta_0^2}{\omega^2}+2C^2\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}-2C\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}$$

$$-\frac{1}{2}k_g\frac{\omega^2}{\beta_0^2}\bigg)\bigg]\times a_\rho^{(1)}(z)a_\mu^{(1)}(z)$$

$$-\frac{1}{\eta_g\nu_g^2}\bigg(\frac{\beta_0^4}{\alpha_0^4}Ck_g^3-3\frac{\beta_0^2}{\alpha_0^2}Ck_g^5\frac{\beta_0^2}{\omega^2}-k_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}+2k_g^5\nu_g^2\frac{\beta_0^4}{\omega^4}+2Ck_g^7\frac{\beta_0^4}{\omega^4}\bigg)$$

$$\times\bigg[\frac{1}{2}\int_0^z dz'a_\mu^{(1)}{}_z\bigg(\frac{(C+1)z-(C-1)\,z'}{2}\bigg)a_\mu^{(1)}(z')+\frac{1}{C+1}a_\mu^{(1)\prime}(z)\int_0^z dz'a_\mu^{(1)}(z')\bigg]$$

$$-\frac{1}{\eta_g\nu_g^2}\bigg(\frac{\beta_0^2}{\alpha_0^2}Ck_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}+2\frac{\beta_0^2}{\alpha_0^2}k_g^5\frac{\beta_0^2}{\omega^2}-2Ck_g^5\nu_g^2\frac{\beta_0^4}{\omega^4}-\frac{\beta_0^2}{\alpha_0^2}k_g^3+k_g^5\frac{\beta_0^2}{\omega^2}-2k_g^7\frac{\beta_0^4}{\omega^4}\bigg)$$

$$\times\bigg[\frac{1}{2}\int_0^z dz'a_\mu^{(1)}{}_z\bigg(\frac{(C+1)z-(C-1)\,z'}{2}\bigg)a_\mu^{(1)}(z')-\frac{1}{C+1}a_\mu^{(1)\prime}(z)\int_0^z dz'a_\mu^{(1)}(z')\bigg]$$

$$-\frac{1}{4\eta_g^2\nu_g}\bigg(6k_g^3-12k_g^5\frac{\beta_0^2}{\omega^2}-k_g\frac{\omega^2}{\beta_0^2}+8k_g^7\frac{\beta_0^4}{\omega^4}+8C^3\nu_g^2k_g^5\frac{\beta_0^4}{\omega^4}-4\frac{\beta_0^2}{\alpha_0^2}C^3\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}\bigg)$$

$$\times\bigg[\frac{1}{2C}\int_0^z dz'a_\mu^{(1)}{}_z\bigg(\frac{(C+1)z+(C-1)\,z'}{2C}\bigg)a_\mu^{(1)}(z')+\frac{1}{C+1}a_\mu^{(1)\prime}(z)\int_0^z dz'a_\mu^{(1)}(z')\bigg]$$

$$+\frac{1}{4\eta_g\nu_g^2}\bigg(4\frac{\beta_0^2}{\alpha_0^2}k_g^3-8k_g^5\frac{\beta_0^2}{\omega^2}-k_g\frac{\omega^2}{\alpha_0^2}+2k_g^3-4C\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}+8C\nu_g^2k_g^5\frac{\beta_0^4}{\omega^4}-4\frac{\beta_0^2}{\alpha_0^2}k_g^5\frac{\beta_0^2}{\omega^2}+8k_g^7\frac{\beta_0^4}{\omega^4}\bigg)$$

$$\times\bigg[\frac{1}{2C}\int_0^z dz'a_\mu^{(1)}{}_z\bigg(\frac{(C+1)z+(C-1)\,z'}{2C}\bigg)a_\mu^{(1)}(z')-\frac{1}{C+1}a_\mu^{(1)\prime}(z)\int_0^z dz'a_\mu^{(1)}(z')\bigg]$$

$$-\frac{k_g\left(Ck_g^2+\nu_g^2\right)}{8\eta_g\nu_g^2}\bigg[\frac{1}{2}\int_0^z dz'a_\rho^{(1)}{}_z\bigg(\frac{(C+1)z-(C-1)\,z'}{2}\bigg)a_\rho^{(1)}(z')+\frac{1}{C+1}a_\rho^{(1)\prime}(z)\int_0^z dz'a_\rho^{(1)}(z')\bigg]$$

$$+\frac{k_g\left(k_g^2+C\nu_g^2\right)}{8\eta_g\nu_g^2}\bigg[\frac{1}{2}\int_0^z dz'a_\rho^{(1)}{}_z\bigg(\frac{(C+1)z-(C-1)\,z'}{2}\bigg)a_\rho^{(1)}(z')-\frac{1}{C+1}a_\rho^{(1)\prime}(z)\int_0^z dz'a_\rho^{(1)}(z')\bigg]$$

$$-\frac{C^3k_g\nu_g^2+k_g^3}{8\eta_g^2\nu_g}\bigg[\frac{1}{2C}\int_0^z dz'a_\rho^{(1)}{}_z\bigg(\frac{(C+1)z+(C-1)\,z'}{2C}\bigg)a_\rho^{(1)}(z')+\frac{1}{C+1}a_\rho^{(1)\prime}(z)\int_0^z dz'a_\rho^{(1)}(z')\bigg]$$

$$+\frac{k_g\left(k_g^2+C\nu_g^2\right)}{8\eta_g\nu_g^2}\bigg[\frac{1}{2C}\int_0^z dz'a_\rho^{(1)}{}_z\bigg(\frac{(C+1)z+(C-1)\,z'}{2C}\bigg)a_\rho^{(1)}(z')-\frac{1}{C+1}a_\rho^{(1)\prime}(z)\int_0^z dz'a_\rho^{(1)}(z')\bigg]$$

$$-\frac{\beta_0^2}{\alpha_0^2}\frac{k_g\left(k_g^2-\nu_g^2\right)}{4\nu_g^3}\bigg[\frac{1}{2}\int_0^z dz'a_\gamma^{(1)}{}_z\bigg(\frac{(C+1)z-(C-1)\,z'}{2}\bigg)a_\mu^{(1)}(z')+\frac{1}{C+1}a_\mu^{(1)\prime}(z)\int_0^z dz'a_\gamma^{(1)}(z')\bigg]$$

$$-\frac{1}{4\eta_g\nu_g^2}\bigg(k_g\frac{\omega^2}{\alpha_0^2}-2\frac{\beta_0^2}{\alpha_0^2}k_g^3\bigg)\bigg[\frac{1}{2}\int_0^z dz'a_\gamma^{(1)}{}_z\bigg(\frac{(C+1)z-(C-1)\,z'}{2}\bigg)a_\mu^{(1)}(z')$$

$$-\frac{1}{C+1}a_\mu^{(1)\prime}(z)\int_0^z dz'a_\gamma^{(1)}(z')\bigg]$$

Non-linear elastic inversion: all four components of data

Continued

$$
\begin{aligned}
&+\frac{k_g\left(k_g^2+\nu_g^2\right)}{8\nu_g^3}\left[\frac{1}{2}\int_0^z dz' a^{(1)}_{\gamma}{}_{,z}\left(\frac{(C+1)z-(C-1)\,z'}{2}\right)a^{(1)}_{\rho}\left(z'\right)+\frac{1}{C+1}a^{(1)\prime}_{\rho}(z)\int_0^z dz' a^{(1)}_{\gamma}(z')\right]\\
&-\frac{k_g\left(k_g^2+\nu_g^2\right)}{8\eta_g\nu_g^2}\left[\frac{1}{2}\int_0^z dz' a^{(1)}_{\gamma}{}_{,z}\left(\frac{(C+1)z-(C-1)\,z'}{2}\right)a^{(1)}_{\rho}\left(z'\right)-\frac{1}{C+1}a^{(1)\prime}_{\rho}(z)\int_0^z dz' a^{(1)}_{\gamma}(z')\right]\\
&-\frac{1}{4\eta_g\nu_g^2}\left(\frac{\beta_0^2}{\alpha_0^2}Ck_g^3+\nu_g^2k_g-2Ck_g^5\frac{\beta_0^2}{\omega^2}-2k_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}\right)\\
&\times\left[\frac{1}{2}\int_0^z dz' a^{(1)}_{\rho}{}_{,z}\left(\frac{(C+1)z-(C-1)\,z'}{2}\right)a^{(1)}_{\mu}\left(z'\right)+\frac{1}{C+1}a^{(1)\prime}_{\mu}(z)\int_0^z dz' a^{(1)}_{\rho}(z')\right]\\
&-\frac{1}{4\eta_g\nu_g^2}\left(2\frac{\beta_0^2}{\alpha_0^2}Ck_g^3-2Ck_g^5\frac{\beta_0^2}{\omega^2}-2k_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}\right)\\
&\times\left[\frac{1}{2}\int_0^z dz' a^{(1)}_{\mu}{}_{,z}\left(\frac{(C+1)z-(C-1)\,z'}{2}\right)a^{(1)}_{\rho}\left(z'\right)+\frac{1}{C+1}a^{(1)\prime}_{\rho}(z)\int_0^z dz' a^{(1)}_{\mu}(z')\right]\\
&+\frac{1}{4\eta_g\nu_g^2}\left(\frac{\beta_0^2}{\alpha_0^2}C\nu_g^2k_g+k_g^3-2k_g^5\frac{\beta_0^2}{\omega^2}-2Ck_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}\right)\\
&\times\left[\frac{1}{2}\int_0^z dz' a^{(1)}_{\rho}{}_{,z}\left(\frac{(C+1)z-(C-1)\,z'}{2}\right)a^{(1)}_{\mu}\left(z'\right)-\frac{1}{C+1}a^{(1)\prime}_{\mu}(z)\int_0^z dz' a^{(1)}_{\rho}(z')\right]\\
&+\frac{1}{4\eta_g\nu_g^2}\left(2\frac{\beta_0^2}{\alpha_0^2}k_g^3-2k_g^5\frac{\beta_0^2}{\omega^2}-2Ck_g^3\nu_g^2\frac{\beta_0^2}{\omega^2}\right)\\
&\times\left[\frac{1}{2}\int_0^z dz' a^{(1)}_{\mu}{}_{,z}\left(\frac{(C+1)z-(C-1)\,z'}{2}\right)a^{(1)}_{\rho}\left(z'\right)-\frac{1}{C+1}a^{(1)\prime}_{\rho}(z)\int_0^z dz' a^{(1)}_{\mu}(z')\right]\\
&-\frac{1}{4\eta_g^2\nu_g}\left(k_g^3-2k_g^5\frac{\beta_0^2}{\omega^2}+\frac{\beta_0^2}{\alpha_0^2}C^3\nu_g^2k_g-2C^3\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}\right)\\
&\times\left[\frac{1}{2C}\int_0^z dz' a^{(1)}_{\rho}{}_{,z}\left(\frac{(C+1)z+(C-1)\,z'}{2C}\right)a^{(1)}_{\mu}\left(z'\right)+\frac{1}{C+1}a^{(1)\prime}_{\mu}(z)\int_0^z dz' a^{(1)}_{\rho}(z')\right]\\
&-\frac{1}{4\eta_g^2\nu_g}\left(-2C^3\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}-\frac{1}{2}k_g\frac{\omega^2}{\beta_0^2}+2C^2\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}\right)\\
&\times\left[\frac{1}{2C}\int_0^z dz' a^{(1)}_{\mu}{}_{,z}\left(\frac{(C+1)z+(C-1)\,z'}{2C}\right)a^{(1)}_{\rho}\left(z'\right)+\frac{1}{C+1}a^{(1)\prime}_{\rho}(z)\int_0^z dz' a^{(1)}_{\mu}(z')\right]\\
&+\frac{1}{4\eta_g\nu_g^2}\left(C\nu_g^2k_g-2C\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}+\frac{\beta_0^2}{\alpha_0^2}k_g^3-2k_g^5\frac{\beta_0^2}{\omega^2}\right)\\
&\times\left[\frac{1}{2C}\int_0^z dz' a^{(1)}_{\rho}{}_{,z}\left(\frac{(C+1)z+(C-1)\,z'}{2C}\right)a^{(1)}_{\mu}\left(z'\right)-\frac{1}{C+1}a^{(1)\prime}_{\mu}(z)\int_0^z dz' a^{(1)}_{\rho}(z')\right]\\
&+\frac{1}{4\eta_g\nu_g^2}\left(2C^2\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}-2C\nu_g^2k_g^3\frac{\beta_0^2}{\omega^2}-\frac{1}{2}k_g\frac{\omega^2}{\beta_0^2}\right)\\
&\times\left[\frac{1}{2C}\int_0^z dz' a^{(1)}_{\mu}{}_{,z}\left(\frac{(C+1)z+(C-1)\,z'}{2C}\right)a^{(1)}_{\rho}\left(z'\right)-\frac{1}{C+1}a^{(1)\prime}_{\rho}(z)\int_0^z dz' a^{(1)}_{\mu}(z')\right],
\end{aligned}
$$

The end

Non-linear elastic inversion: all four components of data

The solutions for the third and fourth equations

$$G_0^S \hat{V}_2^{SP} G_0^P = -\hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P - \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P$$

$$G_0^S \hat{V}_2^{SS} G_0^S = -\hat{G}_0^S \hat{V}_1^{SP} \hat{G}_0^P \hat{V}_1^{PS} \hat{G}_0^S - \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S \hat{V}_1^{SS} \hat{G}_0^S$$

Please see the annual report.

M-OSRP
Mission - Oriented Seismic Research Program

Discrimination between pressure and fluid saturation using direct non-linear inversion method: *an application to time-lapse seismic data*

Haiyan Zhang, Arthur B. Weglein, Robert Keys, Douglas Foster and Simon Shaw

M-OSRP Annual Meeting
University of Houston
May 10 –12, 2006

Statement of the problem

- **Distinguishing pressure changes from reservoir fluid changes is difficult with conventional seismic time-lapse attributes.**
- **Pressure changes or fluid changes?**
  - **shear modulus** ***sensitive to*** **pressure changes**
  - **Vp/Vs** ***sensitive to*** **fluid changes**
- **A direct non-linear inversion method may be useful for accomplishing this goal.**

Introduction of the method

Inverse scattering series	Time-lapse seismic monitoring
Reference medium L_0	**Initial reservoir condition**
Actual medium L	**Current reservoir condition**
Earth property changes in space $V=L_0-L$	**Reservoir property changes in time**
Reference wave field G_0	**Baseline survey**
Actual wave field G	**Monitor survey**
Scattered wave field $D=G-G_0$	**Monitor-Baseline**

Core data tests

Fixing the fluid as 100% water saturation, while the pressure changes from 1000 to 9000psi.

- –Baseline: pressure = 5000psi
- –Monitor: other different pressures

Fixing the pressure at 5000psi, while the fluid changes from 0 to 100 percent.

- **– Baseline: 100% saturation**
- **– Monitor: other different water saturations**

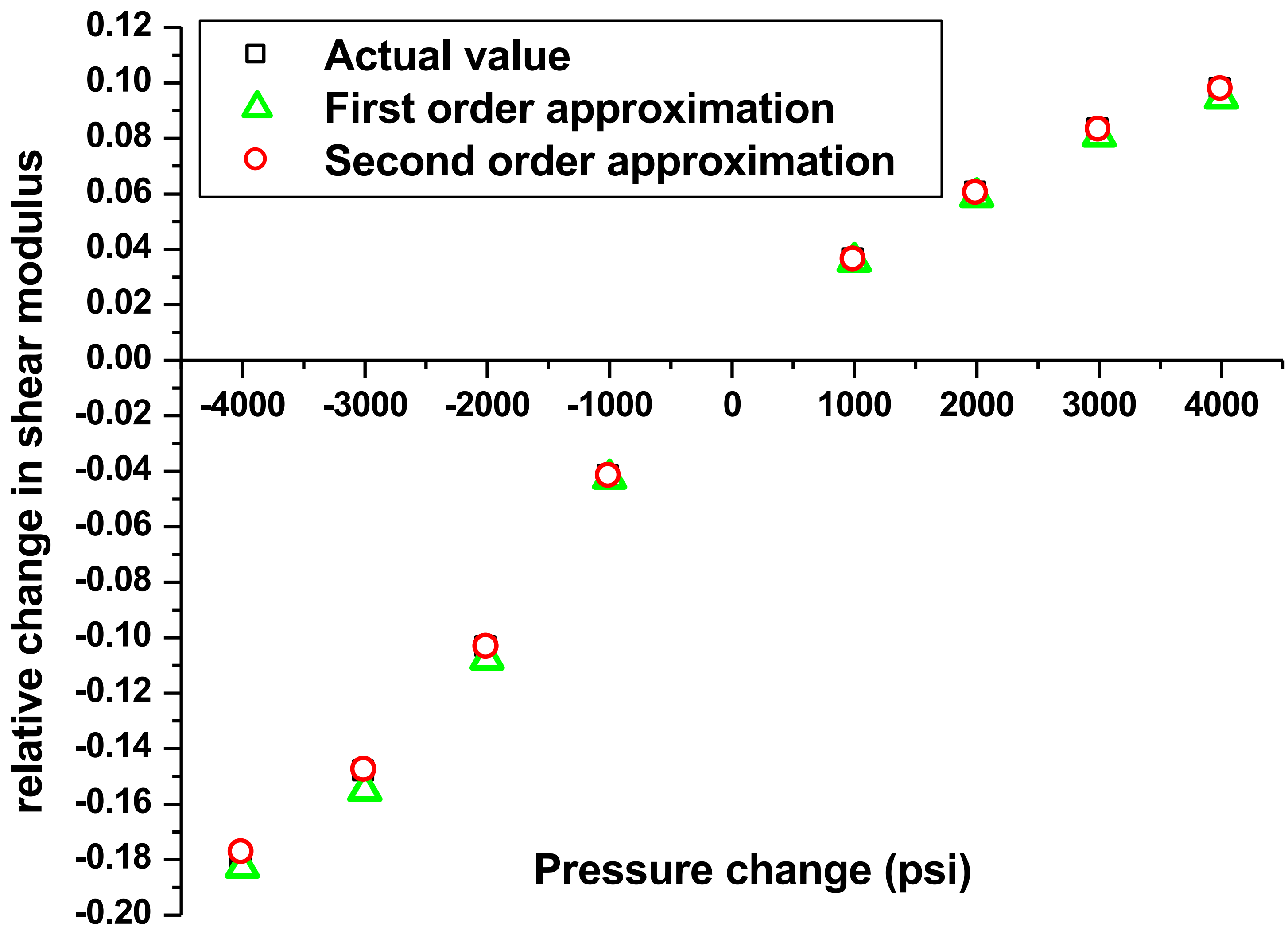
Core data tests

- Compare effects of pressure and fluid changes on the elastic properties.
- Compare first order and second order approximations.

Comparison of 1st and 2nd order approximation for pressure changes

Fluid fixed (100% water saturation)

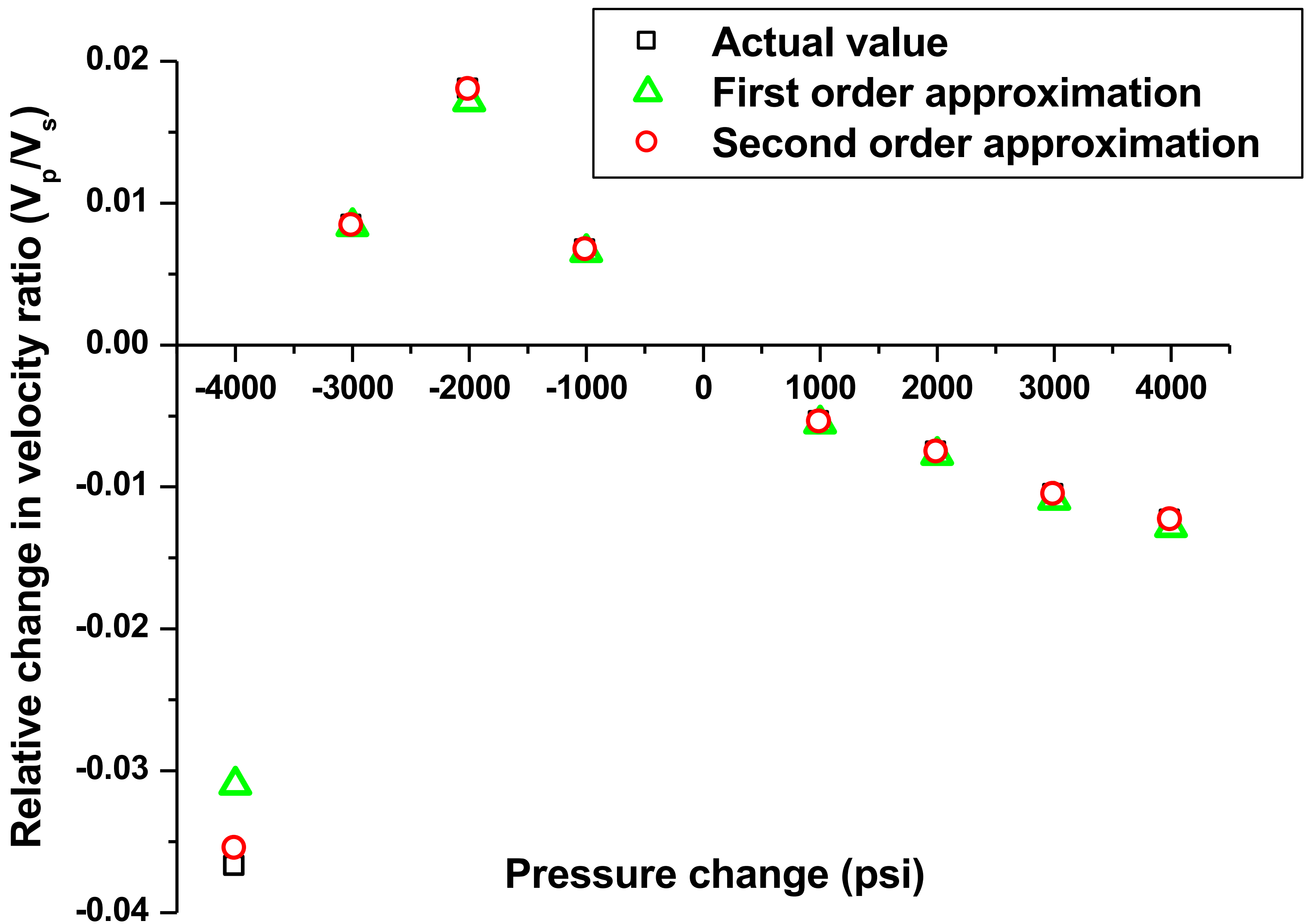
A. R. Gregory 1976



Comparison of 1st and 2nd order approximation for pressure changes

Fluid fixed (100% water saturation)

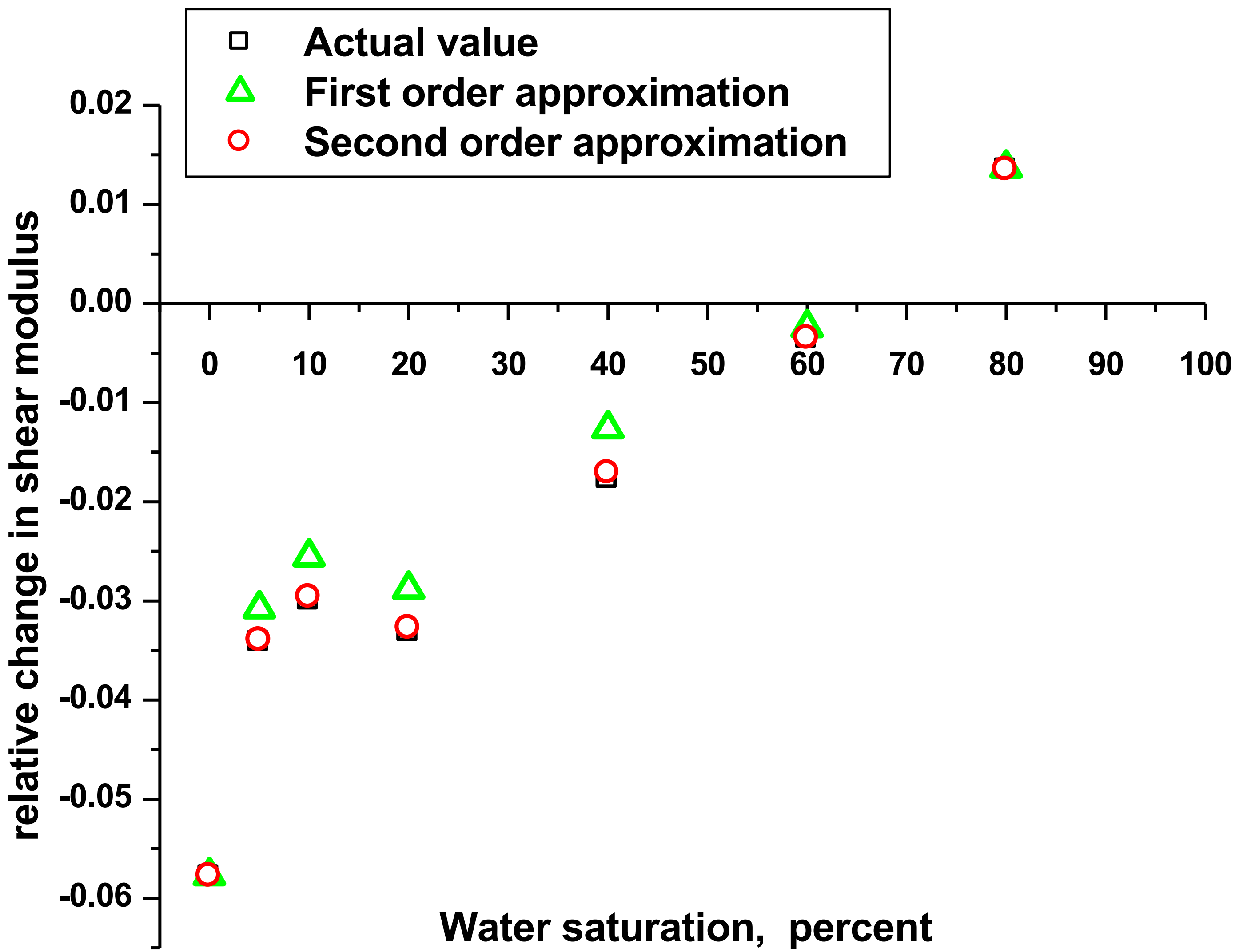
A. R. Gregory 1976



Comparison of 1st and 2nd order approximation for fluid changes

Pressure fixed (5000 psi)

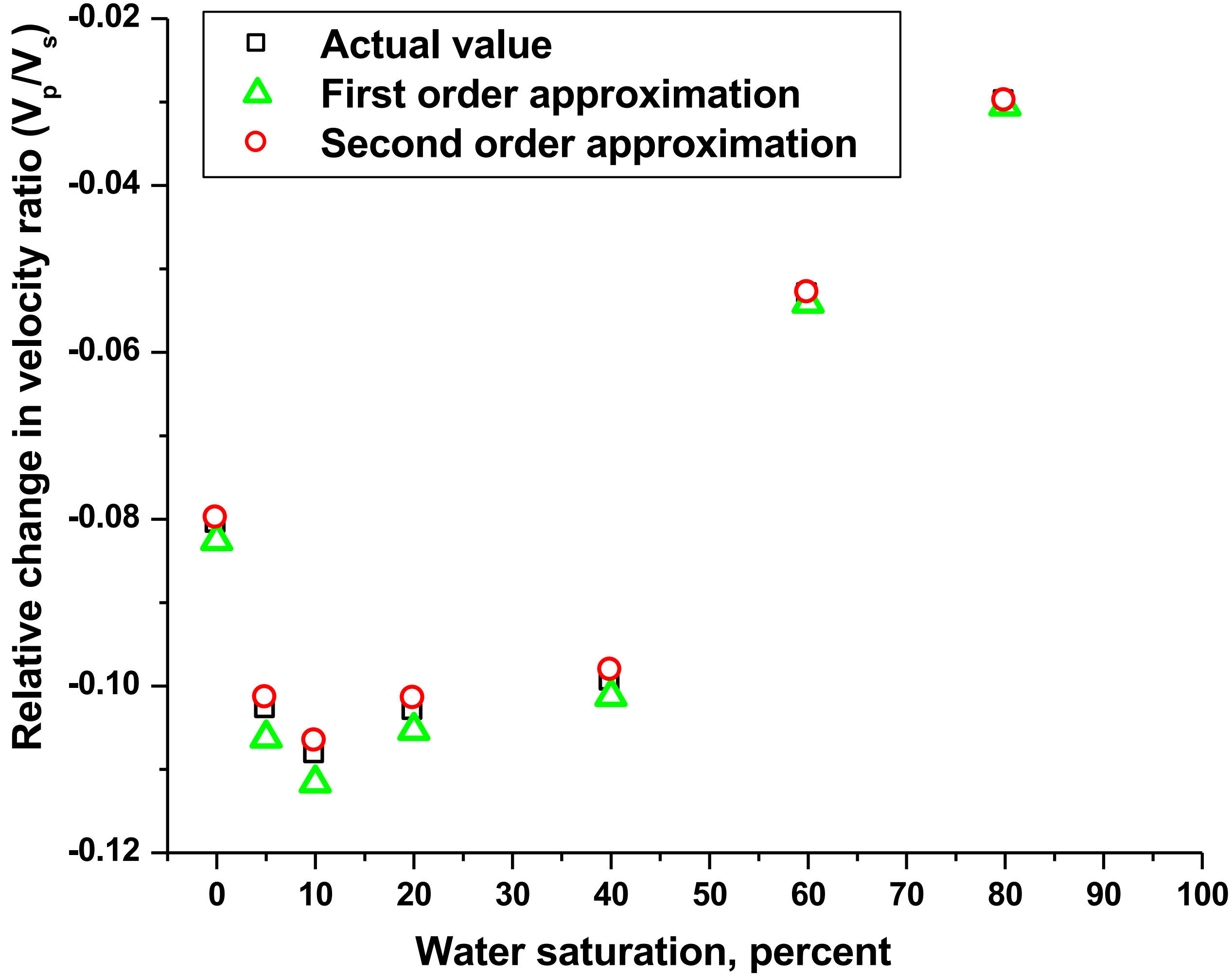
A. R. Gregory 1976



Comparison of 1st and 2nd order approximation for fluid changes

Pressure fixed (5000 psi)

A. R. Gregory 1976

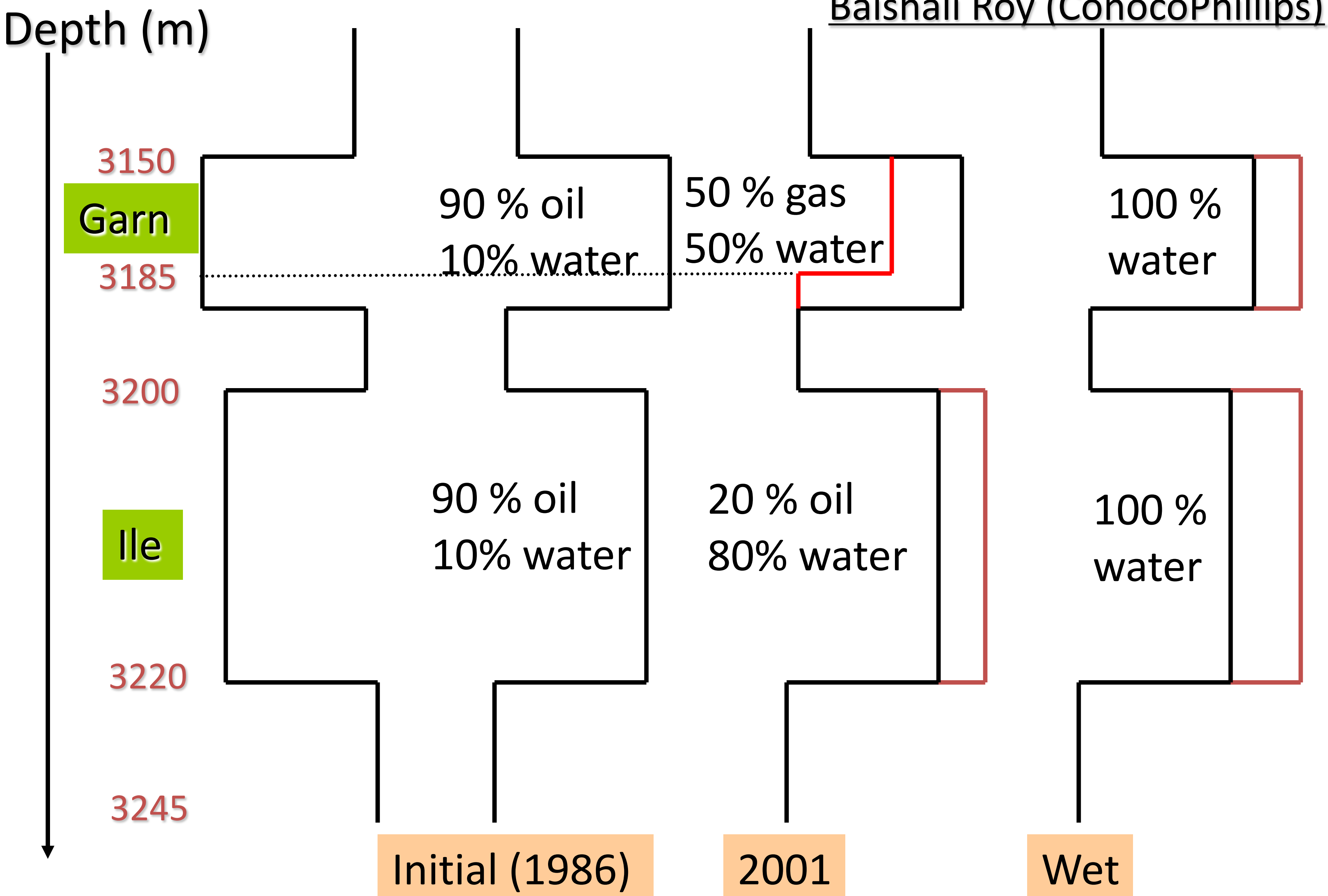


Outline

- Comparing the first and second order algorithms in estimating shear modulus and Vp/Vs contrasts.
  - Core data tests (A. R. Gregory, 1976)
  - Heidrun well log data tests
- Conclusions and Plan
- Acknowledgements

Synthetic Modeling -- A-52

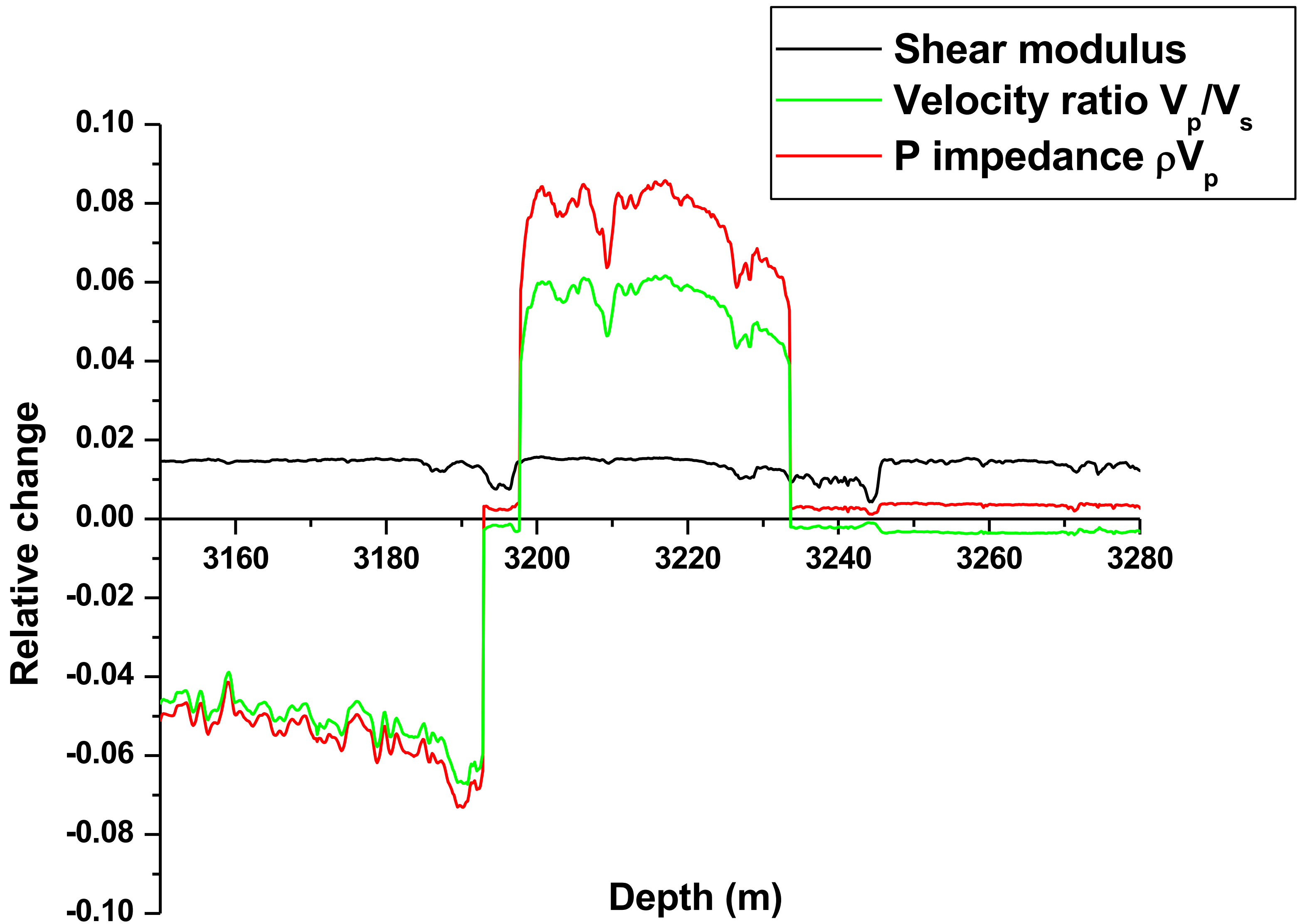
Baishali Roy (ConocoPhillips)



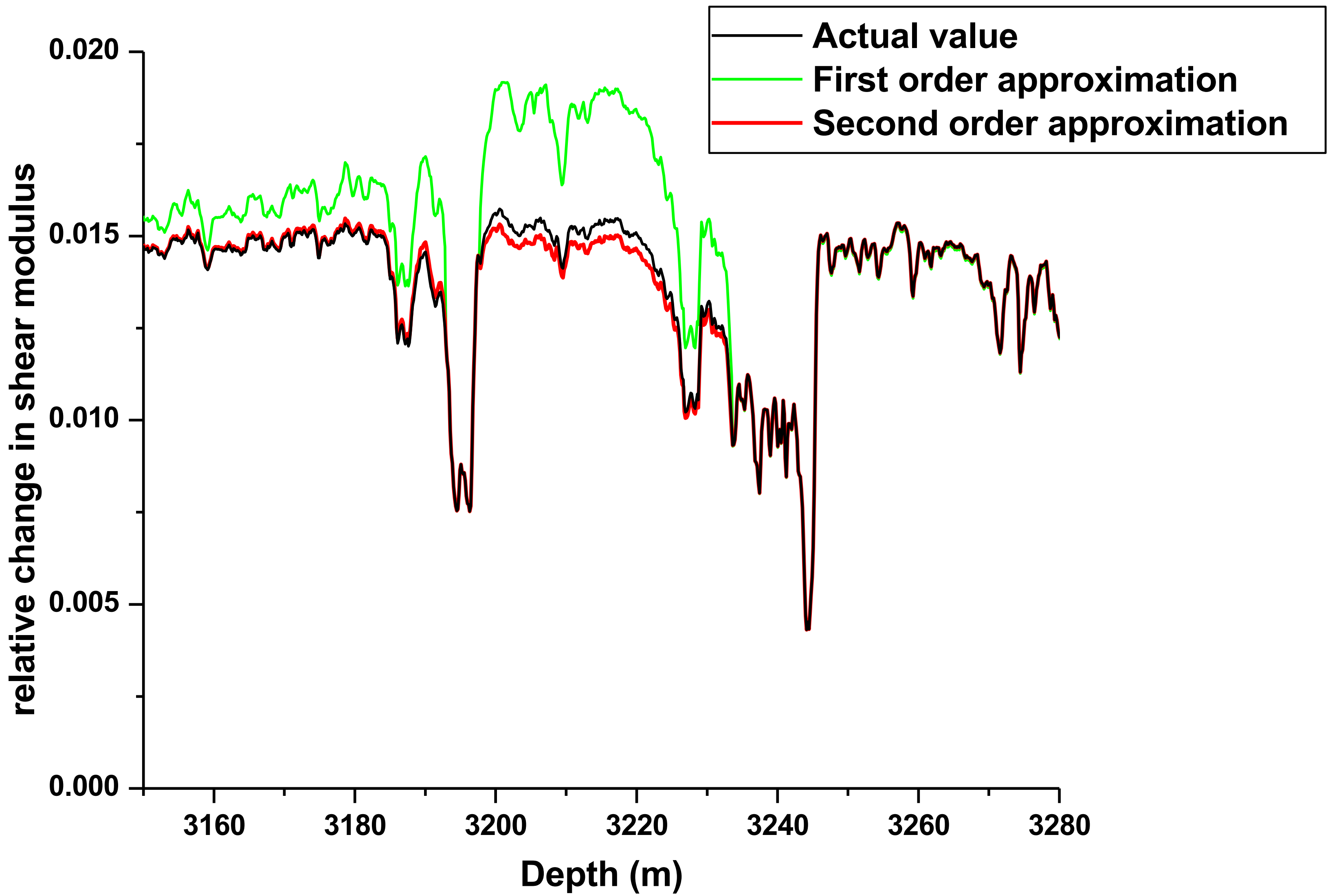
Heidrun well log data tests

- Compare effects of pressure and fluid changes on the elastic properties.
- Compare first order and second order approximations.

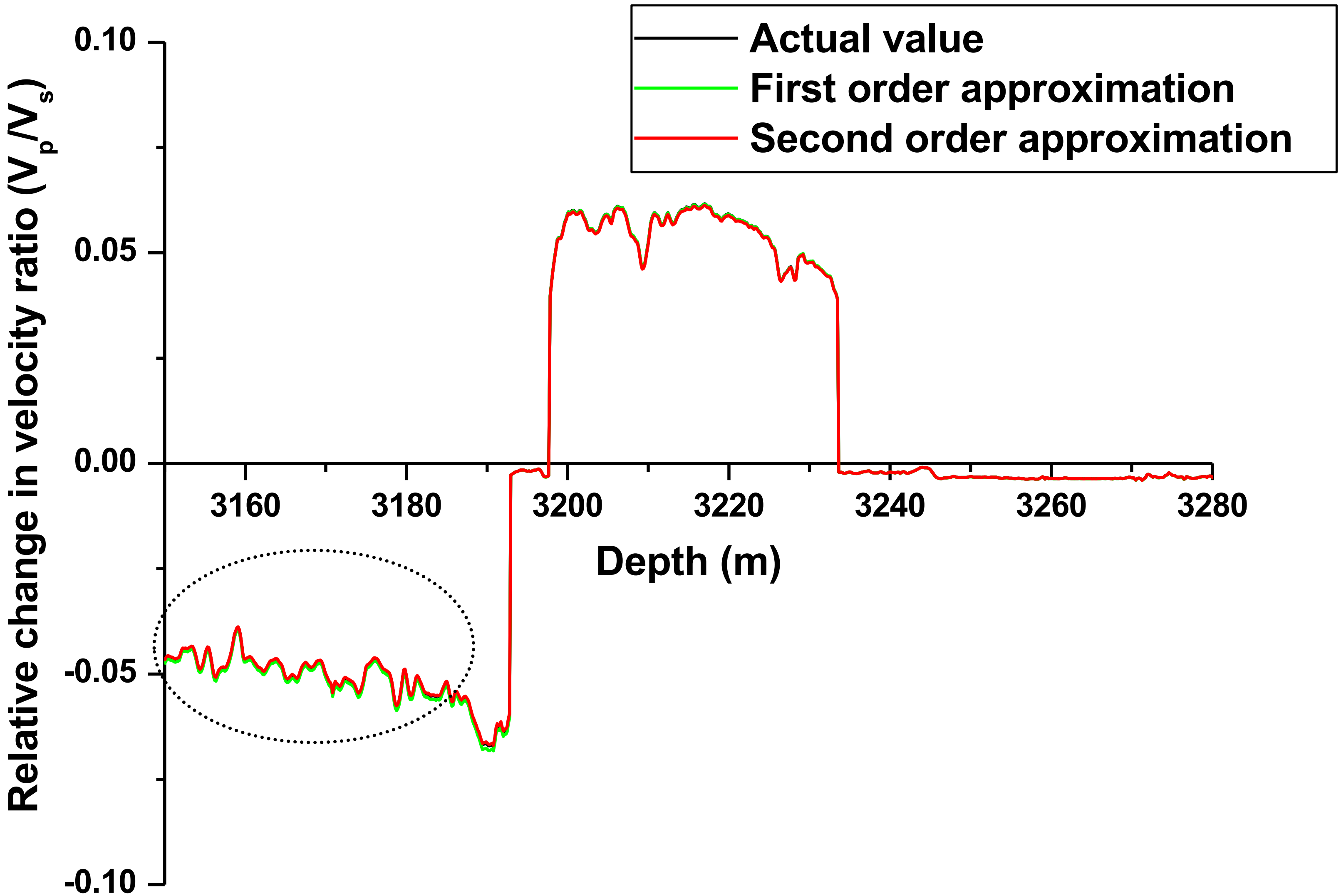
Heidrun well log data tests



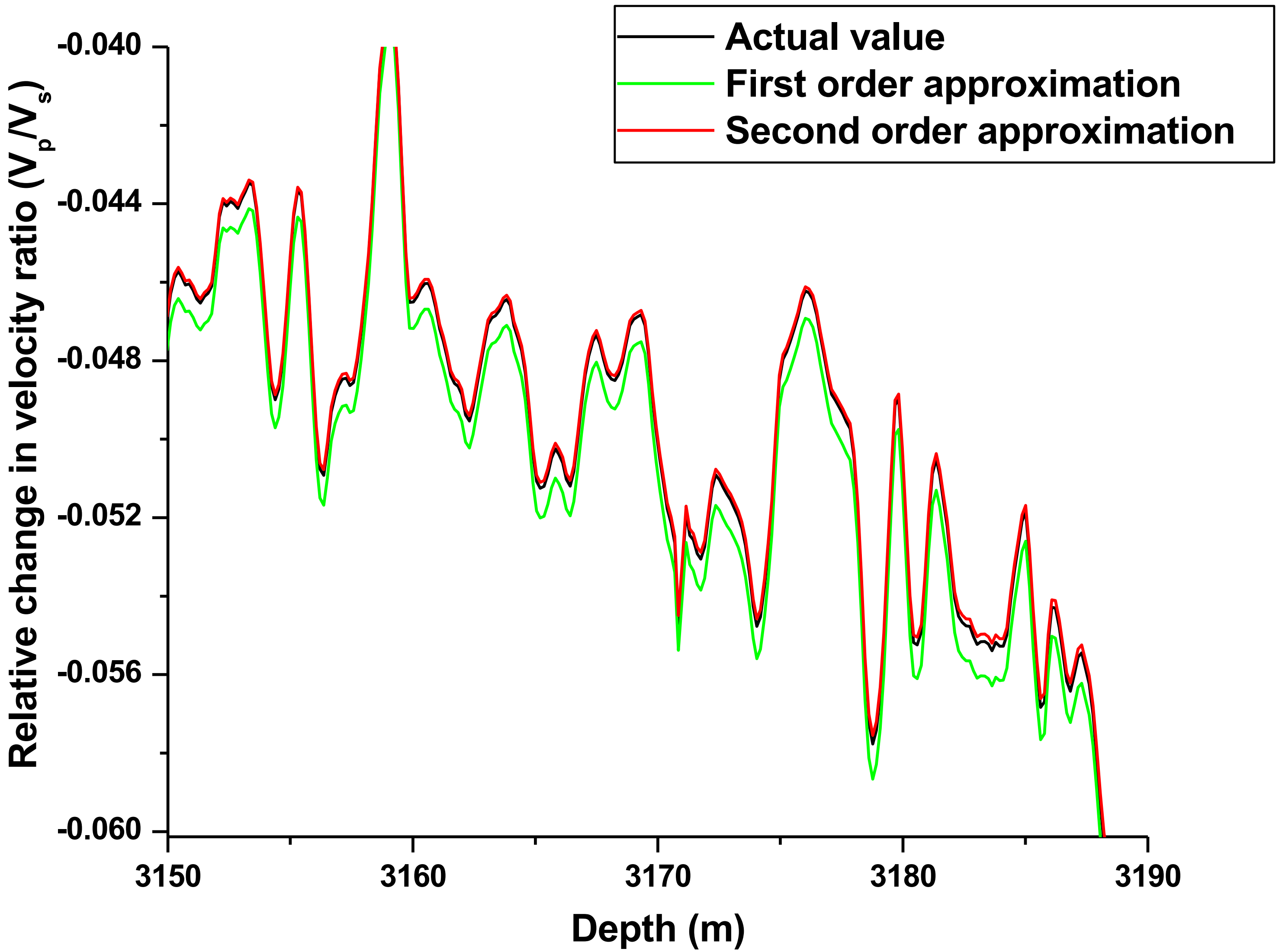
Heidrun well log data tests



Heidrun well log data tests



Heidrun well log data tests



Observations

- **The second order approximation provides improvements in the earth property predictions.**

- **In this well log data case, the second order approximation is more helpful for predicting shear modulus compared to Vp/Vs.**

Plan

- **Comparing the first and second order algorithms in estimating shear modulus and Vp/Vs contrasts.**
  - **Heidrun synthetic data**
  - **Real seismic data tests (Heidrun)**

Acknowledgements

- Baishali Roy (ConocoPhillips)
- Thanks to ConocoPhillips for providing the opportunity.
- The M-OSRP sponsors are thanked for supporting this research.