The first test and evaluation of the inverse scattering series internal multiple attenuation algorithm for an attenuating medium

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SUMMARY

In this paper, the Inverse Scattering Series (ISS) internal multiple attenuation algorithm is analytically and numerically evaluated on reflection data from an attenuating medium. All previous synthetic data tests on this algorithm have involved multidimensional acoustic and elastic media. The results for an attenuating medium show that the method retains its value to directly predict internal multiples (IM) with the exact phase and an approximate amplitude, without knowing the medium, and its anelastic properties.

INTRODUCTION

The inverse scattering series can achieve all processing objectives directly by using distinct isolated task-specific subseries and without subsurface information (Weglein et al. (2003)). ISS internal multiple attenuator has shown stand-alone capabilities on both marine and on-shore plays (e.g., Ferreira, 2011; Fu et al., 2010). To extend attenuation method to elimination, Zou and Weglein (2013) proposes a new algorithm to compensate for transmission loss in the attenuator. This new elimination method requires the input data to be wavelet deconvolved and assumes an elastic subsurface. Obviously, if the data are attenuated and broadened because of their propagation in the anelastic medium, Q compensation is the conventional step to recover the amplitudes before substituting the data into ISS internal multiple elimination algorithm. That can be a difficult step to effectively achieve in practice.

Q compensation based on ISS without Q information of the subsurface has demonstrated an early but encouraging effectiveness (e.g., Innanen and Weglein, 2003, 2005; Innanen and Lira, 2008). ISS Q compensation without Q method supposes that the input data contain primaries only, i.e., the internal multiple has been attenuated or eliminated for best before stepping into Q compensation algorithm.

This paper demonstrates that applying the industry standard ISS internal multiple attenuator to data from an anelastic earth will attenuate the multiples. The data with primary and relatively weak residual internal multiple can be substituted into the ISS Q compensation algorithm to obtain effective elastic data and then insert that data into the new elastic internal multiple elimination algorithm.

In this paper, for the first time the ISS internal multiple attenuator is tested on data from an attenuating medium. A two-reflector model with constant Q in each layer is used for analytical and numerical testing and evaluation. The result indicates that the prediction has the correct phase and an approximate amplitude. That is positive news for the ISS internal multiple attenuator and encourages developing an elimination method for the exploration plays where absorption is significant, e.g., pre-salt plays in the deep water Gulf of Mexico, off-shore Brazil, the Red Sea and the North Sea.

ANALYTICAL TEST OF ISS INTERNAL MULTIPLE ATTENUATION ALGORITHM ON DATA WITH Q

Q Definition

Based on Aki and Richards (2002), Q is used to represent the energy lost for a wave-field propagating, in one wave length, and is defined as

$$Q = \frac{2\pi E}{\Delta E}, \tag{1}$$

where E is the energy of the wave-field, and ΔE is the energy lost in a wavelength of propagation. With the definition of Q, the amplitude of wave-field A along propagation direction x can be represented as

$$A(x) = A_0 e^{-\frac{\omega}{2cQ}x}, \tag{2}$$

where A_0 is the amplitude without an absorption influence, ω is the angular frequency, and c is the velocity of the wave-field. The exponentially decaying term causes the attenuation and results in a wavelet broadening with a finite length, rather than the original spike. It is not difficult to understand that when Q decreases, the amplitude will decrease; on the other hand, when Q increases to infinity, there is no absorption.

Here, we assume that Q is frequency independent. In order to guarantee that the amplitude attenuates for negative frequency, it is convenient to replace ω with $|\omega|$, and then we have

$$A(x) = A_0 e^{-\frac{|\omega|}{2cQ}x}. \tag{3}$$

The dispersion is ignored here. That is convenient for later analytical calculations.

Analytical Test Under 1D Normal Incidence

Following the Q definition, we can express the wave-field in an anelastic medium analytically. In this section, the anelastic data will be used as input to test the ISS internal multiple attenuation algorithm analytically.

For 1D normal incidence, the ISS internal multiple attenuation algorithm (e.g., Araújo, 1994; Weglein et al., 1997, 2003) can be expressed as:

$$b_3(k_z) = \int_{-\infty}^{\infty} b_1(z)e^{ik_z z}dz \int_{-\infty}^{z-\varepsilon} b_1(z_1)e^{-ik_z z_1}dz_1 \int_{z_1+\varepsilon}^{\infty} b_1(z_2)e^{ik_z z_2}dz_2, \tag{4}$$

where the deghosted data, $D(t)$, for an incident spike wave, satisfies $D(\omega) = b_1(2\omega/c_0)$, and $b_1(z) = \int_{-\infty}^{\infty} b_1(k_z)e^{-ik_z z}dk_z$, $k_z = 2\omega/c_0$ is the vertical wavenumber, and $b_1(z)$ corresponds to an uncollapsed FK migration of the normal-incident spike plane-wave data. ε in the formula is used to make sure the events satisfy the lower-higher-lower relationship, and its value is chosen on the basis of the length of the wavelet.

A two-reflector model is provided below as an example, with the parameters listed in Fig.1, and with the depths of source and receiver both assumed to be zero.

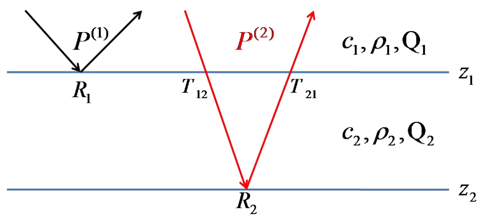


Figure 1: A two-reflector 1D model. $P^{(1)}$ and $P^{(2)}$ are primaries from the first and the second interface, respectively; R_1 and R_2 are reflection coefficients; T_{12} and T_{21} are transmission coefficients; c_1 and c_2 are the velocities; ρ_1 and ρ_2 are densities; and Q_1 and Q_2 are quality factors.

For a 1D model and a 1D normal-incident plane wave, two primaries in the data $D(\omega)$ can be represented as:

$$P^{(1)}(\omega) = R_1 e^{i\omega\frac{2z_1}{c_1}} e^{-|\omega|\frac{z_1}{c_1 Q_1}}, \quad (5)$$

$$P^{(2)}(\omega) = T_{12}R_2T_{21}e^{i\omega(\frac{2z_1}{c_1}+\frac{2(z_2-z_1)}{c_2})} e^{-|\omega|(\frac{z_1}{c_1Q_1}+\frac{z_2-z_1}{c_2Q_2})}. \quad (6)$$

The dispersion effect is not considered here in order to simplify the analytical calculation, i.e., the velocity does not change with frequency.

After migrating the data into the pseudo depth domain to get $b_1(z)$, we can substitute it into eqn.4. We further assume that the two primaries are isolated and ε is chosen reasonably to make sure there is no overlap between the two events among the integrals. The predicted internal multiple $b_3(k_z)$ can be obtained:

$$b_3(k_z) = (T_{12}R_2T_{21})^2 R_1 e^{ik_z(z_1+\frac{2c_1}{c_2}(z_2-z_1))} e^{-|k_z|(\frac{z_1}{2Q_1}+\frac{c_1}{c_2}\frac{z_2-z_1}{Q_2})} e^{-|k_z|\frac{z_1}{Q_1}}. \quad (7)$$

The actual first-order internal multiple in the k_z domain is

$$IM(k_z) = -T_{12}T_{21}R_2{}^2R_1 e^{ik_z(z_1+\frac{2c_1}{c_2}(z_2-z_1))} e^{-|k_z|(\frac{z_1}{2Q_1}+\frac{c_1}{c_2}\frac{z_2-z_1}{Q_2})}. \quad (8)$$

The relation between the predicted internal multiple and the actual internal multiple is

$$b_3(k_z) = -T_{12}T_{21}e^{-|k_z|\frac{z_1}{Q_1}} IM(k_z). \quad (9)$$

By using the ISS internal multiple attenuation algorithm, the multiple can be predicted with the correct phase and an approximate amplitude.

If the data are without the influence of Q absorption, then from Weglein et al. (2003), we can obtain the relation between predicted and actual internal multiple as

$$b_3(k_z) = -T_{12}T_{21}IM(k_z). \quad (10)$$

Comparing eqn.9 and eqn.10, it can be seen that the predicted amplitude is less accurate for input data with Q absorption than it is for data without Q; however, the phases are correct under both conditions.

NUMERICAL TEST OF ISS INTERNAL MULTIPLE ATTENUATION ALGORITHM ON DATA WITH Q

A two-reflector 1D model (Fig.1) will be used as an example to numerically test the effectiveness of ISS internal multiple attenuator on anelastic data. The parameters are listed in Table1.

Layer Number	Velocity (m/s)	Density (kg/m^3)	Travel Times (s)	Q Value
1	1500	1000	0.5	200
2	4000	1000	1.1	100
3	2000	1000		

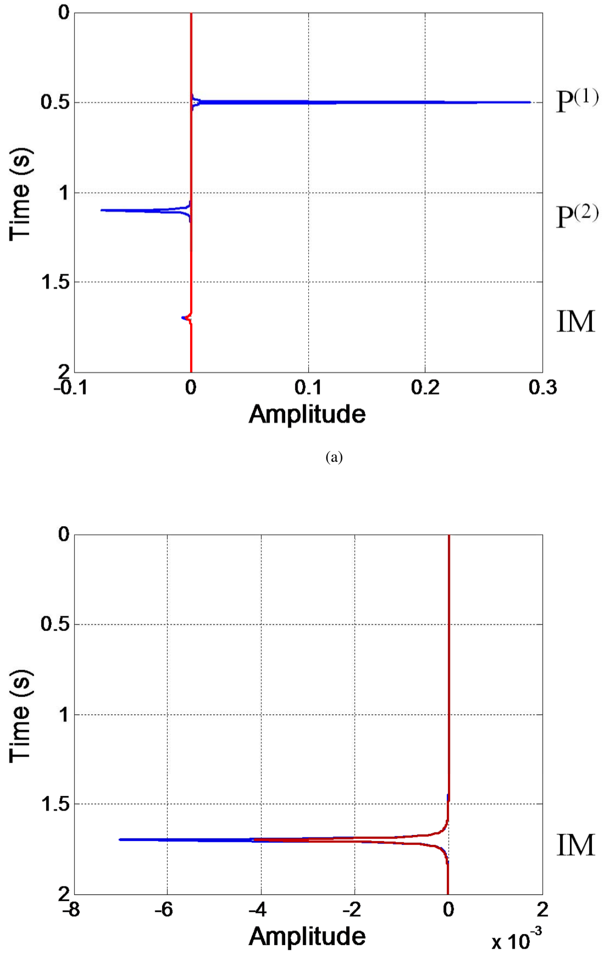
Table 1: The parameters of a two-reflector 1D model

By using the parameters of Table 1, the synthetic data involving the Q value of each layer are generalized analytically without considering dispersion. The data include all the primaries and all the first-order internal multiples.

Substituting the input data b_1, shown as the blue line in Fig.2(a), into ISS internal multiple attenuation algorithm, we can predict internal multiple b_3, shown as the red line in Fig.2(a). Actually, the red line in Fig.2(a) is -b_3. It can be seen from eqn.9 that the polarity of b_3 is opposite to that of the actual internal multiple. In order to show the result more clearly, the predicted internal multiple and the actual internal multiple are compared in Fig.2(b). From the result, we can further establish that the prediction result matches well in phase and approximately in amplitude even with data from an attenuating medium, without knowing Q absorption properties.

DISCUSSION

In this paper, the ISS internal multiple attenuation algorithm is tested analytically and numerically using Q-influenced data, with the conclusion that the prediction will have the correct phase and an approximate amplitude.



(a)

(b)

Figure 2: The numerical result of ISS internal multiple attenuation algorithm with anelastic data. (a): the input data b_1 (blue line) and the predicted multiple -b_3 (red line); (b): the actual internal multiple (blue line) and the predicted internal multiple -b_3 (red line).

The discussion in this paper gives us confidence that even for an attenuating medium, the ISS internal multiple attenuator can provide a result that retains the primary and partially removes the internal multiple. This is an important step in a strategy to eliminate internal multiples for both elastic and anelastic media. That will allow application for exploration plays where the geology exhibits significant absorption, e.g., pre-salt plays in the deep water Gulf of Mexico, off-shore Brazil, the Red Sea and the North Sea.

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APPENDIX A

B3 CALCULATION

The primaries in frequency domain can be expressed from eqn.5 and eqn.6. Since the migrated data in pseudo depth domain are required to substitute into the internal multiple attenuation algorithm, the variable should be changed from ω to $k_z = \frac{2\omega}{c_1}$:

$$P^{(1)}(k_z) = R_1 e^{ik_z z_1} e^{-|k_z|\frac{z_1}{2Q_1}}, \quad \text{(A-1)}$$

$$P^{(2)}(k_z) = T_{12}R_2T_{21} e^{ik_z(z_1+\frac{c_1}{c_2}(z_2-z_1))} e^{-|k_z|(\frac{z_1}{2Q_1}+\frac{c_1}{c_2}\frac{z_2-z_1}{2Q_2})}. \quad \text{(A-2)}$$

Then, Fourier transform is applied over k_z to pseudo depth domain to obtain

$$P^{(1)}(z) = \frac{R_1}{\pi}\frac{\frac{z_1}{2Q_1}}{(\frac{z_1}{2Q_1})^2+(z-z_1)^2}, \quad \text{(A-3)}$$

$$P^{(2)}(z) = \frac{T_{12}R_2T_{21}}{\pi}\frac{\frac{z_1}{2Q_1}+\frac{c_1}{c_2}\frac{z_2-z_1}{2Q_2}}{(\frac{z_1}{2Q_1}+\frac{c_1}{c_2}\frac{z_2-z_1}{2Q_2})^2+(z-(z_1+\frac{c_1}{c_2}(z_2-z_1)))^2}. \quad \text{(A-4)}$$

$b_1(z) = P^{(1)}(z) + P^{(2)}(z)$, which will be substituted into ISS internal multiple attenuation algorithm to predict the internal multiple b_3.

Based on Weglein et al. (2003), the 1D ISS internal multiple attenuation algorithm is

$$b_3(k_z) = \int_{-\infty}^{\infty} b_1(z)e^{ik_z z}dz \int_{-\infty}^{z-\epsilon} b_1(z_1)e^{-ik_z z_1}dz_1 \int_{z_1+\epsilon}^{\infty} b_1(z_2)e^{ik_z z_2}dz_2, \quad \text{(A-5)}$$

where ϵ is used to make sure the events satisfy the lower-higher-lower relationship, and its value is chosen on the basis of the length of the wavelet.

For this model, there are two primaries in the data. Now I suppose these two events are isolated (Fig.A-1). The pseudo depth of the first event is z_1 with a length of 2a, whereas the pseudo depth of the second event is z_2' with a length of 2b. For ϵ in eqn.A-5 , it is chosen to satisfy $\epsilon \geq max(2a, 2b)$ and $\epsilon \leq (z_2' - b - (z_1 + a))$.

Kaplan et al. (2004) change the integral order of eqn.A-5 and rewrite the formula as:

$$b_3(k_z) = \int_{-\infty}^{\infty} b_1(z)e^{-ik_z z}[\int_{z+\epsilon}^{\infty} b_1(z')e^{ik_z z'}dz']^2 dz. \quad \text{(A-6)}$$

Since $b_1(z) = P^{(1)}(z) + P^{(2)}(z)$, eqn.A-6 can be divided into two parts:

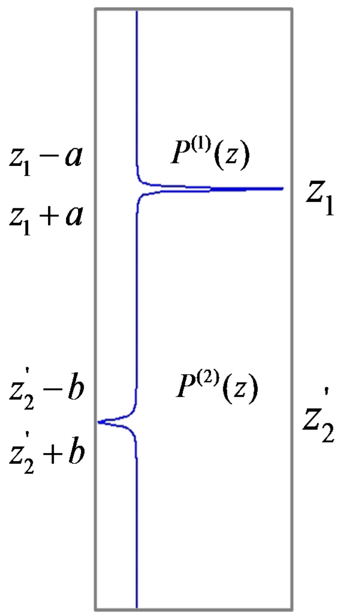

Figure A-1: A two-reflector model reflection record. $P^{(1)}$ and $P^{(2)}$ are primaries from the first and the second interface, respectively.

$$\begin{aligned}
& b_3(k_z) \\
&= \int_{-\infty}^{\infty} P^{(1)}(z) e^{-ik_z z} [\int_{z+\varepsilon}^{\infty} b_1(z') e^{ik_z z'} dz']^2 dz \\
&+ \int_{-\infty}^{\infty} P^{(2)}(z) e^{-ik_z z} [\int_{z+\varepsilon}^{\infty} b_1(z') e^{ik_z z'} dz']^2 dz \\
&= \int_{z_1-a}^{z_1+a} P^{(1)}(z) e^{-ik_z z} [\int_{z+\varepsilon}^{\infty} b_1(z') e^{ik_z z'} dz']^2 dz \qquad (A-7-1) \\
&+ \int_{z_2'-b}^{z_2'+b} P^{(2)}(z) e^{-ik_z z} [\int_{z+\varepsilon}^{\infty} b_1(z') e^{ik_z z'} dz']^2 dz. \qquad (A-7-2)
\end{aligned} \qquad \text{(A-7)}$$

For (A-7-1), the integral limitation of z is $[z_1 - a, z_1 + a]$. Consider the lower limit of the integral of z' and the constraint of ε,

$$z + \varepsilon \geq z_1 - a + \varepsilon \geq z_1 + a + 2a = z_1 + a,$$

and

$$z + \varepsilon \leq z_1 + a + \varepsilon \leq z_1 + a + z_2' - b - (z_1 + a) = z_2' - b.$$

We can see that the lower limit of the second integral should be after the end of the first event and before the beginning of the second event, i.e., in $[z + \varepsilon, \infty)$, the kernel of the second integral is $b_1(z') = P^{(2)}(z')$.

So

$$\begin{aligned}
& (A-7-1) \\
&= \int_{z_1-a}^{z_1+a} P^{(1)}(z) e^{-ik_z z} [\int_{z+\varepsilon}^{\infty} b_1(z') e^{ik_z z'} dz']^2 dz \\
&= \int_{z_1-a}^{z_1+a} P^{(1)}(z) e^{-ik_z z} [\int_{z+\varepsilon}^{\infty} P^{(2)}(z') e^{ik_z z'} dz']^2 dz \\
&= \int_{-\infty}^{\infty} P^{(1)}(z) e^{-ik_z z} [\int_{-\infty}^{\infty} P^{(2)}(z') e^{ik_z z'} dz']^2 dz \\
&= (T_{12} R_2 T_{21})^2 R_1 e^{ik_z (z_1 + \frac{2c_1}{c_2}(z_2 - z_1))} e^{-|k_z| (\frac{z_1}{2Q_1} + \frac{c_1}{c_2} \frac{z_2 - z_1}{Q_2})} e^{-|k_z| \frac{z_1}{Q_1}}.
\end{aligned} \qquad \text{(A-8)}$$

Similarly, for (A-7-2), the integral limitation of z is $[z_2' - b, z_2' + b]$. Consider the lower limit of the integral of z' and the constraint of ε,

$$z + \varepsilon \geq z_2' - b + \varepsilon \geq z_2' + b + 2b = z_2' + b.$$

The lower limit of the second integral should be after the end of the second event, i.e., in $[z + \varepsilon, \infty)$, the kernel the of second integral is $b_1(z') = 0$.

So

$$\begin{aligned}
& (A-7-2) \\
&= \int_{z_2'-b}^{z_2'+b} P^{(2)}(z) e^{-ik_z z} [\int_{z+\varepsilon}^{\infty} b_1(z') e^{ik_z z'} dz']^2 dz \\
&= 0.
\end{aligned} \qquad \text{(A-9)}$$

Now

$$\begin{aligned}
& b_3(k_z) \\
&= (A-7-1) \\
&= (T_{12} R_2 T_{21})^2 R_1 e^{ik_z (z_1 + \frac{2c_1}{c_2}(z_2 - z_1))} e^{-|k_z| (\frac{z_1}{2Q_1} + \frac{c_1}{c_2} \frac{z_2 - z_1}{Q_2})} e^{-|k_z| \frac{z_1}{Q_1}}.
\end{aligned} \qquad \text{(A-10)}$$

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