Comparing the new Inverse Scattering Series (ISS) internal-multiple-elimination algorithm and the industry-standard ISS internal-multiple-attenuation algorithm plus adaptive subtraction when primaries and internal multiples interfere and where we can evaluate the efficacy using wave-theoretical data consisting of only primaries

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SUMMARY

In multiple-removal methods, a multiple prediction is usually followed by an adaptive-subtraction step that attempts to compensate for any shortcoming in the prediction and allows an adjustment for bridging the difference between the predicted multiple and actual multiple. For example, the ISS internal-multiple-attenuation algorithm (which can predict internal multiples with *accurate time* and *approximate amplitude* without requiring any subsurface information) is typically combined with an energy-minimization adaptive subtraction. That combination is the most capable method available today for removing internal multiples. However, an adaptive-subtraction process (e.g., based on energy-minimization criteria) can also damage primaries when removing multiples, especially when the multiples interfere with primaries. Therefore, under that type of circumstance, a more accurate multiple prediction is sought in order to reduce the dependence on the adaptive-subtraction. The objective is to preserve primaries and successfully remove multiples. The ISS internal-multiple-*elimination* algorithm can predict internal multiples with both *accurate time* and *accurate amplitude*, without requiring any subsurface information and without an adaptive-subtraction step. In this paper, using 1D prestack data where internal multiples interfere with primaries, we examine the internal-multiple-removal result by the ISS internal-multiple-elimination algorithm. For the situation where a multiple interferes with a primary, we compare (1) the result from the ISS internal-multiple-elimination algorithm without adaptive subtraction, and (2) the result from the ISS internal-multiple attenuation with adaptive subtraction with (3) the actual primaries in the data. We note that for the third item, modeling primaries, wave-theoretic modeling methods (not ray theory) can produce an accurate data set consisting of only primaries (and is only available) for a 1D subsurface. That is the reason this paper examines the efficacy of different elimination approaches in 1D. The test results demonstrate that the ISS elimination algorithm (with an accurate prediction of both time and amplitude) can directly remove internal multiples that interfere with primaries, without damaging primaries, whereas the current most capable method to remove internal multiples (i.e., the ISS internal-multiple-attenuation algorithm plus an energy-minimization adaptive subtraction) can remove multiples but often at the expense of damaging primaries.

INTRODUCTION

Multiple removal is a long-standing problem in seismic data processing. Methods that attenuate multiples can be classified as belonging to two broad categories (Weglein, 1995): (1) those that seek to exploit a feature or property that differentiates primary from multiple (e.g., Thorson and Claerbout, 1985; Foster and Mosher, 1992) and (2) those that predict and then subtract multiples from seismic data (e.g., Araujo et al., 1994; Weglein et al., 1997; Berkhout and Verschuur, 1997).

For multiple removal methods in the second category, a multiple prediction is usually followed by an adaptive-subtraction step that attempts to compensate for any shortcoming in the prediction and allows an adjustment for bridging the difference between the predicted multiple and the actual multiple. For example, the ISS internal-multiple-attenuation algorithm is typically combined with an energy-minimization adaptive subtraction. The energy-minimization adaptive subtraction can use an L_2 norm and assumes minimum energy after multiple removal. The idea is that the "energy" in some interval of space and time is less without the multiple than with the multiple. This assumption can be violated in complex off-shore and most on-shore plays where primaries and internal multiples often destructively interfere and, as a result, the energy can actually increase, rather than decrease, after multiple removal. Primaries can be easily damaged in that process.

For dealing with this challenging problem, Weglein (2015) proposed a three-pronged strategy including (1) develop the ISS prerequisites for predicting the reference wave field and producing de-ghosted data; (2) develop internal-multiple-elimination algorithms from ISS, and (3) develop a replacement for the energy-minimization criteria for adaptive subtraction.

In this paper, we focus on the second aspect of the three-pronged strategy. For a situation where a multiple interferes with a primary, we compare the result from the ISS internal-multiple elimination without adaptive subtraction and the result from the ISS internal-multiple attenuation with adaptive subtraction to the actual primaries in the data. For the actual primaries in the data, wave-theoretic modeling methods (e.g., reflectivity) can produce an accurate data set consisting of only primaries for a 1D subsurface, hence, the comparison is examined in 1D.

THE ISS INTERNAL-MULTIPLE-ATTENUATION ALGORITHM

The ISS internal-multiple-attenuation algorithm (Araujo et al., 1994; Weglein et al., 1997), for 1D subsurface case and prestack data, is expressed as follows:

$$D(x,t) + D_3(x,t). \tag{1}$$

$D(x,t)$ is the input data in space-time domain, and consists of primaries and internal multiples (the reference wave, ghosts and free-surface multiples are assumed to have been removed, and the wavelet has been deconvolved). $D_3(x,t)$ is the prediction for the first-order internal multiples. It can be calculated using the input data as follows: $D(x,t) \xrightarrow{①} D(k,\omega) \xrightarrow{②} b_1(k,z) \xrightarrow{③} b_3(k,2q) \xrightarrow{④} D_3(k,\omega) \xrightarrow{⑤} D_3(x,t)$. Step 1 is the Fourier transform of the input data from x,t to k,ω. Step 2 is an uncollapsed Stolt migration (Weglein et al., 1997) of the transformed input data using a reference velocity c_0. Step 3 is the ISS internal-multiple prediction, shown below:

$$b_3(k,2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k,z) \int_{-\infty}^{z-\varepsilon} dz' e^{-2iqz'} b_1(k,z') \times \int_{z'+\varepsilon}^{\infty} dz'' e^{2iqz''} b_1(k,z''). \tag{2}$$

In step 4, $b_3(k,2q) = 2iqD_3(k,\omega)$, where $q = sgn(\omega)\sqrt{\omega^2/c_0^2 - k^2}$. Step 5 is the Inverse Fourier transform of the prediction from k,ω back to x,t.

The predicted first-order internal multiples in D_3 have the *accurate time* and *approximate amplitude* (in opposite polarity) compared with the actual internal multiples in the data D:

$$IM_{pred} = -(AF) \times IM_{actual}. \tag{3}$$

The amplitude difference between the actual and predicted internal multiple is represented by *Attenuation Factor* (AF) (Weglein and Matson, 1998; Weglein et al., 2003). The data with their first-order internal multiples attenuated are represented by Equation 1.

THE ISS INTERNAL-MULTIPLE-ELIMINATION ALGORITHM

Weglein and Matson (1998) studied the attenuation factor and provided the initial idea and algorithm to remove the attenuation factor. Following their idea, Ramírez and Weglein (2005), Herrera and Weglein (2013), Y. Zou and Weglein (2013, 2014, 2015), and Y. Zou et al. (2016) developed the ISS internal-multiple-elimination algorithm for a 1D and multi-D earth, respectively. For a 1D subsurface and prestack data, the ISS internal-multiple-elimination algorithm is

$$b_E^{IM}(k,2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k,z) \int_{-\infty}^{z-\varepsilon} dz' e^{-2iqz'} F[b_1(k,z')] \times \int_{z'+\varepsilon}^{\infty} dz'' e^{2iqz''} b_1(k,z''), \tag{4}$$

where

$$F[b_1(k,z)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \frac{1}{[1 - |\int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k,z'') e^{iq'z''}|^2]} \times \frac{e^{-iq'z} e^{iq'z'} b_1(k,z')}{[1 - \int_{-\infty}^{z'-\varepsilon} dz''' b_1(k,z''') e^{iq'z'''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-iq'z'''}]^2}, \tag{5}$$

	Layer 1	Layer 2	Layer 3	Layer 4
Velocity (m/s)	1500	3500	1000	1500
Thickness (m)	150	500	120	∞

Table 1: The 1D acoustic model used to generated the synthetic data in the first example.

	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
V_p (m/s)	1500	2000	1500	1500	2500
V_s (m/s)	400	1200	900	1500	1500
ρ (g/cm^3)	1.0	3.0	2.1	1.5	1.8
Thickness (m)	300	500	1100	200	∞

Table 2: The 1D elastic model used to generated the synthetic data in the second example.

$$g(k,z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dz' dq' \times \frac{e^{-iq'z} e^{iq'z'} b_1(k,z')}{1 - \int_{-\infty}^{z'-\varepsilon} dz''' b_1(k,z''') e^{iq'z'''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-iq'z'''}}. \tag{6}$$

$b_1(k,z)$ in Equation 4, 5, and 6 is the same as in Equation 2. $F[]$ in Equation 4 removes the attenuation factor and turns the attenuation algorithm (i.e., Equation 2) into the elimination algorithm (i.e., Equation 4).

NUMERICAL EXAMPLE

In this section, we show two numerical examples where primaries and internal multiples interfere with each other. We choose a 1D prestack example to test because wave-theoretic (not ray-theory) based modeling can only produce a data set exclusively with primaries when the subsurface is 1D acoustic or 1D elastic.

Table 1 shows the subsurface parameters for the first example of synthetic data generated using the reflectivity method.

Figure 1(a) shows the tested input data. Notice that, only three primaries (P_1, P_2, and P_3) and one first-order internal multiple generated by the first two reflectors (IM_{212}) are considered. Figures 1(b) and 1(c) show the predictions of internal multiples by the ISS attenuation algorithm and elimination algorithm, respectively. Figures 1(d) and 1(f) show the results after internal multiple removal, by direct addition of the two prediction results (i.e., 1(b), 1(c)) to the data. Figure 1(e) shows the result after internal-multiple removal using the prediction result from the attenuation algorithm (i.e., 1(b)) plus energy-minimization adaptive subtraction. Please notice that we use a single-channel matching filter for the adaptive subtraction process and the subtraction operator is calculated assuming that the predicted multiple only has a scalar difference with the actual multiple (Equation (2) and (3) in Wang, 2003). Figure 1(g) shows the wiggle plot of the region where P_3 and IM_{212} interfere (offset: 400m–2500m, time: 0.6s–1.2s). Notice that only P_3 and IM_{212} are plotted in Figure 1(g). Figures 1(h) and 1(i) show the actual P_3 and actual IM_{212} in the data, respectively. Figures 1(j), 1(l), and 1(k) show the wiggle

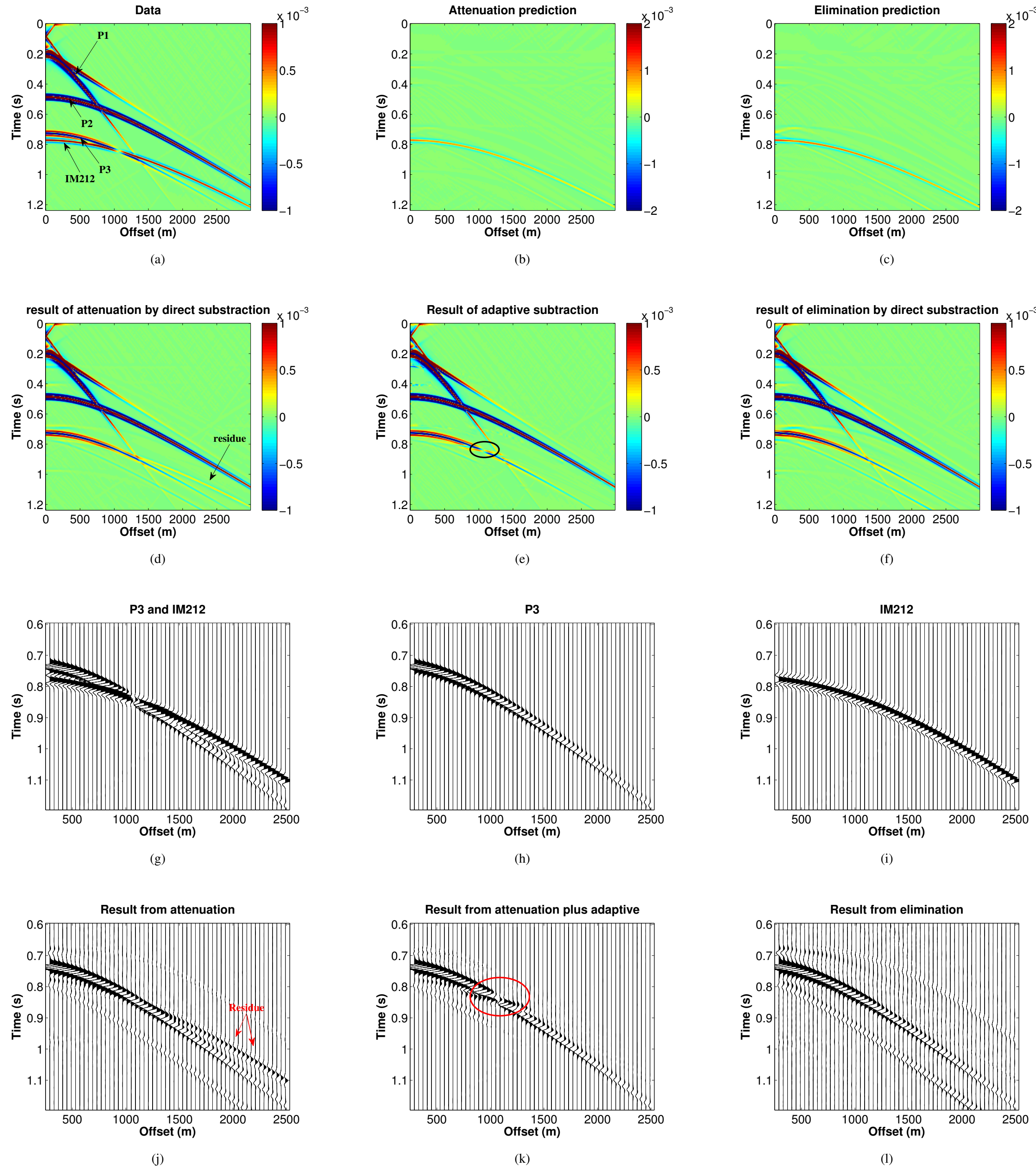

Figure 1: Results in the first example. See text for detailed explanation.

plots of that region corresponding to the Figures in 1(d), 1(f), and 1(e). Examining the results in Figure 1(j), it is noticed that, the ISS internal-multiple-attenuation prediction (with accurate time and approximate amplitude) can be directly added onto the data to attenuate internal multiples and partially recover primaries around 1000m offset (where primaries and internal multiple destructively interfere). However, with this approximate amplitude, there are residues left, especially at far offsets (2000m–2500m). The reason is that the attenuation factor becomes smaller as offset increases, hence, the residue after applying the attenuation increases as offset increases. Combining this prediction result from the attenuation algorithm with the energy-minimization adaptive-subtraction criterion, the residue will be removed in the region where P_3 and IM_{212} do not interfere, as shown in Figure 1(k). However, in the region where they do interfere, the primary is damaged

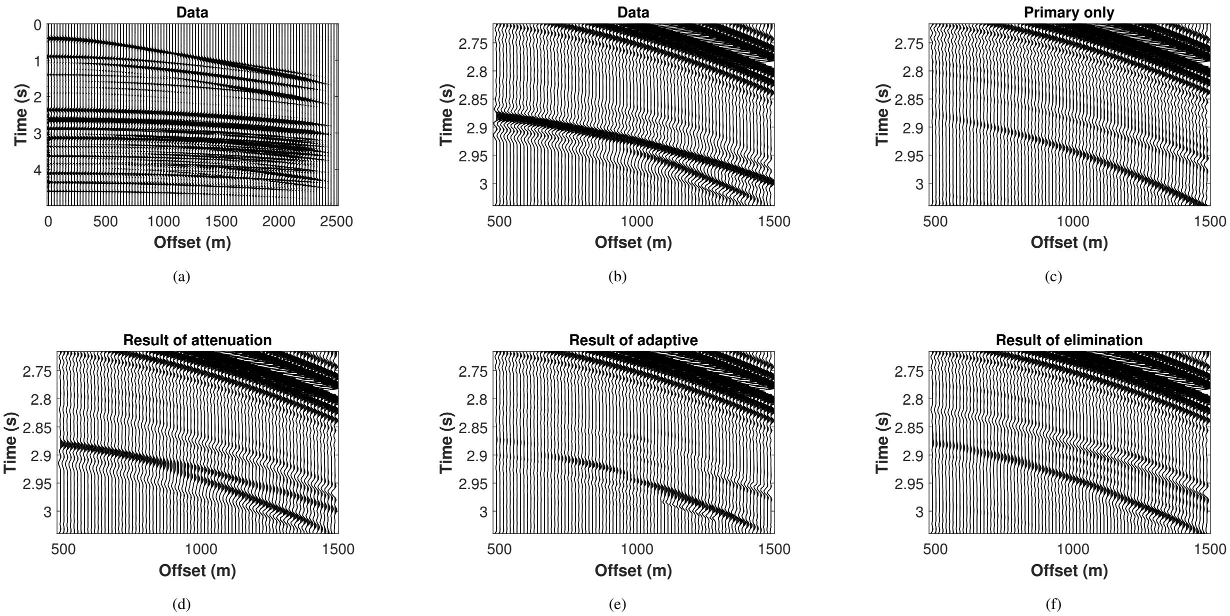

Figure 2: Results in the second example. See text for detailed explanation.

and shows distortions. With the improved amplitude prediction from the ISS internal-multiple-elimination algorithm, the internal multiple is cleanly removed, and the primary can be recovered in the interfering region as shown in Figure 1(l).

In the second example, we generate the synthetic data using a four-reflector elastic model (Table 2). Figure 2(a) shows the synthetic data we used in this example; only the *PP* component of the wavefield is shown. We focus on one of the regions in the data where primaries and internal multiples interfere, shown in Figure 2(b) (Offset: 500m–1500m, Time: 2.75s–3.05s). Figure 2(c) shows the actual primaries in the data in that region. Figures 2(d), 2(e), and 2(f) show the result after internal-multiple removal using the current ISS attenuation algorithm with (Figure 2(d)) and without (Figure 2(e)) adaptive subtraction, and the ISS elimination algorithm (Figure 2(f)) and without adaptive subtraction, respectively. Similar to the first example, the predicted multiple is assumed to only have a scalar difference with the actual multiple, and a single-channel matching filter for the adaptive-subtraction process is used. Examining the results and comparing them with the actual primaries in the data, we can see the elimination algorithm can effectively recover primaries which interfere with internal multiples, whereas the attenuation algorithm plus energy-minimization adaptive subtraction could damage primaries.

CONCLUSIONS

A three-pronged strategy is proposed by (Weglein, 2015) for dealing with the internal-multiple-removal challenges where internal multiples interfere with primaries. In this paper, we focus on the second aspect of the strategy (developing internal-multiple-elimination algorithms from ISS). The ISS internal-multiple-elimination algorithm provides a new capability of predicting exactly the time and amplitude of all first-order internal multiples to directly address the current industry challenge (proximal and interfering multiples and primaries) and surgically remove (eliminate) the multiples directly, and without subsurface information, and without damaging primaries. In this paper, we compare the new ISS internal-multiple-elimination algorithm and the industry-standard ISS internal-multiple-attenuation algorithm plus adaptive subtraction when primaries and internal multiples interfere and where we can evaluate the efficacy by using wave-theoretical accurate data consisting of only primaries. The tests on the 1D prestack data demonstrate this new capability to reduce our dependence on energy-minimization adaptive subtraction in the case of interfering primaries and internal multiples. We provide it as a new internal-multiple-removal capability in the multiple removal toolbox that can remove internal multiples that interfere with primaries without subsurface information and without damaging primaries.

In addition to the second aspect of the three-pronged strategy, progress has been made and will continue to be made for the other two aspects (developing the ISS prerequisites for predicting the reference wave field and producing de-ghosted data and developing a replacement for the energy-minimization criteria for adaptive subtraction). For example, a candidate for a replacement for energy-minimization adaptive subtraction for free-surface multiples is given in Weglein (2012). A similar adaptive criteria will be developed for internal multiples.

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