2D Green's theorem receiver deghosting in the (x-omega) domain using a depth-variable cable towards on-shore and ocean bottom application with variable topography

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SUMMARY

Green's theorem derived deghosting methods in the (x-ω) domain using flat cables have been successfully applied to synthetic and field data. Based on Green's theorem wavefield separation concepts, this paper derives a 2D acoustic receiver deghosting formula for a depth-variable cable assuming the shape of the cable is known. In numerical tests, the air-water boundary is assumed to be horizontal. We use the Cagniard-de Hoop method to generate synthetic data on parabolic cables and on periodically semicircular cables, respectively. The normal derivative of the total field on the cable is assumed to be known and is estimated by finite difference. Numerical results show that current Green's theorem up/down separation formula for a constant depth cable remains useful for a mildly depth-variable cable. When the actual cable deviates significantly from horizontal, the horizontal cable formula produces serious errors and artifacts whereas the new formula produces an effective and satisfactory result. While the analysis and tests in this paper are based on nonhorizontal towed streamers, the motivation (and future work) is for on-shore and ocean bottom acquisition. Under these circumstances, the deviation from horizontal acquisition can be significant and the ability to accommodate a variable topography can have a considerably positive impact on subsequence processing and interpretation objectives.

INTRODUCTION

Deghosting is a long-standing seismic objective and problem (Amundsen (1993); Robinson and Treitel (2008)). It removes downgoing events of the recorded field (receiver ghost) and events first going up from source to air-water boundary (source ghost). Seismic resolution can be enhanced by removing spectrum notches and boosting low frequencies. Also, deghosting has risen in importance as prerequisites for free surface and internal multiple removal as well as for the resolution and delineation of imaged-inverted primaries (Weglein et al. (2002)).

The problem of accounting for the amplitude and phase distortions introduced by the so-called ghost effect was first studied in the context of sources by Van Melle and Weatherburn (1953). They showed that by using more than one source with a delayed firing pattern, it was possible to mitigate the ghost effect. Based on this, different acquisition techniques were proposed to serve deghosting. These techniques include over/under streamers (Senneland et al. (1986); Posthumus (1993); Moldoveanu et al. (2007); Özdemir et al. (2008)), ocean bottom cable (OBC) (Barr et al. (1989)), hydrophone plus geophone (Carlson et al. (2007)) and multicomponent towed-streamers (Robertsson et al. (2008); Vassallo et al. (2013)). Other researchers started from signal characteristics of ghosts and designed specialized acquisition like single linearly slanted (Ray and Moore (1982); Dragoset and William (1991)) or depth-variable streamer (Soubaras et al. (2010)). Because the streamer is nonflat, notches diverse in time from one end of the streamer to the other. Therefore the ghost can be attenuated through stacking and/or migrating data from different parts of the streamer.

Based on these significant progresses in acquisition, different deghosting theories have developed. Green's theorem method for deghosting was first introduced by Weglein et al. (2002), Zhang et al. (2005), Zhang et al. (2006) and developed and tested by Zhang (2007). Tests on field data were done by Mayhan and Weglein (2013). Weglein et al. (2013a) modified this method to get rid of the need for normal derivative of pressure so as to avoid finite difference error. Tang (2014) analyzed the impact of acquisition on deghosting. For on-shore preprocessing, Wu and Weglein (2015a,b) derived elastic Green's theorem wavefield separation methods in pressure space and displacement space, respectively. As these studies demonstrate, the Green's theorem method has characteristics not shared by previous methods. It is consistent with inverse scattering series (ISS) wave theory methods that do not require subsurface information (Weglein et al. (2003)). As pointed out by Weglein et al. (2003), every ISS isolated-task subseries requires (1) the removal of the reference wavefield, (2) estimation of the source signature and radiation pattern, and (3) source and receiver deghosting, and how the ISS has a nonlinear dependence on these preprocessing steps. And Green's theorem can offer a number of useful algorithms (Zhang (2007); Mayhan (2013)) by choosing different reference medium according to different objectives (e.g. deghosting, P_0/P_s separation).

As pointed out by Mayhan and Weglein (2013) and Weglein et al. (2013b), deghosting methods derived from the representation theorem are wave-theoretic algorithms that can be defined in the frequency-space domain, and in principle can succeed in cables of any shape (e.g., slanted). The main objective of nonhorizontal measurement surface methods is to accommodate on-shore and ocean bottom acquisition where deviation from horizontal acquisition can frequently occur. Therefore, based on Weglein et al. (2002) and Zhang (2007), we derive a 2D acoustic receiver deghosting formula in the (x-ω) domain for depth-variable cable, assuming the sea surface is horizontal. Compared to current deghosting methods using seismic pressure wavefields recorded on nonhorizontal streamers (Riyanti et al. (2008); van Borselen et al. (2013); van Borselen et al. (2013); Robertsson and Amundsen (2014); Amundsen and Reitan (2014)), the algorithm in this paper can preserve both time and amplitude information very well.

THEORY

Figure 1 shows the principle of 2D offshore Green's theorem receiver deghosting. It requires a one-shot record. Airgun is at $\mathbf{r}_s$. The receivers (green triangles) lie on a measurement line which can be nonflat. $\mathbf{r}$ is where we want to predict deghosted data using pressure measured by these receivers. According to Green's theorem wavefield separation theory, it can be anywhere above the measurement line. Here we let $\mathbf{r}$ be below the source and not far away from the measurement line. According to the meaning of receiver ghost, the receiver deghosted data at $\mathbf{r}$ are exactly the total upgoing events of the field that we can observe at $\mathbf{r}$ without deghosting. In this way, receiver deghosting is equal to up/down separation.

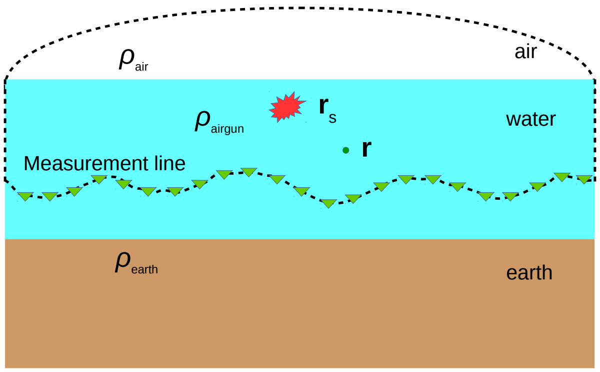

Figure 1: Configuration of Green's theorem receiver deghosting. The integration contour is the dashed line. ρ_{air}, $\rho_{airguns}$ and ρ_{earth} overlay the reference medium (whole space of water) to give the actual medium.

According to perturbation theory, if we choose the reference medium to be whole space of water (with acoustic speed c_0), there are three sources: medium perturbation ρ_{air} due to the difference between air and water, the airgun $\rho_{airguns}$, and medium perturbation ρ_{earth} due to the difference between earth and water. The upgoing wave at $\mathbf{r}$ represents the contribution due to ρ_{earth}. The reference wavefield is $A(\omega)G_0^+(\mathbf{r},\mathbf{r}_s,\omega)$, where $G_0^+(\mathbf{r},\mathbf{r}_s,\omega) = -\frac{i}{4}H_0^{(1)}(kR_+)$ is the 2D causal whole-space Green's function and $H_0^{(1)}$ is the zeroth order Hankel function of the first kind. $A(\omega)$ is source wavelet, $k = \omega/c_0$ and $R_+ = |\mathbf{r}-\mathbf{r}_s|$. From the view of scattering theory, all three sources actually generate outgoing wave straight away to the field point $\mathbf{r}$. Hence the total wavefield at $\mathbf{r}$ can be decomposed into three parts (see Zhang (2007)) using Green's theorem,

$$P(\mathbf{r},\mathbf{r}_s,\omega) - A(\omega)G_0^+(\mathbf{r}_s,\mathbf{r},\omega) - \int_S G_0^+(\mathbf{r}',\mathbf{r},\omega)k_0^2\alpha_{air}(\mathbf{r}')P(\mathbf{r}',\mathbf{r}_s,\omega)d\mathbf{r}'$$
$$= \oint_l [P(\mathbf{r}',\mathbf{r}_s,\omega)\nabla' G_0^+(\mathbf{r}',\mathbf{r},\omega) - G_0^+(\mathbf{r}',\mathbf{r},\omega)\nabla' P(\mathbf{r}',\mathbf{r}_s,\omega)]\cdot d\mathbf{l}' \quad (1)$$

Here we give the formula for 2D case, which can be proved using 2D Green's second identity. The second term on the left is reference wave. The third term represents the downgoing part of scattered wave, which is due to ρ_{air} and whose last motion is downward from the air-water boundary towards $\mathbf{r}$. Therefore, the RHS is the upgoing part of scattered wave, which is due to ρ_{earth} and is the receiver side deghosted wave at $\mathbf{r}$. In fact, we can prove that the line integration on the semicircular goes to zero as the radius goes to infinity, using Sommerfeld radiation condition. Hence the RHS reduces to a line integration,

$$P^{up}(\mathbf{r},\mathbf{r}_s,\omega)$$
$$= \int_{m.l.} [P(\mathbf{r}',\mathbf{r}_s,\omega)\nabla' G_0^+(\mathbf{r}',\mathbf{r},\omega) - G_0^+(\mathbf{r}',\mathbf{r},\omega)\nabla' P(\mathbf{r}',\mathbf{r}_s,\omega)]\cdot d\mathbf{l}'. \quad (2)$$

This is the fundamental receiver deghosting formula. It removes all downgoing wave including direct wave and its ghost, for $\mathbf{r}$ below $\mathbf{r}_s$. When the cable is horizontal, the gradient operator is just the derivative with respect to depth

$$P^{up}(\mathbf{r},\mathbf{r}_s,\omega)$$
$$= \int_{m.l.} [P(\mathbf{r}',\mathbf{r}_s,\omega)\frac{d}{dz'}G_0^+(\mathbf{r}',\mathbf{r},\omega) - G_0^+(\mathbf{r}',\mathbf{r},\omega)\frac{d}{dz'}P(\mathbf{r}',\mathbf{r}_s,\omega)]\cdot dl' \quad (3)$$

When the cable has some regular lateral variation in depth $z'(x')$, where z' is a depth function of offset x', we can derive the normal vector of the cable $\mathbf{n} = (-\frac{\frac{dz'(x')}{dx'}}{\sqrt{1+(\frac{dz'(x')}{dx'})^2}}, \frac{1}{\sqrt{1+(\frac{dz'(x')}{dx'})^2}})$. So there is a new formula

$$P^{up}(\mathbf{r},\mathbf{r}_s,\omega)$$
$$= \int_{m.l.} [P(\mathbf{r}',\mathbf{r}_s,\omega)(-\frac{\frac{dz'(x')}{dx'}}{\sqrt{1+(\frac{dz'(x')}{dx'})^2}}\frac{\partial}{\partial x'} + \frac{1}{\sqrt{1+(\frac{dz'(x')}{dx'})^2}}\frac{\partial}{\partial z'})$$
$$G_0^+(\mathbf{r}',\mathbf{r},\omega) - G_0^+(\mathbf{r}',\mathbf{r},\omega)\frac{\partial}{\partial n'}P(\mathbf{r}',\mathbf{r}_s,\omega)]dx'\sqrt{1+(\frac{dz'(x')}{dx'})^2} \quad (4)$$

where $\frac{\partial}{\partial n'}P(\mathbf{r}',\mathbf{r}_s,\omega)$ is normal derivative of $P(\mathbf{r}',\mathbf{r}_s,\omega)$ with respect to the cable. Numerical examples below test formula (4) and compare it with formula (3).

NUMERICAL EXAMPLES

Velocity model is shown in Figure 2. The velocities of water and earth are 1500m/s and 2250m/s, respectively. The water bottom and the air-water boundary are both horizontal. Two different cables will be used to illustrate the effectiveness of formula (4). 2D synthetic data are generated on the cable using the Cagniard-de Hoop method. The advantage of Cagniard-de Hoop method is that it can generate separately the direct wave and the primary reflected from the water bottom as well as their ghosts and free surface multiples of any order. The $\frac{\partial}{\partial n'}P(\mathbf{r}',\mathbf{r}_s,\omega)$ is estimated using finite difference with data generated on a secondary cable near the original cable.

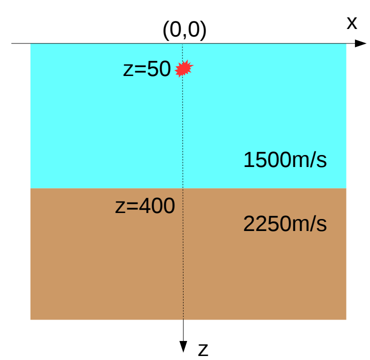

Figure 2: Velocity model and source position. The source is at (0,50). The water bottom is at 400m.

Parabolic Cable

The depth function of the cable is $z = 0.0004x^2 + 100$. The offset is from -400m to 400m. Figure 3a shows the synthetic data along the cable. The 6 events from top to bottom represent the direct wave, ghost of the direct wave, the primary reflected from the water bottom, source ghost of the primary, receiver ghost of the primary and source-receiver ghost of the primary, respectively. Figure 3b shows the synthetic upgoing wave including the primary and its source ghost at depth 95m, which is for comparison with separation result at 95m using Figure 3a. That means we can use Figure 3b to assess the deghosting results.

Figure 3c is the separated upgoing wave at 95m, assuming the cable is flat, i.e. using formula (3). Figure 3d shows the difference between Figure 3c and Figure 3b, using the same color scale as Figure 3c. We can see the difference is very small at near offset but obvious at far offset, especially for direct wave and its ghost. This is probably because the cable at near offset has slight depth gradient and hence the flat cable assumption is valid. Figure 3e and 3f show the separated upgoing wave at 95m using formula (4), and its difference with Figure 3b, respectively. We can see the difference is neglectable, and even the strong direct wave and its ghost are removed well.

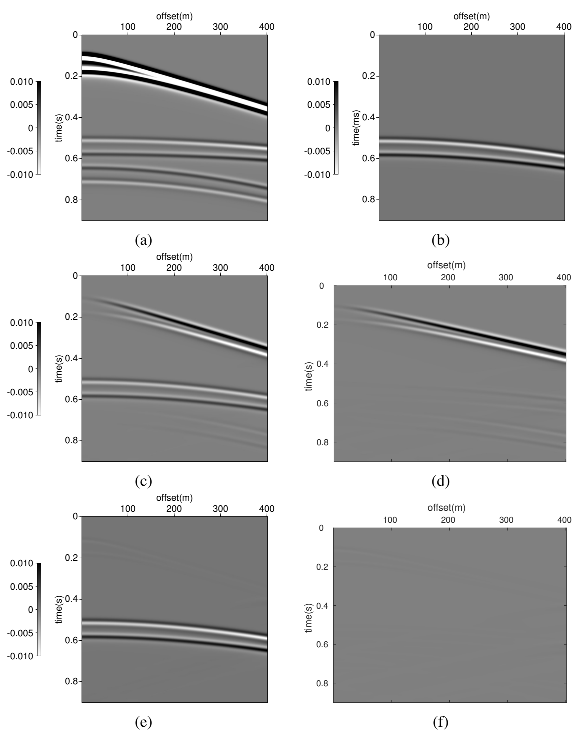

Figure 3: Wave separation results. (a)generated total wavefield by Cagniard-de Hoop method at the cable $z = 0.0004x^2 + 100$, (b)generated primary and it source ghost by Cagniard-de Hoop method at $z = 95m$ for comparison, (c)separated up wave at $z = 95m$ using formula (3), (d)difference between (c) and (b), (e)separated up wave at $z = 95m$ using formula (4), and (f)difference between (e) and (b)

Periodically Semicircular Cable

The subsection above tests formula (4) using a mildly depth-variable cable. The result is good. This subsection tests it using a fluctuating cable, as shown by Figure 4. The solid line is the primary cable with radius 50m, while the dash line is a shallower cable with radius 50.1m or 49.9m. Data are generated on both cables to provide pressure and its normal derivative.

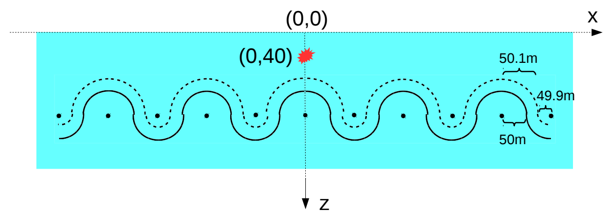

Figure 4: Configuration of the periodically semicircular cable.

Regular spatial sampling interval

Notice that some parts of the cable can be as steep as 90 degree. To avoid as much as possible the deviation of the cable shape in computer implementation, the spatial sampling interval is choosen to be 0.1m. In actual situation, such fine sampling can be achieved by interpolation. Figure 5 compares deghosting results using formula (3) and using formula (4). As shown by figure 5c and 5d, because of the cable varies greatly in depth with offset, formula (3) fails to remove downgoing wave and hence the receiver deghosting result is not satisfactory. Some crossing artifacts appear where there should be no energy events. In contrast, the deghosting result of formula (4) shown by figure 5e and 5f is much better, and only small residues of direct wave and its ghost remain. Receiver ghost and source-receiver ghost are basically removed.

Irregular spatial sampling interval

To further remove the residues of direct wave and its ghost in Figure 5f, smaller spatial sampling interval is adopted from offset 45m to 55m, 95m to 105m and so on, where the cable is very steep, as shown by Figure 6. Other parts of cables still adopt 0.1m interval. Figure 7 shows the deghosting result. We can see, the residues of direct wave and its ghost are removed as expected. This is because such fine spatial sampling provides more accurate dl' for the integration.

In the presence of free surface multiple

Finally, we test formula (4) using the same periodically semicircular cable in the presence of free surface multiples, which are very common in offshore data. For the velocity model shown by Figure 2, the Cagniard-de Hoop method can easily generate free surface multiples of any order. Here we just consider the first order, as shown by Figure 8a, since the energy of higher order multiples is neglectable. The top 2 events are direct wave and its ghost. The next 4 events are respectively the primary reflected from the water bottom, its source ghost, receiver ghost and source-receiver ghost. The 4 events at the bottom are the first order free surface multiples of the primary and its ghosts. Figure 8c and 8d show the deghosting result. We can see the receiver ghost, the source-receiver ghost as well as their free surface multiples are eliminated as expected. And the direct wave and its ghost are removed as well.

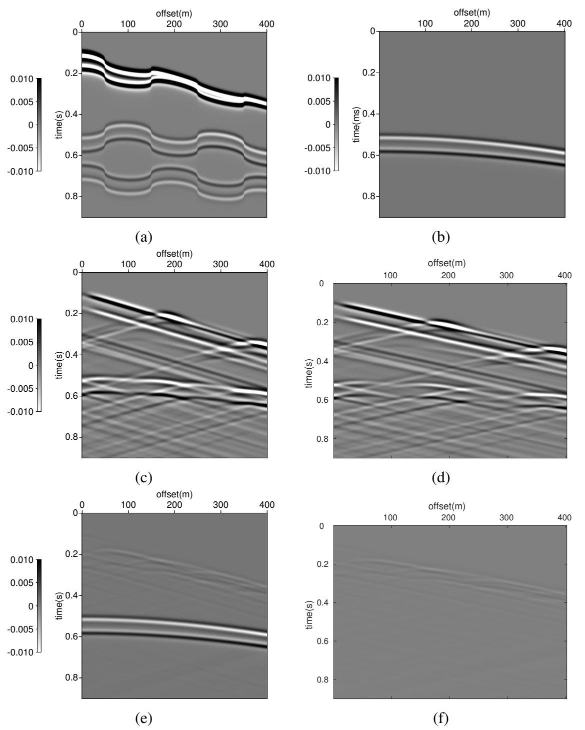

Figure 5: Wave separation results. (a)generated total wavefield by Cagniard-de Hoop method at the primary cable, (b)generated primary and it source ghost by Cagniard-de Hoop method at $z = 95m$ for comparison, (c)separated up wave at $z = 95m$ using formula (3), (d)difference between (c) and (b), (e)separated up wave at $z = 95m$ using formula (4), and (f)difference between (e) and (b)

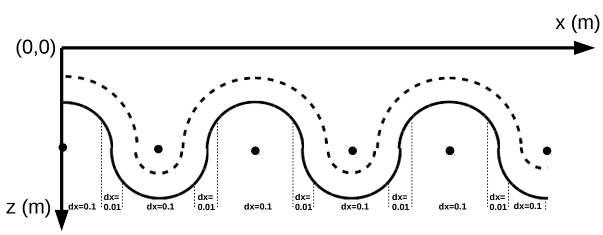

Figure 6: Irregular sampling configuration.

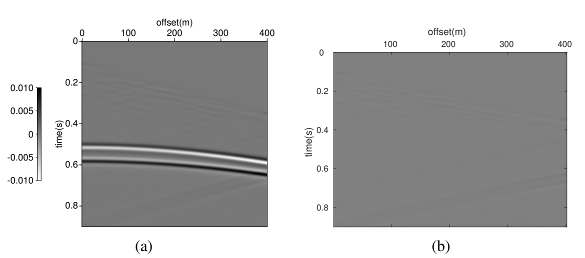

Figure 7: Wave separation results. (a)separated up wave at $z = 95m$ using formula (3), and (b)difference between (a) and Figure 5b

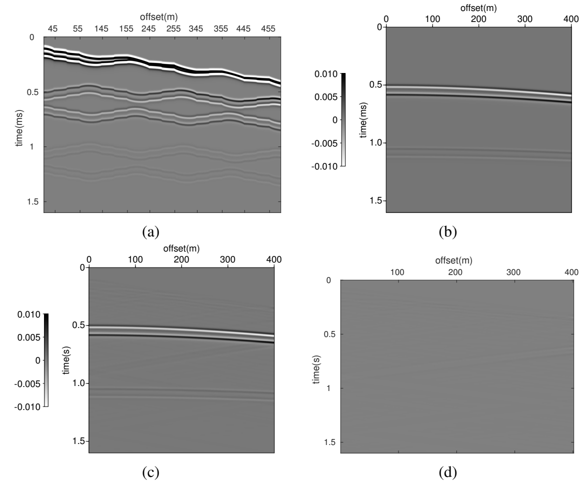

Figure 8: Wave separation results. (a)generated total wavefield by Cagniard-de Hoop method at the primary cable (b)generated primary and it source ghost by Cagniard-de Hoop method at $z = 95m$ for comparison, (c)separated up wave at $z = 95m$ using formula (4), and (d)difference between (c) and (b)

CONCLUSIONS

The Green's theorem deghosting algorithm is developed and tested for depth-variable towed streamers. This is very meaningful for ocean bottom preprocessing when the ocean bottom is not flat, and for on-shore preprocessing since the earth's surface is usually laterally variant. Numerical examples show that the constant depth assumption is valid when the cable is just slightly curved. The formula proposed in this paper avoids such limitation, by incorporating the cable's real depth function of offset. The case of periodically semicircular cable shows that spatial sampling interval is the main factor of a successful deghosting. Since the depth-variable cable version of Green's theorem takes the exact shape function into consideration, we can get satisfactory deghosting result even if the cable is steep. The last numerical case demonstrates that when free surface multiples exist, the free surface multiples can be deghosted. This is encouraging for subsequent processing including multiple attenuation. This would allow the benefit, from Green's theorem deghosting, for towed streamer data to be extended and utilized for on-shore and ocean bottom acquisition, where the measurement can at times be far from horizontal.

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REFERENCES

Amundsen, L., 1993, Wavenumber-based filtering of marine point-source data: Geophysics, **58**, 1335–1348.

Amundsen, L., and A. Reitan, 2014, Deghosting of marine pressure recordings along curved streamers: 76th Annual International Conference and Exhibition, EAGE, Extended Abstracts, Th P04 01.

Barr, F. J., J. I. Sanders, 1989, Attenuation of water-column reverberations using pressure and velocity detectors in a water-bottom cable: 59th Annual International Meeting, SEG, Expanded Abstracts, 653–656.

Carlson, D., A. Long, W. Söllner, H. Tabti, R. Tenghamn, and N. Lunde, 2007, Increased resolution and penetration from a towed dual-sensor streamer: First Break, **25**, 71–77.

Dragoset Jr, William H., 1991, Processing for seismic data from slanted cable: US Patent 4,992,992.

Mayhan, J., 2013, Wave-theoretic preprocessing to allow the inverse scattering series methods for multiple removal and depth imaging to realize their potential and impact: Ph.D. dissertation, University of Houston.

Mayhan, J. D., and A. B. Weglein, 2013, First application of Green's theorem-derived source and receiver deghosting on deep-water gulf of mexico synthetic (seam) and field data: Geophysics, **78**, WA77–WA89.

Moldoveanu, N., L. Combee, M. Egan, G. Hampson, L. Sydora, and W. Abriel, 2007, Seismic-over/under towed-streamer acquisition: A method to extend seismic bandwidth to both higher and lower frequencies: The leading edge, **26**, 41–58.

Morse, P. M., and H. Feshbach, 1953, Methods of theoretical physics: International Series in Pure and Applied Physics, New York: McGraw-Hill, 1953, **1**.

Özdemir, A. K., P. Caprioli, A. Özbek, E. Kragh, and J. O. Robertsson, 2008, Optimized deghosting of over/under towed-streamer data in the presence of noise: The Leading Edge, **27**, 190–199.

Posthumus, B., 1993, Deghosting using a twin streamer configuration: Geophysical Prospecting, **41**, 267–286.

Ray, C. H., and N. A. Moore, 1982, High resolution, marine seismic stratigraphic system: US Patent 4,353,121.

Riyanti, C. D., R. G. Van Borselen, P. M. van den Berg, and J. T. Fokkema, 2008, Pressure wave-field deghosting for non-horizontal streamers: 78th Annual International Meeting, SEG, Expanded Abstracts, 2652-2656.

Robertsson, J. O., and L. Amundsen, 2014, Wave equation processing using finite-difference propagators, part 2: Deghosting of marine hydrophone seismic data: Geophysics, **79**, T301–T312.

Robertsson, J. O., I. Moore, M. Vassallo, K. Özdemir, D. J. van Manen, and A. Özbek, 2008, On the use of multicomponent streamer recordings for reconstruction of pressure wavefields in the crossline direction: Geophysics, **73**, A45–A49.

Robinson, E. A., and S. Treitel, 2008, Digital imaging and deconvolution: The abcs of seismic exploration and processing: Society of Exploration Geophysicists.

Senneland, L., L. Berg, A. Haugen, B. Fotland, and J. Vestby, 1986, 2-D deghosting using vertical receiver arrays: 56th Annual International Meeting, SEG, Expanded Abstracts, 516–519.

Soubaras, R., 2010, Deghosting by joint deconvolution of a migration and a mirror migration: 80th Annual International Meeting, SEG, Expanded Abstracts, 3406–3410.

Tang, L., 2014, Developing and analyzing Green's theorem methods to satisfy prerequisites of inverse scattering series multiple attenuation for different types of marine acquisition and extending prerequisite satisfaction to on-shore exploration: Ph.D. dissertation, University of Houston.

Van Borselen, R. G., J. Fokkema, and P. van den Berg, 2013, Wavefield decomposition based on acoustic reciprocity: Theory and applications to marine acquisition: Geophysics, **78**, WA41–WA54.

Van Melle, F., and K. Weatherburn, 1953, Ghost reflections caused by energy initially reflected above the level of the shot: Geophysics, **18**, 793–804.

Vassallo, M., K. Eggenberger, D.-J. van Manen, A. Özbek, and P. Watterson, 2013, Broadband and beyond with marine towed streamers: The Leading Edge, **32**, 1356–1365.

Weglein, A., S. Shaw, K. Matson, J. Sheiman, R. Stolt, T. Tan, A. Osen, G. Correa, K. Innanen, Z. Guo, et al., 2002, New approaches to deghosting towed-streamer and ocean-bottom pressure measurements: 72th Annual International Meeting, SEG, Expanded Abstracts, 2114–2117.

Weglein, A. B., F. V. Araújo, P. M. Carvalho, R. H. Stolt, K. H. Matson, R. T. Coates, D. Corrigan, D. J. Foster, S. A. Shaw, and H. Zhang, 2003, Inverse scattering series and seismic exploration: Inverse problems, **19**, R27–R83.

Weglein, A. B., H. Liang, J. Wu, J. D. Mayhan, L. Tang, and L. Amundsen, 2013, A new Green's theorem deghosting method: Journal of Seismic Exploration, **22**, 413–426.

Weglein, A. B., J. D. Mayhan, L. Amundsen, H. Liang, J. Wu, L. Tang, Y. Luo, and Q. Fu, 2013, Green's theorem deghosting algorithms in (k, omega)(eg, p-vz deghosting) as a special case of (x, omega) algorithms (based on Green's theorem): Journal of Seismic Exploration, **22**, 389–412.

Weglein, A. B., and B. G. Secrest, 1990, Wavelet estimation for a multidimensional acoustic or elastic earth: Geophysics, **55**, 902–913.

Wu, J. and A. B. Weglein, 2015, Preprocessing in the PS space for on-shore seismic processing: 85th Annual International Meeting, SEG, Expanded Abstracts, 4740–4744.

Wu, J. and A. B. Weglein, 2015, Preprocessing in displacement space for on-shore seismic processing: 85th Annual International Meeting, SEG, Expanded Abstracts, 4626–4630.

Zhang, J., 2007, Wave theory based data preparation for inverse scattering multiple removal, depth imaging and parameter estimation: Analysis and numerical tests of Green's theorem deghosting theory: Ph.D. dissertation, University of Houston.

Zhang, J. and A. B. Weglein, 2005, Extinction theorem deghosting method using towed streamer pressure data: 75th Annual International Meeting, SEG, Expanded Abstracts, 2095-2098.

Zhang, J. and A. B. Weglein, 2006, Application of extinction theorem deghosting method on ocean bottom data: 76th Annual International Meeting, SEG, Expanded Abstracts, 2674-2678.