Green's theorem based onshore preprocessing – a reduced data requirement assuming a vacuum/earth model for the air/earth interface and the evaluation of the usefulness of that assumption

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SUMMARY

The elastic Green's theorem wave separation method can effectively reduce ground roll and ghosts in land data, and most importantly, without harming reflection data. Both tractions and multicomponent displacements are assumed in the original elastic Green's theorem method (Wu and Weglein, 2015a). However, it is not easy to measure tractions directly. In order to reduce the requirement of traction, this paper simplifies the algorithm assuming that the acquisition is on a vacuum/earth surface. The new formula requires the traction at only the source point rather than all the points along the measurement surface. The reason is that traction is zero everywhere except the place where the source is located. Synthetic tests show: (1) successful processing results for data that are generated assuming a vacuum/earth boundary, and (2) very useful results when the data are generated from a more realistic air/earth model.

INTRODUCTION

Onshore seismic exploration and processing seek to use reflection data (the scattered wavefield) to make inferences about the subsurface. However, besides the reflection data, the measured total wavefield also contains direct wave and surface wave (also called ground roll), which can interfere with reflections. As a main type of coherent noises, ground roll dominates the energy of measured data, and it can seriously mask the reflections. Therefore, ground roll removal is an essential processing step for land data. The current ground roll removal methods, e.g., filtering methods in the frequency-wavenumber (ω, k_x) domain or the frequency-offset (ω, x) domain (Yilmaz, 2001), may damage the reflection data, particularly when ground roll interferes with the reflection. In addition, for buried sources or buried receivers, not only are upgoing waves in the reflection data, but also ghosts, which correspond to the downward reflection at air/earth boundary. On one hand, ghosts can cause notches at very low frequencies if measurements are deeply beneath the air/earth surface; on the other hand, they can seriously interfere with the up waves when the measurement surface is close to the air/earth surface. The upgoing reflected data are usually pursued for subsequent processing and imaging. Ground roll removal and deghosting problems are discussed in this paper.

As a flexible and useful tool, Green's theorem provides a method to achieve these two goals – removal of the ground roll and removal of the ghosts, without damaging the upgoing reflected data. The distinct advantages of applying the wave-separation method based on Green's theorem have been demonstrated by Weglein et al. (2002); Zhang (2007); Mayhan and Weglein (2013); Tang et al. (2013); Yang et al. (2013). The application of this method is mature for marine exploration. In a further step, we extended this method to accommodate elastic land data, where ground roll becomes a strong noise (Wu and Weglein, 2014, 2015a,b).

For onshore plays, the extended elastic Green's theorem method is applicable for displacement data (Pao and Varatharajulu, 1976; Weglein and Secrest, 1990; Wu and Weglein, 2015a). Both the ground roll and the receiver-side ghosts can be removed in one step, when a homogeneous elastic whole space is chosen as the reference medium. However, the method demands the multicomponent traction and displacement as input, in which the traction is not directly available in general. This requirement limits the practical application of this method.

In this paper, we get rid of the reliance of traction for wider application of this method in practice. The original formula can be modified to ask for the traction only at the source point instead of all the points along the measurement surface, assuming (1) the world to be a half space of vacuum over a half space of earth, and (2) the acquisition is at vacuum/earth boundary. The traction at the source point can be calculated by the displacement measurements at a point and at neighboring points, using the stress-displacement relationship. One extra geophone put below the source point can support the calculation of displacement gradient.

The numerical tests on vacuum/elastic-earth data, which is consistent with the assumption, show promising results that remove ground rolls and ghosts, and, meanwhile, restore the upgoing reflection wavefield. Furthermore, the method also provides positive results for air/elastic-earth data, which disagree with the assumption but can be considered as a more realistic situation. This success gives us confidence to apply the elastic Green's theorem wave separation method to field experiment.

ELASTIC GREEN'S THEOREM WAVE SEPARATION THEORY

Figure 1 shows a generic onshore model consisting of an air half-space and an elastic-earth half-space. Receivers are buried in the earth, and source (e.g., vibroseis) in the form of a force is applied on the free surface (F. S.). Therefore, ghosts exist at the receiver side only. The measurement surface (M. S.) can be close to the free surface as in the case of on-surface-receiver acquisition; it can be also meters below the free surface for buried receivers. However, the receivers are coupled with the elastic medium for both situations. Assuming that the portion of earth along the measurement surface is homogeneous and known, the reference medium can be chosen as a homogenous elastic whole space (Figure 2), whose property agrees with the actual earth along the measurement surface.

There are three sources (Figure 3) acting on the selected homogeneous reference medium: the energy source (the force S_1), and two passive sources (the perturbations S_2 and S_3). S_1 and S_2 collaborate to produce the ground rolls, the direct wave and

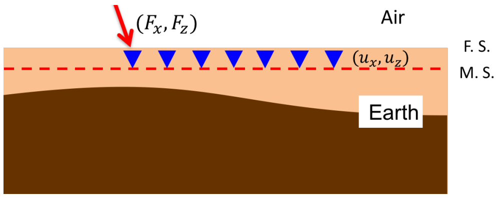


Figure 1: A generic model describing the onshore experiment. The blue triangles represent the receivers.

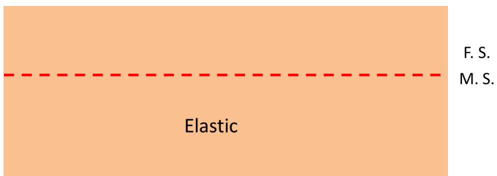


Figure 2: A homogeneous whole-space reference medium.

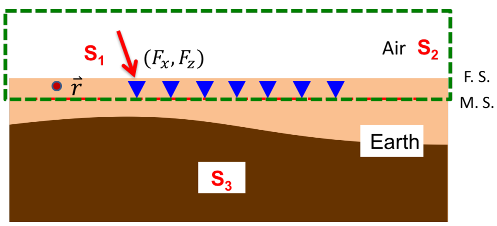


Figure 3: Three sources act on the reference medium, and the surface integral will extract the contributions from S_3, for evaluation point $\vec{r}$ inside the volume.

the receiver ghosts. S_3 generates upgoing reflection from the earth. The upgoing waves due to S_3 are expected to be separated out from those produced by both S_1 and S_2. Choosing a closed semi-infinite surface bounded below by the measurement surface, and evaluating the surface integration inside the volume, then the portion of the wavefield due to the source outside the volume can be produced; i.e., the contribution of S_3, which is the upgoing wave, can be predicted inside the volume.

The Green's theorem wave separation formula is

$$\vec{u}^{up}(\vec{r},\omega)$$
$$= -\int_{m.s.} [\vec{t}(\vec{r}',\omega)\cdot \mathbf{G}_0(\vec{r}',\vec{r},\omega) - \vec{u}(\vec{r}',\omega)\cdot(\hat{n}\cdot\Sigma_0(\vec{r}',\vec{r},\omega))]d\vec{r}', \tag{1}$$

where $\vec{r}'$ is receiver point, $\vec{r}$ is evaluation point, $\vec{u}$ is the displacement, and $\vec{t}$ is the traction along the measurement surface; $\mathbf{G}_0$ is the Green's displacement tensor, Σ_0 is the Green's stress tensor; $\vec{u}^{up}$ is the predicted upgoing wave, and $\hat{n}$ is the normal outside vector along the measurement surface.

By applying Equation 1, we can remove ground rolls, direct waves, and ghosts simultaneously. There are three important points with respect to Equation 1: (1) No source information is required, neither amplitude nor radiation pattern. (2) The integrals are carried out at receiver side only and work on one experiment at a time, resulting in relatively small data requirement and low computational cost. (3) There is no assumption about the shape of the measurement surface – it can be flat, inclined, or undulating.

FORMULA SIMPLIFICATION

Equation 1 calls for both the multicomponent displacements and traction. We derive the simplification of the equation in this section with the assumption that the experiment works on a vacuum/elastic-earth model rather than air/elastic-earth, and measurements are on the free surface (Figure 4).

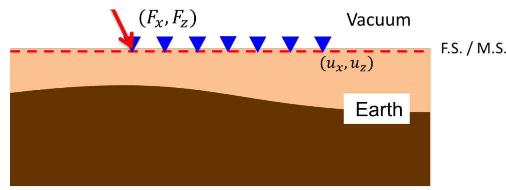


Figure 4: A vacuum/earth experiment at the free surface.

With force equilibrium relationship,

$$\vec{t}(x',z'=0,\omega) = -\vec{F}(\omega)\delta(x'-x_s)/\sqrt{1+(dz'/dx')^2|_{x'=x_s}}, \tag{2}$$

where (x_s,z_s) is the source location, and $\sqrt{1+(dz'/dx')^2|_{x'=x_s}}$ is a factor related to the shape of free surface at the source point. Substituted it into Equation 1, the wave separation formula is transformed to be

$$\vec{u}^{up}(\vec{r},\omega) = \vec{F}(\omega)\cdot\mathbf{G}_0(\vec{r}_s,\vec{r},\omega) + \int_{m.s.} \vec{u}(\vec{r}',\omega)\cdot(\hat{n}\cdot\Sigma_0(\vec{r}',\vec{r},\omega))d\vec{r}', \tag{3}$$

The first integral in Equation 1 turns out to be a product, and only the traction at the source point is required, which is an immediate data requirement reduction.

TRACTION COMPUTATION AT SOURCE POINT

With the definition of stress,

$$\vec{T} = \hat{n}\cdot\tau, \tag{4}$$

where τ is the stress tensor. Considering an isotropic medium, and applying Hooke's law,

$$\tau = \lambda\nabla\cdot\vec{u}\,\mathbf{I} + \mu(\nabla\vec{u}+\vec{u}\nabla), \tag{5}$$

where λ and μ are Lamé's parameters of the elastic medium.

The derivatives of the multicomponent displacements at the source points are required along both x- and z-directions to compute the traction there. The neighboring geophones can be used for the calculation of x-direction; whereas, an extra geophone below the source point can be used to measure displacement, and to facilitate the calculation of z-direction.

NUMERICAL EVALUATION ON A VACUUM/EARTH MODEL

To evaluate the simplified formula of Equation 3, we conduct three experiments, with different inputs as listed in Table 1. $\surd$ represents the input is available at all the receiver points. We expect the same result from the first two experiments, since the traction is always zero except at the source point for these experiments.

Experiment	Displacement	Traction
1	$\surd$	$\surd$
2	$\surd$	one at source
3	$\surd$	none

Table 1: Inputs for the experiments and comparisons.

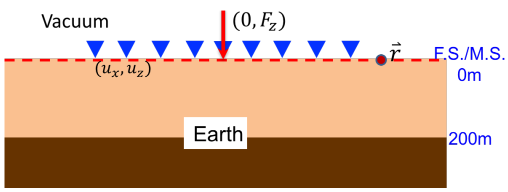


Figure 5: A vacuum/elastic-earth model for the numerical test.

Layer's Number	P Velocity (m/s)	S Velocity (m/s)	Density (kg/m^3)
1	1000	500	1500
2	3000	1800	1800

Table 2: The parameters of the model in Figure 5.

Figure 5 describes the specific vacuum/earth model, with parameters shown in Table 2. All the source and receivers are on the free surface, which agree with the new formula's assumption. The output point **r** is arranged on the measurement surface to be part of the volume above, by implementing the formula in the (k_x, ω) domain (see, e.g., Weglein et al. (2013)). With a vertical force as the excitation source, the product of Equation 3 equals to $(0, F_z * G_{zz})$, as $G_{zx}(x_s, x, z_s = z = 0) = 0$. Thereby, the different inputs of traction can only affect the results of z component, but not x component.

With receivers at the free surface, the displacements (see Figure 6(a) for x component of total wave and Figure 6(e) for z component) display a strong Rayleigh waves and a relatively weak reflected waves. In addition, The interference between the ghosts and the upgoing waves starts at zero offset. Figure 6(b,c,d) show results of x component, with the three different traction inputs, which are tractions at all receiver points, at only the source point, and not available at all, respectively. All effectively predict the x component of upgoing waves, which agree with the previous analysis; i.e., for a vertical source experiment, the wave separation results along x direction are independent of traction. On the other hand, the first two experiments (Figure 6(f,g)) both produce satisfying results for z component. However, the third one (Figure 6(h)) that assumes the traction is zero everywhere leaves a strong residual, particularly for ground roll. The comparisons demonstrate that the traction information at the source point is necessary and adequate to produce an accurate wave separation result.

EVALUATION ON AN AIR/EARTH MODEL

Problem description

The previous evaluation confirms the effectiveness of the modified wave-separation formula in removing both ground roll and ghosts, assuming a vacuum/elastic-earth model and a free-surface acquisition. However, the real world is better to be modeled as air/elastic-earth. A further evaluation of the reliability to solve a realistic problem using this new formula is necessary from a practical point of view; i.e., the usefulness of a vacuum/earth-interface assumption applying to an air/earth world should be examined.

Numerical evaluation

Another three experiments (settled in Table 1) are carried out, with parameters in Table 3. Both source and receivers are on the air/earth surface, and the output point is on the surface to be part of the volume above. Due to the reality that the traction is not zero at air/earth boundary, these three experiments lead to three different assumptions. The first one uses the traction everywhere along the measurement surface, assuming an air/earth model; the second one uses the traction only at the source point, assuming a vacuum/earth model; the last one doesn't use traction at all, assuming traction is zero everywhere.

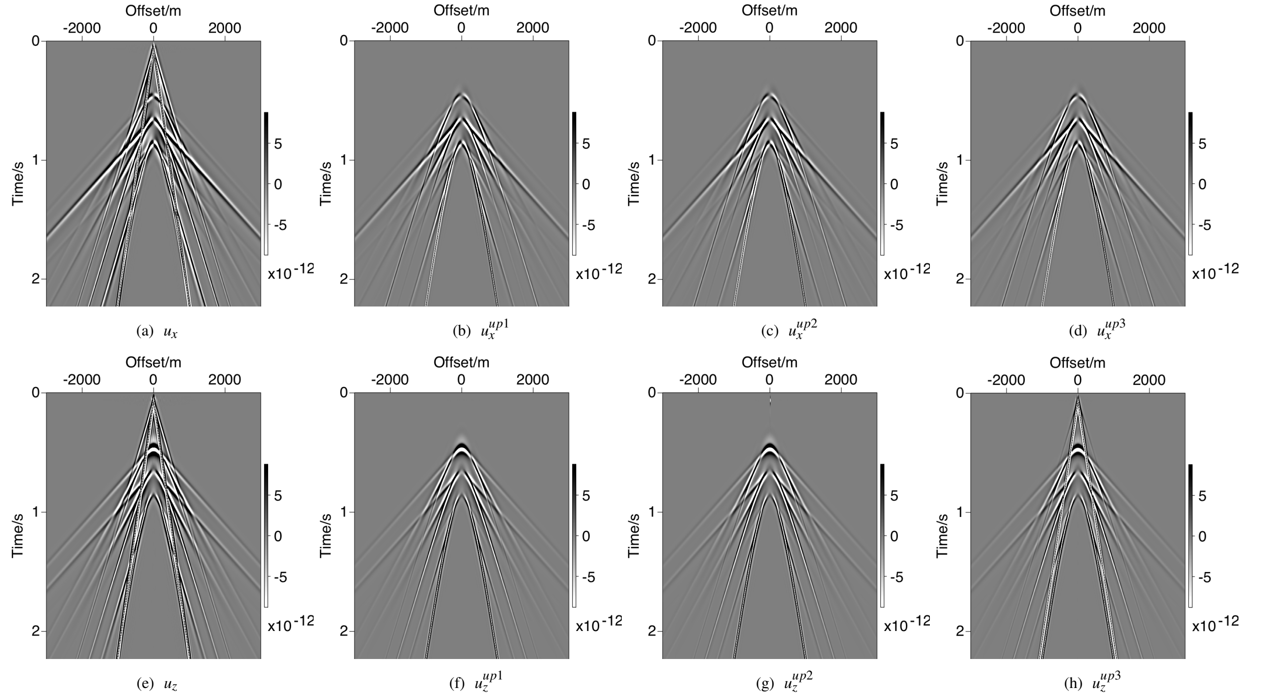
Layer's Number	P Velocity (m/s)	S Velocity (m/s)	Density (kg/m^3)
1	340	0	3
2	700	400	600
3	1500	800	1000

Table 3: The parameters of the air/earth model.

The results of onshore components can be seen in Figure 7. The first one produces the most accurate result; the second has a small residual for the ground roll at near offset; the last one shows a very strong residual. We select the traces at offset 400 m from Figure 7 (b,c,d) for a further comparison. The single-trace plot (in Figure 8) concludes that the separation result from the third experiment (black line) is not accurate; whereas the result from the second one (blue line) is much closer to the first experiment (red line), which is most accurate. The error analysis by averaging over all offsets shows that the second one has an error of 4%, while the third with 76%. The results of x component are not shown in this example, because the accurate results can be produced by all these experiments for a vertical force excitation.

CONCLUSION

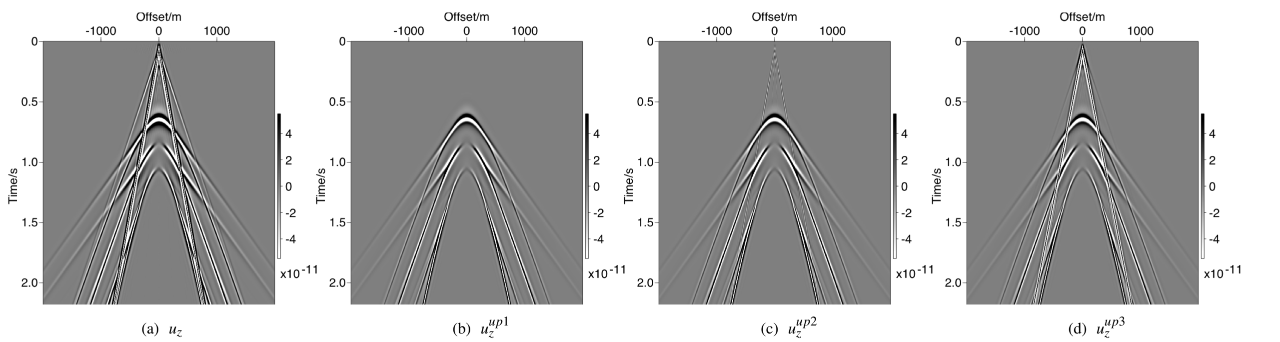
The elastic Green's theorem based wave separation method has the potential to remove ground roll and ghosts for onshore exploration. The original algorithm demands both displacements and tractions. In this paper, we obtain a simplified formula



(a) u_x (b) u_x^{up1} (c) u_x^{up2} (d) u_x^{up3}

(e) u_z (f) u_z^{up1} (g) u_z^{up2} (h) u_z^{up3}

Figure 6: Wave separation on vacuum/earth model. (a) for x component total wave; (b) for x component separated up wave from case 1; (c) for x component result from case 2; (d) for x component result from case 3; (e–h) for corresponding z component results.



(a) u_z (b) u_z^{up1} (c) u_z^{up2} (d) u_z^{up3}

Figure 7: Wave separation on air/earth model for z component. (a) for total wave; (b) for separated up wave from case 1; (c) for the result from case 2; (d) for the result from case 3.

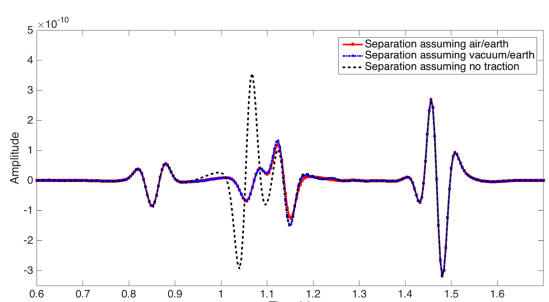


Figure 8: Comparison at offset 400 m of Figure 7 (b,c,d).

that requires displacements along the surface and traction only at the source point, using a vacuum/earth model and putting measurements at vacuum/earth surface. The traction at the source point can be calculated by the displacement measurements at a point and at neighboring point. One extra geophone below the source can assist the computation. Although the new formula is derived under vacuum/earth-model assumption, the synthetic test indicates that it is still valuable for an air/earth model, which is closer to the real exploration environment.

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