A new Inverse Scattering Series (ISS) internal-multiple-attenuation algorithm that predicts the accurate time and approximate amplitude of the first-order internal multiples and addresses spurious events: Analysis and Tests in 2D

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SUMMARY

The ISS internal-multiple-attenuation algorithm assumes that the free-surface multiples have been removed from the input of this algorithm, and the input consists of only primaries and internal multiples. The algorithm selects three events by a "longer-shorter-longer" relationship in the *vertical-travel-time* domain and the *primaries* selected in that procedure predict the accurate time and approximate amplitude of all first-order internal multiples without any subsurface information (Weglein et al., 2003). However, the input data contain both *primaries* and *internal multiples*. When internal multiples themselves are selected in that procedure, two different types of events will be produced. The first type is higher-order internal multiples (e.g., second-order internal multiples (Zhang and Shaw, 2010)), and these predicted higher-order internal multiples will *cooperatively assist and benefit* the attenuating of higher-order internal multiples in the data. The second type is spurious events (events that do not exist in the data). For the second type of events, Weglein et al. (2011), Liang et al. (2013) and Ma and Weglein (2014) show that the spurious events are *fully anticipated* by the ISS, and specific higher-order terms from ISS will precisely address that spurious-event issue. The inclusion of higher-order terms provides a new ISS internal-multiple-attenuation algorithm that does not generate the spurious events and, at the same time, retains the strength of the original algorithm. That original contribution (i.e., the aforementioned new algorithm) was for a one dimensional subsurface. In this paper, we extend the previous work on addressing the spurious events to a multi-D case and show more realistic synthetic test results in 2D. Those tests exemplify the relevant and practical benefit provided by this new internal-multiple algorithm.

INTRODUCTION

The inverse scattering series (ISS) communicates that it is possible to achieve all seismic data processing objectives directly and without subsurface information. The current ISS internal-multiple-attenuation algorithm was first developed by Araujo et al. (1994) and Weglein et al. (1997). The unique strength (always present independent of the circumstances and complexity of the geology and the play) of the ISS internal-multiple-attenuation algorithm is that this algorithm is able to predict internal multiples without any subsurface information. Hence, the ISS internal-multiple-attenuation algorithm is often called upon in the cases in which the multiple-removal is a challenging problem and it is difficult to find the subsurface information for other multiple-suppression methods to be effective. The tests on ISS internal-multiple-attenuation algorithm have shown promising results and unique value compared with other multiple-suppression methods (e.g., K.Maston et al. (1999); Fu et al. (2010); Hsu et al. (2010); Ferreira (2011); Terenghi et al. (2011); Luo et al. (2011); Weglein et al. (2011); Kelamis et al. (2013)).

Early analysis of the current ISS internal-multiple-attenuation algorithm focused on selecting primaries in the input to predict internal multiples. However, the input data contain both primaries and internal multiples and all events in the data will be selected. Internal multiples selected in this algorithm can generate spurious events under the circumstances where three or more reflectors are involved in the data being processed as shown by Weglein et al. (2011), Liang et al. (2013) and Ma and Weglein (2014).

The work of Ma and Weglein (2014) also demonstrates that spurious-event issue is serious and significant when there are tens, hundreds (or even thousands) of internal-multiple generators (e.g., Middle East and North Sea), and addressing the spurious events under those circumstances is essential by applying a new ISS algorithm that addresses the spurious-event prediction and retains the strength of the original algorithm simultaneously. In this paper, we extend the analysis of addressing the spurious events to a multi-D case, and also provide a more realistic numerical test in 2D.

THE CURRENT ISS INTERNAL-MULTIPLE-ATTENUATION ALGORITHM

The current ISS internal-multiple-attenuation algorithm starts with the input data, $D(k_g, k_s, \omega)$, in 2D case, which are the Fourier transform of the deghosted prestack data, and with the wavelet deconvolved and direct wave and free-surface multiples removed. The second term, $D_3(k_g, k_s, \omega)$, is the attenuator of the first-order internal multiples. In a 2D earth, $D_3(k_g, k_s, \omega)$ is obtained from $b_3(k_g, k_s, \omega) = -2iq_s D_3(k_g, k_s, \omega)$, where $b_3(k_g, k_s, \omega)$ is (Weglein et al., 2003)

$$b_3(k_g, k_s, q_s + q_s) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 e^{-iq_1(z_g - z_s)} e^{iq_2(z_g - z_s)}$$
$$\times \int_{-\infty}^{\infty} dz_1 b_1(k_g, k_1, z_1) e^{i(q_g + q_1)z_1}$$
$$\times \int_{-\infty}^{z_1 - \varepsilon} dz_2 b_1(k_1, k_2, z_2) e^{-i(q_1 + q_2)z_2}$$
$$\times \int_{z_2 + \varepsilon}^{\infty} dz_3 b_1(k_2, k_s, z_3) e^{i(q_2 + q_s)z_3}, \quad (1)$$

where k_s and k_g are the horizontal wavenumbers for the source and receiver coordinates, respectively; q_g and q_s are the vertical source and receiver wavenumbers defined by $q_i = sgn(\omega)\sqrt{\frac{\omega^2}{c_0^2} - k_i^2}$ for $i \in \{g, s\}$ (ω is the temporal freqency);

z_s and z_g are source and receiver depths; and z_j ($i \in \{1,2,3\}$) represents pseudo-depth by using a reference velocity migration. The quantity $b_1(k_g, k_s, z)$ corresponds to an uncollapsed migration (Weglein et al., 1997) of effective plane-wave incident data.

The data with their first-order internal multiple attenuated are

$$D(k_g, k_s, \omega) + D_3(k_g, k_s, \omega). \tag{2}$$

For a 1-D earth and a normal incident plane wave, equation 1 reduces to

$$b_3(k) = \int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} b_1(z_2)$$
$$\times \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} b_1(z_3). \tag{3}$$

The deghosted data, $D(t)$, for an incident plane wave, satisfy $D(\omega) = b_1(\frac{2\omega}{c_0})$, $D(\omega)$ is the temporal Fourier transform of $D(t)$, $b_1(z) = \int_{-\infty}^{\infty} e^{ikz} b_1(k) dk$, and $k = \frac{2\omega}{c_0}$ is the vertical wavenumber.

Equation 2 then reduces to

$$D(t) + D_3(t), \tag{4}$$

where $D_3(t)$ is Inverse Fourier transform of $D_3(\omega)$, and $D_3(\omega) = b_3(\frac{2\omega}{c_0})$, where $k = \frac{2\omega}{c_0}$.

Figure 1 illustrates the idea behind using equation 1 or equation 3 to predict the first-order internal multiple by selecting primaries (events that experience only one upward reflection) in the data as subevents, and combining different subevents that satisfy the "longer(A)-shorter(B)-longer(C)" relationship in vertical-travel-time domain (or equivalently, "lower(A)-higher(B)-lower(C)" relationship in pseudo-depth domain (Nita and Weglein, 2007)).
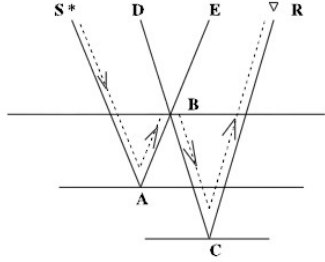

Figure 1: An internal multiple (dashed line) constructed by the lower-higher-lower pattern of three primary subevents (solid line). Figure adapted from Weglein et al. (2003)

THE ORIGIN OF SPURIOUS EVENTS AND ITS RESOLUTION IN 1D

The work of Araujo et al. (1994) and Weglein et al. (1997) focuses on the analysis of the prediction of first-order internal multiples (i.e., equation 1) by using primaries in the data as
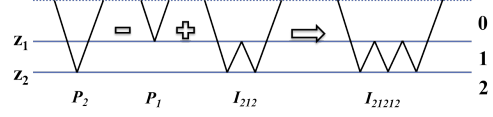

Figure 2a: In a two-reflector example, a "Primary – Primary – Internal multiple" combination predicts a second-order internal multiple.
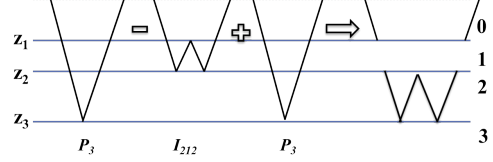

Figure 2b: In a three-reflector example, a "Primary – Internal multiple – Primary" combination predicts a spurious event.

subevents (see Figure 1). However, data consist of both primaries and internal multiples. Zhang and Shaw (2010) show that higher-order internal multiples will be predicted by b_3 when internal multiples themselves are selected as a subevents in a two-interface case. For example, in Figure 2a, a second-order internal multiples will be predicted when a first-order internal multiple is selected as a subevent.

In addition, the situation is considerably more complicated when the data from three or more reflectors are considered. For instance, spurious events can also be generated when an internal multiple is selected as a subevent in a three-reflector example, as shown in Figure 2b. However, these spurious events are also entirely anticipated by the inverse scattering series and there are terms in the series that can exactly address those false event prediction.

After identifying all the terms in the series that address the spurious events, a new ISS internal-multiple-attenuation algorithm is provided by including those terms. The new algorithm boils down as follows,

$$D(t) + D_3^{New}(t), \tag{5}$$

where $D_3^{New}(t)$ is the Inverse Fourier transform of $D_3^{New}(\omega)$, $D_3^{New}(\omega) = b_3^{New}(k = \frac{2\omega}{c_0})$, and $b_3^{New}(k)$ is obtained from

$$b_3^{New}(k) = \int_{-\infty}^{\infty} dz_1 e^{ikz_1} (b_1(z_1) + b_3(z_1))$$
$$\times \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} (b_1(z_2) + b_3(z_2))$$
$$\times \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} (b_1(z_3) + b_3(z_3)). \tag{6}$$

The new algorithm will address the spurious events by reducing the internal multiples using $(b_1(z) + b_3(z))$ as the new input.

$$D(x_g, x_s; t)$$

① (step 1)

$$b_1(k_g, k_s, z)$$

② (step 2)

$$b_3(k_g, k_s, q_g + q_s) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 e^{iq_1(\varepsilon_g - \varepsilon_s)} dk_2 e^{iq_2(\varepsilon_g - \varepsilon_s)}$$
$$\times \int_{-\infty}^{\infty} dz_1 e^{i(q_g + q_1)z_1} b_1(k_g, k_1, z_1)$$
$$\times \int_{-\infty}^{z_1 - \varepsilon} dz_2 e^{iq_1(-q_1 - q_2)z_2} b_1(k_1, k_2, z_2)$$
$$\times \int_{z_2 + \varepsilon}^{\infty} dz_3 e^{i(q_2 + q_s)z_3} b_1(k_2, k_s, z_3)$$
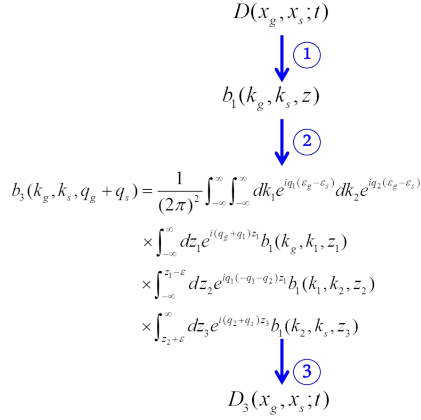
③ (step 3)

$$D_3(x_g, x_s; t)$$

Figure 3a: In a 2D case, the workflow of the current ISS internal-multiple-attenuation algorithm. Step 1 is the uncollapsed Stolt migration, step 2 is the prediction from the attenuator of the first-order internal multiples, step 3 transforms the prediction back to space-time domain.

A NEW ISS INTERNAL-MULTIPLE-ATTENUATION ALGORITHM THAT PREDICTS THE ACCURATE TIME AND APPROXIMATE AMPLITUDE OF THE FIRST-ORDER INTERNAL MULTIPLE AND ADDRESSES SPURIOUS EVENTS

By understanding the generation of the spurious events and its resolution in 1D case, we provide a new multi-D ISS internal-multiple-attenuation algorithm (Figure 3b) that addresses the spurious events and preserves the strength of the current algorithm. For the purpose of comparison, we show the current ISS internal multiple attenuation algorithm in Figure 3a.

We test the new algorithm using a synthetic 2D data set. Figure 4 shows a three-reflector model used to generate the 2D synthetic data set by finite-difference method. The data consist of 251 shots × 251 receivers, with both shot- and receiver-interval 25 m, each trace has 500 samples with a total duration 4s. The internal multiples will be strong because of the big impedance contrast between layers. Figure 5 and 6 show one shot and trace comparison between the test data and the prediction results **with** (Figure 5b and 6b) and **without** (Figure 5a and 6a) addressing the spurious-event prediction. In Figure 5, black and red arrows point to the primaries and internal multiples, respectively. The numbers in the subscript indicate the reflectors where the reflection happens. The blue arrows in Figure 5 indicate the places where the prediction result **with** the addressing of spurious events better matches the test data than that **without** addressing the spurious events (see black arrows in Figure 6 for details). Compared with Figure 5a (or 6a), the prediction of higher-order internal multiples (e.g., I_{21212}) in Figure 5b (or 6b) gets reduced (see red arrows in Figure 6 for details) because of the reduced internal multiple in the input data.

$$\boxed{D_3(x_g, x_s; t) \quad + \quad D(x_g, x_s; t)}$$

① (step 1)

$$b_1^{New}(k_g, k_s, z)$$

② (step 2)

$$b_3^{New}(k_g, k_s, q_g + q_s) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 e^{iq_1(\varepsilon_g - \varepsilon_s)} dk_2 e^{iq_2(\varepsilon_g - \varepsilon_s)}$$
$$\times \int_{-\infty}^{\infty} dz_1 e^{i(q_g + q_1)z_1} b_1^{New}(k_g, k_1, z_1)$$
$$\times \int_{-\infty}^{z_1 - \varepsilon} dz_2 e^{iq_1(-q_1 - q_2)z_2} b_1^{New}(k_1, k_2, z_2)$$
$$\times \int_{z_2 + \varepsilon}^{\infty} dz_3 e^{i(q_2 + q_s)z_3} b_1^{New}(k_2, k_s, z_3)$$
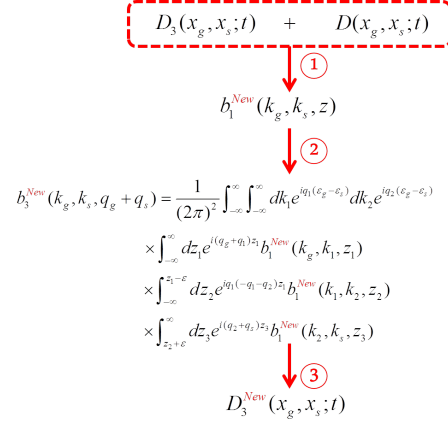
③ (step 3)

$$D_3^{New}(x_g, x_s; t)$$

Figure 3b: In a 2D case, the workflow of the new ISS internal-multiple-attenuation algorithm with the addressing of the spurious events. The steps are same as in Figure 3a, but using a new input. $D_3(x_g, x_s; t)$ is the output from the current algorithm in Figure 3a.
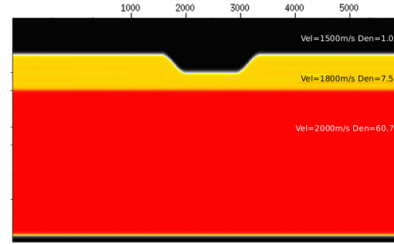

Figure 4: Synthetic velocity and density model used to generate the test data in this section (courtesy of WesternGeco). The average dip of the walls of the trench featuring in the center of the model is approximately 20 degree (Figure adapted from Terenghi and Weglein (2011)).

CONCLUSIONS

In this paper, we analyze, develop and test a new multi-D ISS internal-multiple-attenuation algorithm that anticipates and removes the spurious events that are generated by the current ISS internal-multiple-attenuation algorithm. The numerical test on a synthetic 2D data set in this paper shows the added value of applying the new algorithm to address the spurious events that can be significant many real-world applications. This issue will arise when many reflectors generate the multiples in complex on-shore and off-shore plays, and can be serious impediment to interpretation and making effective drilling decisions. The new algorithm in this paper addresses that issue.
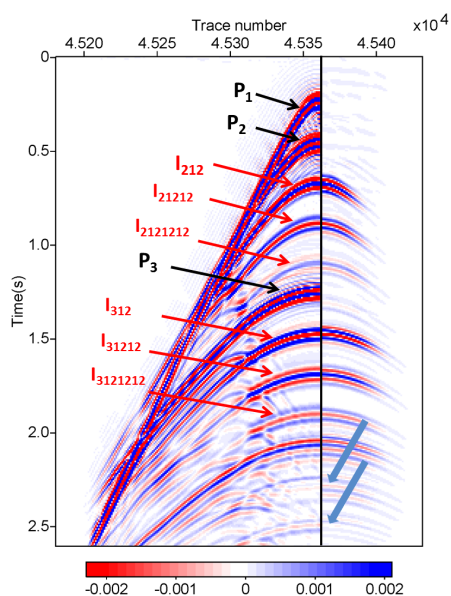
ACKNOWLEDGMENTS

Figure 5a: A shot comparison between the test data (left part) and the ISS internal multiple prediction (right part) **without** addressing the spurious events.
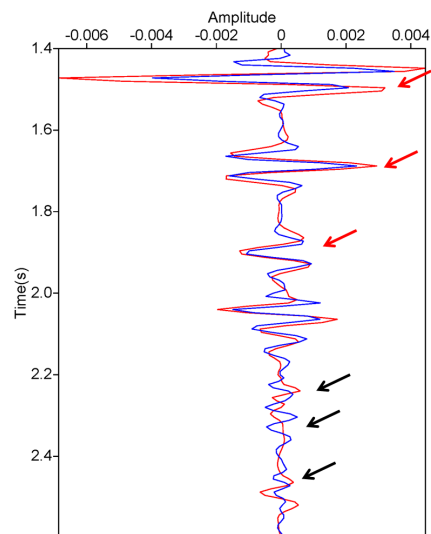

Figure 6a: A trace comparison (from 1.4s to 2.6s) between the test data (red line) and the ISS internal multiple prediction (blue line) **without** addressing the spurious events.
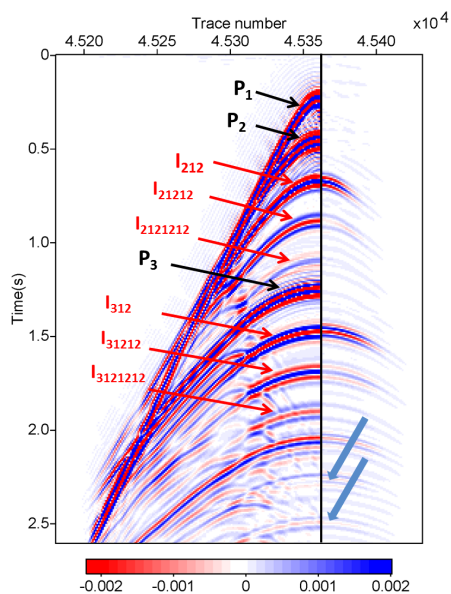

Figure 5b: A shot comparison between the test data (left part) and the ISS internal multiple prediction (right part) **with** addressing the spurious events.
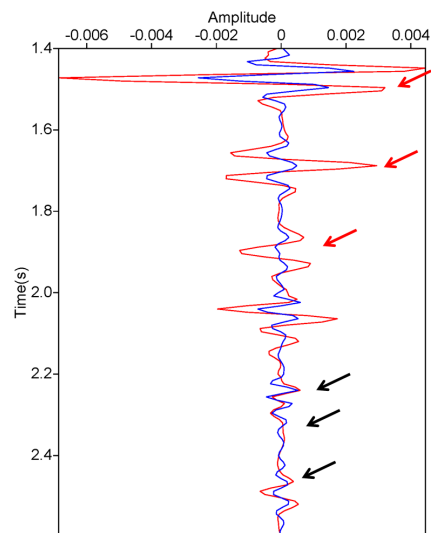

Figure 6b: A trace comparison (from 1.4s to 2.6s) between the test data (red line) and the ISS internal multiple prediction (blue line) **with** addressing the spurious events.

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