Multiples can be useful (at times) to enhance imaging, by providing an approximate image of an unrecorded primary, but its always primaries that are migrated or imaged

Arthur B. Weglein, M-OSRP, Physics Department, University of Houston

SUMMARY

Primaries are seismic reflection events with one reflection in their history, whereas multiples are events that have experienced more than one reflection. Migration was originally, and remains today, basically and unequivocally about taking a primary event on a recorded seismic trace in time, and to locate where in space that reflection event was generated by a reflector; that concept assumes the event in time has only one reflection in its history. Since, by definition, only primaries have experienced one reflector in their history, migration relates to and only has meaning for primaries. Migration has no meaning for multiples. We will see in this paper that not only did the original definition of migration only have meaning for primaries, but, in addition, when using the most complete physically interpretable and quantitative imaging condition for wave equation migration that only primaries contribute to the image at any reflector, in depth, and both free surface and internal multiples do not. However, we also show that multiples can be useful (at times) by providing an approximate image of an unrecorded primary.

INTRODUCTION

In this paper, we briefly review methods for migrating data where waves are: (1) one way propagating and (2) two way propagating. Methods that use wave theory to migrate data have two ingredients, a wave propagation component and an imaging condition. There were three landmark imaging conditions introduced by Claerbout (1971); Loewenthal et al. (1985) and Stolt (1978) and their colleagues in the 1970's. Those three imaging conditions are: (1) the exploding reflector model, for zero offset data, (2) the space and time coincidence of up and down-going waves, and (3) predicting a coincident source and receiver experiment at depth and asking for time equals zero. We will refer to these three imaging conditions as Claerbout imaging I, II, and III, respectively. The third imaging condition predicts an actual seismic experiment at depth, and that predicted experiment consists of all the events that experiment would record, if you had a source and receiver at that subsurface location. That experiment would have its own recorded events, the primaries and multiples for that predicted experiment. All of the recorded primaries and multiples contribute to the events in the predicted coincident source and receiver experiment at depth. But only the recorded primaries contribute to the coincident source and receiver experiment at time equals zero. Hence, only recorded primaries contribute to seismic imaging.

SUMMARY OF WAVE EQUATION MIGRATION FOR ONE WAY AND TWO WAY PROPAGATING WAVES

For one-way wave propagation, the experiment at depth is

$$D(\text{at depth}) = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} D \, dS_g \, dS_s, \tag{1}$$

where D in the integrand is equal to the data on the measurement surface. G_0^{-D} is the anticausal Green's function with Dirichlet boundary condition on the measurement surface, s = shot, and g = receiver. For two-way wave propagation, the experiment at depth is:

$$D(\text{at depth}) = \int_{S_s} \left[\frac{\partial G_0^{DN}}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g + G_0^{DN} \frac{\partial}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g \right] dS_s, \tag{2}$$

where D in the integrand is equal to the data on the measurement surface. G_0^{DN} is the Green's function for the model of the finite volume that vanishes along with its normal derivative on the lower surface and the walls (Weglein et al., 2011a,b).

Liu and Weglein (2014) and Weglein (2015) take the next step towards our goal and objective. The role of recorded primaries and multiples in contributing first to the predicted source and receiver experiment at depth, and then to the (Claerbout Imaging III) coincident source and receiver experiment at time equals zero provides a definitive response to whether or not multiples contribute to seismic imaging.

We summarize the conclusion of those references (Liu and Weglein (2014) and Weglein (2015))

1. All recorded events, primaries, internal multiples and free surface multiples contribute to the predicted coincident source and receiver experiment at depth
2. Only the recorded primaries contribute to the image, that is once the time equal zero imaging condition is called on, only recorded primaries contribute to the image at any depth.
3. The location of each reflector is determined, along with the reflection coefficient for the experiment both from above and from below each reflector (Figure 1). The latter is not achievable using Claerbout Imaging II.

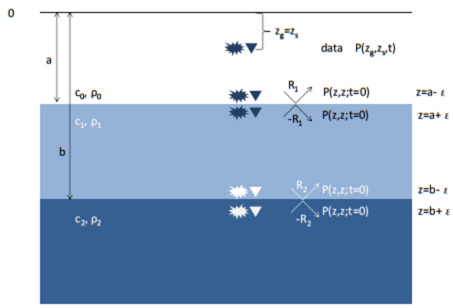

Figure 1: Green's theorem predicts the wavefield at an arbitrary depth z between the shallower depth a and deeper depth b. The experiment illustrated here corresponds to a plane wave normal incident on a layered medium with two reflectors. The measurement coordinates are z_g and z_s , the coincident source and receiver depths. $a-\varepsilon$, $a+\varepsilon$, $b-\varepsilon$, $b+\varepsilon$ are the depth of the predicted source and receiver experiment at depths above and below the first reflector at $z=a$ and the second reflector at $z=b$.

If you remove the multiples in the recorded data, the coincident source and receiver experiment at depth would change, but once the imaging condition is applied, the image's location at the correct depth and its amplitude, the reflection coefficient, will not be affected. If, in these examples, your data consisted of only multiples, you will have no image at any depth. These conclusions are all shown in full detail in the above cited references (Liu and Weglein, 2014; Weglein, 2015).

Hence, for the purposes of imaging and inversion (and employing the most capable and quantitative imaging condition Claerbout imaging III), primaries are the events that contribute to imaging and inversion and multiples are not.

CLAERBOUT II AND CLAERBOUT III IMAGING RESULTS

In Claerbout imaging II, the time and space coincidence of up and down waves is formulated as

$$I(\vec{x}) = \sum_{\vec{x}_s} \sum_{\omega} D^*(\vec{x}, \vec{x}_s, \omega) U(\vec{x}, \vec{x}_s, \omega), \tag{3}$$

where D is the downgoing wave and U is the upgoing wave, respectively, and $*$ represents the complex conjugate.

The sum over receivers for a given shot record realizes the Claerbout II imaging concept. The sum over sources is "introduced" in an ad hoc manner to mitigate the inconsistent amplitude and phase of images, that can be clearly seen from imaging results with exact data and imaging a single horizontal reflector (please see the example in Ma and Zou (2015); Zou and Weglein (2015)). A comparison with a Claerbout imaging III result for the same reflector and the same data, produces an accurate and consistent reflection coefficient at every point on the reflector, for a single shot record.

For Claerbout III, the sum over receivers predicts the receiver experiment at depth for a source on the measurement surface, and then the sum over sources then precisely predicts the experiment with the source at depth, as well. The integrations over receivers and over sources bring the source and receiver experiment to depth. There is nothing ad hoc or designed to fix something amiss (as though the data had random noise, to be mitigated by stacking). The noise is algorithmic, within Claerbout imaging II and is present with exact, analytic noise free data in the first integral over receivers in Claerbout imaging II. That is the reason we state that Claerbout III is on the firmest physics foundation, with an interpretable, quantitative and consistent meaning to the image. We adopt Claerbout III for the analysis of the role of primaries and multiples in imaging (in Liu and Weglein (2014) and Weglein (2015)).

For our immediate purpose of examining how multiples can be used to provide an approximate image of an unrecorded primary, we look at Claerbout II with a few examples since the "migrating of multiples" activity is inspired and motivated by that algorithm with different up and down going waves chosen for different uses/objectives/purposes.

IMAGING PRIMARIES WITH CLAERBOUT IMAGING CONDITION II

1D normal incident analytic example

In this section, we use a 1D normal incident analytic example to illustrate the idea of imaging a primary with Claerbout imaging condition II. Assume a down-going spike data that starts at $z=\varepsilon_s$ at $t=t_0=0$. The down-going wavefield from the source side that is being forward propagated to depth z is $D=e^{i\omega[\frac{z-\varepsilon_s}{c_0}]}$ whereas the up-going wavefield from the receiver side that is being back propagated to depth z is $U=R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{d-z}{c_0}]}$, where R_1 and d are the reflection coefficient and the depth of the reflector, respectively (see Figure 2).

Applying the Claerbout imaging condition II we have

$$\begin{aligned} I_p &= \int (e^{-i\omega[\frac{z-\varepsilon_s}{c_0}]}) \times (R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{d-z}{c_0}]}) d\omega \\ &= \int R_1 e^{-i\omega[\frac{2d-2z}{c_0}]} d\omega = \pi c_0 R_1 \delta(z-d) \end{aligned} \tag{4}$$

We obtain the correct image location at depth d with an amplitude of $\pi c_0 R_1$.

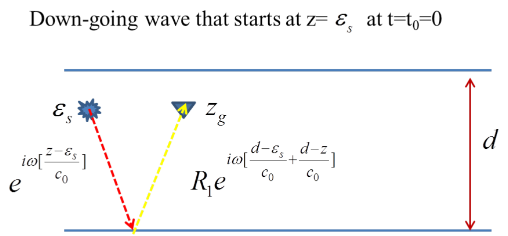

Figure 2: Migrating a primary with Claerbout II to find an image.

USING A MULTIPLE TO APPROXIMATELY IMAGE AN UNRECORDED PRIMARY

1D normal incident analytic example

In this section, we apply Claerbout imaging condition II to a

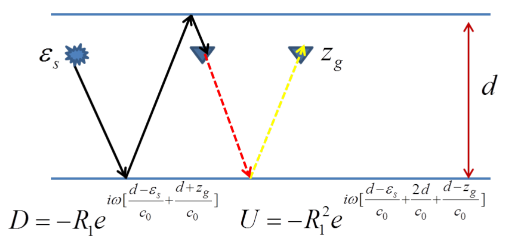

Figure 3: Use of a multiple to find an approximate image of an unrecorded primary.

seismic data set that contains a first-order free-surface multiple. Similarly, assuming a down-going spike data starts at $z = \varepsilon_s$ at $t = t_0 = 0$ (see Figure 3). A first-order free-surface multiple is recorded at z_g. The down-going wavefield from a "virtual source" (represented by the dashed red line in Figure 3) that is being forward propagated to depth z is $D = -R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{d+z}{c_0}]}$, whereas the up-going wavefield from the receiver side (represented by the yellow dashed line in Figure 4) that is being back propagated to depth z is $U = -R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{2d}{c_0}+\frac{d-z}{c_0}]}$, where we have assumed the downward reflection coefficient at the free-surface to be -1 in deriving the up and down wavefield (see Figure 3). Applying the Claerbout imaging condition II, we have

$$
\begin{aligned}
I_M &= \int (-R_1 e^{-i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{d+z}{c_0}]}) \times (-R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{2d}{c_0}+\frac{d-z}{c_0}]}) d\omega \\
&= \int R_1^3 e^{-i\omega[\frac{2d-2z}{c_0}]} = \pi c_0 R_1^3 \delta(z-d) \qquad (5)
\end{aligned}
$$

We obtain the correct image location at depth d, with a different amplitude of $\pi c_0 R_1^3$, Hence, this use of a multiple can produce an approximate image of an unrecorded primary.

The methods that seek to use multiples today as "signal" are really seeking to approximate images due to primaries that have not been recorded, due to limitations in acquisition. They are not really using the multiple itself as an event to be followed into the subsurface for imaging purposes. Figure 4 illustrates the idea.

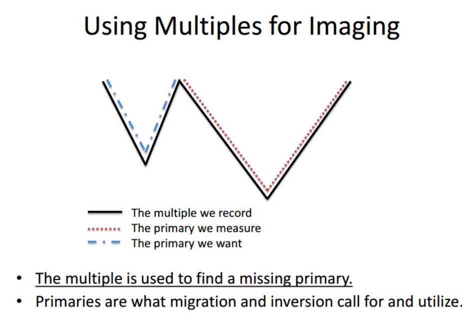

Figure 4: Using multiples for imaging.

In a Recent Advances and the Road Ahead presentation, "Multiples: signal or noise?", Weglein (2014a) (please see Weglein (2014b)) showed field data examples, from PGS, where there was clear added-value demonstrated for the enhanced image from using multiples.

However, there is another issue: in order to predict a free surface or internal multiple, the primary sub-events that constitute the multiple must be in the data. If a primary is not recorded, the multiple that contains that unrecorded primary will not be predicted as a multiple. That issue and basic contradiction within the method is recognized by those who practice this method, and instead of predicting the multiple, they use all the events in the recorded data, primaries and multiples, and while the multiples can be useful for predicting approximate images of missing primaries, the primaries in the data will cause artifacts. There are other artifacts that also come along with this method (from the inability to isolate primaries from multiples with unrecorded primaries) that have been noted in the literature (see Figure 5).

Values has been demonstrated for using multiples to enhance imaging (e.g., Berkhout and Verschuur (1994); Guitton (2002); Shan (2003); Muijs et al. (2007); Whitmore et al. (2010); Lu et al. (2011); Liu et al. (2011), Valenciano et al. (2014) and Weglein (2014a)).

Since the procedure is itself ad hoc, depending on an (Claerbout II) imaging condition for primaries which starts off as somewhat ad hoc (summing over sources), it cannot be easily or naturally improved because there is no starting point or framework without the artifacts that utilizes multiples for an

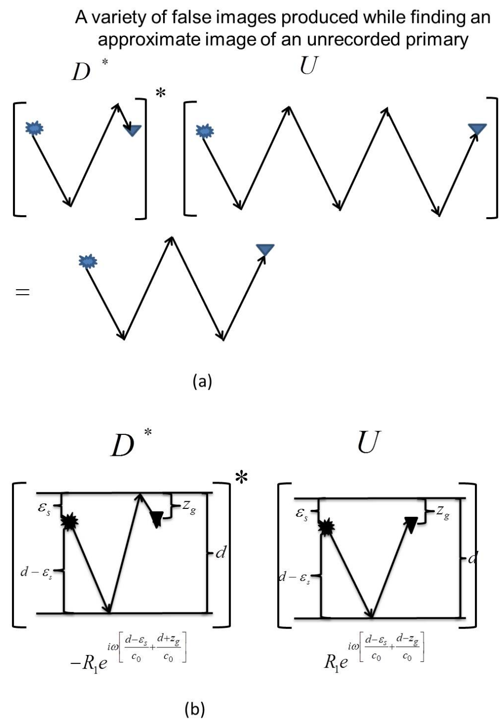

Figure 5: Examples of different types of false images generated by the use of multiples to predict the approximate image of an unrecorded primary. Figure 5a will produce an artifact due to an image of a multiple and figure 5b will produce an artifact at $z = 0$ (the origin) that is beyond false image due to output mages of multiples.

enhanced image. One response to the artifacts is to collect the required primaries.

CONCLUSIONS

Hence, primaries are signal and multiples can be useful, at times, for predicting the image of missing primaries. But it's primaries that are signal, that we use for structure and inversion.

Primaries are signal for all methods that seek to locate and identify targets.

Given an accurate discontinuous velocity and density model, and data with primaries and multiples, then Liu and Weglein (2014) and Weglein (2015) demonstrated that only primaries contributed to the images at every depth. If you predicted the source and receiver experiment at depth with a smooth velocity, it is possible to correctly locate (but not invert) each recorded primary event but with a smooth velocity model every free surface and internal multiple will then produce a false image/artifact/event. If you removed the multiples first you can correctly locate structure from recorded primaries using a smooth velocity model. The methods that are using multiples to enhance imaging require a velocity model. All velocity analysis methods require multiples to have been effectively removed. Hence, an effective multiple removal step is a prerequisite for the methods that utilize multiples.

We emphasize that the inability, in practice, to provide an accurate discontinuous velocity model is why multiples need to be removed before imaging. That reality has been the case, is the case, and will remain true for the foreseeable future. Multiples need to be removed before velocity analysis and they need to be removed before imaging. Many things are useful for creating primaries: money, the seismic boat, the air-guns, the observer, the cable, computers, etc., but we don't call all useful things signal.

One serious problem and real danger is not in the procedure itself, but the serious misuse of the term "migration" as in referring to multiples being migrated. What's the problem with the label? We all know that primaries are migrated, and if multiples are now migrated as well, they must be on equal footing with primaries, and since they are now rehabilitated as good seismic citizens, we should no more seek to remove multiples than we seek to remove primaries. That is part of the danger of the misuse of the term migration in this process of trying to have a more complete and approximate set of primaries.

The danger in this mislabeling and overselling in this case is two-fold, one is a discounting of the actual substantive value represented by the method, and avoiding disappointment and an inevitable back-lash, and the second is it can advertently or inadvertently distract from serious matters of substance (e.g., internal multiple **elimination** for offshore and onshore applications).

All methods that provide a more complete set of primaries are to be supported and encouraged. Those methods include: (1) advances, in and more complete, acquisition, (2) interpolation and extrapolation methods, and (3) using multiples to predict missing primaries. However, a recorded primary is still the best and most accurate way to provide a primary, and the primary is the seismic signal. On balance, the value that multiples can provide to improve imaging can often outweigh issues resulting from artifacts.

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REFERENCES

Berkhout, A. J., and D. J. Verschuur, 1994, Multiple technology: Part 2, migration of multiple reflections: SEG Technical Program Expanded Abstracts, 1497–1500.

Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: Geophysics, **36**, 467–481.

Guitton, A., 2002, Shot-profile migration of multiple reflections: SEG Technical Program Expanded Abstracts, 1296–1299.

Liu, F., and A. B. Weglein, 2014, The first wave equation migration RTM with data consisting of primaries and internal multiples: theory and 1D examples: Journal of Seismic Exploration, **23**, 357–366.

Liu, Y., X. Chang, D. Jin, R. He, H. Sun, and Y. Zheng, 2011, Reverse time migration of multiples for subsalt imaging: Geophysics, 209–216.

Loewenthal, D., L. Lu, R. Roberson, and J. W. C. Sherwood, 1985, The wave equation applied to migration: Geophysical Prospecting, **24**, 380–399.

Lu, S., N. Whitemore, A. Valenciano, and N. Chemingui, 2011, Imaging of primaries and multiples with 3d seam synthetic: SEG Technical Program Expanded Abstracts, 3217–3221.

Ma, C., and Y. Zou, 2015, A clear example of using multiples to enhance and improve imaging; a comparison of two imaging conditions that are relevant to this analysis: Submitted to The Leading Edge.

Muijs, R., J. O. A. Robertsson, and K. Holliger, 2007, Prestack depth migration of primary and surface-related multiple reflections: Part i imaging: Geophysics, S59–S69.

Shan, G., 2003, Source-receiver migration of multiple reflections: SEG Technical Program Expanded Abstracts, 1008–1011.

Stolt, R. H., 1978, Migration by Fourier transform: Geophysics, **43**, 23–48.

Valenciano, A. A., S. Crawley, E. Klochikhina, N. Chemingui, S. Lu, and D. Whitemore, 2014, Imaging complex structures with separated up- and down-going wavefields: SEG Technical Program Expanded Abstracts, 3941–3945.

Weglein, A. B., 2014a, Multiples: Signal or noise? Invited presentation given at the SEG Convention special session on Recent advances and the road ahead, Denver, Colorado.

———, 2014b, Multiples: Signal or noise?: `https://vts.inxpo.com/scripts/Server.nxp?LASCmd=L:0&AI=1&ShowKey=21637&LoginType=0&InitialDisplay=1&ClientBrowser=0&DisplayItem=NULL&LangLocaleID=0&RandomValue=1415030021699`. ([Online; accessed 2014-10]).

———, 2015, Multiples: signal or noise?: Geophysics. (Submitted).

Weglein, A. B., R. H. Stolt, and J. D. Mayhan, 2011a, Reverse-time migration and Green's theorem: Part I — The evolution of concepts, and setting the stage for the new RTM method: Journal of Seismic Exploration, **20**, 73–90.

———, 2011b, Reverse time migration and Green's theorem: Part II — A new and consistent theory that progresses and corrects current RTM concepts and methods: Journal of Seismic Exploration, **20**, 135–159.

Whitmore, N. D., A. Valenciano, W. Sllner, and S. Lu, 2010, Imaging of primaries and multiples using adual-sensor towed streamer: SEG Technical Program Expanded Abstracts, 3187–3192.

Zou, Y., and A. Weglein, 2015, A 1d pre-stack example examining the differences in two important imaging conditions: the space-time coincidence of up and down waves and the predicted coincident source and receiver experiment at depth at time zero: submitted to SEG Technical Program Expanded Abstracts.