The impact of accommodating the source radiation pattern on the inverse scattering series free-surface multiple elimination algorithm

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SUMMARY

The inverse scattering series (ISS) free-surface multiple elimination algorithm has certain prerequisites: (1) removing the reference wavefield, (2) estimation of source wavelet and radiation pattern, and (3) source and receiver deghosting. Weglein and Secrest (1990) describe a method to separate the reference wavefield from the scattered wavefield (reflection data) without subsurface information. In this abstract, the impact of prerequisites (2) and (3) on the ISS free-surface multiple elimination algorithm (Carvalho, 1992; Weglein et al., 1997) is discussed and the algorithm is modified and extended to accommodate the source radiation pattern. That radiation pattern accommodation can provide added value compared to previous methods that assumed an isotropic point source for predicting amplitude and phase of free-surface multiples. All these prerequisites can be provided by Green's theorem methods. Green's theorem methods for wave separation do not require subsurface information. They are consistent with the ISS free-surface multiple elimination algorithm. The extended ISS free-surface multiple elimination algorithm that accommodates the source radiation pattern is tested on a 1D acoustic model, and the results indicate that the new and extended ISS free-surface multiple elimination algorithm can predict more accurate results in comparison with methods without that accommodation when the source has a radiation pattern. This increased effectiveness in prediction is essential for removing free-surface multiples that are proximal or interfering with primaries (or other multiples).

INTRODUCTION

In seismic exploration, preprocessing of seismic data, including the removal of reference waves, wavelet estimation, and removal of ghosts, are very important. The reference wave should be removed because it does not experience reflection from the earth, which is our interest. Both the active source and the properties of the earth contribute to the amplitude and phase of recorded seismic events. To identify subsurface properties from seismic data, we need to identify and remove the source's effect from the seismic data (Weglein and Secrest, 1990). Source and receiver deghosting will remove the ghost notches and enhance the low-frequency content of the seismic data (Mayhan et al., 2011, 2012; Mayhan and Weglein, 2013). These are the prerequisites of the ISS free-surface multiple elimination algorithm (Weglein et al., 2003). All three of these processing steps can be achieved by using Green's theorem methods without requiring subsurface information. Green's theorem methods have been pioneered by J. Zhang (Weglein et al., 2002; Zhang and Weglein, 2005, 2006; Zhang, 2007) and developed by J. Mayhan (Mayhan et al., 2011, 2012; Mayhan and Weglein, 2013). Wu and Weglein (2014) extended Green's theorem reference wave prediction algorithm from the off-shore acoustic to the on-shore elastic wavefield separation.

In addition, multiple removal is a classic long-standing problem. Various methods (e.g., Carvalho, 1992; Verschuur et al., 1992; Weglein et al., 1997, 2003; Berkhout and Verschuur, 1999; Dragoset et al., 2008) have been developed to either attenuate or eliminate free-surface multiples, and each method has different assumptions, advantages, and limitations. Among these methods, the ISS free-surface multiple elimination algorithm (Carvalho, 1992; Weglein et al., 1997, 2003) is fully data-driven and does not need any subsurface information, which is a big advantage, especially under conditions of complex geology. Given its prerequisites, the ISS free-surface multiple elimination algorithm (Carvalho, 1992; Weglein et al., 1997, 2003) can predict the exact amplitude and phase of all free-surface multiples at all offsets and remove them through a simple subtraction without adaptively subtraction using certain criteria (energy minimization, for example). Methods, like SRME, do not provide this ability and they rely on an adaptive subtraction to fill that gap. The adaptive subtraction can be reasonable at times, but at other times, it can have issues with proximal and interfering events, e.g., damaging a primary and failing to remove the multiples.

However, for data generated by a general source with a radiation pattern, the ISS free-surface multiple elimination algorithm assumes an isotropic point source, where the source has no variation of amplitude or phase with take-off angle. In towed marine acquisition, a source array is commonly used to increase the power of the source, broaden the bandwidth, and cancel the random noise. The source array exhibits directivity in take-off angle (Loveridge et al., 1984). That directivity is an issue for multiple removal and attenuation and AVO analysis. In seismic processing, it is essential that we characterize the source array's effect on any seismic processing methods. Therefore, to improve the accuracy of the predicted multiples, the ISS free-surface multiple elimination algorithm is modified and extended by accommodating a general source with a radiation pattern. That accommodation can enhance the fidelity of amplitude and phase prediction of free-surface multiples at all offsets when the source has a radiation pattern.

THEORY

The ISS free-surface multiple elimination algorithm in 2D case starts with the input data $D_1'(k_g,k_s,\omega)$, that is source and receiver deghosted. For an isotropic point source, it is proposed by (Carvalho, 1992; Weglein et al., 1997, 2003):

$$D_n'(k_g,k_s,\omega) = \frac{1}{i\pi A(\omega)}\int dk D_1'(k_g,k,\omega) q e^{iq(\varepsilon_g+\varepsilon_s)} D_{n-1}'(k,k_s,\omega), \quad (1)$$

where k_g, k_s and ω represent the Fourier conjugates of receiver, source, and time, respectively. The parameters ε_g and ε_s are the receivers' and sources' depth below the free surface, respectively. q is the obliquity factor $q = sgn(\omega)\sqrt{\omega^2/c_0^2 - k^2}$, and c_0 is the reference velocity. $A(\omega)$ is the source signature, which is a function of time or ω in different domains. The free-surface multiples are predicted order-by-order and then added together give the deghosted and free-surface demultipled data $D'(k_g,k_s,\omega) = \sum_{n=1}^{\infty} D'_n(k_g,k_s,\omega)$.

For the data generated by a general source with a radiation pattern, the ISS free-surface multiple elimination algorithm can only predict multiples approximately. To accommodate the source's effect, the ISS free-surface multiple elimination algorithm is modified and extended from an isotropic point source to a general source ρ with a radiation pattern (Yang et al., 2013; Yang, 2014)

$$D'_n(k_g,k_s,\omega) = \frac{1}{i\pi}\int \frac{dk}{\rho(k,q,\omega)} D'_1(k_g,k,\omega) q e^{iq(\varepsilon_g+\varepsilon_s)} D'_{n-1}(k,k_s,\omega), \quad (2)$$

where $\rho(k,q,\omega)$ is the projection of source signature in the f-k domain and $k^2+q^2 = \omega^2/c_0^2$ The projection of the source signature $\rho(k,q,\omega)$ can be achieved from the reference wavefield that is separated from the total wavefield by using Green's theorem methods (Weglein and Secrest, 1990; Mayhan and Weglein, 2013; Tang et al., 2013).

In this paper, we assume that the source array is invariant from one shot to the next. In other words, the geometry or the distribution of the source array is the same for each shot. The direct reference wavefield P_0^d for a 2D case can be expressed as an integral of the direct reference Green's function G_0^d over all air-guns in an array,

$$P_0^d(x,z,x_s,z_s,\omega) = \iint dx'dz' \rho(x',z',\omega) G_0^d(x,z,x'+x_s,z'+z_s,\omega), \quad (3)$$

where (x,z) and (x_s,z_s) are the prediction point and source point, respectively. (x',z') is the distribution of the source with respect to the source locator (x_s,z_s). Using the bilinear form of Green's function and Fourier transforming over x, we obtain the relationship between ρ and P_0^d as

$$P_0^d(k,z,x_s,z_s,\omega) = \rho(k,q,\omega)\frac{e^{iq|z-z_s|}}{2iq}e^{-ikx_s}. \quad (4)$$

Since $k^2+q^2 = \omega^2/c_0^2$, q is not a free variable, hence, we can not obtain $\rho(x,z,\omega)$ in space-frequency domain by taking an inverse Fourier transform on $\rho(k,q,\omega)$. However, the projection of the source signature $\rho(k,q,\omega)$ can always be achieved directly from the direct reference wavefield P_0^d in the f-k domain, where the variable k or q represent the amplitude variations of the source signature with angles. Ikelle et al. (1997) also proposed a similar quantity $A(k,\omega)$, the inverse source wavelet, and solved it indirectly using the energy minimization criterion, while we apply Green's theorem wave separation methods to find the generalized source signature directly.

Substituting the projection of the source signature $\rho(k,q,\omega)$ into the inverse scattering free-surface multiple removal subseries, the ISS free-surface multiple elimination algorithm can be modified and extended (Yang, 2014). The extended algorithm accommodates a general source with a radiation pattern and can provide added value for the fidelity of amplitude and phase prediction of the free-surface multiples at all offsets. The extended ISS free-surface multiple elimination algorithm is fully multidimensional and does not require any subsurface information. Therefore, it is consistent with Green's theorem methods that provide all the data requirements. The extended free-surface multiple elimination algorithm (equation 2) is also consistent with the previous free-surface multiple elimination algorithm (equation 1) when the general source (e.g., source array) reduces to an isotropic point source. In addition, this modification can be easily extended into 3D case.

NUMERICAL TESTS

In this section, the effects of satisfying and not satisfying the prerequisites of the ISS free-surface multiple elimination algorithm are exemplified and tested. In each test, we will show the impact of each prerequisite on the ISS free-surface multiple elimination algorithm separately and compare its result with that after accommodating this prerequisite.

We will show the impact of ghosts, source wavelet, and radiation pattern on free-surface multiple removal. The numerical tests are based on a 1D acoustic model with varying velocity and constant density, as shown in Figure 1. The model has

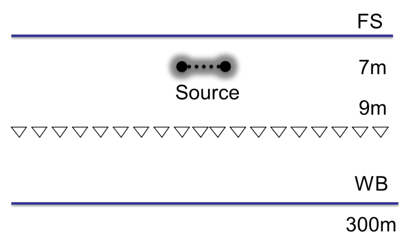


Figure 1: One-dimensional acoustic constant-density medium.

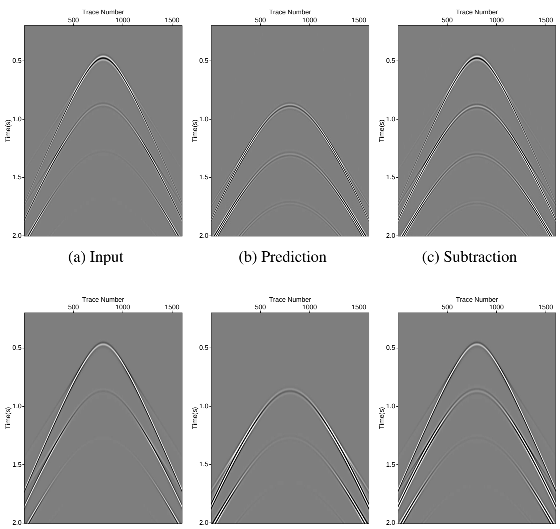
one reflector at 300m. The depths of the source and receiver are 7m and 9m, respectively. The synthetic data are generated by using the Cagniard-de Hoop method (de Hoop and van der Hijden, 1983; Aki and Richards, 2002), which can accurately produce any specific event that we are interested in. For exemplifying the impact of ghosts and source wavelet on the free-surface multiple removal, a point source is applied to generate the data. For exemplifying the impact of source radiation pattern on the free-surface multiple removal, a source array is applied to generate the data.

The tests are organized as follows: First, we test the impact of ghosts and source wavelet on the free-surface multiple removal. If both prerequisites are satisfied, the result of the free-surface multiple removal is also shown. Second, the impact of the source radiation pattern on free-surface multiple removal is presented. The results of the free-surface multiple removal by using the previous algorithm (equation 1) and the extended algorithm (equation 2) are compared.

The impact of ghosts and source wavelet on free-surface multiple removal

Free-surface multiple elimination algorithm

For exemplifying the impact of ghosts and source wavelet, a point source is applied to generated the data. We apply the ISS free-surface multiple elimination algorithm (equation 1) to predict and remove the free-surface multiples from the point-source data. Figure 2 shows the impact of ghosts and source wavelet on the free-surface multiple removal. Figure 2a is the input data with ghosts. Inputting it into the ISS free-surface multiple elimination algorithm gives the free-surface multiple prediction as shown in Figure 2b. After subtracting the prediction from the input data, Figure 2c shows the results of after free-surface multiple removal through a simple subtraction. From this example, we can see that if the input data are not deghosted, the ISS free-surface multiple elimination algorithm can only predict the correct phase but approximate amplitude of the free-surface multiples. After removing the ghosts, the



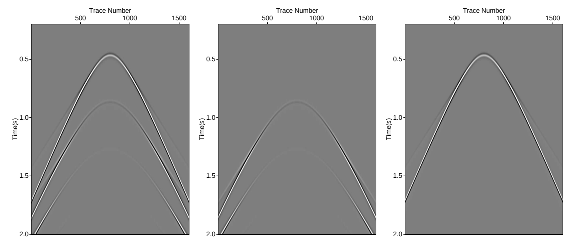
(a) Input (b) Prediction (c) Subtraction

(d) Input (e) Prediction (f) Subtraction

Figure 2: (a)&(d) Input data with and without ghosts; (b)&(e) Free-surface multiple prediction using the ISS free-surface multiple elimination algorithm with and without source wavelet deconvolution; (c)&(f) After free-surface multiple removal through a simple subtraction.

input data is shown in Figure 2d. Figure 2e shows the predicted free-surface multiples using the ISS free-surface multiple elimination algorithm without source wavelet deconvolution. Figure 2f illustrates the results of free-surface multiple removal by a simple subtraction. Without incorporating the source wavelet deconvolution, the ISS free-surface multiple elimination algorithm can only predict the correct phase but approximate amplitude of the free-surface multiples. Therefore, without removing the ghosts and incorporating the source wavelet deconvolution, the ISS free-surface multiple elimination algorithm can only predict approximate free-surface multiples and can not remove through a simple subtraction.

If all the prerequisites are satisfied, i.e., the ghosts have been removed and the source wavelet has been deconvolved, Figure 3 shows the results of after free-surface multiple removal. Figure 3a is the input data and Figure 3b is the free-surface multiple prediction. After subtracting from the input data through a simple subtraction, Figure 3c shows the result of after free-surface multiple removal. From this example, we can see that if all the prerequisites are satisfied, the free-surface multiples are predicted exactly by using the ISS free-surface multiple elimination algorithm and removed through a simple subtraction. Most importantly, all primaries are not touched, as shown in Figure 3c. Therefore, the ISS free-surface multiple elimination algorithm has the ability to predict accurately the phase and amplitude of multiples if its prerequisites (incorporating the source wavelet deconvolution and deghosting) are satisfied.



(a) Input (b) Prediction (c) Subtraction

Figure 3: Free surface multiple removal with all prerequisites are satisfied

The impact of the source radiation pattern on free-surface multiple removal

To evaluate the significance of the source radiation pattern, a source array (Figure 4) with nine air-guns are applied to generated the data. Here, we assume that the source array only varies laterally with identical source signatures, but this assumption is not necessary in the ISS free-surface multiple elimination algorithm. In the source-array data, only the primary and free-surface multiples are generated by the Cagniard-de Hoop method.

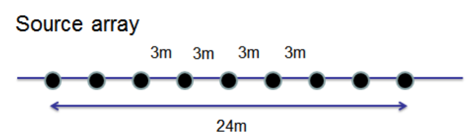
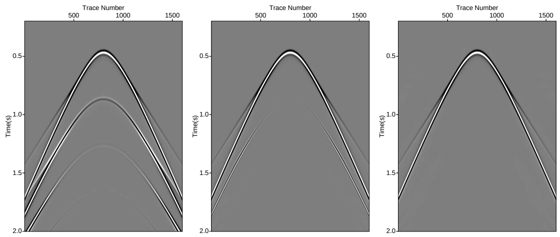


Figure 4: Source array with nine air-guns.

Figure 5a is the source-array data with nine point sources. Figures 5b and 5c are the results of after the free-surface multiple removal by using the current (equation 1) and extended (equation 2) ISS free-surface multiple elimination algorithms. It can be seen that for the source-array data with radiation pattern, the current ISS free-surface multiple elimination algorithm can effectively remove the free-surface multiples, but there are still some residual multiples due to the effect of the source radiation pattern. While the extended ISS free-surface multiple elimination algorithm can remove the free-surface multiples completely through a simple subtraction.

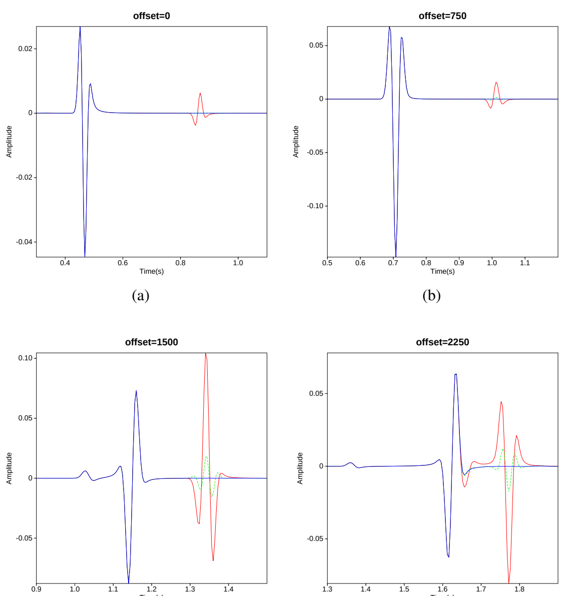
For details, we pick four traces from the source-array data and compare the results after free-surface multiple removal, by using both the current and extended ISS free-surface multiple elimination algorithms. Here, only the results of after first-order free-surface multiple removal are compared with the in-



(a) Input (b) current (c) Extended

Figure 5: The impact of source radiation pattern on free-surface multiple removal. (a) the source-array data; (b) following free-surface multiple removal using the current ISS free-surface multiple elimination algorithm, there are some residual multiples; (c) following free-surface multiple removal using the extended ISS free-surface multiple elimination algorithm, all the multiples are completely eliminated.

put data. At zero offset, both the current and extended ISS free-surface multiple elimination algorithms can predict the accurate amplitude and phase of the free-surface multiples and remove them completely through a simple subtraction, as shown in Figure 6a. At large offsets, the current ISS free-surface mul-



(a) (b) (c) (d)

Figure 6: Comparisons between the input data and the results of after free-surface multiple removal at four different offsets (a) 0m, (b) 750m, (c) 1500m, and (d) 2250m. Red line: The input data; Green dash line: after the free-surface multiple removal by the current ISS free-surface multiple elimination algorithm; Blue line: after the free-surface multiple removal by the extended ISS free-surface multiple elimination algorithm.

tiple elimination algorithm can still predict the correct phase of the free-surface multiples, while the amplitude of the predicted free-surface multiples has some errors. The green dash line in Figures 6b, 6c, and 6d shows the residual free-surface multiples after the free-surface multiple removal using the current ISS free-surface multiple elimination algorithm. With the increasing offset, the residual multiples are larger. The extended ISS free-surface multiple elimination algorithm can predict the accurate amplitude and phase of the free-surface multiples at both zero and large offsets. The blue line in Figure 6 shows the results after the free-surface multiple removal. The multiples are totally removed at any offsets. This is in contrast to SRME that has amplitude and phase errors at all offsets, and relies on adaptive subtraction to fix the errors in prediction.

For the source-array data, the current ISS free-surface multiple elimination algorithm can well predict and remove the free-surface multiples with some small residues, while the extended ISS free-surface multiple elimination algorithm can accurately predict and completely eliminate the free-surface multiples without damaging the primaries.

CONCLUSIONS

We discussed and examined the impact of accommodating prerequisites (ghosts, source wavelet and radiation pattern) on the ISS free-surface multiple elimination algorithm. The ISS free-surface multiple elimination algorithm is modified and extended by accommodating a general source with radiation pattern. The extended algorithm can provide added value compared to previous methods for the fidelity of amplitude and phase prediction of free-surface multiples at all offsets. It is multidimensional and does not require any subsurface information. The numerical tests show that if the prerequisites are provided, the ISS free-surface multiple elimination algorithm can, in principle, have the ability to predict more accurate free-surface multiples and then remove them more effectively. This is particularly important for the case, where free-surface multiples are proximal or interfering with other events and we can not rely on adaptive subtraction to fix the errors in amplitude and phase of the prediction.

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