A 1D pre-stack example examining the differences in two important imaging conditions: the space-time coincidence of up and down waves and the predicted coincident source and receiver experiment at depth at time zero.

Yanglei Zou, Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston

SUMMARY

In the 1970's Claerbout, Lowenthal and their colleagues (Claerbout, 1971; Riley and Claerbout, 1976; Lowenthal et al., 1985) introduced three imaging conditions : (1) the exploding reflector model for zero offset data (2) the space and time coincidence of up and down going waves and (3) predicting a coincide source and receiver experiment at depth at time equals zero. We refer to these as Claerbout Imaging Condition I, II and III, respectively. For a normal incident plane wave on a single horizontal reflector they are equivalent. For a shot record recorded above a single horizontal reflector or more complicated situations they are no longer equivalent. Claerbout III is superior to Claerbout I and II in that it provides the most quantitative and interpretable image amplitude. Claerbout III is also extendable/generalizable to provide an angle dependent reflection coefficient. Stolt and his colleagues (Clayton and Stolt, 1981; Stolt and Weglein, 1985; Stolt and Benson, 1986) originally formulated Claerbout III for one-way waves. For imaging two way propagating waves, Whitmore and his colleagues (Whitmore (1983)) launched from Claerbout II. Weglein, Fang and their colleagues (Weglein et al., 2011a,b; Liu, 2013) extended Claerbout III for two way propagating waves. In this paper, the first direct and detailed comparison of Claerbout II and III is carried out for the simplest circumstance where they will produce a different result. The differences are significant and substantive, with implications far beyond the simple example that allows for transparent analysis and analytic evaluation and conclusions.

INTRODUCTION

Methods that use the wave equation to perform seismic migration have two ingredients: (1) a wave propagation component and (2) an imaging principle or concept. Claerbout (Claerbout, 1971; Riley and Claerbout, 1976) was the initial and key wave-equation-migration imaging-concept pioneer and algorithm developer, together with Stolt (Stolt (1978)) and Lowenthal (Lowenthal et al. (1985)) and their colleagues, they introduced imaging conditions for locating reflectors at depth from surface-recorded data. The three key imaging conditions that were introduced are:

I. the exploding-reflector model
II. time and space coincidence of up and downgoing waves
III. predicting a source and receiver experiment at a coincident-source-and-receiver subsurface point, and asking for time equals zero (the definition of wave-equation migration)

For a normal-incident spike plane wave on a horizontal reflector, these three imaging concepts are totally equivalent. However, a key point to make clear for this paper, is that for a non-zero-offset surface seismic-data experiment, with either a one-dimensional or a multi-dimensional subsurface, they are no longer equivalent. Wave-equation migration is defined as using the Claerbout Imaging Condition III, predicting a source and receiver experiment at depth at time equals zero. Stolt and his colleagues (Clayton and Stolt, 1981; Stolt and Weglein, 1985; Stolt and Benson, 1986; Stolt and Weglein, 2012; Weglein and Stolt, 1999) extended and formulated the experiment-at-depth concept to allow a separated source and receiver experiment at time equals zero for one way propagating waves. Weglein, Fang and their colleagues (Weglein et al., 2011a,b; Liu, 2013) extended Claerbout III for two way propagating waves. Claerbout III is superior to Claerbout I and II in that it provides the most quantitative and interpretable image amplitude. Claerbout III is also extendable/generalizable to provide an angle dependent reflection coefficient. For the purpose of determining quantitative information on the physical meaning of the image, the clear choice is Claerbout Imaging Condition III.

In this paper, we will compare the imaging results obtained by Claerbout Imaging Condition II and III. The Claerbout Imaging Condition III predicts a physical experiment with both source and receiver at depth, allowing it to provide the imaging definitiveness and physical interpretation that Claerbout Imaging Condition II cannot match.

STOLT MIGRATION

Stolt migration represents Claerbout Imaging Condition III for one-way propagating waves. Following Stolt and Weglein (2012), given a 2D data $D(x_g, x_s, t)$ with source location $(x_s, z_s = 0)$, receiver location $(x_g, z_g = 0)$, and time t, we can perform a Fourier transform over all coordinates:

$$D(k_{gx}, k_{sx}, \omega) = \int dx_g \int dx_s \int dt D(x_g, x_s, t) e^{i(k_{sx}x_s - k_{gx}x_g + \omega t)}. \tag{1}$$

where k_{gx}, k_{sx} and ω are Fourier conjugates of x_g x_s and t, respectively.

Then we can predict the data from an experiment where the sources and receivers are all at depth z,

$$P(k_{gx}, z, k_{sx}, z, \omega) = D(k_{gx}, k_{sx}, \omega) e^{i(k_{gz} - k_{sz})z}, \tag{2}$$

where the vertical wavenumber component k_{gz} and k_{sz} are defined as

$$
\begin{aligned}
k_{gz} &= -\frac{\omega}{c}\sqrt{1-\frac{{k_{gx}}^2 c^2}{\omega^2}} \\
k_{sz} &= \frac{\omega}{c}\sqrt{1-\frac{{k_{sx}}^2 c^2}{\omega^2}}.
\end{aligned} \tag{3}
$$

If we make two inverse Fourier transform of k_{gx} and k_{xs} to the same x, we can predict the data of an experiment where a source and a receiver are both at location (x,z),

$$
\begin{aligned}
P(x,z,x,z,\omega) =& \frac{1}{(2\pi)^2}\int dk_{sx} e^{-ik_{sx}x}\int dk_{gx} e^{ik_{gx}x} P(k_{gx},z,k_{sx},z,\omega) \\
=& \frac{1}{(2\pi)^2}\int dk_{sx} e^{-i(k_{sz}z+k_{sx}x)} \\
& \times \int dk_{gx} e^{i(k_{gz}z+k_{gx}x)} D(k_{gx},k_{sx},\omega)
\end{aligned} \tag{4}
$$

Next, predict the coincident source and receiver at time equals zero, we obtain the 2D Stolt migration result,

$$
\begin{aligned}
M^{Stolt}(x,z) =& \frac{1}{2\pi}\int d\omega e^{-i\omega t} P(x,z,x,z,\omega)|_{t=0} \\
=& \frac{1}{(2\pi)^3}\int d\omega \int dk_{sx} e^{-i(k_{sz}z+k_{sx}x)} \\
& \times \int dk_{gx} e^{i(k_{gz}z+k_{gx}x)} D(k_{gx},k_{sx},\omega)
\end{aligned} \tag{5}
$$

where $M^{Stolt}(x,z)$ is the image function [*].

REVERSE TIME MIGRATION

RTM (Reverse Time Migration) is a kind of migration adopting Claerbout Imaging Condition II for primaries in a medium where waves are two way propagating. In RTM, the source wavefield is forward propagated to the subsurface and the receiver wavefield is backward propagated to the subsurface; the imaging result is obtained by cross-correlation, i.e., the space and time coincidence of up and down waves. The Claerbout Imaging Condition II RTM formula (Baysal et al., 1983; Whitmore, 1983; McMechan, 1983) is

$$
I(\vec{r}) = \sum_{x_s}\sum_{\omega} S^*(\vec{r},x_s,\omega) R(\vec{r},x_s,\omega) \tag{6}
$$

where R is the back-propagated reflection data, S is the forward-propagated source wavefield, S^* is the complex conjugate of S. The zero-lag cross-correlation is indicated by the sum over angular frequency, ω, and the sum over sources adds candidate-image travel-time trajectories.

[*] In Stolt and Weglein (2012), the image function $M^{Stolt}(x,z)$ has a half-integral filter. In this section we do not include the half-integral filter.

EXAMPLE

In this section, we will show the images generated by Reverse Time Migration (Claerbout Imaging Condition II) and Stolt migration (Claerbout Imaging Condition III) for a single horizontal reflector.

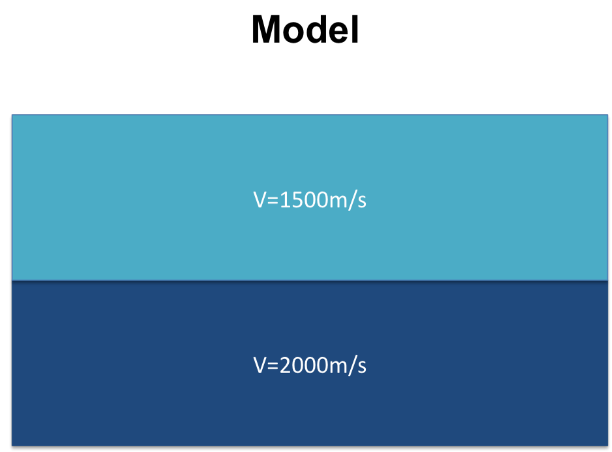

Figure 1: model

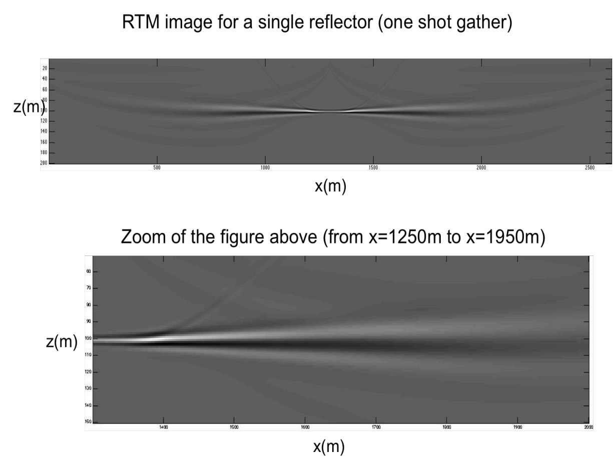

Figure 2: image result (one shot gather) following Claerbout Imaging Conidtion II. The figure below is a zoom of the upper figure. The Claerbout II image shows an inconsistent amplitude and shape of the image along the single reflector.

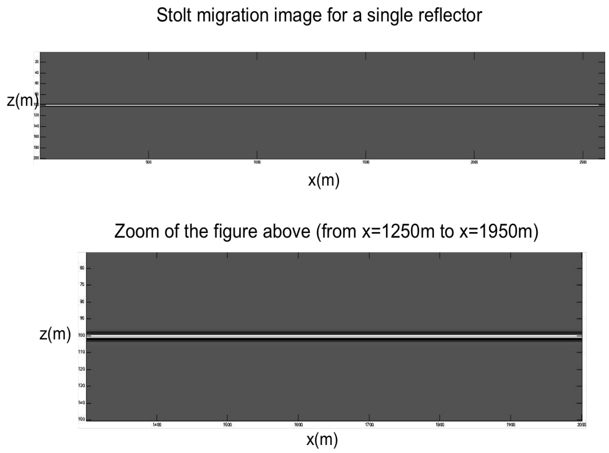

Figure 3: image result following Claerbout Imaging Condition III. The figure below is a zoom of the upper figure. The Claerbout III image in this figure shows an amplitude and shape consistent image. The exact same data was used in the simplest 1D earth prestack Claerbout II and Claerbout III tests and comparisons, indicating their intrinsic and substantive differences even in the simplest circumstances. As pointed out in Weglein (2015) the differences are much more serious when the target is complicated and imaging through and beneath a rapidly changing velocity.

Figure 1 shows the one-reflector model we used for this test. Figure 2 shows the image generated by Reverse Time Migration with a single shot gather (one source); we observe that there is a blur on the image as well as some artifacts generated by the limited aperture[†]. In practice, a sum over all sources is taken with the assumption that the blur and artifacts will go away. However, summing over all sources does not have a clear physical meaning and it is not guaranteed that all the blur and artifacts will go away. Figure 3 shows the image generated by Stolt migration with exactly the same data. We can observe that the image is flat and with few artifacts. The sum over sources in Stolt migration brings the source down to a point in the subsurface, while the sum over sources in RTM (equation 6) seeks to mitigate intrinsic artifacts in Claerbout II imaging. More importantly, every step in Stolt migration has a clear physical meaning. We can readily obtain interpretable amplitude information, such as angle dependent reflection coefficient, from Stolt migration (see Zou (2015) for more detail).

CONCLUSION

In this paper we compared the Claerbout Imaging Conidtion III - Stolt migration, and Claerbout Imaging Conidtion II- utilized in Reverse Time Migration, in the simplest 1D earth, with exactly the same prestack data. This result shows that the Claerbout Imaging Conidtion III (wave equation migration) - Stolt migration, and Claerbout Imaging Conidtion II- Reverse Time Migration, are intrinsically and substantively different even in the simplest circumstances. The Claerbout II image (one shot gather) shows an inconsistent amplitude and shape of the image along the single reflector. The Claerbout III image shows an amplitude and shape consistent image. As pointed out in Weglein (2015) the differences are much more serious when the target is complicated and imaging through and beneath a rapidly changing velocity. Claerbout Imaging Condition III (the wave equation migration) can provide a clear physics meaning with predicting a source and receiver experiment at depth. Thus we can readily obtain interpretable amplitude information, such as angle dependent reflection coefficient, from Claerbout imaging conidtion III. And in evaluating the role of multiples in imaging in Weglein (2015) a two way-wave propagation form of Claerbout III was called upon to provide a definition response to the question ”multiples: signal or noise?”. (see also (Weglein et al., 2011a,b; Liu, 2013))

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[†] In this test, we used a very large aperture to minimize the aperture artifacts. However, we can still observe this kind of artifacts on the image.

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