Dec. 8, 2011 A.B.W.

Notes on a single channel P equation that allows for its behavior and its amplitude and phase fidelity in an inhomogeneous elastic medium. We will illustrate this for a 2D isotropic inhomogeneous case, where there are only SV and P waves, but the extension to 3D and anisotropic media is straightforward. We choose (here for convenience, although not necessary) to describe the medium as a homogeneous background, plus perturbations in background properties that result in actual medium properties. For this specific case the elastic equations for P and S waves, ϕ_P and ϕ_S, respectively can be written as:

$$\left(\nabla^2 + \frac{\omega^2}{\alpha_0^2}\right) \Phi_P = V_{PP}\,\Phi_P + V_{SP}\,\Phi_S + f_P \qquad (1)$$

$$\left(\nabla^2 + \frac{\omega^2}{\beta_0^2}\right) \Phi_S = V_{SS}\,\Phi_S + V_{SP}\,\Phi_P + f_S \qquad (2)$$

where α_0, β_0 are the reference velocities for P and S waves, respectively, V_{PP}, V_{PS}, V_{SS}, ... are the perturbation operators that contain the perturbations in mechanical properties that arrange for reference properties to become actual, and $\begin{pmatrix} f_P \\ f_S \end{pmatrix}$ is the source function in P-S coordinates.

Rewrite equ (2) as:

$$\left(\nabla^2 + \frac{\omega^2}{\beta_0^2} - V_{SS}\right) \Phi_S = V_{SP}\,\Phi_P + f_S \qquad (2)'$$

and define G_S to satisfy

$$\left(\nabla^2 + \frac{\omega^2}{\beta_0^2} - V_{SS}\right) G_S = \delta \qquad (3)$$

and the causal solution to (2)' is

$$\phi_S = \int G_S V_{SP} \Phi_P + G_S * f_S \qquad (4)$$

and substitute (4) into (1)

$$\left(\nabla^2 + \frac{\omega^2}{\alpha_0^2}\right)\phi_P = V_{PP}\,\phi_P + V_{SP}\left\{\int G_S V_{SP}\,\phi_P + G_S f_S\right\} + f_P \qquad (5)$$

for the case where

$\vec{f} = \begin{pmatrix} f_P \\ 0 \end{pmatrix}$ and the source

is entirely P wave, $f_S = 0$ and (5)

can be rewritten :

$$\boxed{\phi_P = G_P^0 V_{PP} \phi_P + G_P^0 V_{SP} \int G_S V_{SP} \phi_P + G_0^P f_P \qquad (6)}$$

where G_P^0 is the causal solution to

$$\left(\nabla^2 + \frac{\omega^2}{\alpha_0^2}\right) G_P^0 = \delta$$

or

$$\boxed{\left\{\nabla^2 + \frac{\omega^2}{\alpha_0^2} - V_{PP} \underset{\substack{\uparrow \\ \text{minus}}}{-} V_{SP} G_S V_{SP}\right\} \phi_P = f_P \qquad (7)}$$

this is a single channel equation for ϕ_P that allows all of the S interactions that influence P, without solving for ϕ_S.

The solution for Φ_p is an integral equation in (6) or as a differential equation in (7).

G_S can be found by solving

$$\left(\nabla^2 + \frac{\omega^2}{\beta_0^2} - V_{SS}\right) G_S = \delta$$

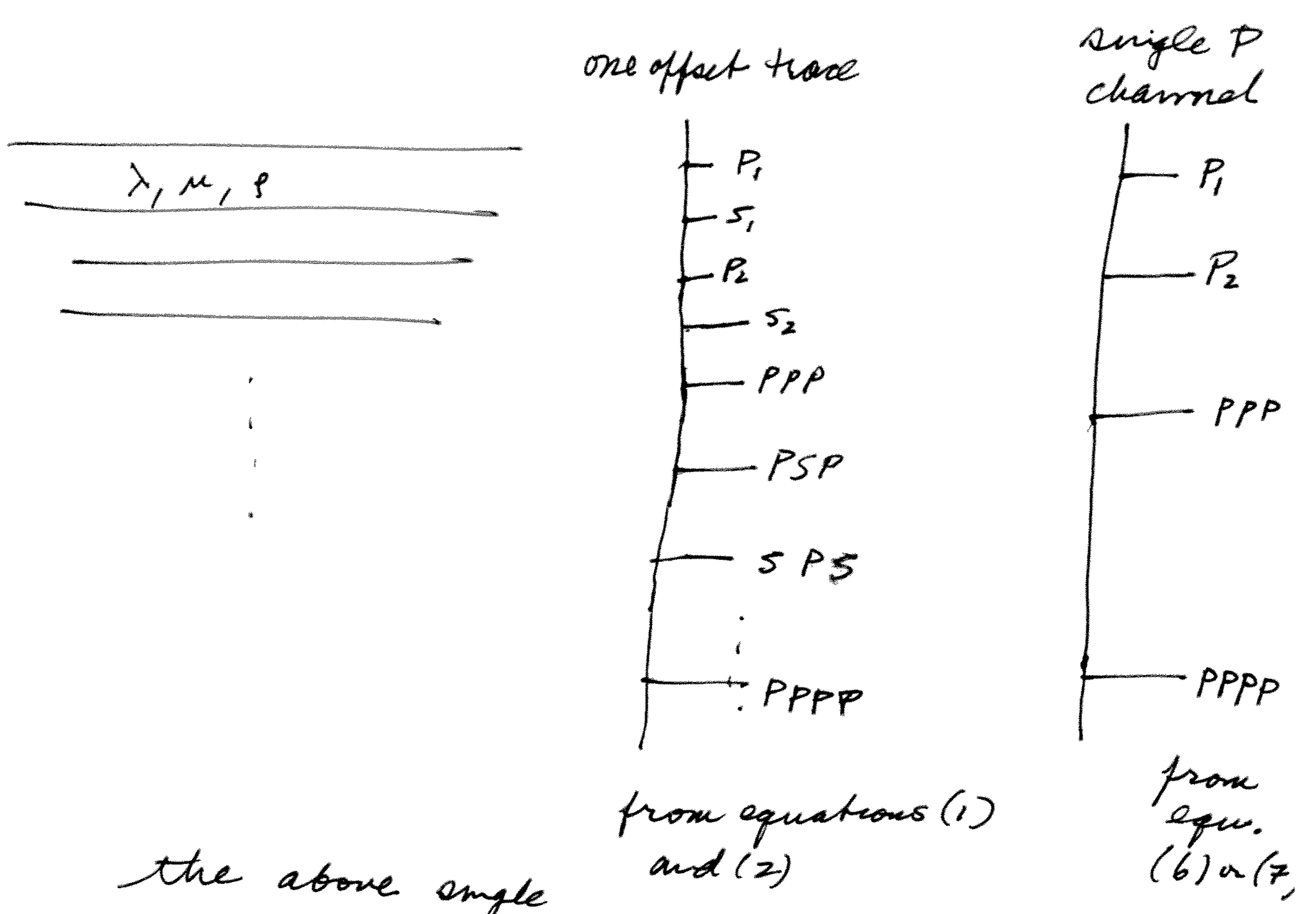
directly by, e.g., finite difference methods or

$$G_S = G_S^{\,0} + G_S^{\,0} V_{SS} G_S = G_S^{\,0}\left(1 + \sum_{n=1}^{\infty}\left(V_{SS} G_S^{\,0}\right)^n\right),$$

where $G_S^{\,0}$ satisfies

$$\left(\nabla^2 + \frac{\omega^2}{\beta_0^2}\right) G_S^{\,0} = \delta$$

and one might choose a 'Born' form.



the above single channel Ponly Calculation is being performed by Dr. Jujan Pang (a new post-doc in our group and Mason Biamonte (a sophmore undergraduate who already published papers in Phys. Rev and a chapter in a Quantum Field Theory book)

If we write a variable background theory then a small perturbation could be realized and a Born form for G_S would be reasonable. As a modeling tool we know the medium properties and using a smooth but proximal background could allow a WKBJ solution with P and S remaining uncoupled in the background.

We could also develop a discontinuous background version where P and S are coupled in the background, as well as in the actual medium.

This single effective and complex channel for a multi-channel problem was pioneered for election atom, projectile-nucleus problems by Herman Feshbach where there are infinite coupled channels (for each excited state of the target) and a single effective elastic channel is sought. The 2 channel or 3 channel (in 3D) solution we propose are also derivable by the Feshbach project operator approach that he developed.