Target identification using the inverse scattering series; inversion of large-contrast, variable velocity and density acoustic media

Haiyan Zhang and Arthur B. Weglein
University of Houston

Abstract

A new task specific multiparameter estimation subseries of the inverse scattering series is derived and tested for a velocity and density varying 1D earth. Tests are encouraging and indicate that one term beyond linear provides significant improvement beyond standard practice.

1 Introduction

The original inverse series research aimed at separating imaging and inversion tasks on primaries (Weglein et al., 2002) was developed for a 1D acoustic constant density medium and a plane wave at normal incidence. In this work we move a step closer to seismic exploration relevance by extending that earlier work to variations in both velocity and density and allowing for point sources and receivers over a 1-D earth. Tests with analytic data indicate significant added value, beyond linear estimates, in terms of both the proximity to actual value and the increased range of angles over which the improved estimates are useful.

This work is another step towards using the task specific parameter estimation inverse series for identifying large contrast targets with either specular or diffractive wave responses.

2 Inverse scattering and seismic processing objectives

Consider the basic wave equations

$$LG = \delta \tag{1}$$

$$L_0G_0 = \delta \tag{2}$$

where L and L_0 are respectively the differential operators that describe wave propagation in the actual and reference medium, and G and G_0 are the corresponding Green's functions.

We define the perturbation $V = L_0 - L$ (Weglein et al., 2002). The Lippmann-Schwinger equation

$$G = G_0 + G_0 V G \tag{3}$$

relates G, G_0 and V (see, e.g., Taylor, 1972). Iterating this equation back into itself generates the Born series

$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots . \tag{4}$$

Then the scattered field $\psi_s \equiv G - G_0$ can be written as

$$\begin{aligned} \psi_s &= G_0 V G_0 + G_0 V G_0 V G_0 + \cdots \\ &= (\psi_s)_1 + (\psi_s)_2 + \cdots \end{aligned} \tag{5}$$

where $(\psi_s)_n$ is the portion of ψ_s that is n^{th} order in V. The measured values of ψ_s are the data, D, where

$$D = (\psi_s)_{ms} = (\psi_s)_{on\ the\ measurement\ surface}.$$

Expanding V as a series in orders of D (Weglein et al. 1997)

$$V = V_1 + V_2 + \cdots \tag{6}$$

then substituting Eq.(6) into Eq.(5) and evaluating Eq.(5) on the measurement surface yields

$$D = [G_0(V_1 + V_2 + \cdots)G_0]_{ms} + [G_0(V_1 + V_2 + \cdots)G_0(V_1 + V_2 + \cdots)G_0]_{ms} + \cdots . \tag{7}$$

Setting terms of equal order in the data equal, leads to the equations that determine V_1, V_2, ... from D and G_0.

$$D = [G_0 V_1 G_0]_{ms} \tag{8}$$

$$0 = [G_0 V_2 G_0]_{ms} + [G_0 V_1 G_0 V_1 G_0]_{ms} \tag{9}$$

$$\vdots$$

3 Derivation of α_1, β_1, α_2 and β_2

To find V is to perform 'inversion', i.e., medium identification. If we associate tasks with inversion: (1) Removal of free-surface multiples (2) Removal of internal multiples (3) Image primaries to correct spatial locations and finally (4) Identify medium properties, then these tasks are directly achievable in terms of data, D, and reference information only.

To illustrate task (4), we will consider a 1-D acoustic two-parameter earth model (e.g. bulk modulus and density or velocity and density). Beginning with the 3-D acoustic wave equations in the actual and reference medium (Weglein et al. 1997, Clayton and Stolt, 1981)

$$(\frac{\omega^2}{K(\mathbf{r})}+\nabla\cdot\frac{1}{\rho(\mathbf{r})}\nabla)G(\mathbf{r},\mathbf{r}^{'};\omega)=\delta(\mathbf{r}-\mathbf{r}^{'}) \tag{10}$$

$$(\frac{\omega^2}{K_0(\mathbf{r})}+\nabla\cdot\frac{1}{\rho_0(\mathbf{r})}\nabla)G_0(\mathbf{r},\mathbf{r}^{'};\omega)=\delta(\mathbf{r}-\mathbf{r}^{'}) \tag{11}$$

Then the perturbation is

$$V=L_0-L=\frac{\omega^2\alpha}{K_0}+\nabla\cdot\frac{\beta}{\rho_0}\nabla \tag{12}$$

Where $\alpha=1-\frac{K_0}{K}$, $\beta=1-\frac{\rho_0}{\rho}$, K is P-bulk modulus, c is P-wave velocity and K, c and density ρ have the relation $K=c^2\rho$.

Now we assume both ρ_0 and c_0 are constants, then Eq.(11) becomes

$$(\frac{\omega^2}{c_0^2}+\nabla^2)G_0(\mathbf{r},\mathbf{r}^{'};\omega)=\rho_0\delta(\mathbf{r}-\mathbf{r}^{'}) \tag{13}$$

and for the 1-D case, the perturbation V has the following form

$$V(z,\nabla)=\frac{\omega^2\alpha(z)}{K_0}+\frac{1}{\rho_0}\beta(z)\frac{\partial^2}{\partial x^2}+\frac{1}{\rho_0}\frac{\partial}{\partial z}\beta(z)\frac{\partial}{\partial z}. \tag{14}$$

Similarly, we expand $V(z,\nabla)$, $\alpha(z)$ and $\beta(z)$ respectively as

$$V(z,\nabla)=V_1(z,\nabla)+V_2(z,\nabla)+\cdots, \tag{15}$$

$$\alpha(z)=\alpha_1(z)+\alpha_2(z)+\cdots, \tag{16}$$

$$\beta(z)=\beta_1(z)+\beta_2(z)+\cdots. \tag{17}$$

Then we have

$$V_1(z, \nabla) = \frac{\omega^2 \alpha_1(z)}{K_0} + \frac{1}{\rho_0}\beta_1(z)\frac{\partial^2}{\partial x^2} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\beta_1(z)\frac{\partial}{\partial z} \tag{18}$$

$$V_2(z, \nabla) = \frac{\omega^2 \alpha_2(z)}{K_0} + \frac{1}{\rho_0}\beta_2(z)\frac{\partial^2}{\partial x^2} + \frac{1}{\rho_0}\frac{\partial}{\partial z}\beta_2(z)\frac{\partial}{\partial z} \tag{19}$$

$$\vdots$$

Substitute Eq.(18) into Eq.(8) and we can get the linear solution for $\alpha_1(z)$ and $\beta_1(z)$ as a function of data D

$$\widetilde{D}(q_g, \theta, z_g, z_s) = -\frac{\rho_0}{4}e^{-iq_g(z_s+z_g)}[\frac{1}{\cos^2\theta}\widetilde{\alpha_1}(-2q_g) + (1 - \tan^2\theta)\widetilde{\beta_1}(-2q_g)] \tag{20}$$

where the subscripts s and g denote source and receiver respectively, and q_g, θ and $k = \omega/c_0$ are shown in Fig.1, and they have the following relations (Matson, 1997)

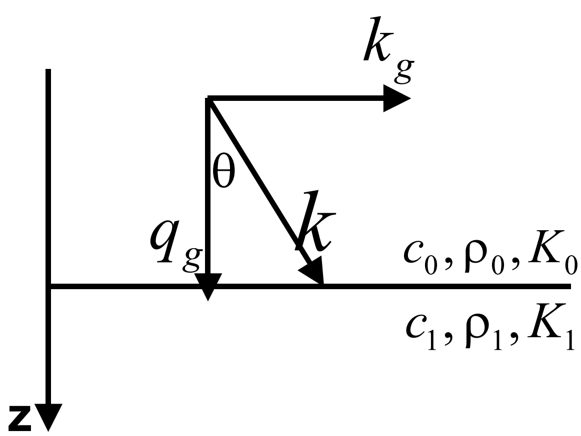

Figure 1: The relation between θ, k and q_g.

$$q_g = q_s = k\cos\theta$$

$$k_g = k_s = k\sin\theta$$

Similarly, substitute Eq.(19) into Eq.(9), and we can get the solution for $\alpha_2(z)$ and $\beta_2(z)$ as a function of $\alpha_1(z)$ and $\beta_1(z)$

$$\begin{aligned}
\frac{1}{\cos^2\theta}\alpha_2(z) + (1-\tan^2\theta)\beta_2(z) &= -\frac{1}{2\cos^4\theta}\alpha_1^2(z) \\
&+ \frac{1}{\cos^4\theta}\alpha_1(z)\beta_1(z) \\
&- (\frac{3}{2} + \tan^2\theta + \frac{1}{2}\tan^4\theta)\beta_1^2(z) \\
&- \frac{1}{2\cos^4\theta}\alpha_1^{'}(z)\int_0^z dz^{'}\alpha_1(z^{'}) \\
&+ \frac{1}{2\cos^4\theta}\alpha_1^{'}(z)\int_0^z dz^{'}\beta_1(z^{'}) \\
&+ \frac{1}{2}(\tan^4\theta - 1)\beta_1^{'}(z)\int_0^z dz^{'}\alpha_1(z^{'}) \\
&- \frac{1}{2}(\tan^4\theta - 1)\beta_1^{'}(z)\int_0^z dz^{'}\beta_1(z^{'}) \qquad (21)
\end{aligned}$$

For a single-interface example, we can see that only the first three terms on the right hand side contribute to amplitude, the other four terms are the “image moving” terms. As shown in Eq.(20) and Eq.(21), for two different angles of θ, we can determine α_1, β_1 and then α_2, β_2.

4 Numerical test

Consider a one-interface example (shown as Fig.2), the interface surface is at $z = a$, and suppose $z_s = z_g = 0$.

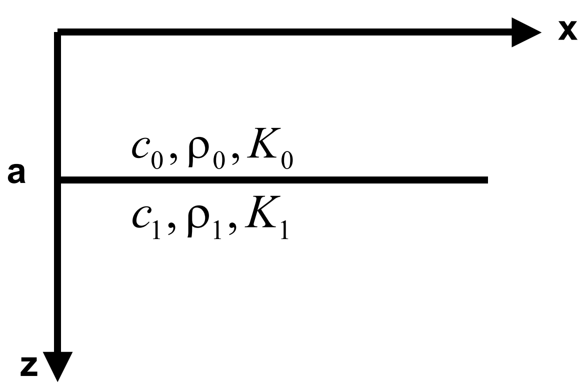

Figure 2: One interface example.

In this case, the reflection coefficient has the following form (Keys, 1989)

$$R(\theta)=\frac{(\rho_1/\rho_0)(c_1/c_0)\sqrt{1-\sin^2\theta}-\sqrt{1-(c_1^2/c_0^2)\sin^2\theta}}{(\rho_1/\rho_0)(c_1/c_0)\sqrt{1-\sin^2\theta}+\sqrt{1-(c_1^2/c_0^2)\sin^2\theta}}. \tag{22}$$

Using perfect data (Clayton and Stolt, 1981)

$$\widetilde{D}(q_g,\theta)=\rho_0 R(\theta)\frac{e^{2iq_g a}}{2iq_g}, \tag{23}$$

and substituting Eq.(23) into Eq.(20), we get

$$\frac{1}{\cos^2\theta}\alpha_1(z)+(1-\tan^2\theta)\beta_1(z)=4R(\theta)H(z-a). \tag{24}$$

Then, choosing two different angles to solve for α_1 and β_1

$$\beta_1(\theta_1,\theta_2)=4\frac{R(\theta_1)\cos^2\theta_1-R(\theta_2)\cos^2\theta_2}{\cos 2\theta_1-\cos 2\theta_2} \tag{25}$$

$$\alpha_1(\theta_1,\theta_2)=\beta_1(\theta_1,\theta_2)+4\frac{R(\theta_1)-R(\theta_2)}{\tan^2\theta_1-\tan^2\theta_2}. \tag{26}$$

Similarly, we can get α_2 and β_2

$$\begin{aligned}\frac{1}{\cos^2\theta}\alpha_2(z)+(1-\tan^2\theta)\beta_2(z)=&-\frac{1}{2\cos^4\theta}\alpha_1^2(z)+\frac{1}{\cos^4\theta}\alpha_1(z)\beta_1(z)\\&-(\frac{3}{2}+\tan^2\theta+\frac{1}{2}\tan^4\theta)\beta_1^2(z)\end{aligned} \tag{27}$$

$$\begin{aligned}\beta_2(\theta_1,\theta_2)=&\{[-\frac{1}{2}\alpha_1^2(\theta_1,\theta_2)+\alpha_1(\theta_1,\theta_2)\beta_1(\theta_1,\theta_2)][\frac{1}{\cos^2\theta_1}-\frac{1}{\cos^2\theta_2}]-\frac{1}{2}\beta_1^2(\theta_1,\theta_2)\\&[\cos^2\theta_1-\cos^2\theta_2+\frac{\sin^4\theta_1}{\cos^2\theta_1}-\frac{\sin^4\theta_2}{\cos^2\theta_2}]\}/\{\cos 2\theta_1-\cos 2\theta_2\}\end{aligned} \tag{28}$$

$$\begin{aligned}\alpha_2(\theta_1,\theta_2)=&\beta_2(\theta_1,\theta_2)+\{[-\frac{1}{2}\alpha_1^2(\theta_1,\theta_2)+\alpha_1(\theta_1,\theta_2)\beta_1(\theta_1,\theta_2)][\frac{1}{\cos^4\theta_1}-\frac{1}{\cos^4\theta_2}]\\&-\beta_1^2(\theta_1,\theta_2)[\tan^2\theta_1-\tan^2\theta_2+\frac{1}{2}\tan^4\theta_1-\frac{1}{2}\tan^4\theta_2]\}/\{\tan^2\theta_1-\tan^2\theta_2\}\end{aligned} \tag{29}$$

Now, for a specific model, $\rho_0=1.0g/cm^3$, $\rho_1=1.1g/cm^3$, $c_0=1500m/s$, $c_1=1700m/s$, let's see how the nonlinear terms contribute to the changes in the P-wave bulk modulus, density, impedance and velocity.

In the figures, we give the results corresponding to different pairs of θ_1 and θ_2. From Fig.3, we can see that when we add α_2 to α_1, the result is much closer to the exact value of α. Furthermore, the result is better behaved over a larger range of precritical angles; the values of $\alpha_1+\alpha_2$ change slowly.

Similarly, from Fig.4, we can also see the results of $\beta_1+\beta_2$ are much better than those of β_1. And also the results of $\frac{\Delta I}{I}$ (see Fig.5) and $\frac{\Delta c}{c}$ (see Fig.6). Especially, we noticed that values of $(\frac{\Delta c}{c})_1$ are always greater than zero, that is, the sign of $(\Delta c)_1$ is positive, which is same as that of the exact value Δc. We will state about it in the next section.

5 Special parameters for linear inversion

In general, linear inversion will produce errors in earth property prediction since the relationship between data and earth property changes is nonlinear.

Manifestation:

When $\Delta(property)=0$, linear prediction of $\Delta(property)\neq 0$

$\Rightarrow$ When $\Delta(property)>0$, linear prediction of $\Delta(property)$ *can be* <0.

There is a special parameter for linear inversion of acoustic media, that never suffers the latter problem.

From Eq.(22) we can see when $c_0=c_1$, the reflection coefficient is independent of θ, then from the linear Eq.(26), we have

$$\left(\frac{\Delta c}{c}\right)_1=\frac{1}{2}(\alpha_1-\beta_1)=0 \; when \; \Delta c=0$$

i.e., when $\Delta c=0$, $(\Delta c)_1=0$. This generalizes to when $\Delta c>0$, then $(\Delta c)_1>0$, or when $\Delta c<0$, then $(\Delta c)_1<0$, as well. Which can be shown as below:

The reflection coefficient is

$$R(\theta)=\frac{(\rho_1/\rho_0)(c_1/c_0)\sqrt{1-\sin^2\theta}-\sqrt{1-(c_1^2/c_0^2)\sin^2\theta}}{(\rho_1/\rho_0)(c_1/c_0)\sqrt{1-\sin^2\theta}+\sqrt{1-(c_1^2/c_0^2)\sin^2\theta}}.$$

Let

$$A(\theta)=(\rho_1/\rho_0)(c_1/c_0)\sqrt{1-\sin^2\theta},$$

$$B(\theta) = \sqrt{1 - (c_1^2/c_0^2)\sin^2\theta}.$$

Then

$$R(\theta_1) - R(\theta_2) = 2\frac{A(\theta_1)B(\theta_2) - B(\theta_1)A(\theta_2)}{[A(\theta_1) + B(\theta_1)]\,[A(\theta_2) + B(\theta_2)]}$$

where the denominator is greater than zero. The numerator is

$$2\,[A(\theta_1)B(\theta_2) - B(\theta_1)A(\theta_2)] = 2(\rho_1/\rho_0)(c_1/c_0)\left[\sqrt{1-\sin^2\theta_1}\sqrt{1-(c_1^2/c_0^2)\sin^2\theta_2} - \sqrt{1-\sin^2\theta_2}\sqrt{1-(c_1^2/c_0^2)\sin^2\theta_1}\right].$$

Then we let

$$C = \sqrt{1-\sin^2\theta_1}\sqrt{1-(c_1^2/c_0^2)\sin^2\theta_2} > 0$$

$$D = \sqrt{1-\sin^2\theta_2}\sqrt{1-(c_1^2/c_0^2)\sin^2\theta_1} > 0$$

Then

$$C^2 - D^2 = \left(\frac{c_1^2}{c_0^2} - 1\right)(sin^2\theta_1 - sin^2\theta_2)$$

Then, as $c_1 > c_0$ and $\theta_1 > \theta_2$, we have

$$\left(\frac{c_1^2}{c_0^2} - 1\right)(sin^2\theta_1 - sin^2\theta_2) > 0$$

Then

$$R(\theta_1) - R(\theta_2) > 0$$

and as $c_1 < c_0$ and $\theta_1 > \theta_2$, we have

$$\left(\frac{c_1^2}{c_0^2} - 1\right)(sin^2\theta_1 - sin^2\theta_2) < 0$$

Then

$$R(\theta_1) - R(\theta_2) < 0$$

For the single interface example, we have

$$\alpha_1(\theta_1,\theta_2) - \beta_1(\theta_1,\theta_2) = 4\frac{R(\theta_1) - R(\theta_2)}{\tan^2\theta_1 - \tan^2\theta_2}$$

So as $c_1 > c_0$, $\alpha_1(\theta_1, \theta_2) - \beta_1(\theta_1, \theta_2) > 0$, then $(\Delta c)_1 > 0$; similarly, as $c_1 < c_0$, $\alpha_1(\theta_1, \theta_2) - \beta_1(\theta_1, \theta_2) < 0$, then $(\Delta c)_1 < 0$.

We are currently generalizing these equations to an elastic Earth model. The strategy is to combine information from special linear parameters with added value from inversion beyond linear. We also note that when the velocity doesn't change across an interface, the Born inverse doesn't change and looking at the integrand of "image moving" terms (see Eq.(21)) $\alpha_1 - \beta_1$, the image doesn't move.

The imaging and inversion subseries automatically accommodate an adequate velocity model. They determine the degree of adequacy of the velocity model from all the data, and act accordingly.

6 Conclusion

Including terms beyond linear in earth property identification subseries provides added value. We are encouraged by these results. The next step is to study the elastic case using three parameters (see, e.g., Boyse, 1986 and Boyse and Keller, 1986).

Acknowledgements

We thank Bogdan Nita and Simon Shaw for valuable discussions, and we are grateful that R. Keys, R. Stolt, T.H. Tan and Jingfeng Zhang are working with us on this project.

References

Boyse, W. E., Wave propagation and inversion in slightly inhomogeneous media, p.40. (1986)

Boyse, W.E. and Keller J.B., Inverse elastic scattering in three dimensions, 1986: J. Acoust. Soc. Am., **79**, 215–218.

Clayton, R.W., R.H. Stolt, A Born-WKBJ inversion method for acoustic reflection data for attenuating multiples in seismic reflection data, Geophysics **46**, 11, 1559-1567 (1981).

Keys, R.G., Polarity reversals in reflections from layered media, 1989: Geophysics, **54**, 7, 900–905.

Matson, K. H., An inverse scattering series method for attenuating elastic multiples from multicomponent land and ocean bottom seismic data, Ph.D thesis, University of British Columbia, p. 18. (1997)

Taylor, J. R., Scattering theory: the quantum theory of nonrelativistic collisions, 1972: John Wiley & Sons, Inc., p. 133.

Weglein, A.B., F.A. Araújo Gasparotto, P.M. Carvalho and R.H. Stolt, An inverse-scattering series method for attenuating multiples in seismic reflection data, Geophysics **62**, 6, 1975-1989 (1997).

Weglein, A.B., D.J. Foster, K.H. Matson, S.A. Show, P.M. Carvalho and D. Corrigan, Predicting the correct spatial location of reflectors without knowing or determining the precise medium and wave velocity: initial concept, algorithm and analytic and numerical example, Journal of Seismic Exploration 10, 367-382 (2002).

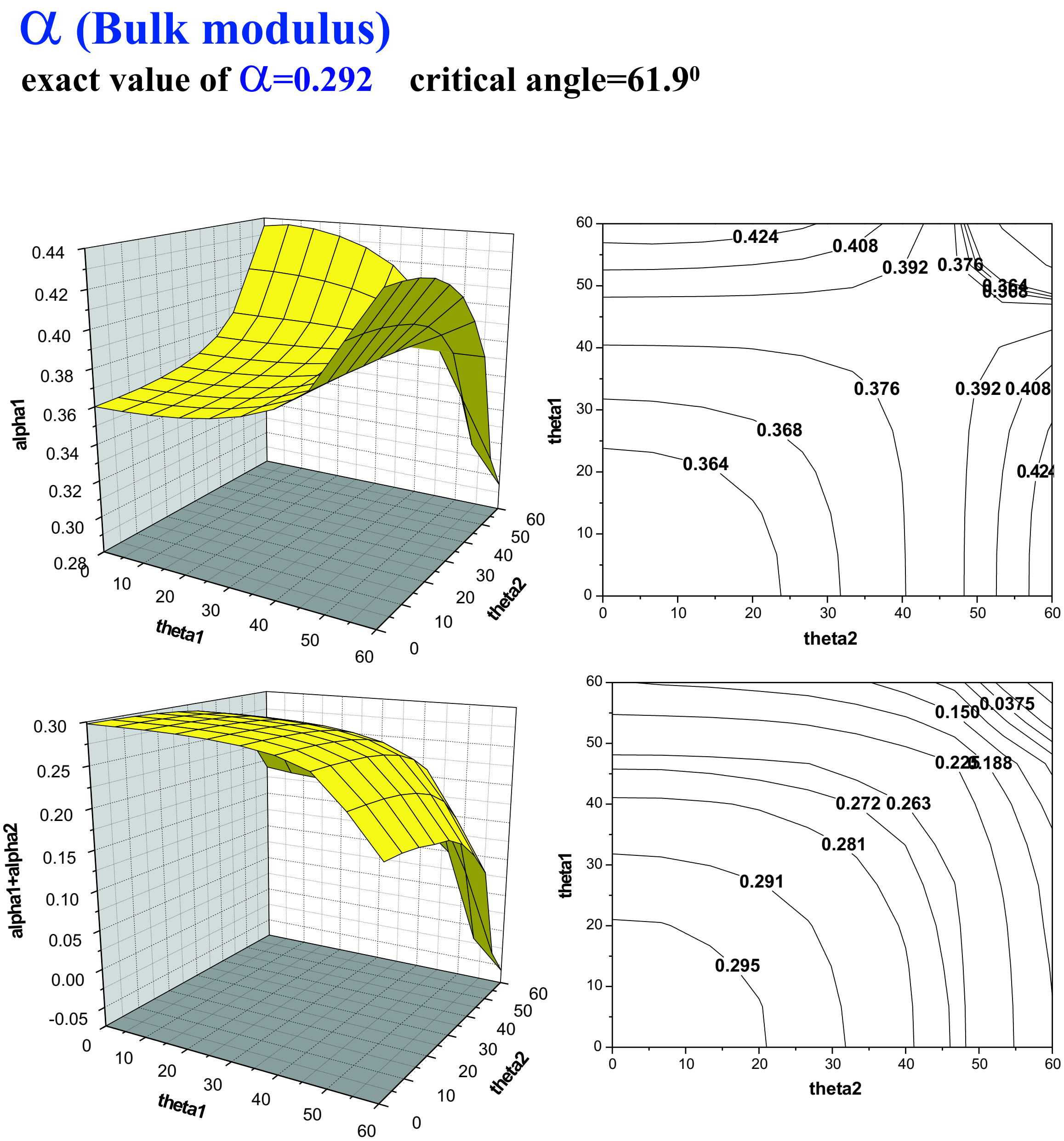

Figure 3: α_1 (top) and $\alpha_1 + \alpha_2$ (bottom).

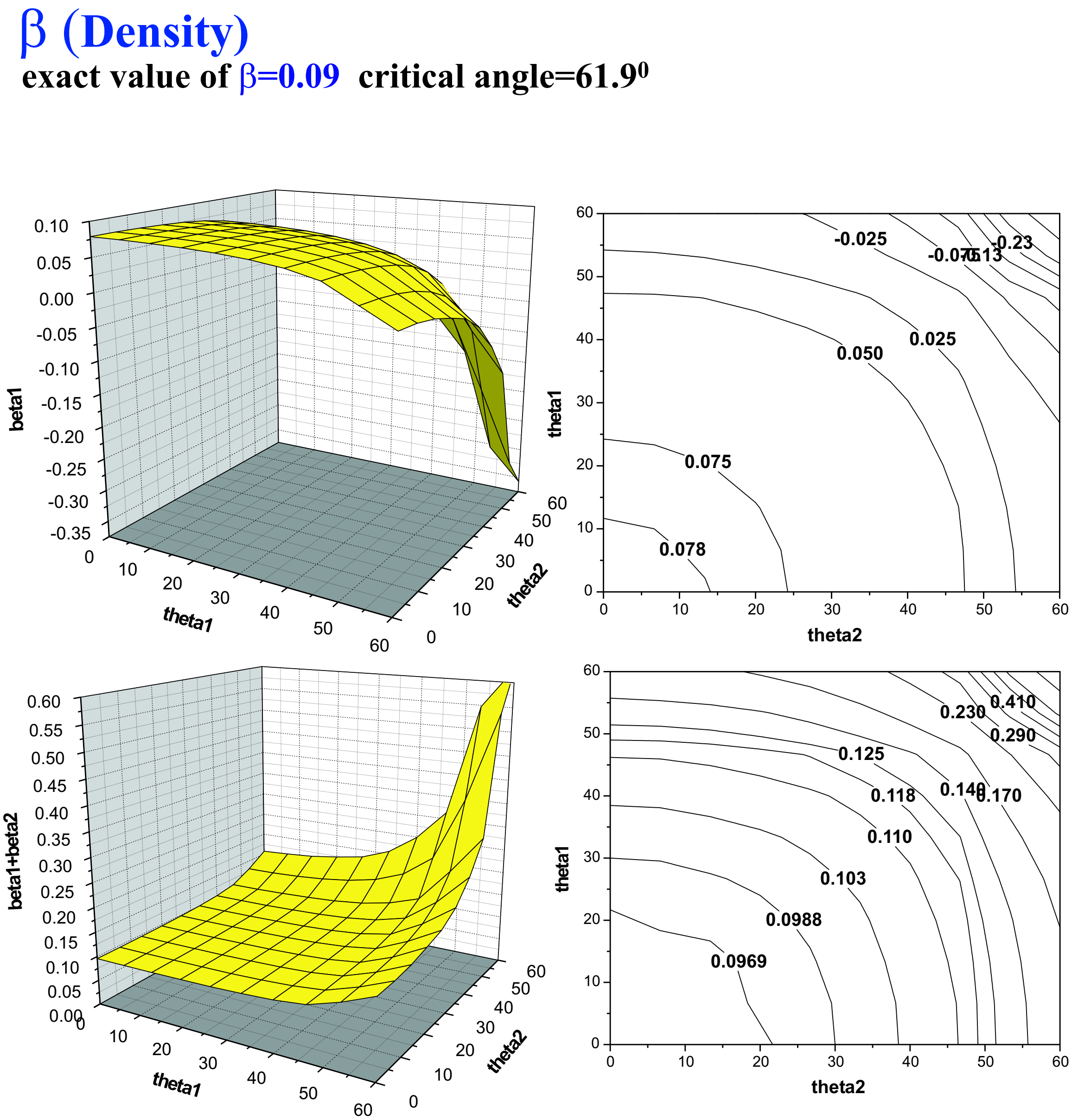

Figure 4: β_1 (top) and $\beta_1 + \beta_2$ (bottom).

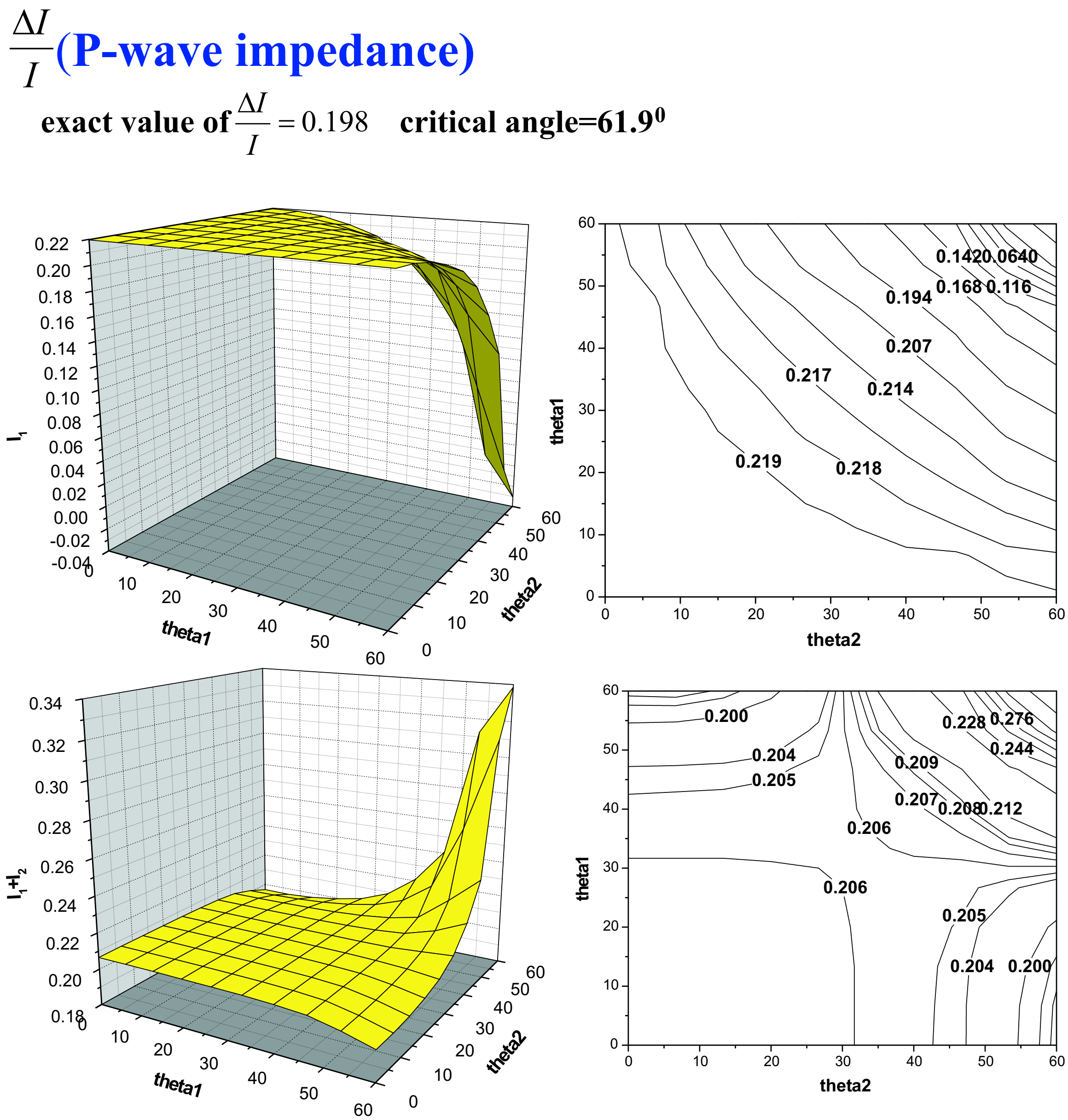

Figure 5: Linear approximation to change in impedance $\left(\frac{\Delta I}{I}\right)_1 = \frac{1}{2}(\alpha_1 + \beta_1)$ (top). Sum of linear and first non-linear terms $\left(\frac{\Delta I}{I}\right)_1 + \left(\frac{\Delta I}{I}\right)_2 = \left(\frac{\Delta I}{I}\right)_1 + \frac{1}{2}\left[\frac{1}{4}(\alpha_1 - \beta_1)^2 + (\alpha_2 + \beta_2)\right]$ (bottom).

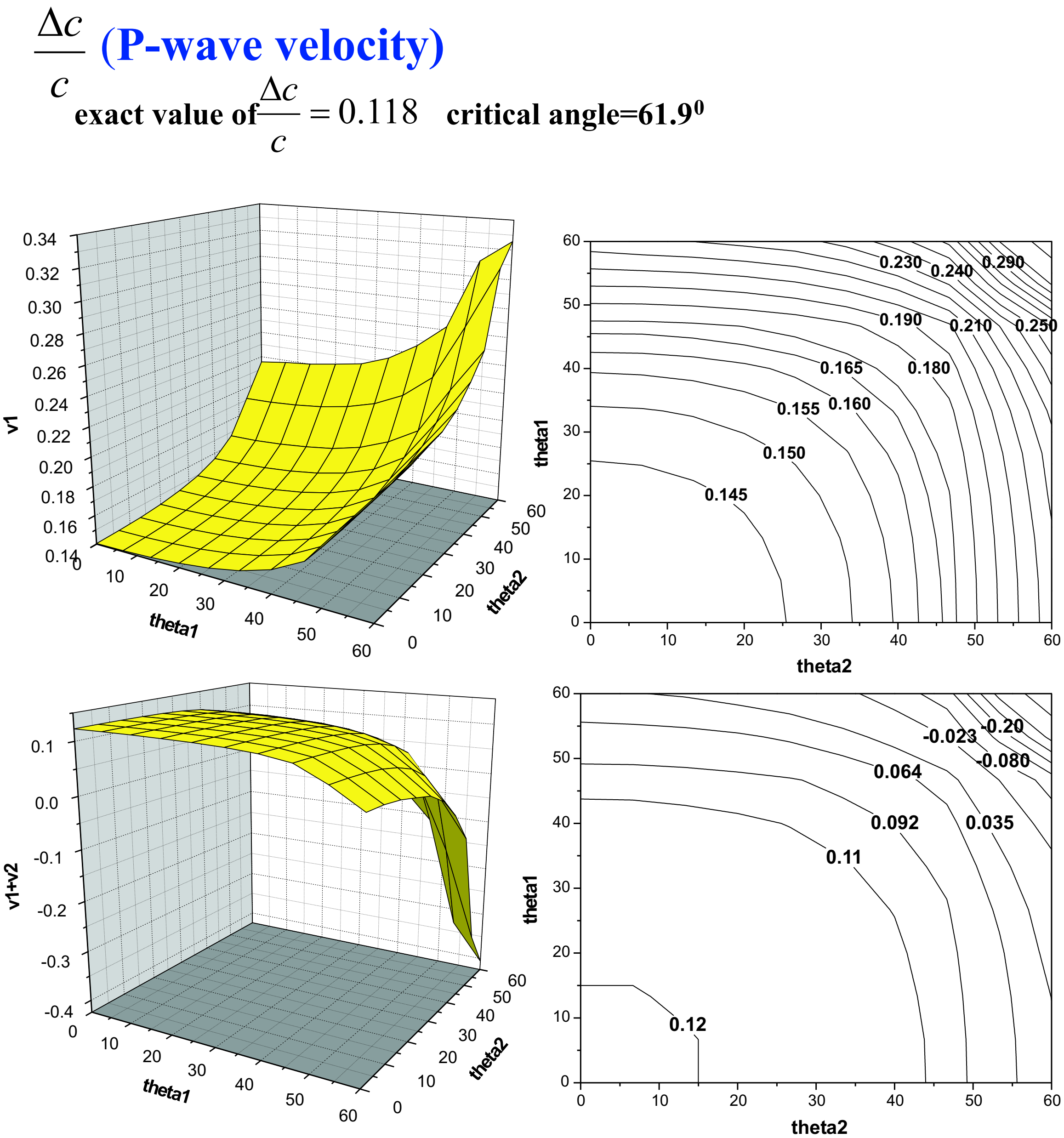

Figure 6: Linear approximation to change in velocity $\left(\frac{\Delta c}{c}\right)_1 = \frac{1}{2}(\alpha_1 - \beta_1)$ (top). Sum of linear and first non-linear terms $\left(\frac{\Delta c}{c}\right)_1 + \left(\frac{\Delta c}{c}\right)_2 = \left(\frac{\Delta c}{c}\right)_1 + \frac{1}{2}\left[\frac{1}{4}(\alpha_1 + \beta_1)^2 - \beta_1^2 + (\alpha_2 - \beta_2)\right]$ (bottom).