Wavelet estimation below towed streamers

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Abstract

Weglein and Secrest (1990) present a method for computing the scattered wavefield between the measurement surface (M.S.) and free surface (F.S.), and the reference wavefield below the measurement surface, given both the pressure and its normal derivative along the cable. Osen et al. (1998) and Tan (1999) show that the wavelet due to an isotropic source can be determined from pressure on the measurement surface and an extra hydrophone between the measurement surface and the free surface. Tan (1999) observes that in practice it is possible to well-estimate the wavefield above single towed streamer for points not directly under the source. Using the Tan (1999) wavefield prediction the wavelet can in principle be estimated from only a single cable (Weglein et al., 2000). However, the integral required for wavelet estimation requires data along the cable including the region excluded by the Tan's prediction. An approach to addressing that problem is presented here that adopts a generalized inverse viewpoint to find a good approximation to the wavelet. Empirical tests indicate that the particular alteration produces an accurate and stable estimated wavelet. Plans include testing the method in conjunction with free-surface multiple removal and then providing a comparison to the current industry-standard energy minimization method. Tests will include examples with interfering primaries and multiples.

1 Introduction

In wave-theoretic multiple attenuation methods (e.g. Carvalho (1992), Weglein et al. (1997), Verschuur et al. (1992)), knowledge of the source wavelet is one of the requirements. The energy-minimization criterion is often applied in practice to estimate the wavelet. Current methods based on the energy minimization criterion have proven to be useful under many circumstances. However, under complex conditions, e.g., with interfering events and weak internal multiples proximal to weak subsalt primaries, experience suggests that the energy minimization criterion is too blunt an instrument for that degree of subtlety. This is the motivation for deriving new methods to provide the source wavelet.

The free-surface and internal multiple methods predict the time and amplitude of multiples without subsurface information. However, if the dimension of data acquisition is not consistent with the dimension of the subsurface, e.g. a 3D subsurface requires 3D acquisition. When that requirement is lacking, the inverse scattering free surface and internal demultiple algorithm experiences a diminishment of its effectiveness. However over the past years, Shell, PGS/DELFT, and Statoil have presented talks at the EAGE and SEG Annual Meetings showing 3D acquisition and extrapolations. That means the amplitude effects, e.g., obliquity, wavelet, and deghosting are all now high priority. Results of the research in this report directly addresses one of these key amplitude requirements. If the requirements of these free-surface and interval multiple methods are satisfied, they can surgically remove multiples without damaging proximal or interfering primaries.

The industry trend towards complex and costly plays raises the bar of required effectiveness for wave theoretic multiple removal and imaging-inversion techniques, and the prerequisites, such as the wavelet, that need to be provided.

The research described here is to test and progress the development of a new source wavelet estimation algorithm that requires only the pressure on the cable. In the following, we will first discuss the Extinction Theorem. We then show how to predict normal derivatives of the wavefield above the measurement surface and finally use these predictions to obtain an estimate of the source wavelet below the measurement surface.

2 Extinction Theorem

The acoustic wave equation can be written in the following form in the frequency domain, where $\vec{r}'$ is any point in a half space below the free surface, $\vec{r}_0$ is the source location, $A(\omega)$ is the source signature, ω is the angular frequency, c is the actual velocity, and $P(\vec{r}', \vec{r}_0, \omega)$ is the pressure field.

$$\nabla^2 P(\vec{r}', \vec{r}_0, \omega) + \frac{\omega^2}{c^2(\vec{r}')} P(\vec{r}', \vec{r}_0, \omega) = A(\omega)\delta(\vec{r}' - \vec{r}_0). \tag{1}$$

Using scattering theory, the actual earth can be parameterized as a homogeneous reference medium with embedded reflectors. Hence we replace with c with c_0,

$$\frac{1}{c^2(\vec{r}')} = \frac{1}{c_0^2}[1 - \alpha(\vec{r}')], \tag{2}$$

where c_0 is the reference medium velocity, and $\alpha(\vec{r}')$ is called the perturbation, which is used to characterize the difference between the actual and reference media. Considering the Green's function in a homogenous medium with Dirichlet boundary conditions at both the free surface and the measurement surface due to a point source at $\vec{r}$, such that

$$\nabla^2 G_0^{DD}(\vec{r}, \vec{r}', \omega) + \frac{\omega^2}{c_0^2} G_0^{DD}(\vec{r}, \vec{r}', \omega) = \delta(\vec{r} - \vec{r}') \tag{3}$$

Applying Green's theorem to equations $P(\vec{r}', \vec{r}_0, \omega)$ and $G_0^{DD}(\vec{r}, \vec{r}', \omega)$:

$$\iint_S \mathrm{d}s' \left[P(\vec{r}', \vec{r}_0, \omega) \frac{\partial G_0^{DD}(\vec{r}, \vec{r}', \omega)}{\partial n} - G_0^{DD}(\vec{r}, \vec{r}', \omega) \frac{\partial P(\vec{r}', \vec{r}_0, \omega)}{\partial n} \right]$$
$$= \iiint_V \mathrm{d}\vec{r}' \left[P(\vec{r}', \vec{r}_0, \omega) \nabla^2 G_0^{DD}(\vec{r}, \vec{r}', \omega) - G_0^{DD}(\vec{r}, \vec{r}', \omega) \nabla^2 P(\vec{r}', \vec{r}_0, \omega) \right] \tag{4}$$

Multiplying equation (3) by $P(\vec{r}', \vec{r}_0, \omega)$, and equation (1) by $G_0^{DD}(\vec{r}, \vec{r}', \omega)$, and then substituting them into the right hand side of equation (4), we have

$$\iint_S \mathrm{d}s' \left[P(\vec{r}', \vec{r}_0, \omega) \frac{\partial G_0^{DD}(\vec{r}, \vec{r}', \omega)}{\partial n} - G_0^{DD}(\vec{r}, \vec{r}', \omega) \frac{\partial P(\vec{r}', \vec{r}_0, \omega)}{\partial n} \right]$$
$$= \iiint_V \mathrm{d}\vec{r}' \left[P(\vec{r}', \vec{r}_0, \omega) \delta(\vec{r} - \vec{r}') \right] + \iiint_V \mathrm{d}\vec{r}' \left[-\frac{\omega^2}{c_0^2} G_0^{DD}(\vec{r}, \vec{r}', \omega) \alpha(\vec{r}') P(\vec{r}', \vec{r}_0, \omega) \right]$$
$$+ \iiint_V \mathrm{d}\vec{r}' \left[-A(\omega) G_0^{DD}(\vec{r}, \vec{r}', \omega) \delta(\vec{r}' - \vec{r}_0) \right] \tag{5}$$

If we choose the integral volume V to be between the free surface (F.S.) and the measurement surface (M.S.), then the second term on the right hand side of equation will be zero since the scatterer $\alpha(\vec{r}')$ (i.e. Earth) is outside of the integral volume V. We then choose $\vec{r}$ above M.S., and applying the sifting property of the delta function,

$$\iiint_v \mathrm{d}\vec{r}' \left[\delta(\vec{r} - \vec{r}') f(\vec{r}') \right] = f(\vec{r}) \tag{6}$$

we have

$$\iint_S \mathrm{d}s' \left[P(\vec{r}', \vec{r}_0, \omega) \frac{\partial G_0^{DD}(\vec{r}, \vec{r}', \omega)}{\partial n} - G_0^{DD}(\vec{r}, \vec{r}', \omega) \frac{\partial P(\vec{r}', \vec{r}_0, \omega)}{\partial n} \right]$$
$$= P(\vec{r}, \vec{r}_0, \omega) - A(\omega) G_0^{DD}(\vec{r}, \vec{r}_0, \omega) \tag{7}$$

Finally, since we have chosen the Green's function $G_0^{\mathrm{DD}}(\vec{r}, \vec{r}', \omega)$ to satisfy Dirichlet boundary conditions on both F.S. and M.S., then

$$P(\vec{r}, \vec{r}_0, \omega) = A(\omega) G_0^{DD}(\vec{r}, \vec{r}_0, \omega) + \iint_{MS} \mathrm{d}s' \left[P(\vec{r}', \vec{r}_0, \omega) \frac{\partial G_0^{DD}(\vec{r}, \vec{r}', \omega)}{\partial n} \right] \tag{8}$$

where $\vec{r}$ is between F.S. and M.S.

3 Normal derivative

As shown in equation (8), $G_0^{DD}(\vec{r}, \vec{r}_0, \omega)$ is critical in order to compute the wavefield above M.S. It is a function of the frequency and the depth of the measurement surface. Tan (1999) discovered that $G_0^{DD}(\vec{r}, \vec{r}_0, \omega)$ is vanishingly small for typical marine streamer depths of approximately 6 m and seismic frequencies less than 125 Hz. Therefore, the first term on the right hand side of equation (8) can be ignored in comparison with the other terms. Also we choose the Green's function to satisfy Dirichlet boundary conditions on both F.S. and M.S., and we assume that the pressure at F.S. will be vanishing. This results in the key observation:

$$P(\vec{r}, \vec{r}_0, \omega) \approx \iint_{MS} \mathrm{d}s' \left[P(\vec{r}\,', \vec{r}_0, \omega) \frac{\partial G_0^{DD}(\vec{r}, \vec{r}\,', \omega)}{\partial n} \right] \tag{9}$$

For the normal derivative

$$\frac{\partial P(\vec{r}, \vec{r}_0, \omega)}{\partial n} \approx \iint_{MS} \mathrm{d}s' \left[P(\vec{r}\,', \vec{r}_0, \omega) \frac{\partial^2 G_0^{DD}(\vec{r}, \vec{r}\,', \omega)}{\partial z' \partial z} \right] \tag{10}$$

Equation (9) will be used to predict the wavefield above M.S. through an integral over the measurement surface. Once we obtain the Green's function, then equation (10) will produce normal derivatives directly.

4 Wavelet estimation

Since we require the normal derivatives under the source for wavelet estimation, we modify the idea of calculating the normal derivatives above the cable without dropping the wavelet term $A(\omega) G_0^{DD}(\vec{r}, \vec{r}_0, \omega)$ in equation (8). Hence

$$\frac{\partial}{\partial z} P(\vec{r}, \vec{r}_0, \omega) = A(\omega) \frac{\partial}{\partial z} G_0^{DD}(\vec{r}, \vec{r}_0, \omega) + \frac{\partial}{\partial z} \iint_{MS} \mathrm{d}s' \left[P(\vec{r}\,', \vec{r}_0, \omega) \frac{\partial G_0^{DD}(\vec{r}, \vec{r}\,', \omega)}{\partial n} \right] \tag{11}$$

We rewrite the wavelet estimation formula (Weglein and Secrest, 1990) as follows:

$$-A(\omega) G_0^{D}(\vec{r}_b, \vec{r}_0, \omega) = \iint_{MS} \mathrm{d}s' \left[P(\vec{r}\,', \vec{r}_0, \omega) \frac{\partial G_0^{D}(\vec{r}_b, \vec{r}\,', \omega)}{\partial n} - G_0^{D}(\vec{r}_b, \vec{r}\,', \omega) \frac{\partial P(\vec{r}\,', \vec{r}_0, \omega)}{\partial n} \right] \tag{12}$$

where the Green's function $G_0^{D}(\vec{r}_b, \vec{r}\,', \omega)$ satisfies the Dirichlet condition on the free surface and where $\vec{r}_b$ represents a location below M.S.

As the wavelet estimation equation (12) is the same as equation (8) on the cable (Weglein and Amundsen, 2003), we alternate the two equations by deliberately introducing some error.

In our case we choose the surface above the cable, and obtain the normal derivatives there, regarding them as the derivatives at cable, and then substitute them into the Weglein-Secrest equation (12). This we approximate

$$\frac{\partial}{\partial z}P(\vec{r},\vec{r}_0,\omega) \approx \frac{\partial}{\partial z'}P(\vec{r}',\vec{r}_0,\omega) \tag{13}$$

which will be used to estimate the normal derivatives required in wavelet estimation formula (12).

Substituting equation (11) into equation (12), we can arrive at

$$A(\omega) \approx \frac{\iint\limits_{MS} \mathrm{d}s' \left[P(\vec{r}',\vec{r}_0,\omega)\frac{\partial G_0^D(\vec{r}_b,\vec{r}',\omega)}{\partial z} - G_0^D(\vec{r}_b,\vec{r}',\omega)\frac{\partial T(\vec{r},\vec{r}_0,\omega)}{\partial z}\right]}{-G_0^D(\vec{r}_b,\vec{r}_0,\omega) + \iint\limits_{MS} \mathrm{d}s' G_0^D(\vec{r}_b,\vec{r}',\omega)\frac{\partial G_0^{DD}(\vec{r},\vec{r}_0,\omega)}{\partial z}} \tag{14}$$

where

$$\frac{\partial T(\vec{r},\vec{r}_0,\omega)}{\partial z} = \frac{\partial}{\partial z}\iint\limits_{S} \mathrm{d}\vec{r}' \left[P(\vec{r}',\vec{r}_0,\omega)\frac{\partial G_0^{DD}(\vec{r},\vec{r}',\omega)}{\partial n}\right]$$

The triangle relationship states that measured values of $P(\vec{r},\vec{r}_0,\omega)$ and its normal derivative along a cable and $A(\omega)$ satisfy the exact equation (12). One might think equation (11), when evaluated on the cable, provides a second independent relationship that would allow to be directly determined from along the cable. However, Weglein and Amundsen (2003) demonstrate that these are the same relationship. If you temporarily ignore this fact, and substitute equation (11) into equation (12) to eliminate $\frac{\partial}{\partial n}P(\vec{r},\vec{r}_0,\omega)$, then when $\vec{r}$ approaches cable, the expression in the denominator of equation (14) will be zero. The inverse is "unstable". To avoid this instability in the inversion, what is being suggested here is that values above the cable for $\frac{\partial}{\partial n}P(\vec{r},\vec{r}_0,\omega)$ and $\frac{\partial}{\partial n}G_0^{DD}(\vec{r},\vec{r}_0,\omega)$ are substituted for those at the cable in the integral to avoid the singularity. This has the effect of avoiding a singular division by solving a nearby perturbed problem with the anticipation that this will lead to a stable approximate solution.

5 Synthetic tests

We test the method in a homogenous model with three scatters. Figure 1 shows all estimated wavelet results with prediction surface changing from 5.3 m to 3.5 m from F.S. by equation (14). Figure 2 indicates the energy of the error with respect to the prediction surface and the ratio of the depth to wavelet. We can see that the least error location is at about 5.3 m from F.S, the ratio is about 0.02. When you get closer to M.S., the error rapidly increases. This means the equation here approximates an unstable state. When it is far from the M.S., the error also increases, because the normal derivative is in greater error.

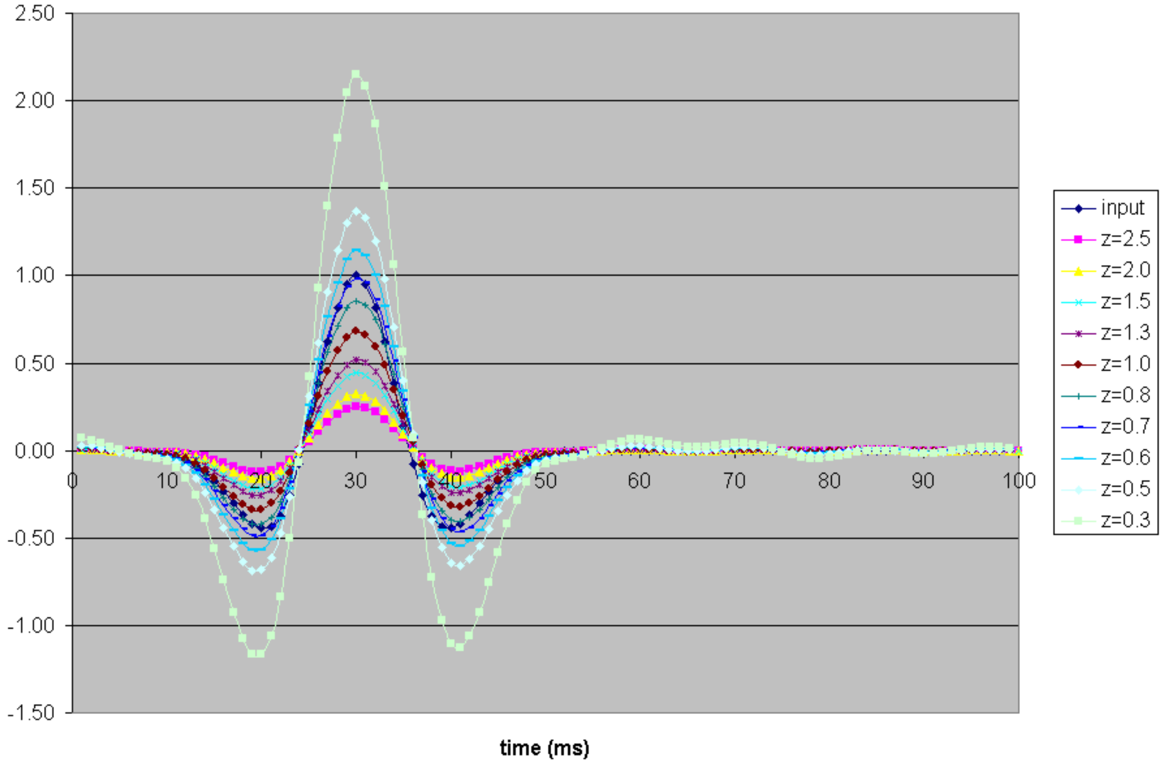

Figure 1: *Wavelet estimation for different prediction depths. Wavelet at* $z = 0.7$ *m matches the input wavelet very closely.*

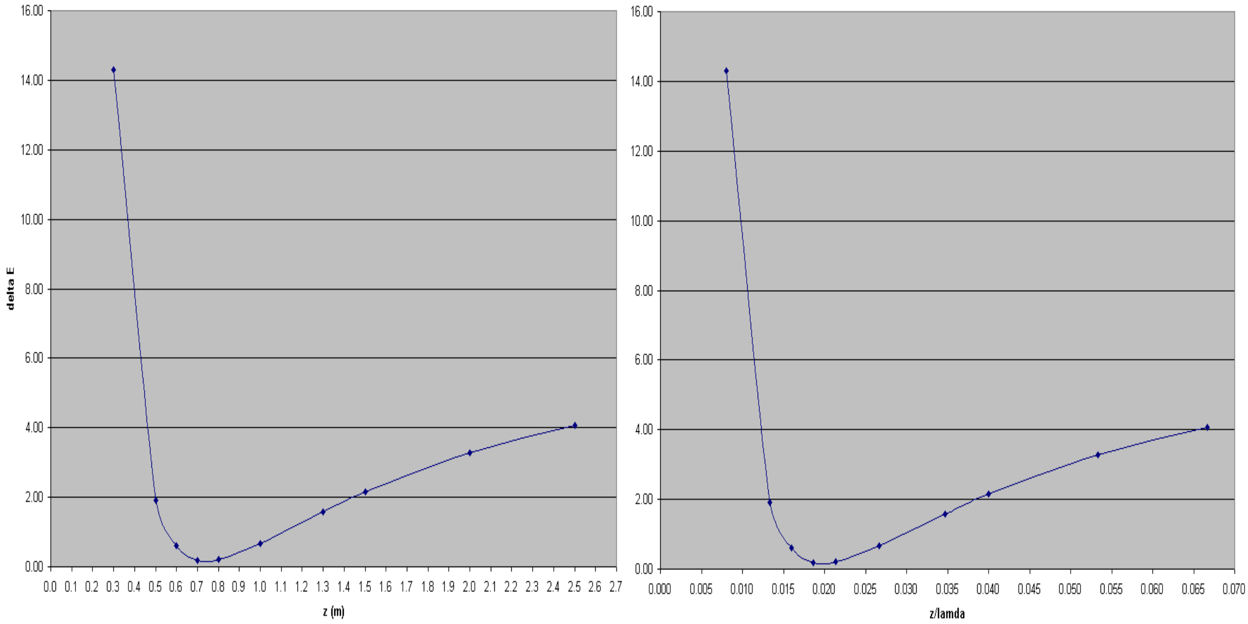

Figure 2: *Energy Error analysis. Left: Wavelet at* $z = 0.7$ *m has least error; the closer to M.S., the bigger the error gets, as it is close to unstable state. Right: Error vs. ratio of depth to wavelength.*

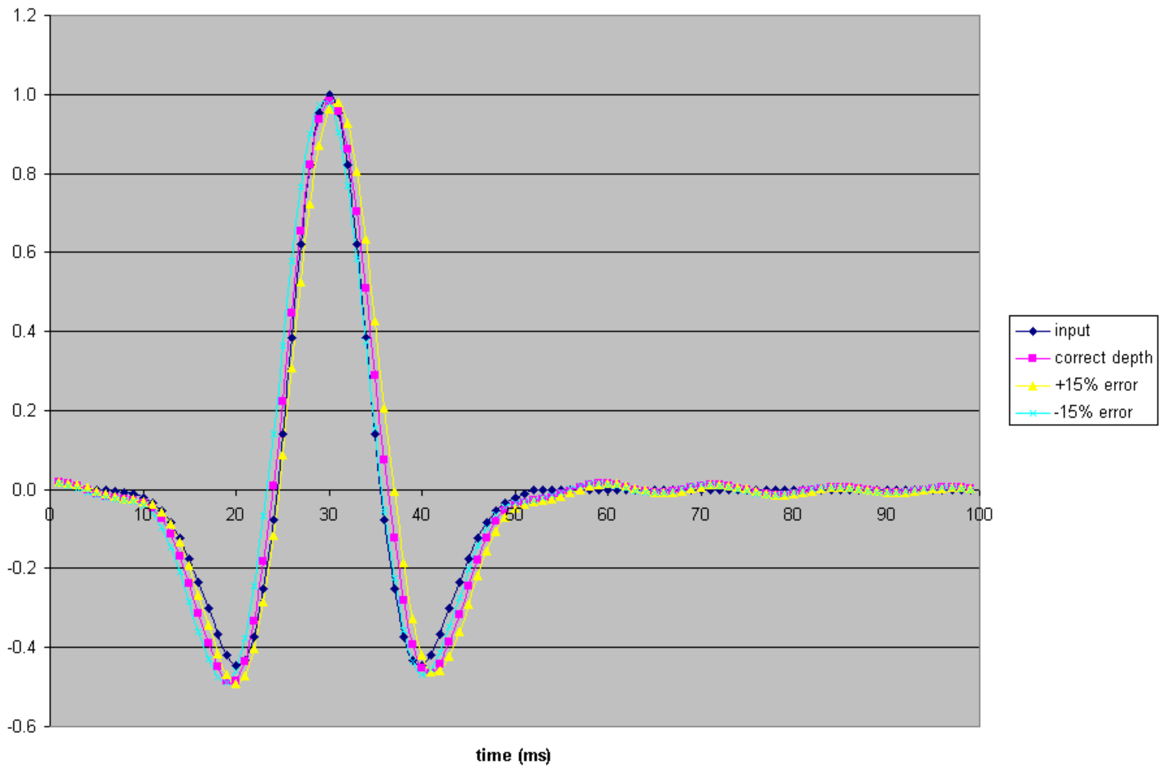

Figure 3: *Wavelet estimation with cable depth ±15% error, the estimated wavelet is close to the input even if the cable depth has ±15% error.*

To test the stability of the cable depth, we assume the cable has ± 15% error. The result is shown in Figure 3. All of the three estimated wavelets are close to the actual wavelet. We further test synthetic datasets with water depth 40 m. The source is 2 m below the free surface, receivers are 6 m below the free surface, and the receiver interval is 2 m. Then equation (14) is used to estimate the wavelet (Figure 4). We compared three cases of weighting of the estimated wavelet. The weighting results are better then the averaged.

6 Conclusions

A method for estimating the wavelet directly from the data on a towed streamer was recently proposed by Weglein et al. (2002). That method proposed using the Tan (1999) wavefield prediction method to approximate the needed normal derivative along cable. However, the wavelet method requires an integral over all receivers for a given shot, and the Tan (1999) prediction is not accurate under the source. In this paper, we propose addressing this problem by not dropping the term, which is small only away from the source to achieve an algorithm that is valid for all offsets needed in the integral.

An intrinsic instability in this approach is addressed by seeking an approximate solution that replaces the unstable inversion by a "nearby" (i.e., perturbed) operation. Tests for different

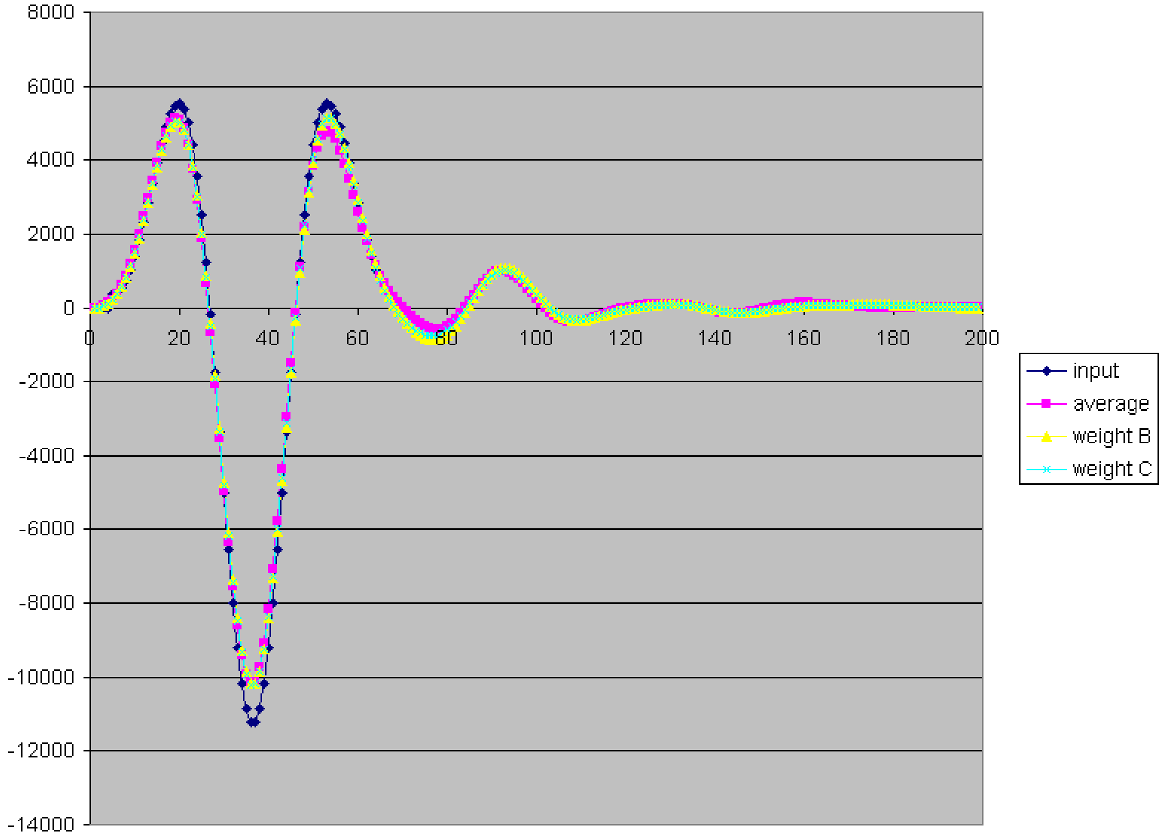

Figure 4: *Wavelet estimation using weighting: average means weighting by 1; B and C have different weighting.*

prediction depths and noise stability on synthetic data are encouraging; further tests are planned for noise stability and the impact on wave-theoretic demultiple methods.

Acknowledgments

The authors would like to thank the M-OSRP sponsors and the members of the wavelet estimation working team for their support and helpful discussions.

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