
Depth consistent PP and PS seismic angle tomography

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PP and PS tomography

- PP / PS differential semblance
- co-depthing (isotropic)
- anisotropy, TIV
 - v_{P0} , v_{S0} , δ , ε
- field data example

PP - Differential semblance

$$\mathcal{J}_{PP}(\mathbf{m}) = \frac{1}{2} \iiint |\partial_{\theta, \psi} \mathcal{I}_{PP}(\theta, \psi; \mathbf{y}, \mathbf{m})|^2 d\theta d\psi d\mathbf{y}$$

$$\mathbf{y} = (y_1, y_2, y_3)$$

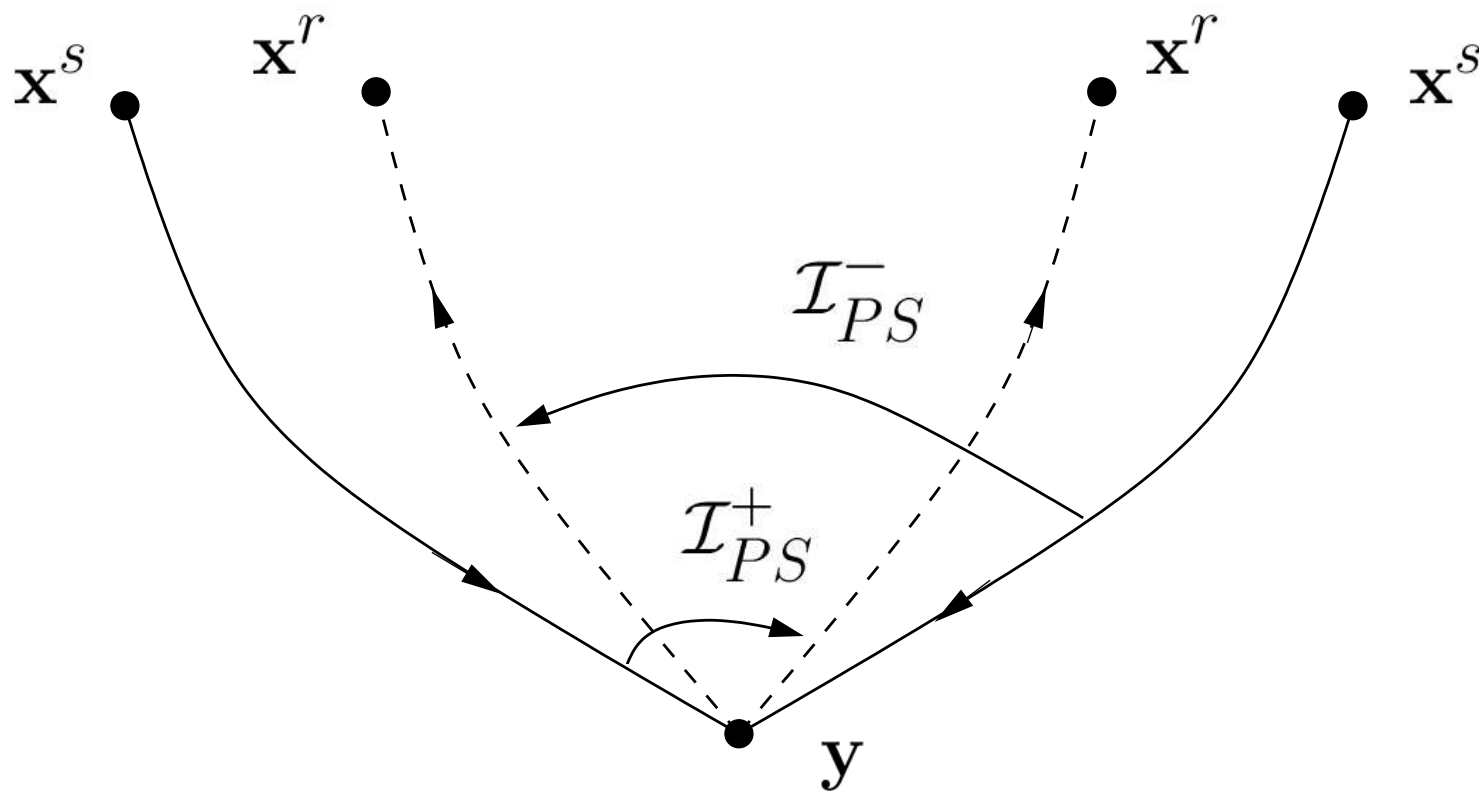
Gradient

$$\nabla_{\mathbf{m}} \mathcal{J}_{PP}(\mathbf{m}) = \iiint [\partial_{\theta, \psi} \mathcal{I}_{PP}(\theta, \psi; \mathbf{y}, \mathbf{m})] \\ \cdot \partial_{\theta, \psi} \nabla_{\mathbf{m}} \mathcal{I}_{PP}(\theta, \psi; \mathbf{y}, \mathbf{m}) d\theta d\psi d\mathbf{y}$$

PS - Differential semblance

$$\mathcal{J}_{PS}(\mathbf{m}) = \frac{1}{2} \iiint \left\{ \left| \partial_{\theta, \psi} \mathcal{I}_{PS}^+(\theta, \psi; \mathbf{y}, \mathbf{m}) \right|^2 \right. \\ \left. + \left| \partial_{\theta, \psi} \mathcal{I}_{PS}^-(\theta, \psi; \mathbf{y}, \mathbf{m}) \right|^2 \right\} d\theta d\psi d\mathbf{y}$$

Positive and negative cigs

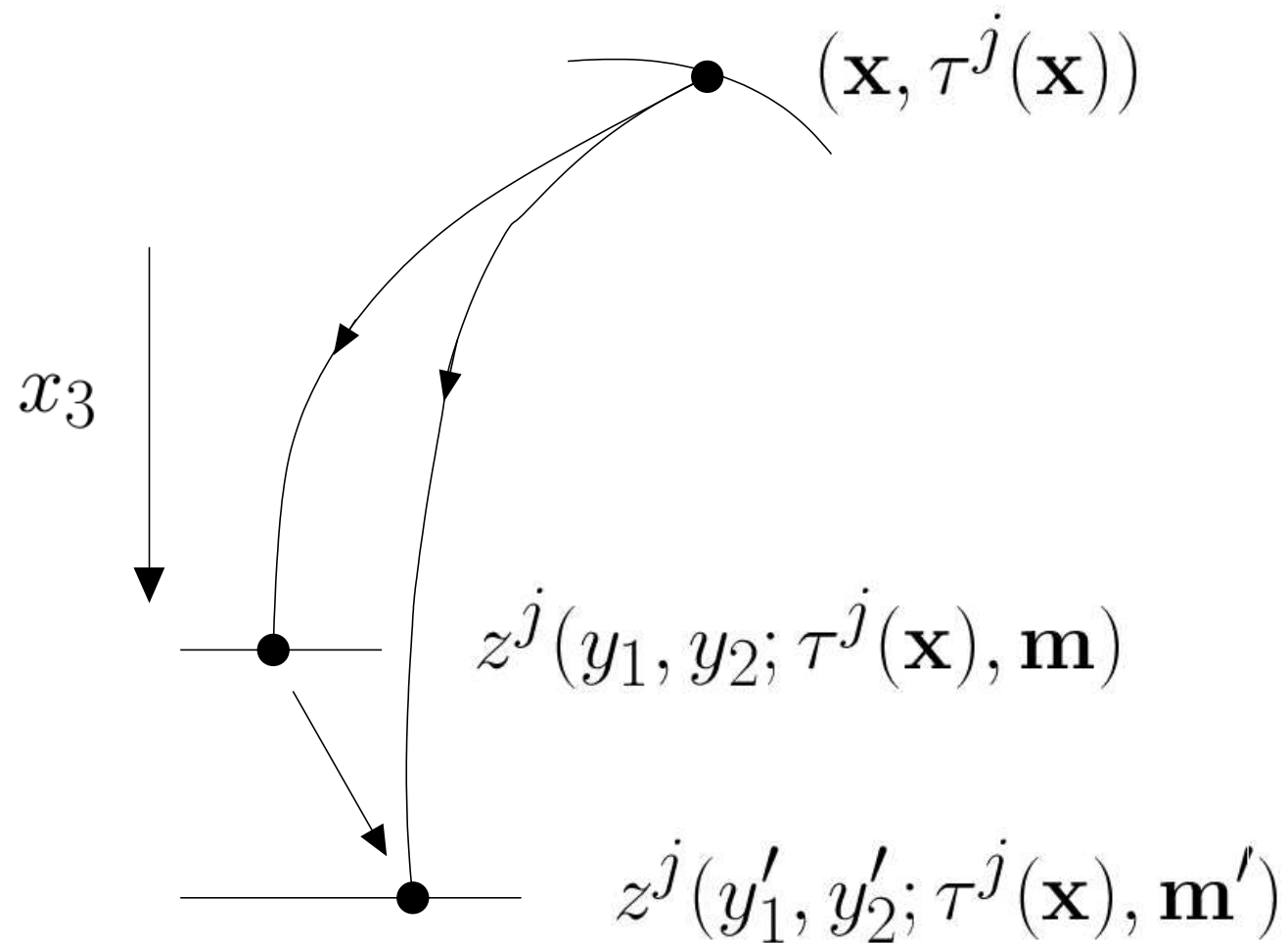


Co-depthing PP and PS

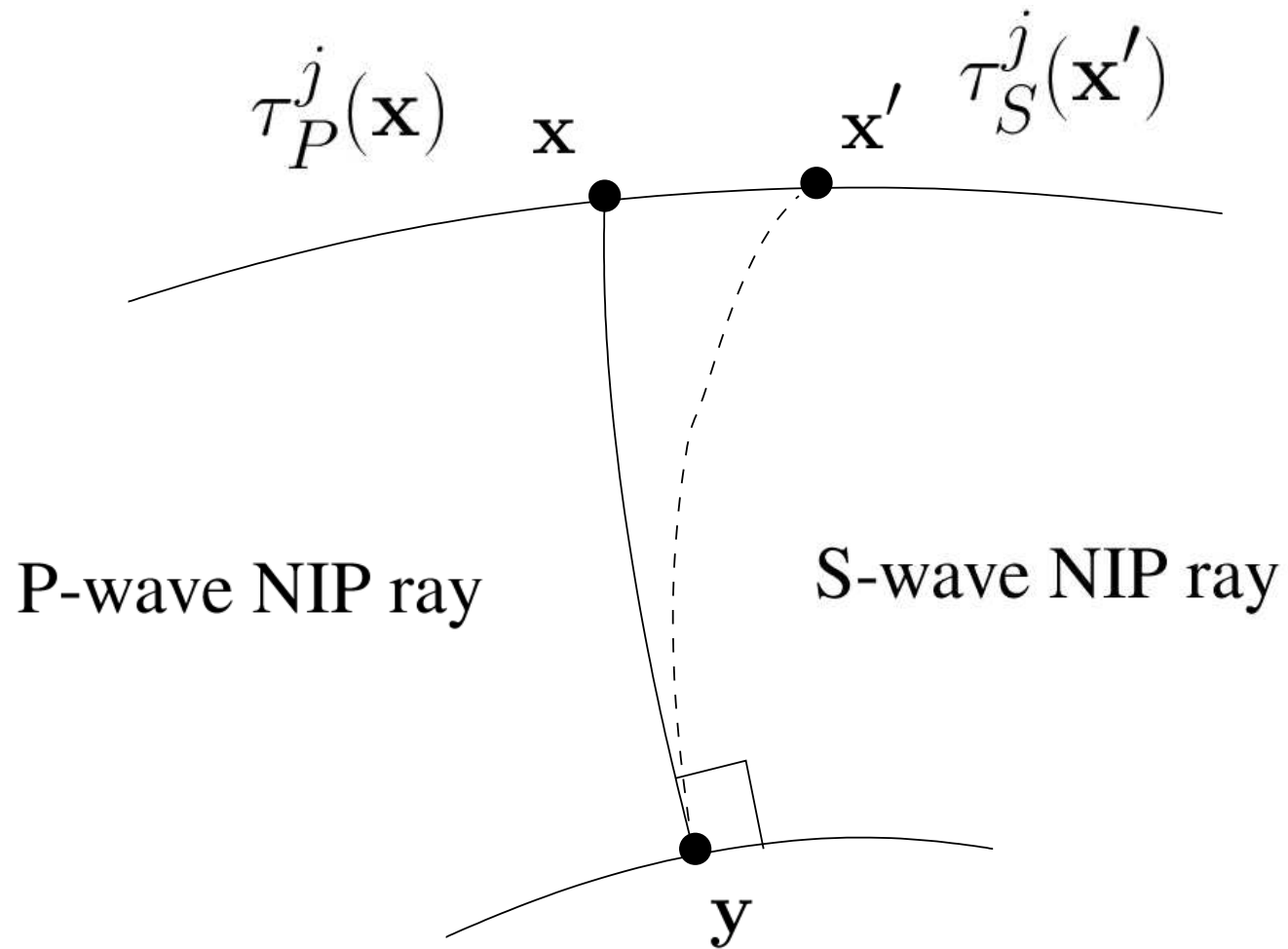
NIP ray map migration :

$$(\mathbf{x}, \tau^j(\mathbf{x})) \xrightarrow{\mathbf{m}} (y_1, y_2, z^j(y_1, y_2; \tau^j(\mathbf{x}), \mathbf{m}))$$

Zero-offset map migration



Zero-offset PS traveltimes



SS - travelttime

$$\left. \begin{array}{l} \tau_{PP}^j(\mathbf{x}) = 2\tau_P^j(\mathbf{x}) \\ \tau_{PS}^j(\mathbf{x}, \mathbf{x}') = \tau_P^j(\mathbf{x}) + \tau_S^j(\mathbf{x}') \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \tau_P^j(\mathbf{x}) \\ \tau_S^j(\mathbf{x}') \end{array} \right.$$

Depth consistency $\tau_S^j(\mathbf{x})$

$$\mathcal{J}_D(\mathbf{m}) =$$

$$\sum_j \int |z_S^j(y_1, y_2; \tau_S^j(\mathbf{x}), \mathbf{m}) - z_P^j(y_1, y_2; \tau_P^j(\mathbf{x}))|^2 dy_1 dy_2$$

Total misfit functional

$$\mathcal{J}(\mathbf{m}) = \lambda_1 \mathcal{J}_{PP}(\mathbf{m}) + \lambda_2 \mathcal{J}_{PS}(\mathbf{m}) + \mu \mathcal{J}_D(\mathbf{m})$$

Model representation

$$v_P(\mathbf{x}) = \sum_k (v_{Pk} + g_k x_3) I_k(\mathbf{x}) + \sum_i a_i B_i(\mathbf{x})$$

$I_k(\mathbf{x})$ layer indicator function by map migration

$B_i(\mathbf{x})$ B-spline

Strategy TI media :

- PP (isotropic)
- PS (isotropic)
- co-depthing (isotropic)
- anisotropic update (v_{P0} , v_{S0} , δ , ε)
 - $\delta = 0$
 - well logs, large offset data

Initial P-wave velocity

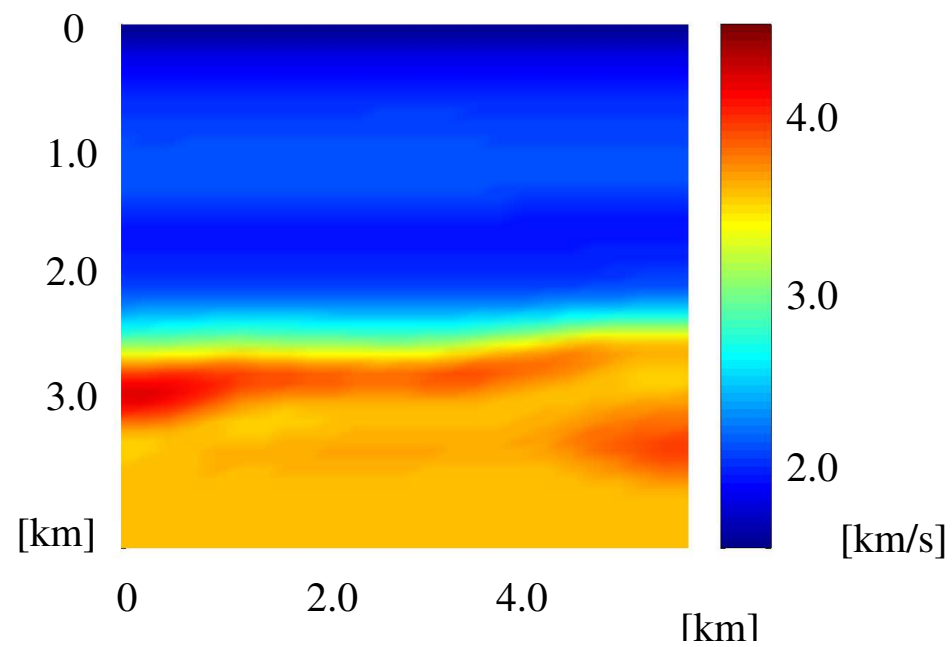
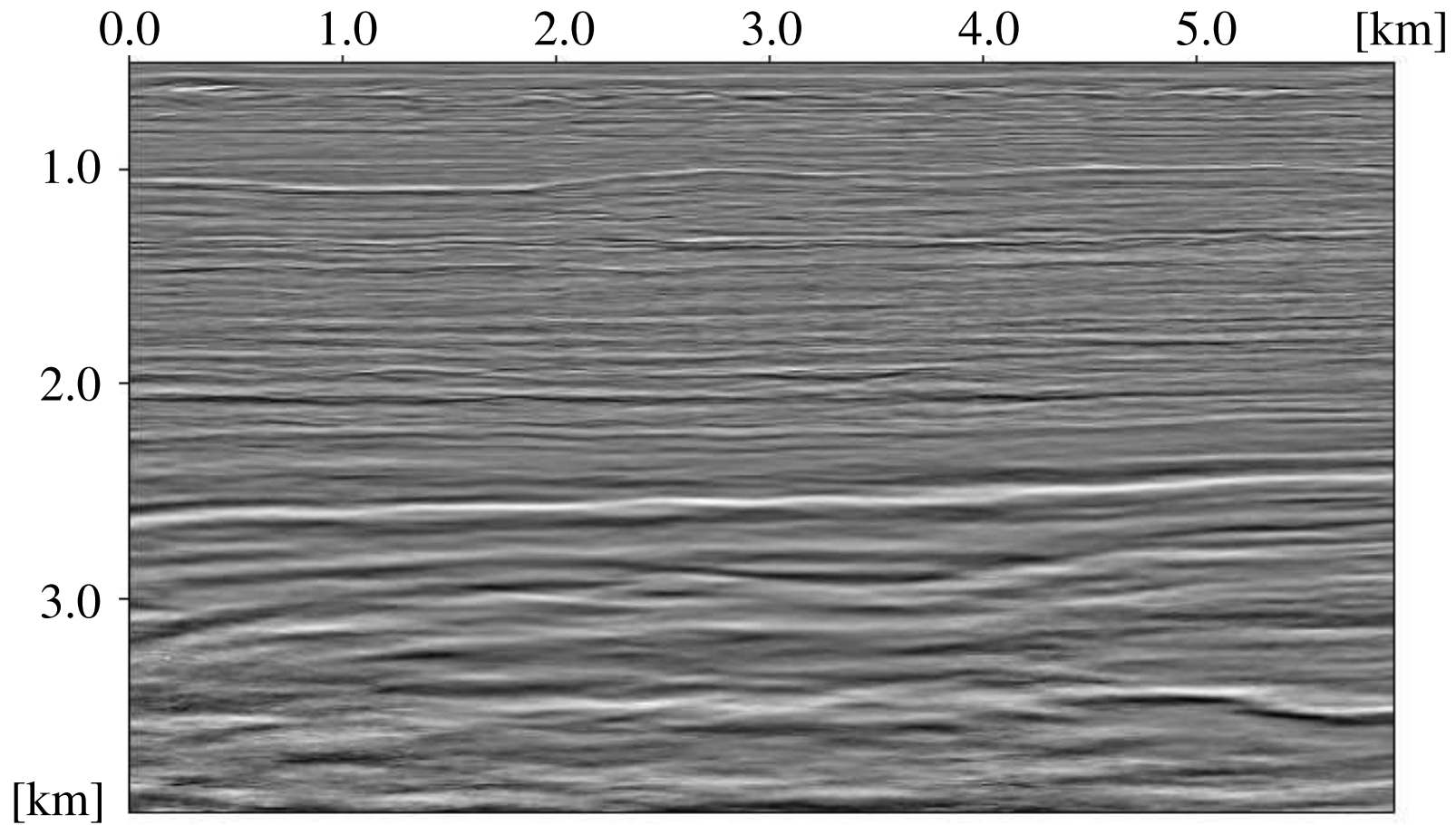
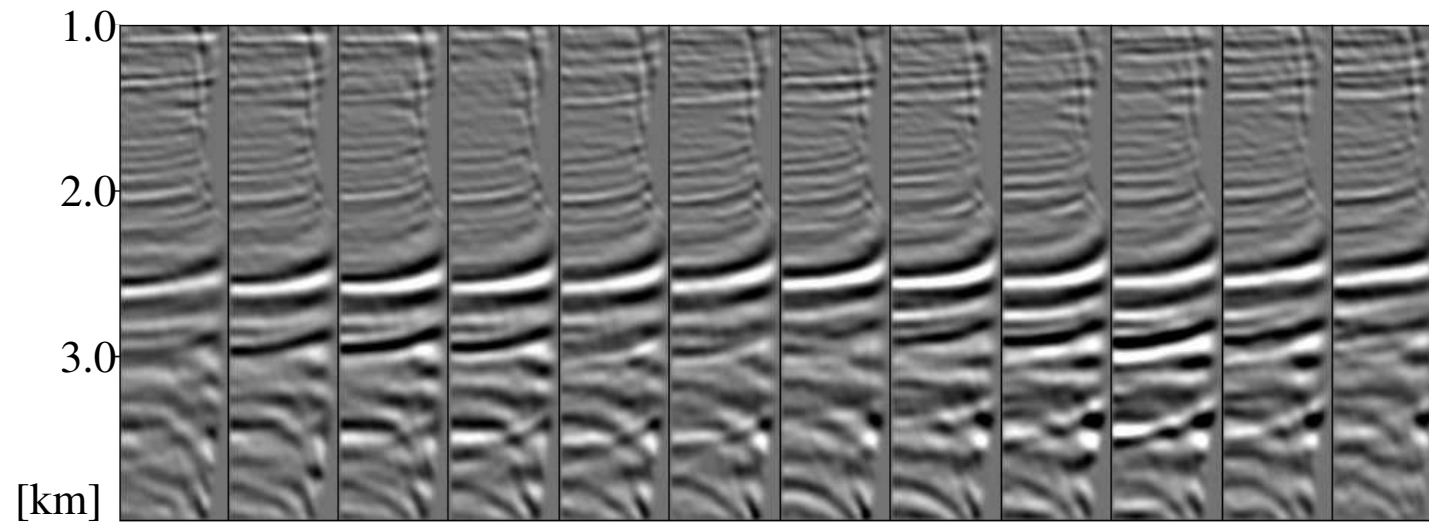


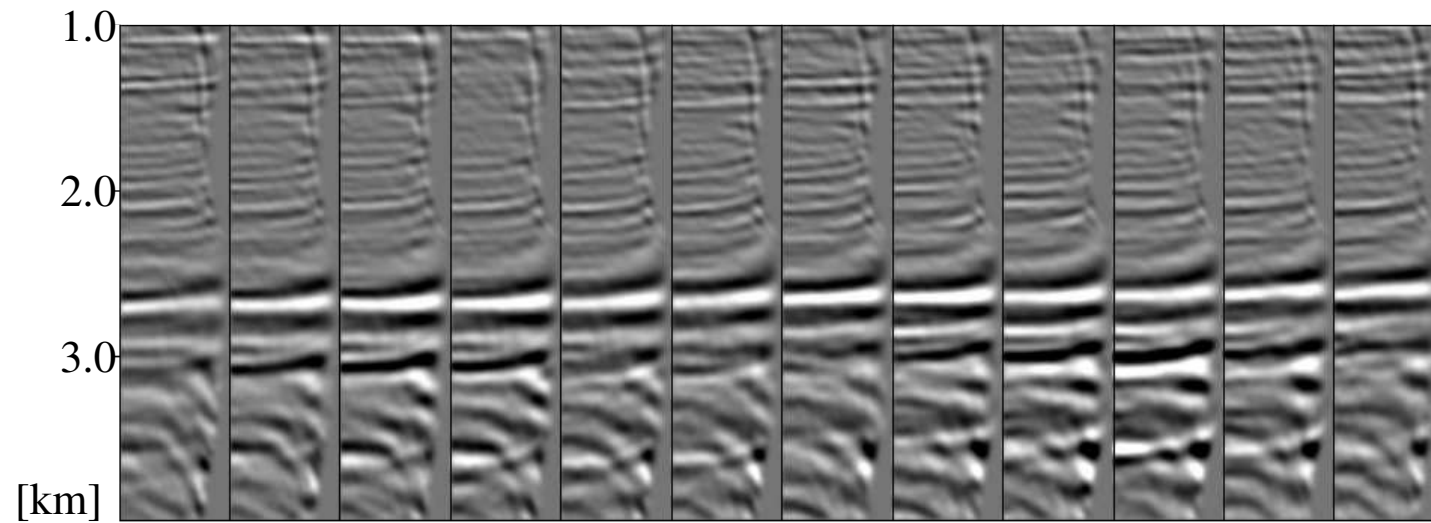
Image with initial velocity



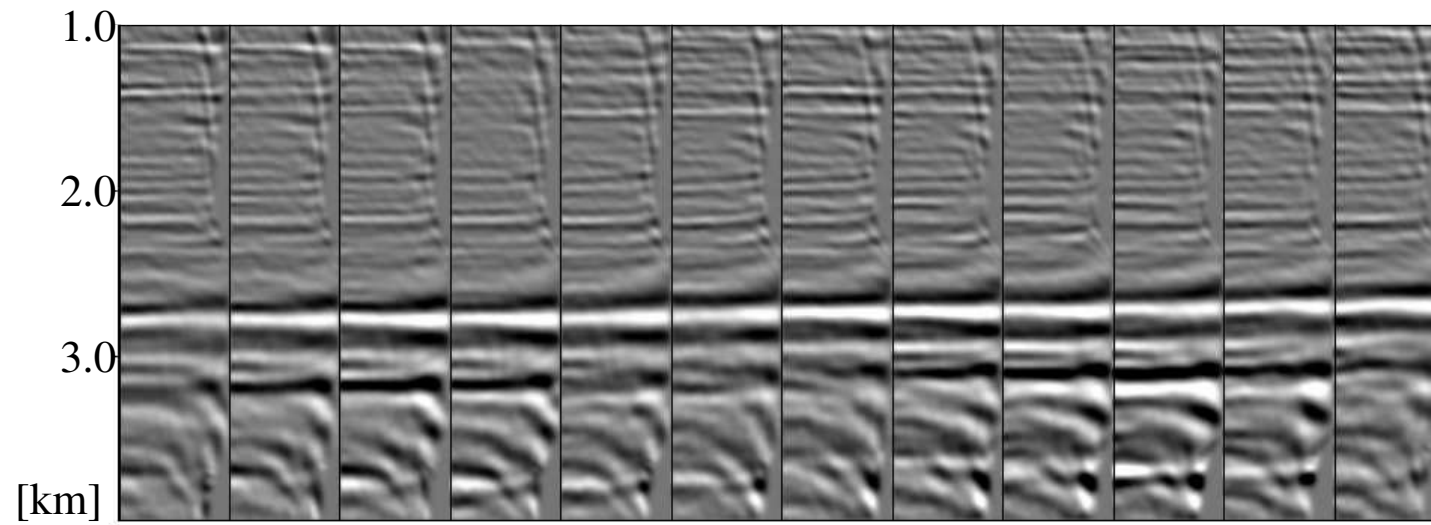
Initial cigs



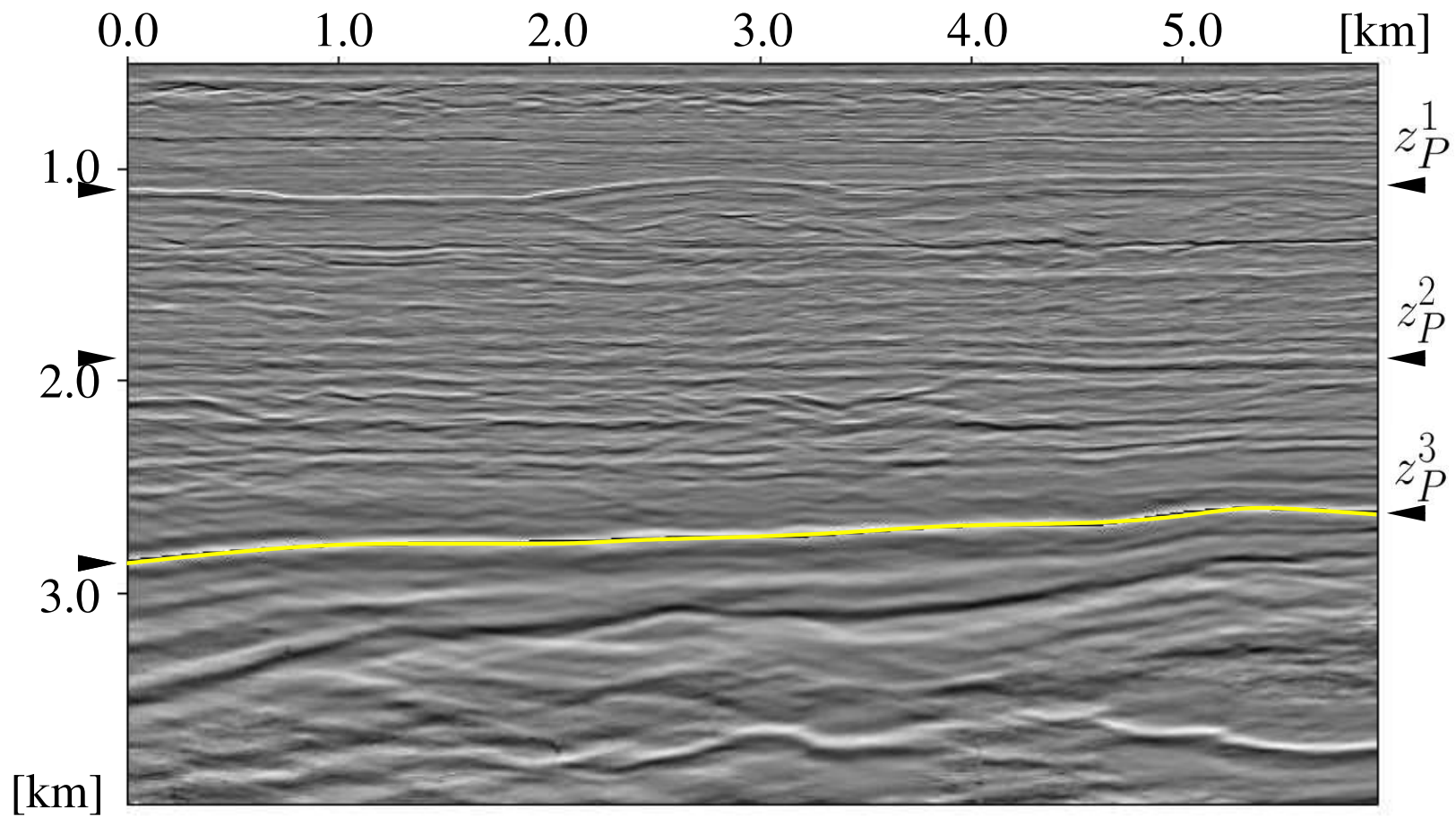
Intermediate cigs



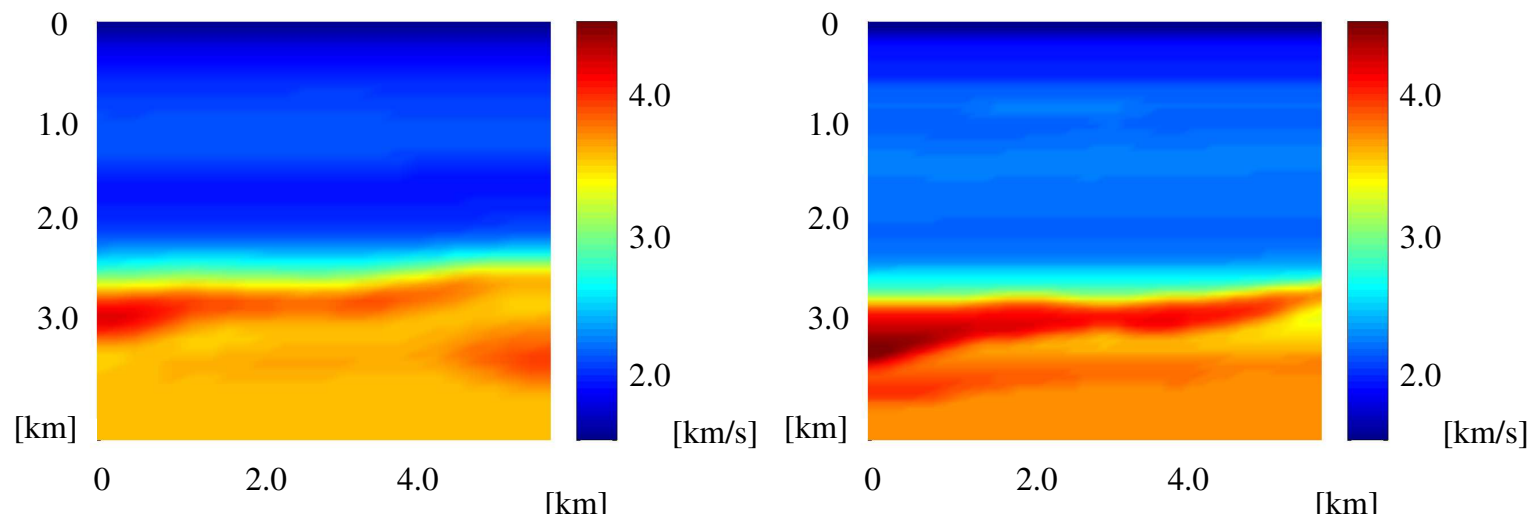
Final cigs



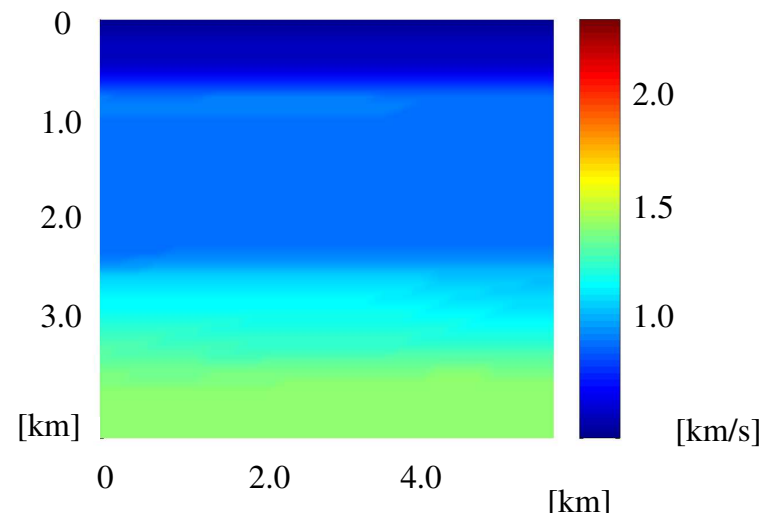
Final PP image



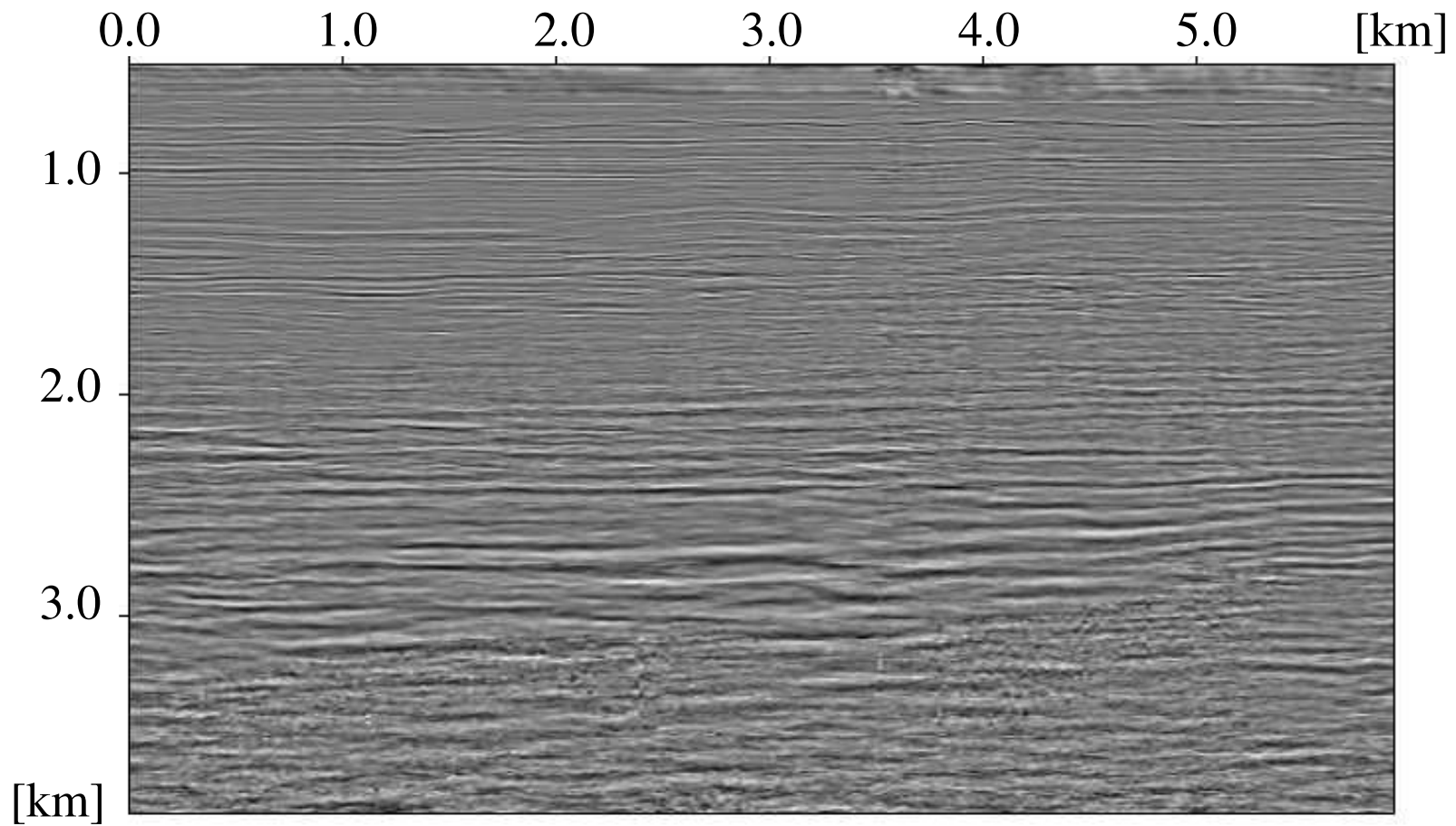
Initial and final P-wave velocity



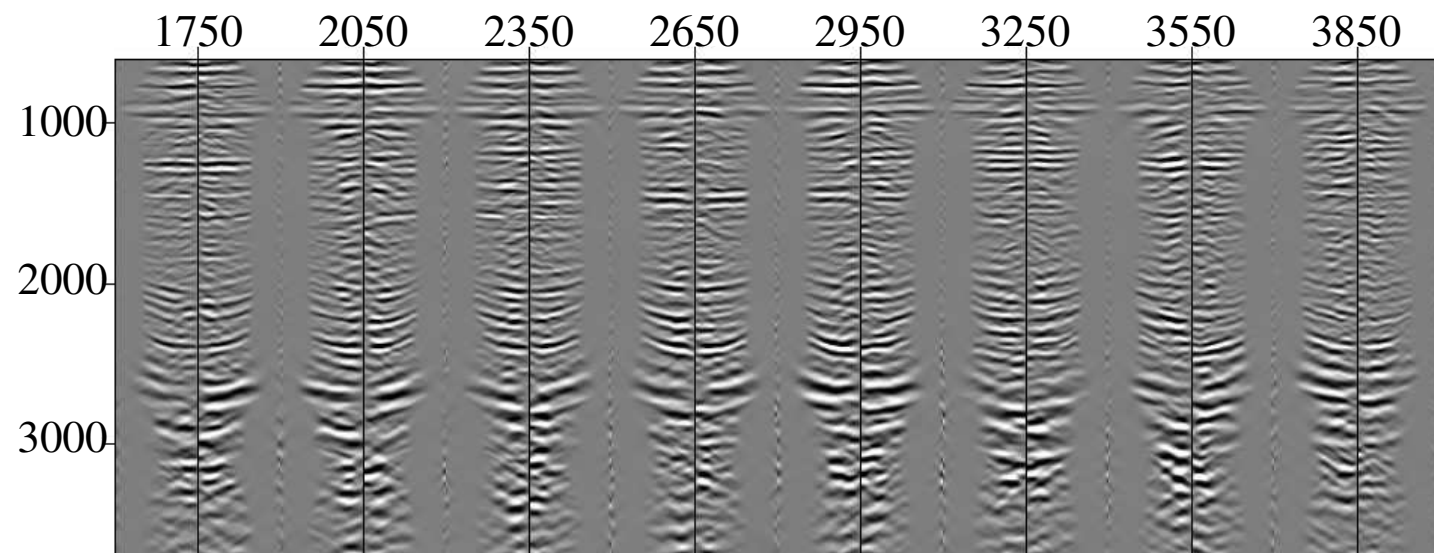
Initial S-wave velocity



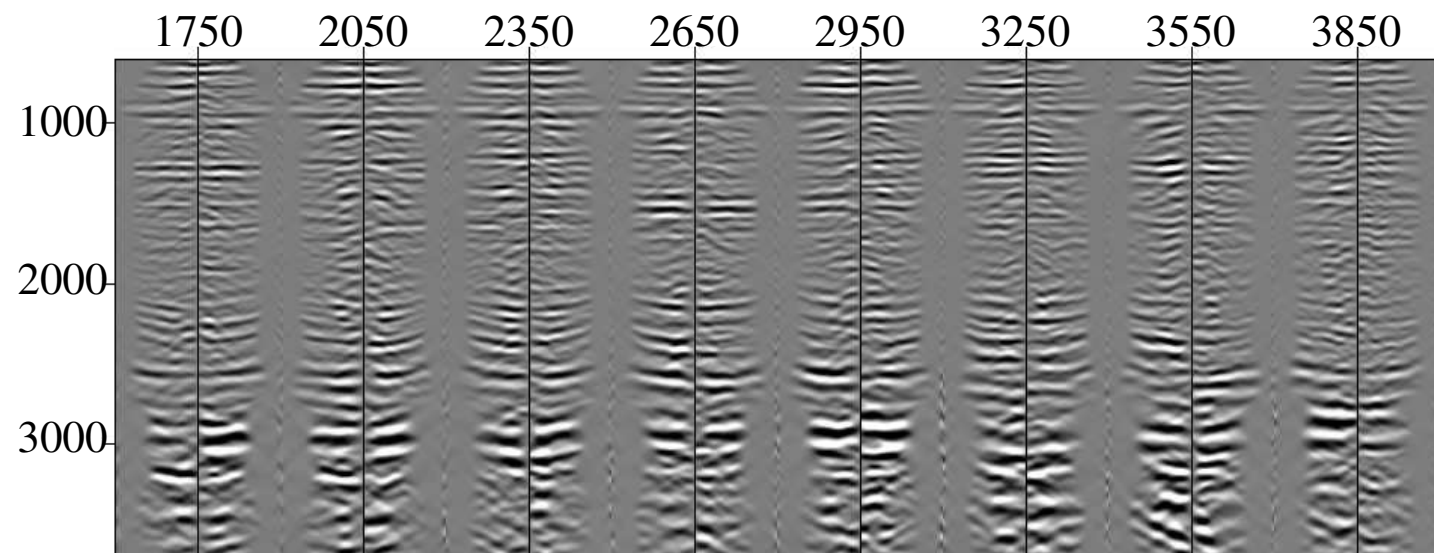
PS image with initial velocity



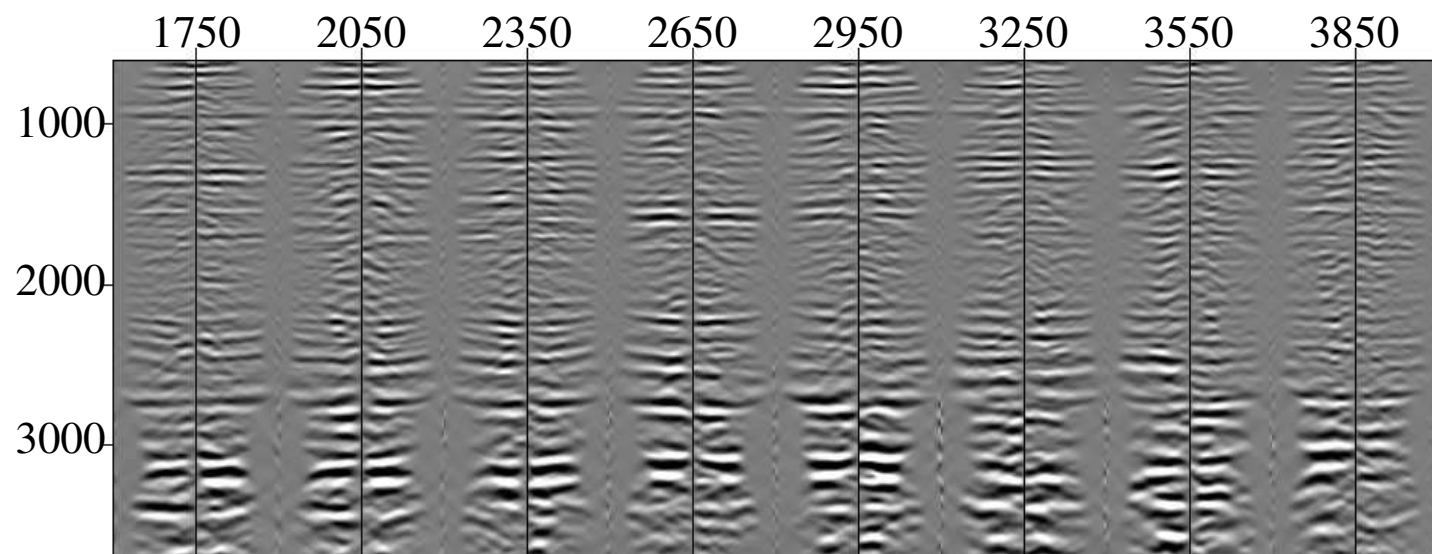
Initial cigs



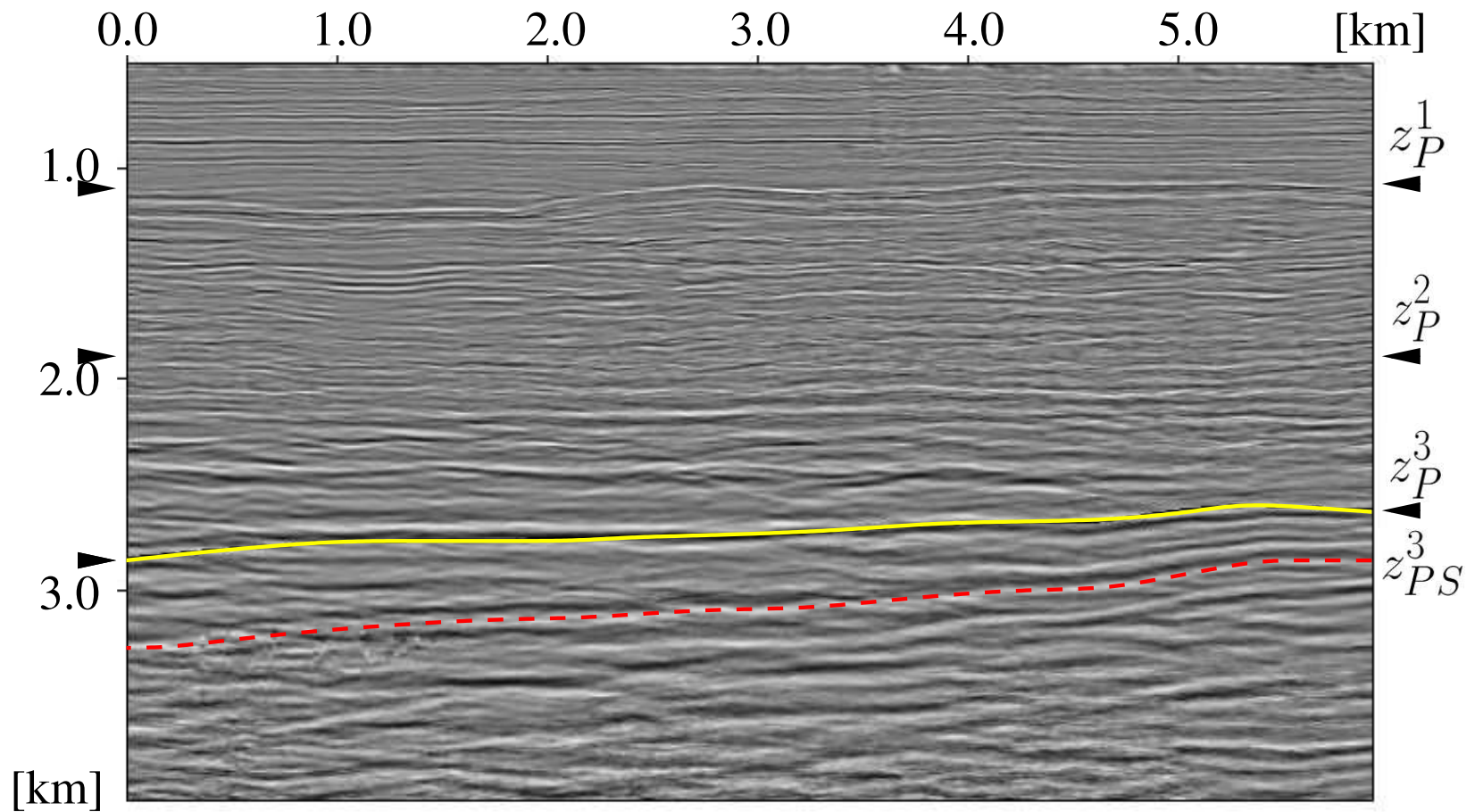
Intermediate cigs



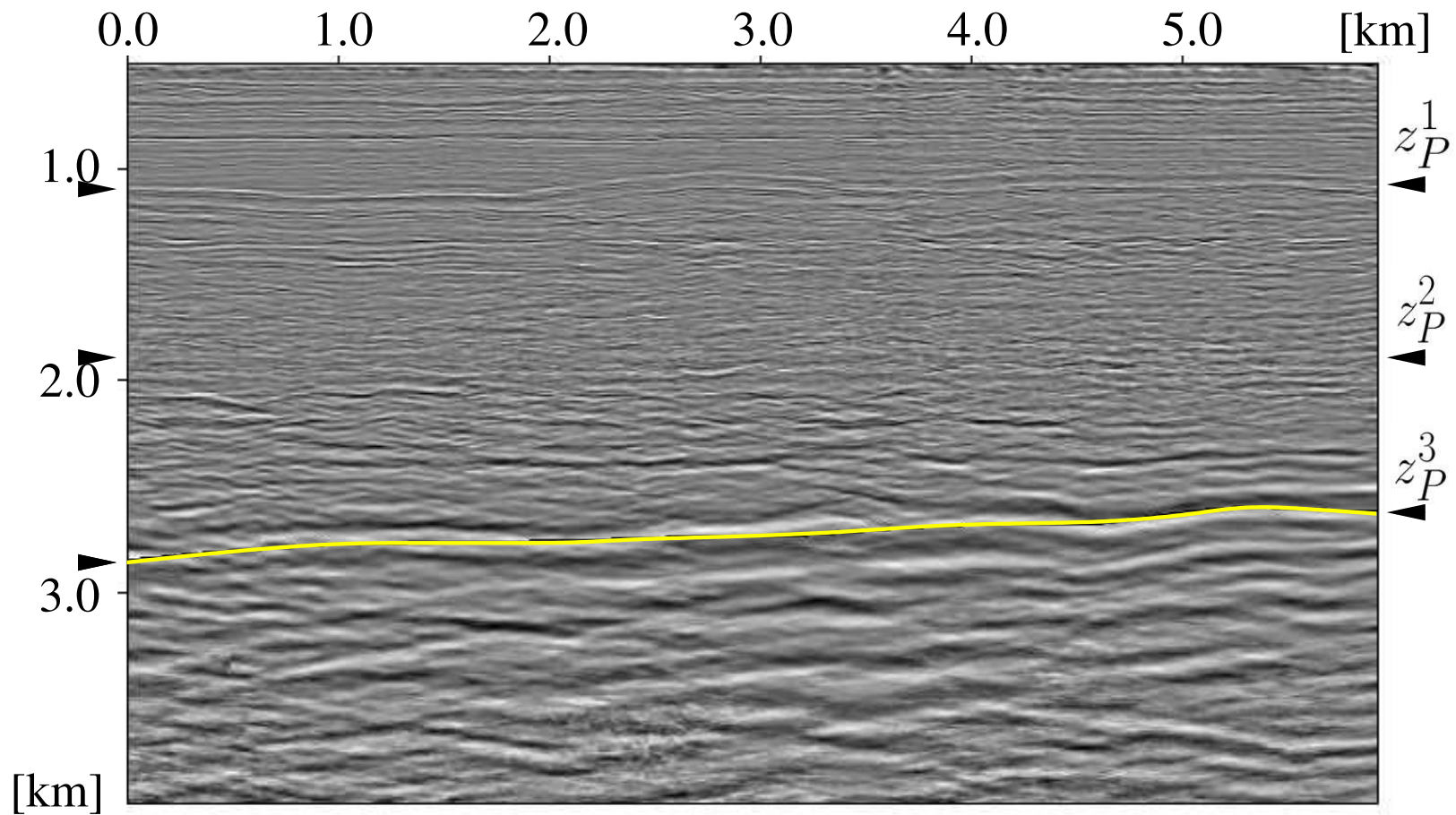
Final cigs



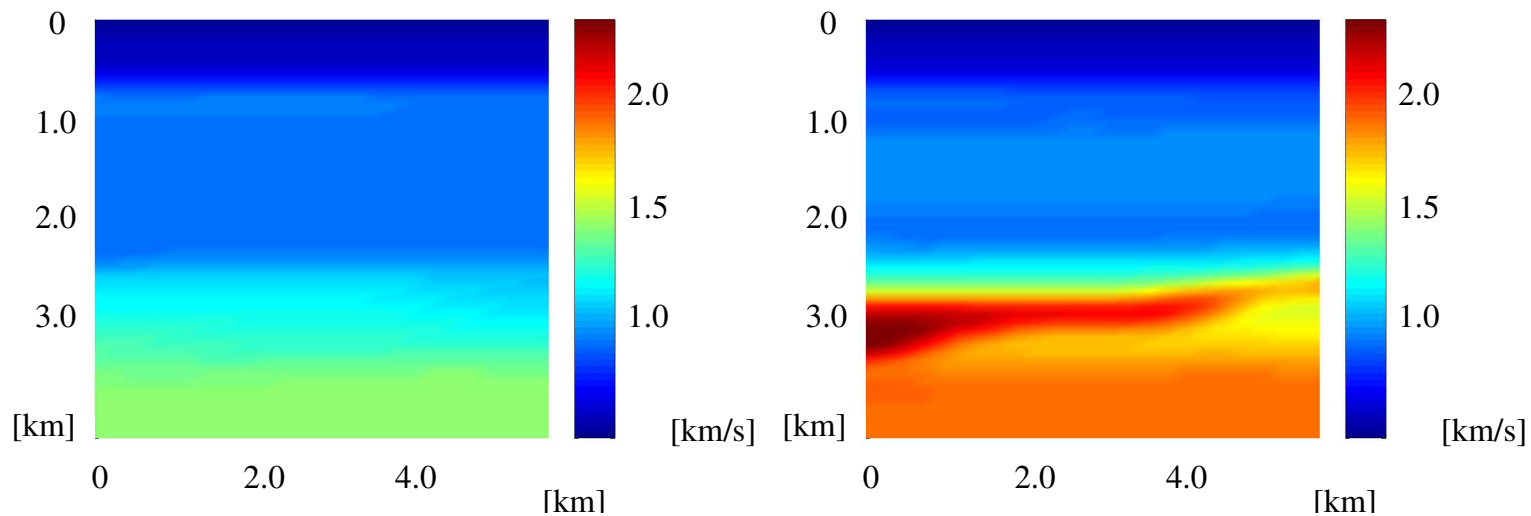
PS image with uniform cigs



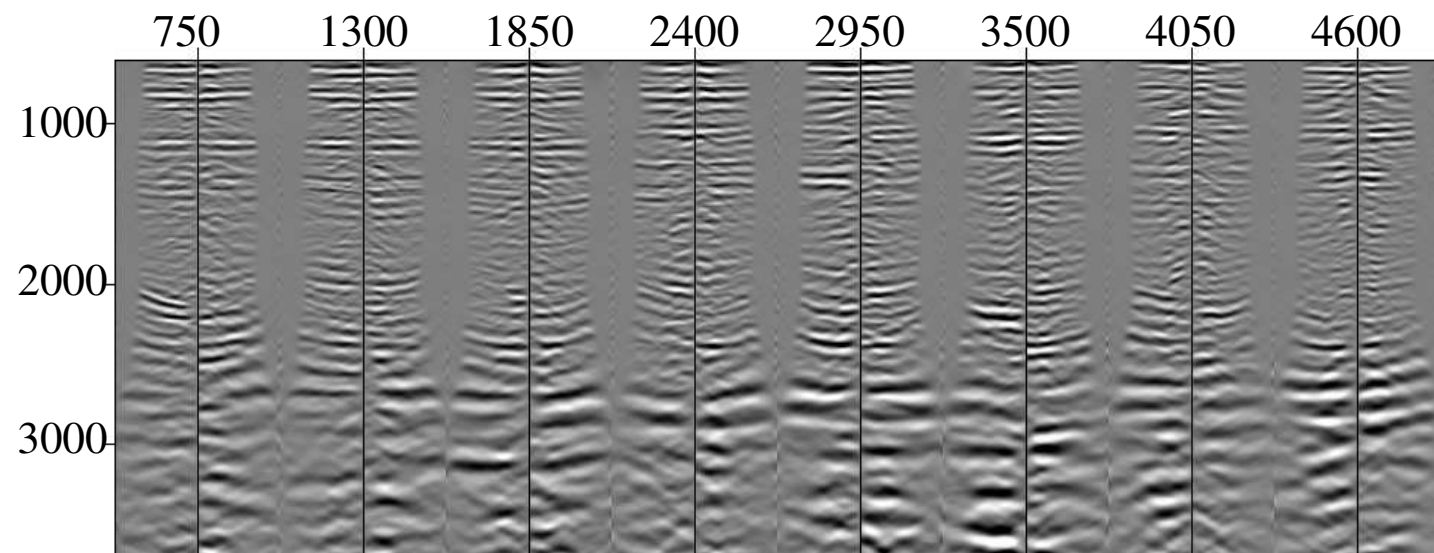
Co-depthed PS image (isotropic)



Initial and final S-wave velocity



PS cigs after isotropic co-depthing



NMO velocities

$$v_{P,NMO} = v_{P0} \sqrt{1 + 2\delta}$$

$$v_{S,NMO} = v_{S0} \sqrt{1 + 2\sigma}$$

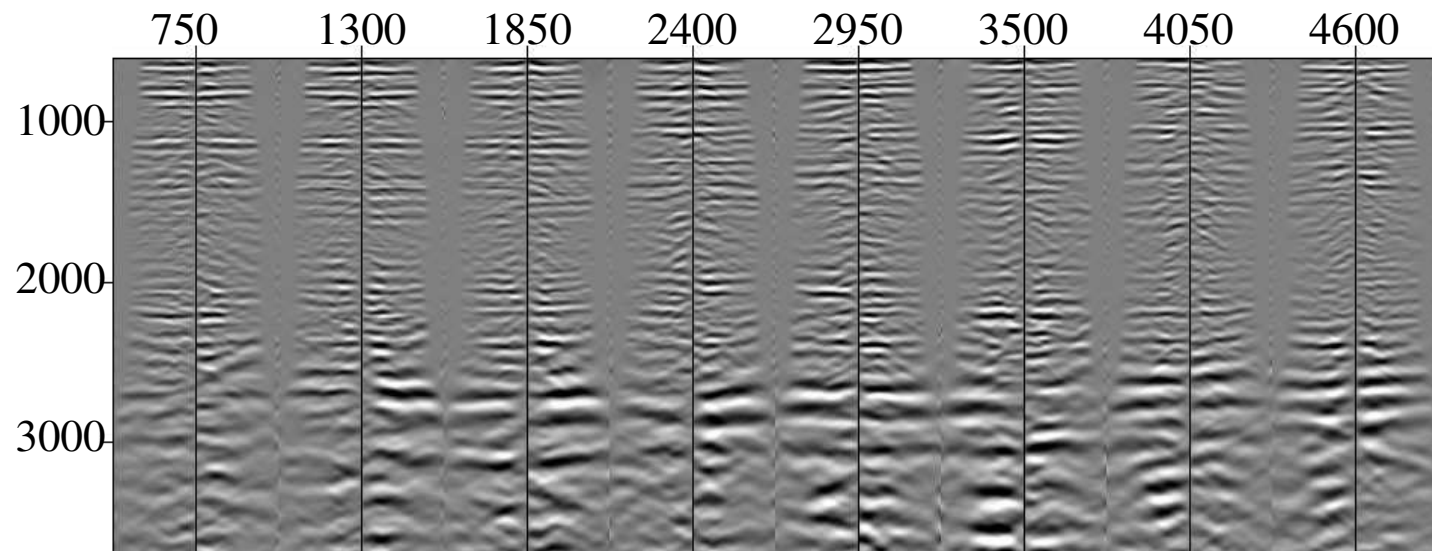
$$\sigma = \left(\frac{v_{P0}}{v_{S0}} \right)^2 (\varepsilon - \delta)$$

Status after isotropic processing

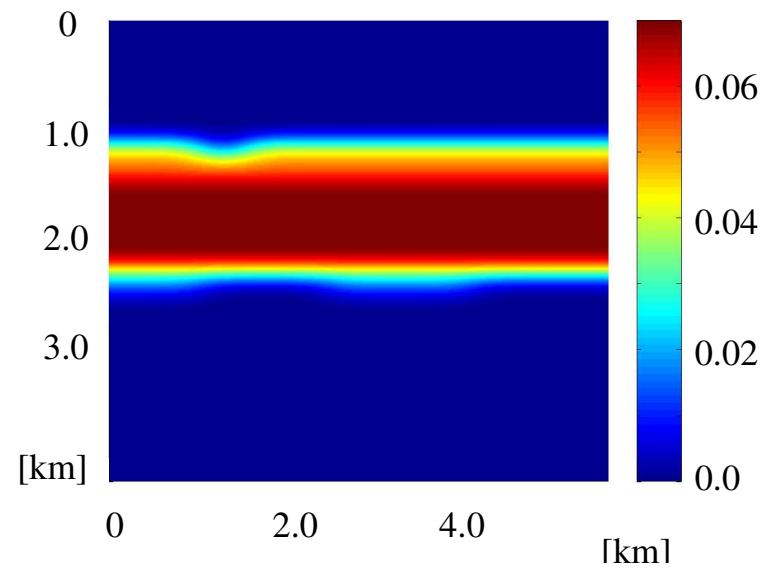
$$\text{(PP)} \quad v_P \approx v_{P,NMO} = v_{P0} \sqrt{1 + 2\delta}$$

$$\text{(PS)} \quad v_S \approx v_{S0} \sqrt{1 + 2\delta}$$

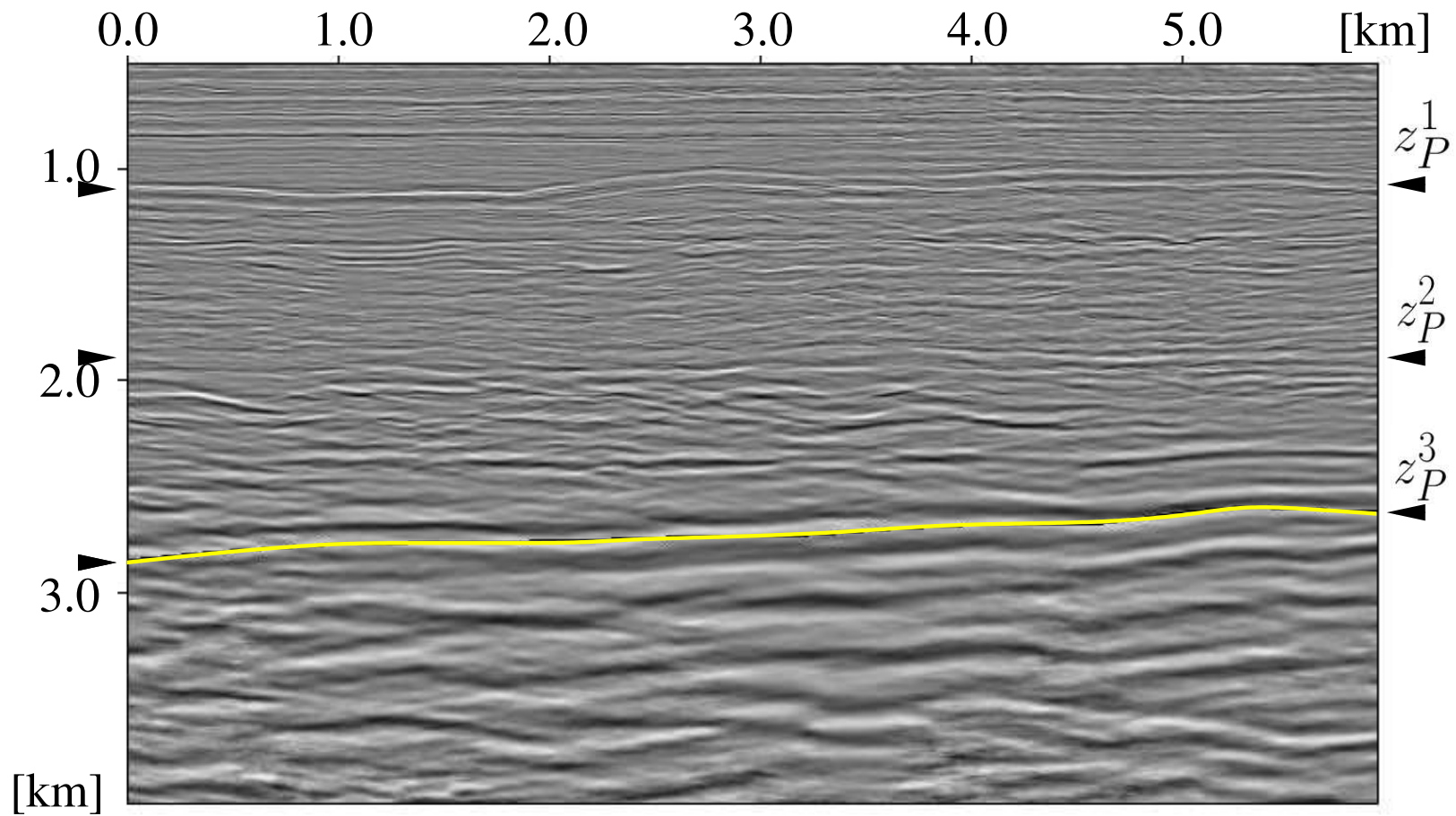
Final PS cigs (ε -update)



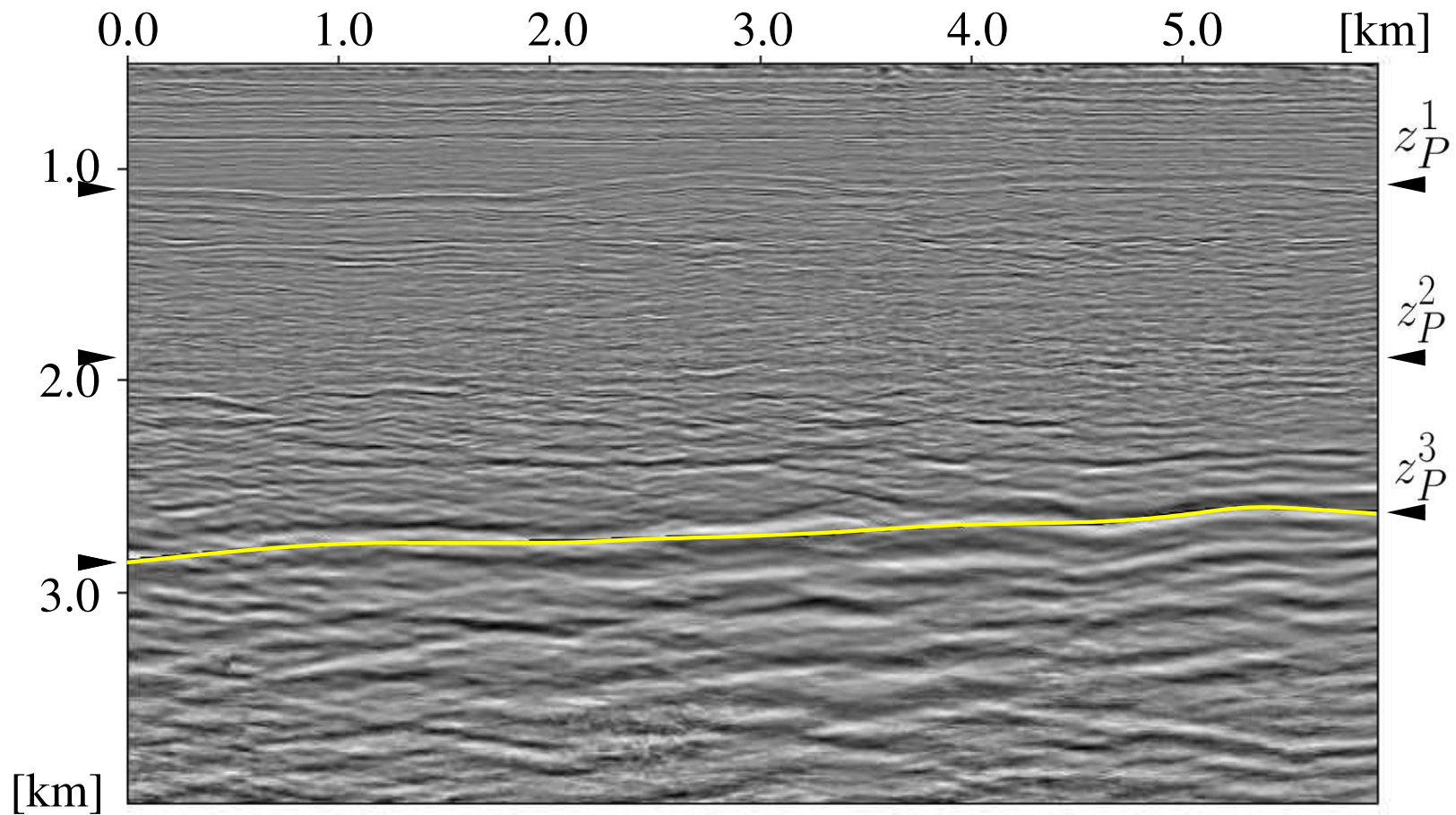
ε -values



PS image after ε -update



PS co-depthed image (isotropic)



Strategy TI media :

- PP (isotropic)
- PS (isotropic)
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 - $\delta = 0$
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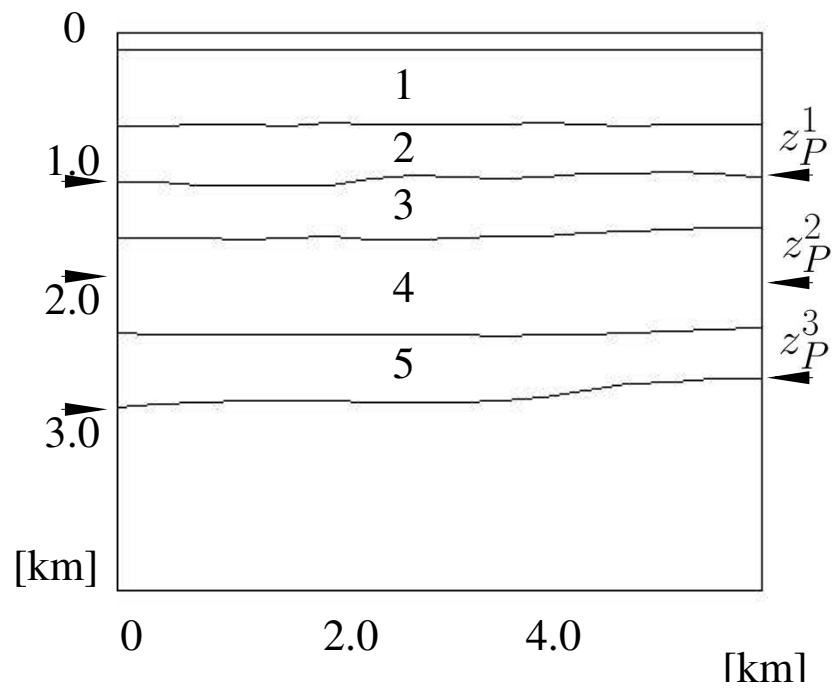
Preserving depth consistency

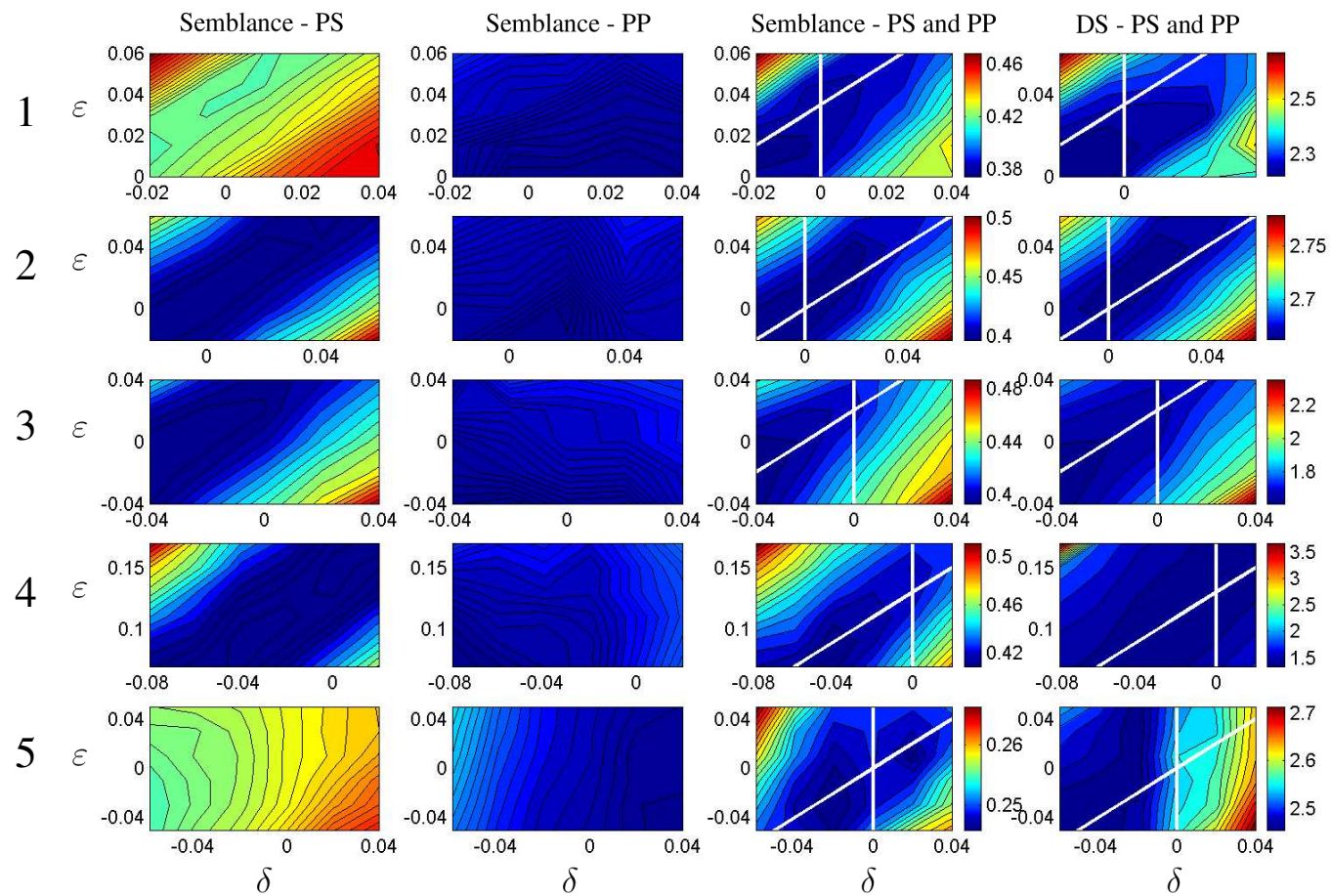
$$\frac{v_P}{v_S} = \textit{const.}$$

$$v_{P0} = \frac{v_P}{\sqrt{1 + 2\delta}}$$

$$v_{S0} = \frac{v_S}{\sqrt{1 + 2\delta}}$$

Geometry of δ , ε -values



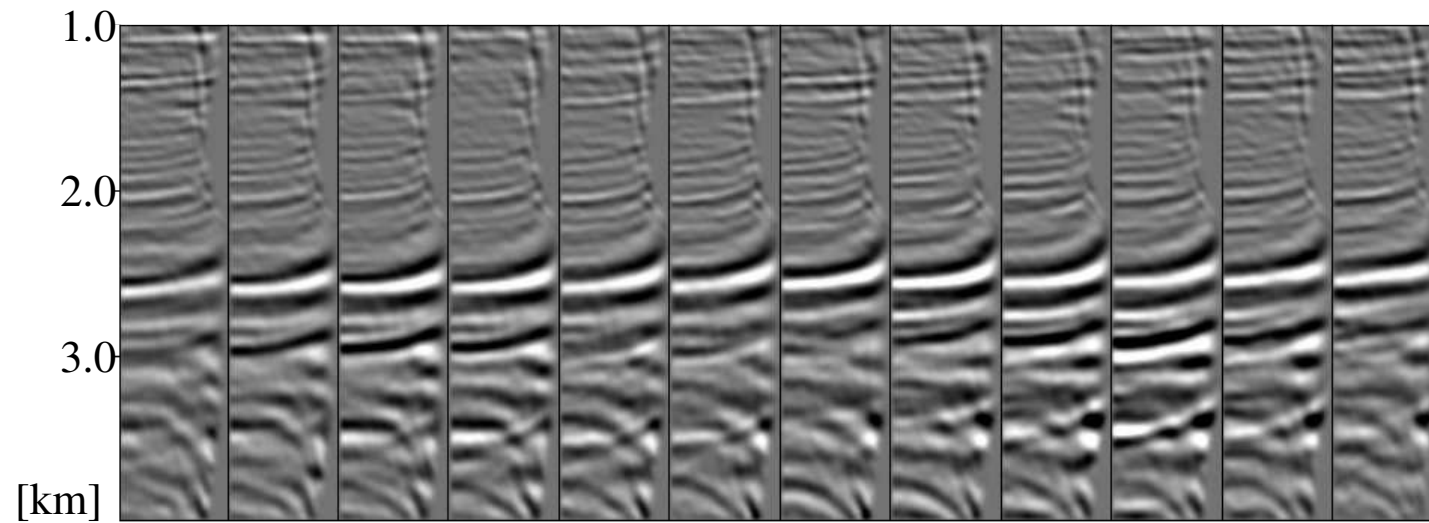


Picked ε , δ -values

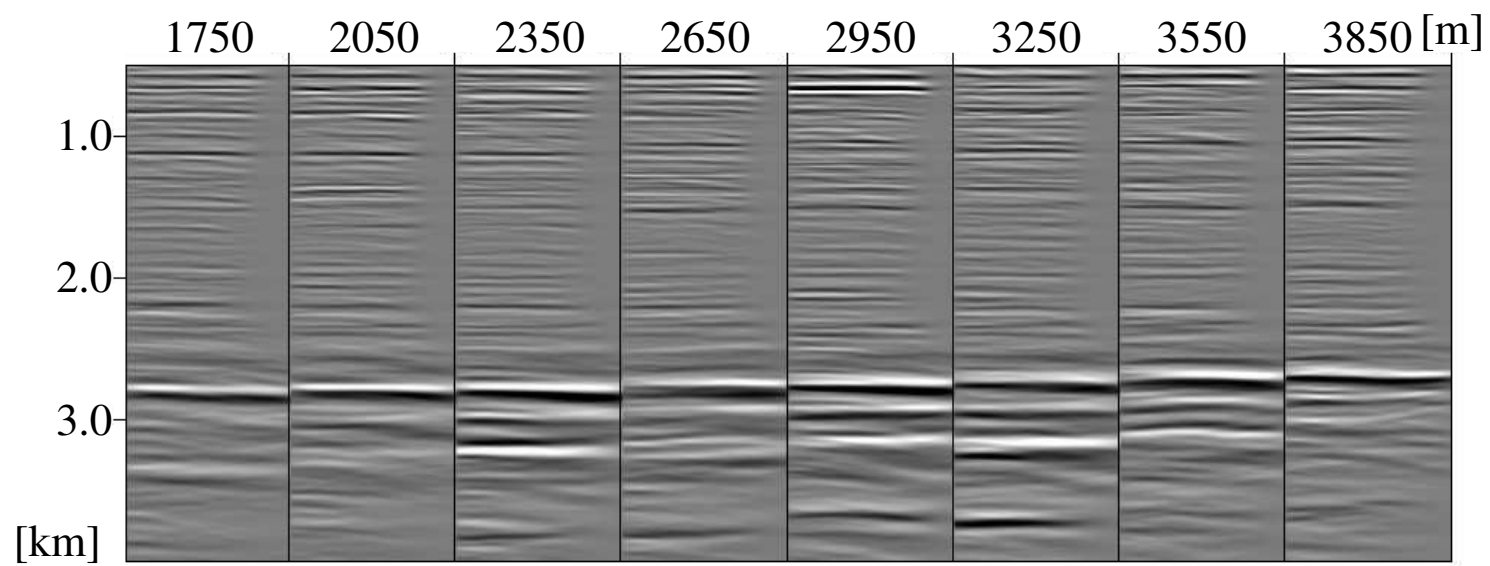
$$\varepsilon - \hat{\varepsilon} = \delta - \hat{\delta}$$

Layers	$\hat{\varepsilon}$	$\hat{\delta}$
1	0.035	0.0
2	0.0	0.0
3	0.02	0.0
4	0.09	-0.04
5	-0.02	-0.02

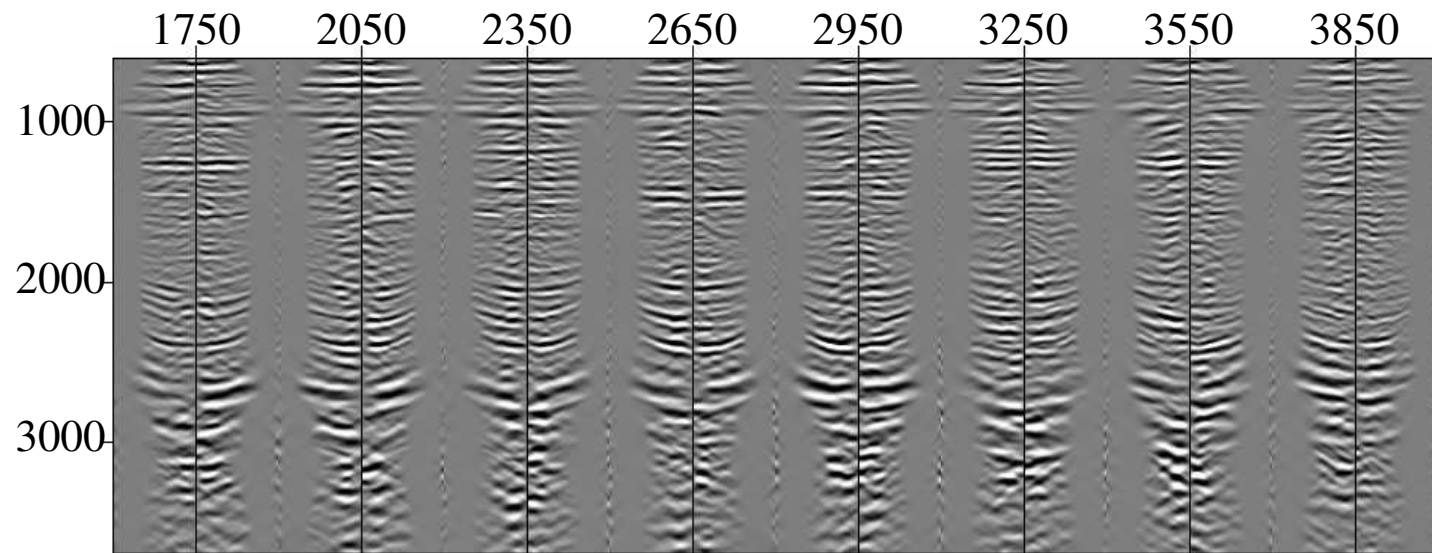
Initial PP cigs



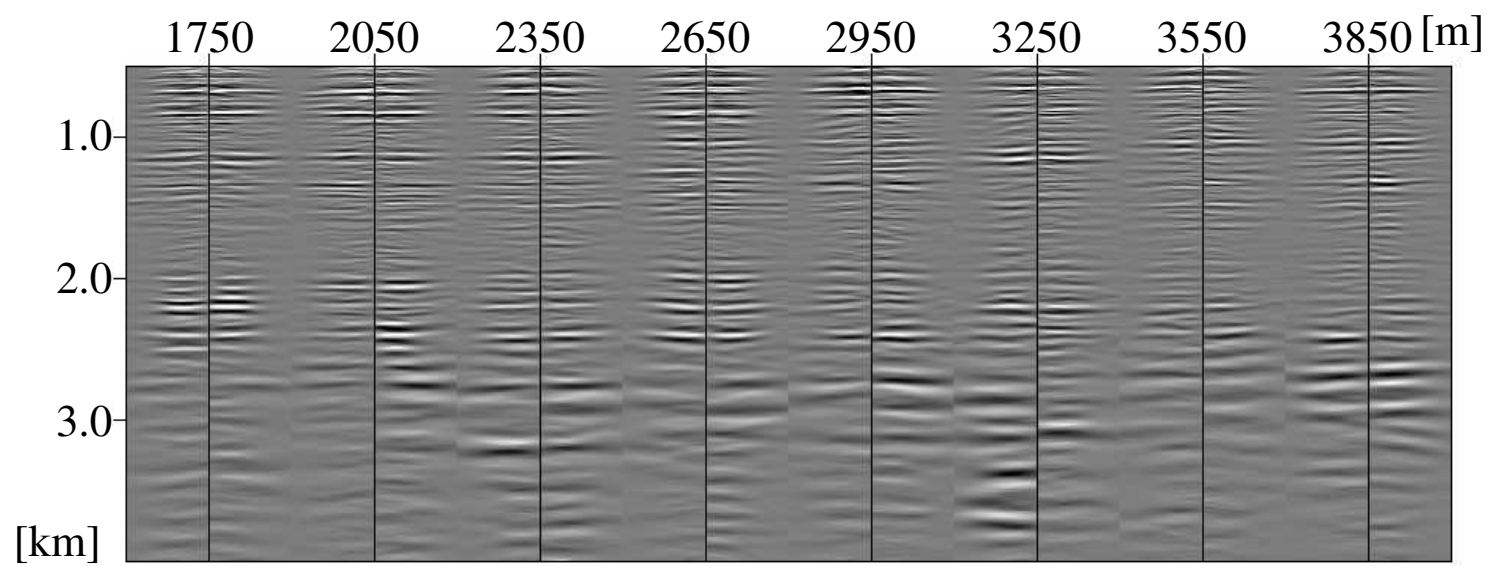
Final PP cigs



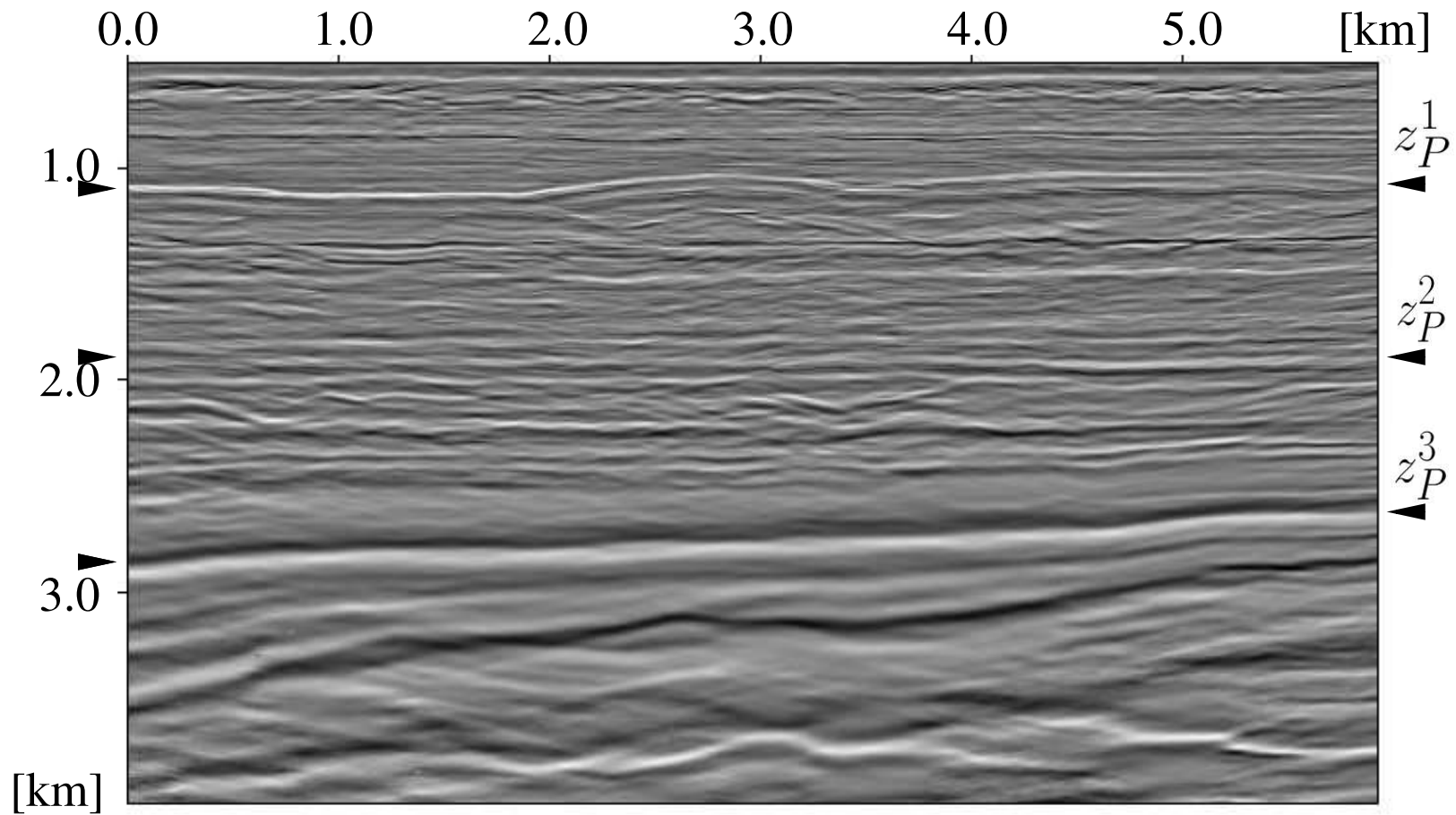
Initial PS cigs



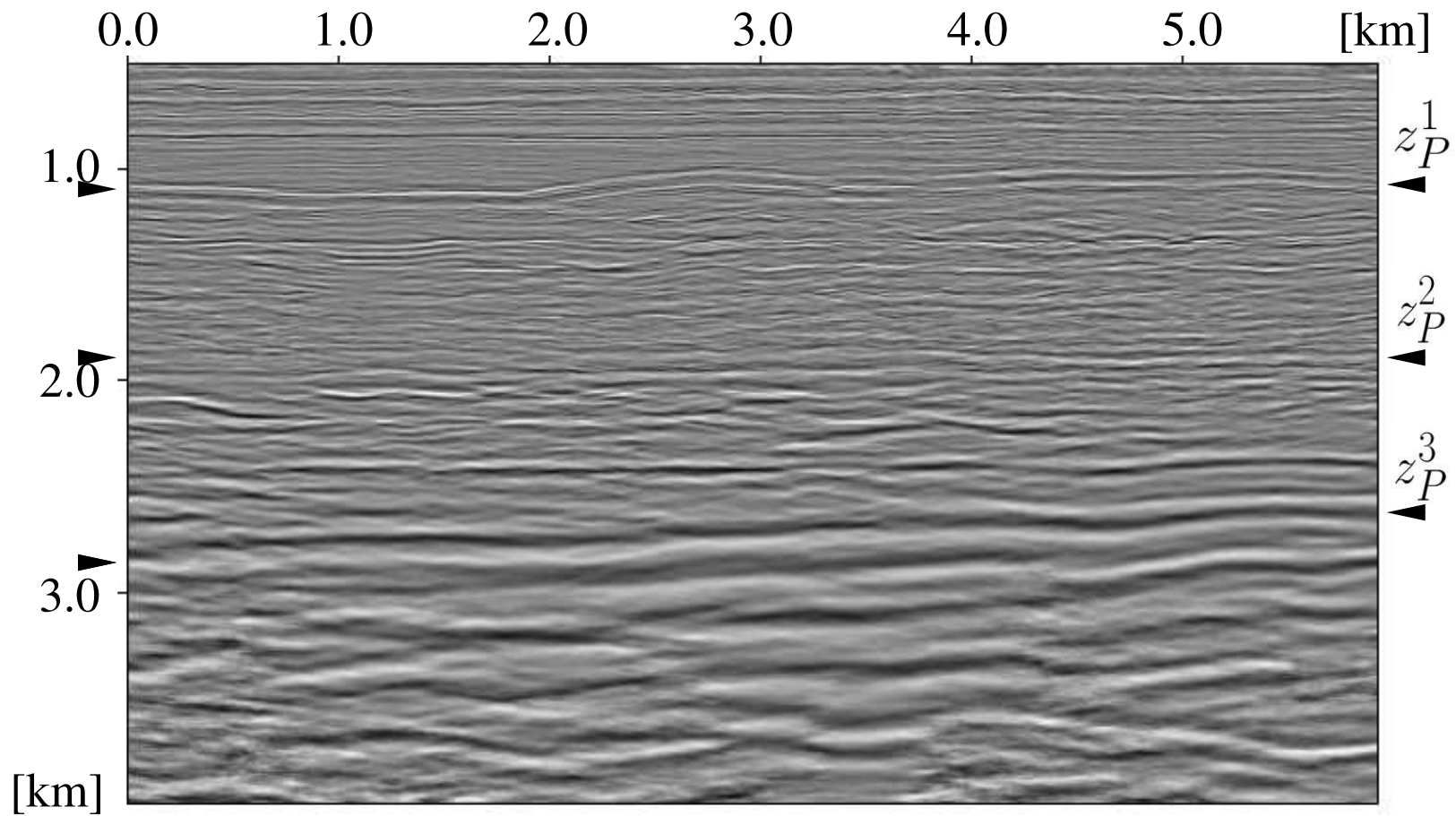
Final PS cigs



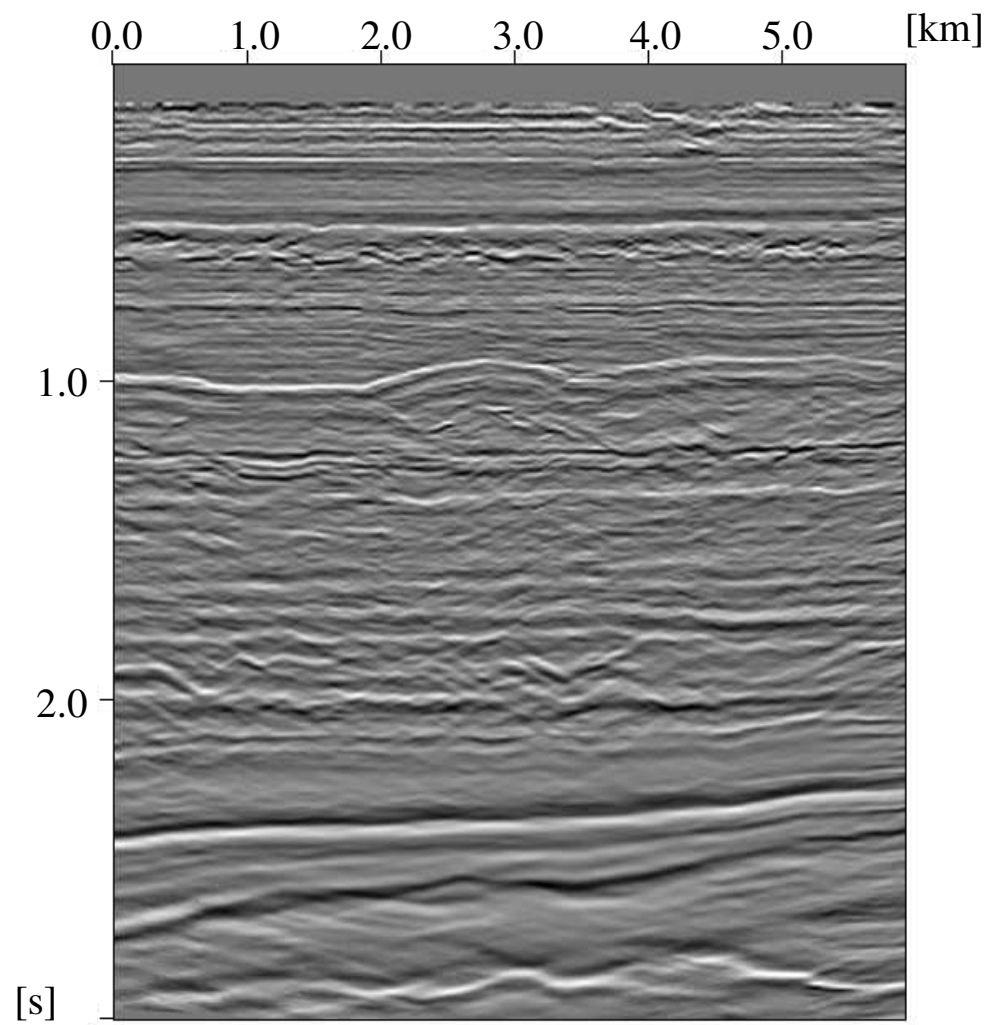
Final PP image (δ, ε)



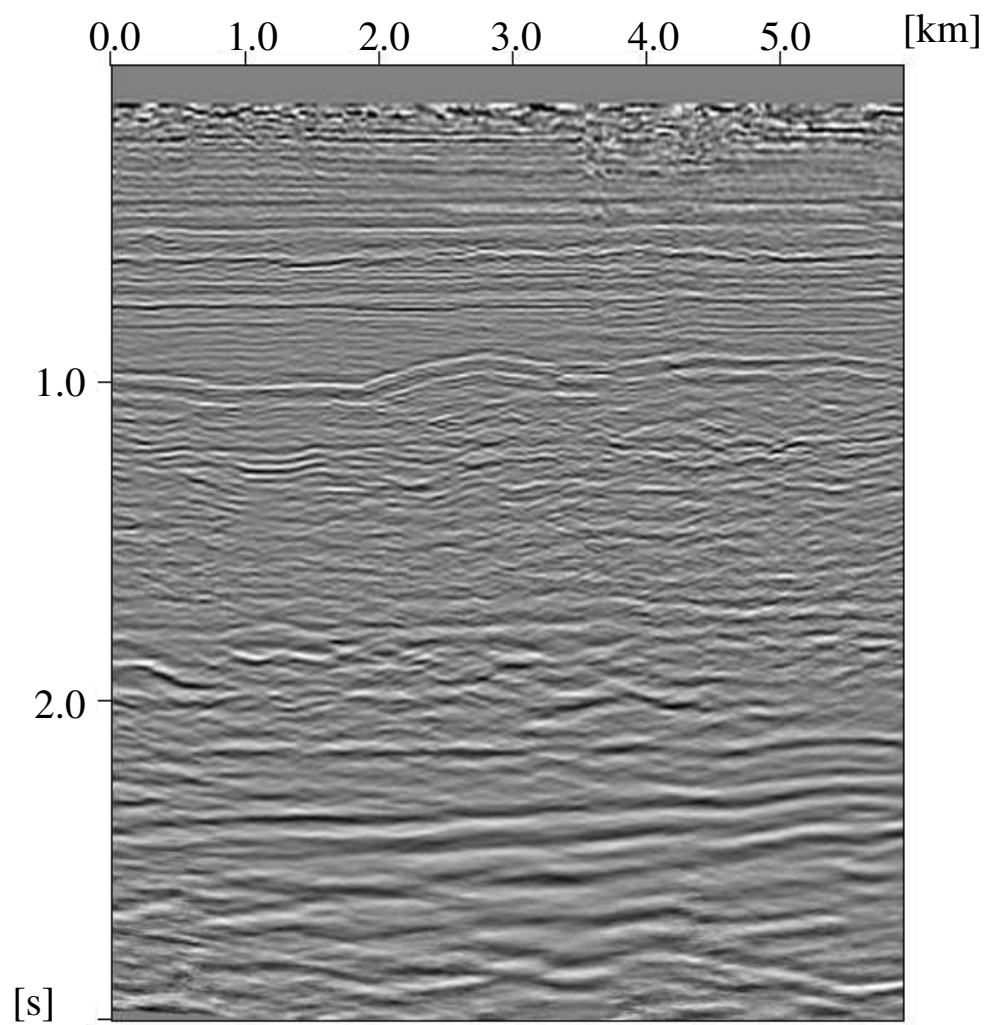
Final PS image (δ, ε)



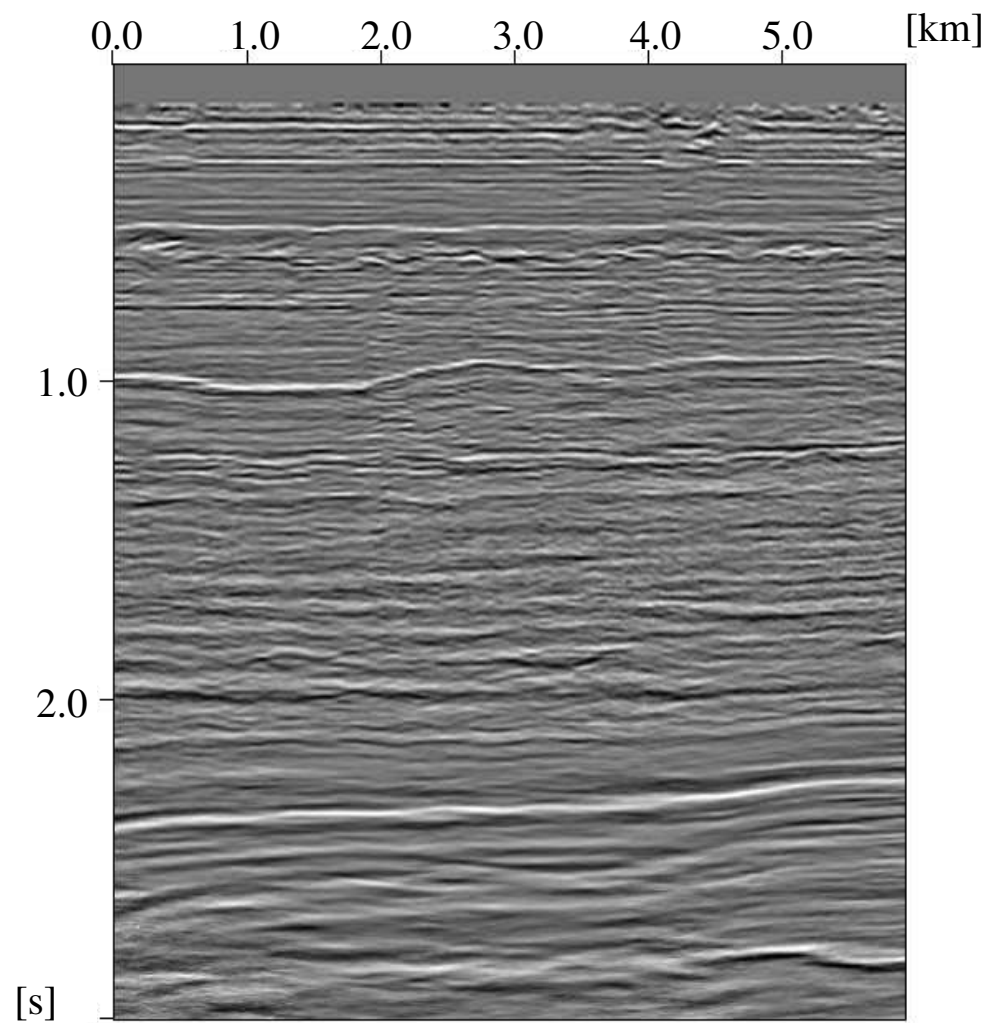
Final PP time image



Final PS time image



Initial PP time image



Strategy TI media :

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Conclusions

- PP and PS depth consistent angle tomography
- strategy for anisotropic migration
- problems:
 - multiples
 - picking on initial PS image

Acknowledgement

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