Green's theorem for source and receiver deghosting with Cagniard-de Hoop, SEAM, and field data tests and impact on multiple attenuation

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Key takeaways

1. Documented Green's theorem deghosting code available on M-OSRP website
   — Code handles 2D and 3D data
2. Code has been tested on deep-water Gulf of Mexico synthetic (SEAM) and field data
3. Input: measured data, analytic Green's functions

Put Green's theorem deghosting in context

M-OSRP concerned with complete seismic processing chain:

Removing reference wavefield Estimating wavelet Deghosting Reverse time migration (RTM)	} Green's theorem

Put Green's theorem deghosting in context

Removing free surface multiples Removing internal multiples Imaging Inverting (for medium properties) Q compensation	}	ISS
Modeling	}	Finite difference Cagniard de Hoop Reflectivity
Adaptive subtraction	}	Kaplan and Innanen (2008)

Put Green's theorem deghosting in context

- Each step in processing chain assumes previous steps have been done
- For example, ISS free surface multiple removal algorithm assumes
  — Input data has no reference wavefield or ghosts
  — Estimate of wavelet is available
- ISS internal multiple removal algorithm assumes free surface multiples have been removed

Put Green's theorem deghosting in context

- Lin Tang, Qi Huang, Jinlong Yang, Jing Wu, and I are working to ensure that input to ISS multiple removal algorithms satisfies their assumptions
- Lin is studying sensitivity (in water) of wavefield separation (another Green's theorem application) to
  — Depth between over/under cables
  — Depth between input cable and predicted cable
  — Reference velocity
- Qi is studying wavelet estimation on land

Put Green's theorem deghosting in context

- Jinlong has modified multiple removal algorithms to handle source arrays (vs. isotropic point sources)
- Jing Wu is comparing Green's theorem deghosting with standard industry method
- My code is first application of Green's theorem deghosting to deep water synthetic (SEAM) and field data

Put Green's theorem deghosting in context

- Green's theorem integral
  $P' = \int_{m.s.} dS\, \hat{\mathbf{n}} \cdot [P\nabla G_0 - G_0 \nabla P]$
  can be used for
  — Deghosting (receiver and source)
  — Wavefield prediction
  — Wavefield separation ($P = P_0 + P_s$)
  — Wavelet estimation

- Choose reference medium (and corresponding Green's function), choose prediction point above or below cable

- I've used it for wavefield separation, wavelet estimation, and wavefield prediction (and tried near-offset extrapolation), but this talk will focus on deghosting

Overview of my research

- I joined M-OSRP in May 2007
- M-OSRP has 3D ISS free surface multiple elimination code and leading order ISS internal multiple attenuation code available on its website (http://mosrp.uh.edu/ > Research > Coding Projects)
  — I was asked to develop Green's theorem code to meet their data requirements (3D deghosted data, estimated wavelet)

Overview of my research

- I wrote Green's theorem code (Jun-Sep 2009) in Fortran and C (with technical assistance provided by Fang Liu)
- Release 0 used $P' = \int_{m.s.} dS\, \hat{\mathbf{n}} \cdot [P \nabla G_0^+ - G_0^+ \nabla P]$ for receiver and source deghosting (Weglein et al., 2002; Zhang and Weglein, 2005; 2006; Zhang, 2007), wavefield separation ($P = P_0 + P_s$) and wavelet estimation (Weglein and Secrest, 1990), near-offset extrapolation

Internship at ExxonMobil (Sep-Dec 2009)

- I tested Release 0 on ExxonMobil synthetic data
- Wavelet estimation worked
- Near-offset extrapolation didn't work, but led to better understanding of local nature of integrand
  $P\nabla G_0^+ - G_0^+\nabla P$
  — Implications: reduced sensitivity to aperture, not necessary to integrate over entire measurement surface

Between internships

- Third author on two papers (Weglein et al., 2011a;b) showing that Green's functions can be constructed that enable wave theoretic RTM
- Recall that Green's theorem wavefield **separation** ($P = P_0 + P_s$ and up/down separation) **does not** require subsurface information, whereas Green's theorem wavefield **prediction** (RTM) **does**
- I rewrote code in C for integration with Seismic Unix (with technical assistance provided by Paolo Terenghi)
  — Release 1 to sponsors April 2011

First internship at PGS (Feb-Jul 2011)

- I tested Release 1 on deep-water Gulf of Mexico synthetic (SEAM) and field data
- Found that numerical algorithm sensitive to vertical distance Δz between input cable and predicted cable
  — If Δx is inline distance between traces, need $\Delta z \gtrsim 1/2\,\Delta x$ for good results (deghosting, wavefield separation ($P = P_0 + P_s$), wavelet estimation, wavefield prediction)
  — This is numerical (not theoretical) artifact (i.e., discrete vs. continuous sampling)
  — Lin Tang will cover this in more detail

First internship at PGS (Feb-Jul 2011)

- PGS receiver deghosting results published in my first SEG Expanded Abstract (Mayhan et al., 2011)
  — Presented at SEG Annual Meeting (San Antonio, Sep 2011)
  — Ranked by SEG "in the top 31 papers presented at the San Antonio meeting"
- PGS source deghosting results published in my second SEG Expanded Abstract (Mayhan et al., 2012)
  — Presented at SEG Annual Meeting (Las Vegas, Nov 2012)

Second internship at PGS (Jul-Oct 2012)

- Prior to this internship, I rewrote code to improve efficiency and add wavefield prediction for source deghosting (Osen et al., 1998; Tan, 1999) (again with technical assistance provided by Paolo Terenghi)
  — Release 2 to sponsors July 2012
- At PGS, I tested Release 2 on deep-water Gulf of Mexico synthetic (SEAM) and field data and deep-water Mediterranean Sea field data

This year

- My first *Geophysics* paper (Mayhan and Weglein, 2013)
- I updated code to add wavefield prediction for receiver deghosting (it's probably time for Release 3)

Receiver deghosting

- Item 3 in Key takeaways (code requirements)
- Theory of Green's theorem deghosting developed in Weglein et al. (2002); Zhang and Weglein (2005; 2006); Zhang (2007).

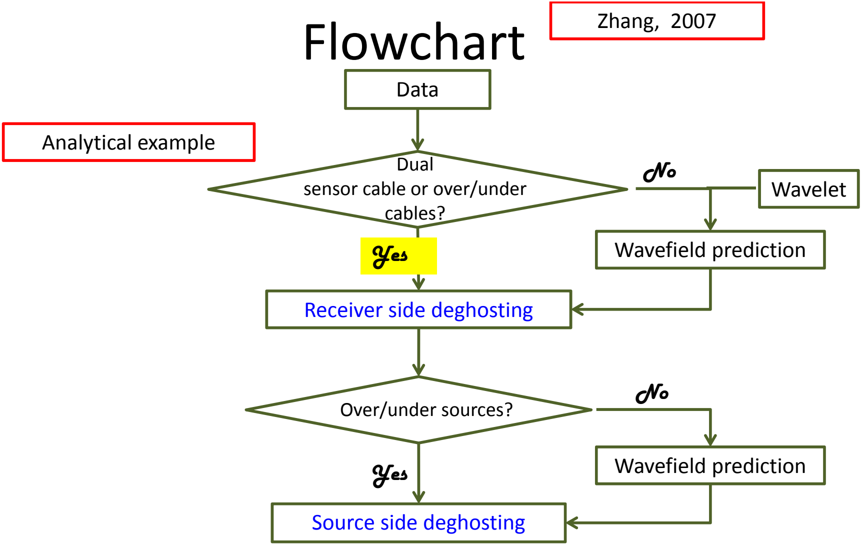


Figure 1: *Green's theorem-derived deghosting can accommodate various configurations of sources and receivers (Wang, 2012, slide 10).*

Receiver deghosting: Math

- **If we have over/under cable or dual-sensor cable**, receiver deghost using

$$P'_R = \int_{m.s.} dS\, \hat{\mathbf{n}} \cdot [P \nabla G_0^{d+} - G_0^{d+} \nabla P], \qquad (1)$$

  where $P, \nabla P$ are measured data, G_0^{d+} is causal whole-space Green's function, $\hat{\mathbf{n}}$ is outward-pointing normal.
- Integration over measurement surface for a common shot gather.
- In all cases discussed here, prediction point lies above input.

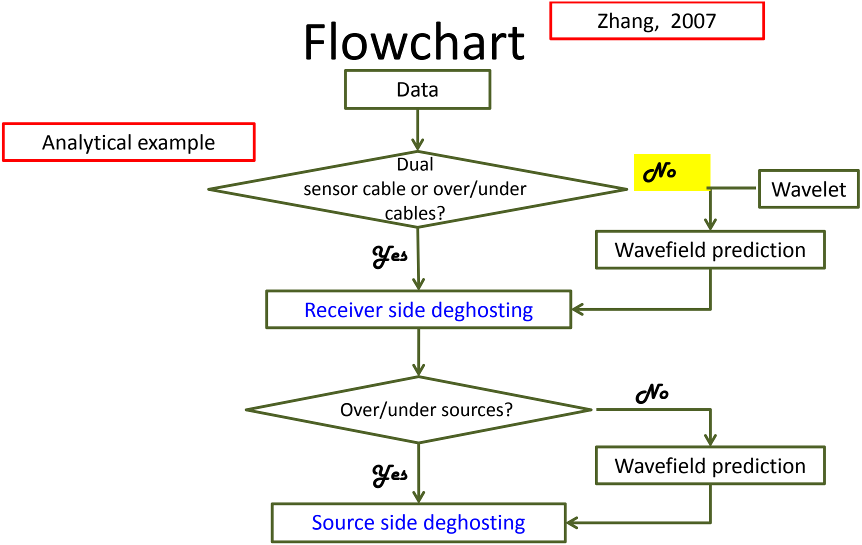


Figure 2: *Green's theorem-derived deghosting can accommodate various configurations of sources and receivers (Wang, 2012, slide 10).*

Receiver deghosting: Math

If we have a single cable measuring P and can estimate the isotropic wavelet $A(\omega)$, receiver deghost using

$$P' = AG_0^{DD} + \int_{m.s.} dS\,\hat{\mathbf{n}} \cdot P\nabla G_0^{DD} \qquad (2)$$

$$\frac{\partial P'}{\partial z'_g} = A\frac{\partial G_0^{DD}}{\partial z'_g} + \int_{m.s.} dS\,\hat{\mathbf{n}} \cdot P\nabla \frac{\partial G_0^{DD}}{\partial z'_g} \qquad (3)$$

$$P'_R = \int_{m.s.} dS\,\hat{\mathbf{n}} \cdot [P'\nabla G_0^{d+} - G_0^{d+}\nabla P'] \qquad (1),$$

Receiver deghosting: Math

where equation 3 is the derivative of equation 2, and G_0^{DD} is a "double Dirichlet" Green's function constructed to vanish on both free surface and measurement surface (Osen et al., 1998; Tan, 1999).

How does Green's theorem deghost?

- We can discuss the event picture, but theory of Green's theorem deghosting derived from scattering picture, so we'll focus on latter.
- Choose whole space of water as reference medium.
- Measured pressure wavefield P created by three spatially distributed sources overlaid on reference medium: air, air guns, and earth.
- Define surface S of integration.

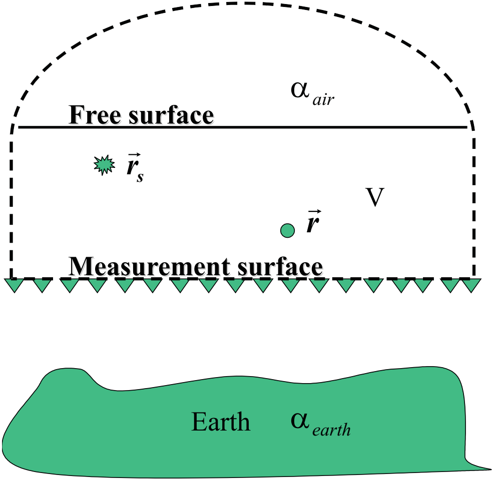


Figure 3: *Configuration for Green's theorem-derived deghosting (Zhang, 2007, Figure 2.10).*

How does Green's theorem deghost?

- Surface S of integration divides all space into two regions — one inside and one outside volume V of integration.
- Per extinction theorem (Born and Wolf, 1964, pages 101-102), equation 1
  $P'_R = \int_{m.s.} dS\, \hat{\mathbf{n}} \cdot [P \nabla G_0^{d+} - G_0^{d+} \nabla P]$
  gives the contribution to the field inside (outside) V due to sources outside (inside) V.
  — This can be seen by putting acoustic wave equation and corresponding differential equation for G_0^{d+} into Green's second identity.

How does Green's theorem deghost?

- Said another way, for a prediction point inside (outside) V, equation 1 extinguishes the field due to sources inside (outside) V (hence the name).
- In deghosting, equation 1 gives field inside V due to sources outside V (the earth).
- The contribution the earth makes at a prediction point inside V is up going.

Summary of flowchart: Receiver deghosting

Over/under cables or dual-sensor cable available?

Yes: R deghost using equation 1

$$P'_R = \int_{m.s.} dS\, \hat{\mathbf{n}} \cdot [P \nabla G_0^{d+} - G_0^{d+} \nabla P]$$

No: R deghost using equations 2, 3, and 1

$$P' = A G_0^{DD} + \int_{m.s.} dS\, \hat{\mathbf{n}} \cdot P \nabla G_0^{DD}$$

$$\frac{\partial P'}{\partial z'_g} = A \frac{\partial G_0^{DD}}{\partial z'_g} + \int_{m.s.} dS\, \hat{\mathbf{n}} \cdot P \nabla \frac{\partial G_0^{DD}}{\partial z'_g}$$

What does receiver deghosting look like?

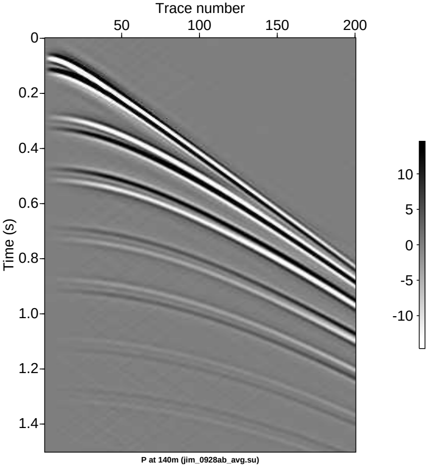
- How do we know when the code is deghosting, given different degrees of interference?
  — By testing on synthetic data
- Please note: In the following numeric examples, I need models to create test data.
  — I DON'T need models to run Green's theorem algorithm.
- More detail on test data in Appendix.

Synthetic data: Separated events

- Data created using reflectivity code
- Model: free surface, water bottom at 300m, acoustic earth ($c_0 = 2250m/s$, $\rho = 1.667g/cm^3$)
- Source at 30m, cables at 139m and 141m — Source and cable depths chosen unrealistically deep to separate events
- $\partial P/\partial z \simeq [P(141m) - P(139m)]/2m$

Synthetic data: Separated events

- Toggle between this slide and next to see receiver ghost attenuation
- 1st event is reference wave
- 3rd event is water bottom primary's receiver ghost and source/receiver ghost
- 5th event is 1st order free surface multiple's receiver ghost and source/receiver ghost


Trace number
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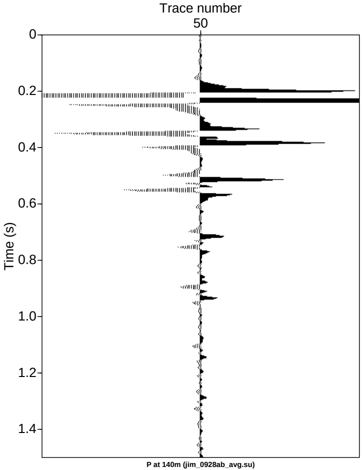
100
150
200
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Time (s)
10
5
0
-5
-10
P at 140m (jim_0928ab_avg.su)


Trace number
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1.0
1.2
1.4
Time (s)
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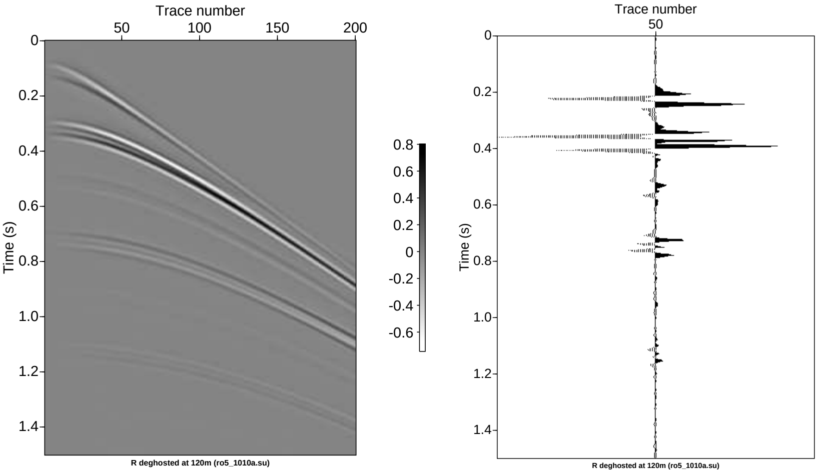
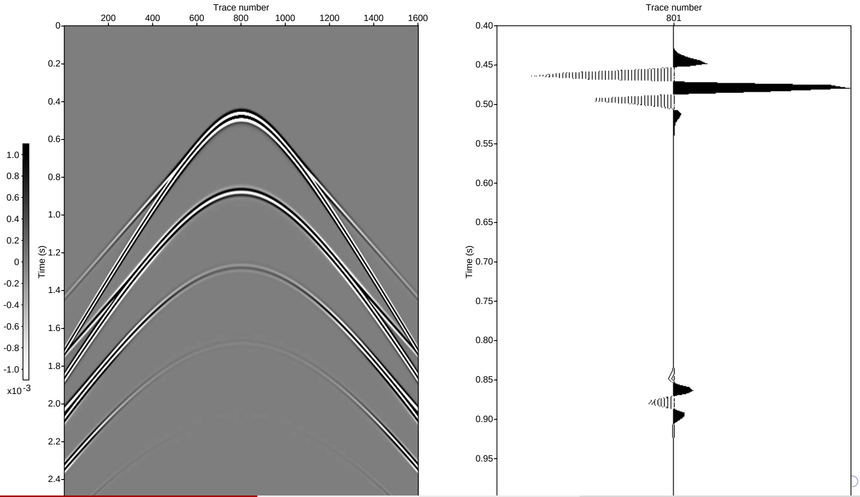

Trace number
50
100
150
200
Time (s)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
R deghosted at 120m (ro5_1010a.su)
Trace number
50
Time (s)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
R deghosted at 120m (ro5_1010a.su)

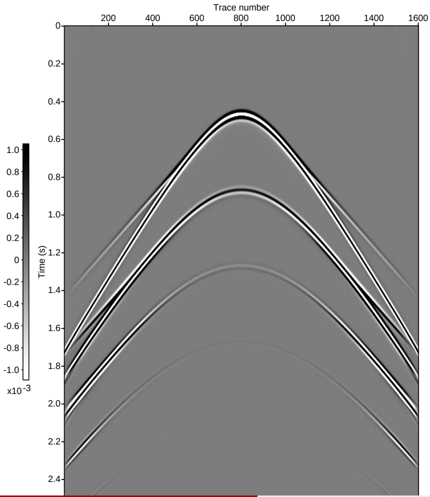
Synthetic data: Overlapping events

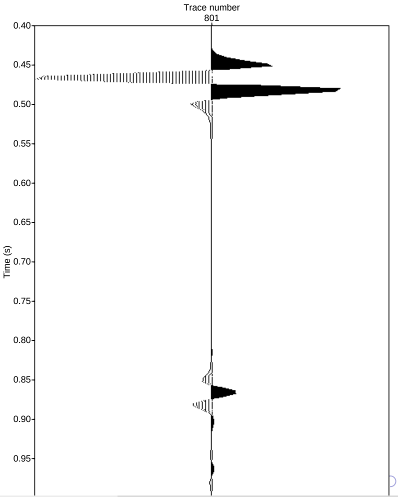
- Data created by Jinlong Yang using Jingfeng Zhang's Cagniard-de Hoop code
- Same model as that used for Separated events
- Source at 7m, cables at 9m and 11m
- $\partial P/\partial z \simeq [P(11m) - P(9m)]/2m$

Toggle between this slide and next to see receiver ghost attenuation



Toggle between this slide and previous to see receiver ghost attenuation





Synthetic data: SEAM Phase I deep-water Gulf of Mexico model

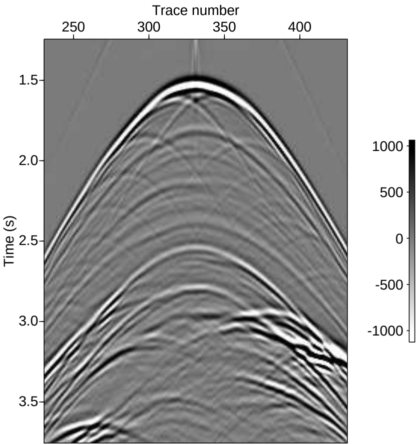
- SEAM (and field) data are Item 2 in Key takeaways (code has been tested)
- Thanks to PGS and Nizar Chemingui for granting access to deep-water Gulf of Mexico synthetic (SEAM) and field data

Synthetic data: SEAM Phase I deep-water Gulf of Mexico model

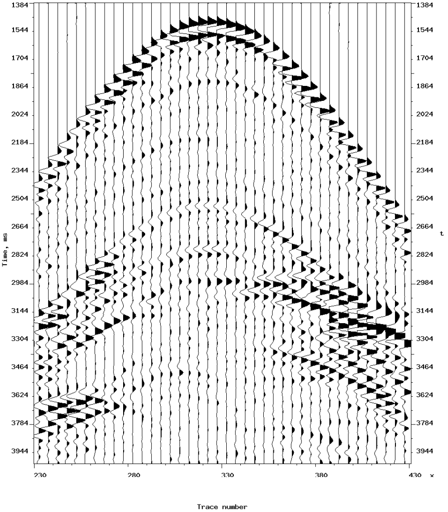
- Source at 15m, cables at 15m and 17m
- $\partial P/\partial z \simeq [P(17m) - P(15m)]/2m$

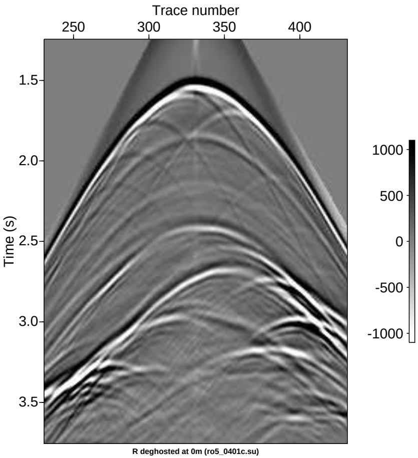
Synthetic data: SEAM Phase I deep-water Gulf of Mexico model

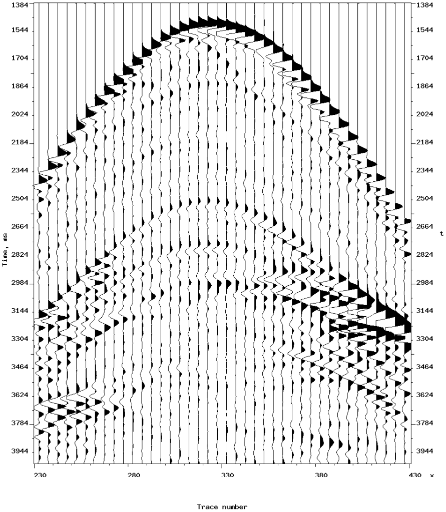
- Toggle between this slide and next to see receiver ghost attenuation
- Bottom of 1st event is water bottom primary's receiver ghost



P at 17m (SOURCE_131307-131439.gx17575_0402_131373.su)




Trace number
250
300
350
400
Time (s)
1.5
2.0
2.5
3.0
3.5
1000
500
0
-500
-1000
R deghosted at 0m (ro5_0401c.su)


Time, ms
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1544
1704
1864
2024
2184
2344
2504
2664
2824
2984
3144
3304
3464
3624
3784
3944
t
230
280
330
380
430
x
Trace number

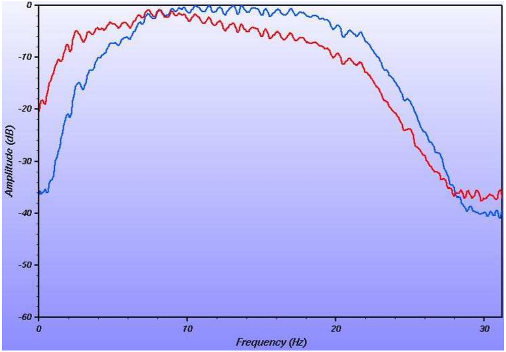


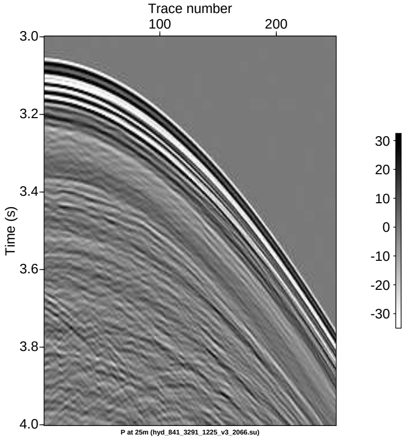
Figure 4: *SEAM data: shot 130305: recorded data (blue), receiver side deghosted (red). Note the shift of the spectrum towards lower frequencies (first receiver notch is at 50 Hz).*

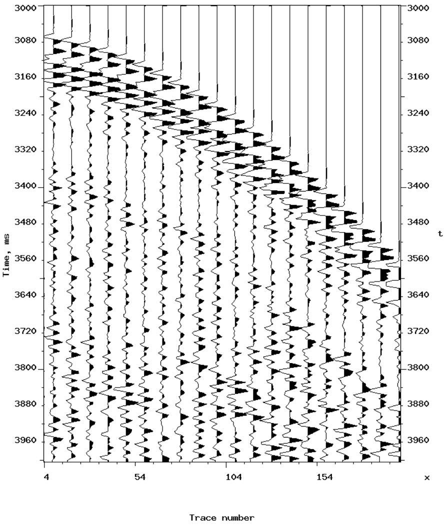
Field data: deep water Gulf of Mexico

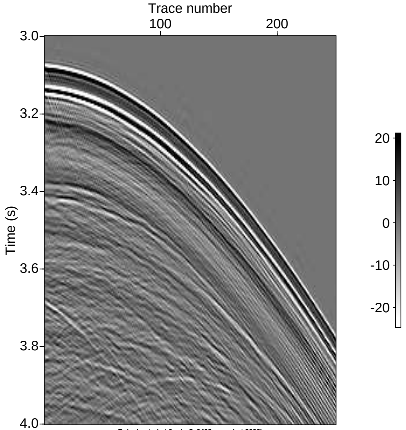
- Source at 9m, 1 dual-sensor towed streamer at 25m
- $\partial P/\partial z = i\omega\rho V_z$ where ρ is density of reference medium (seawater)

Field data: deep water Gulf of Mexico

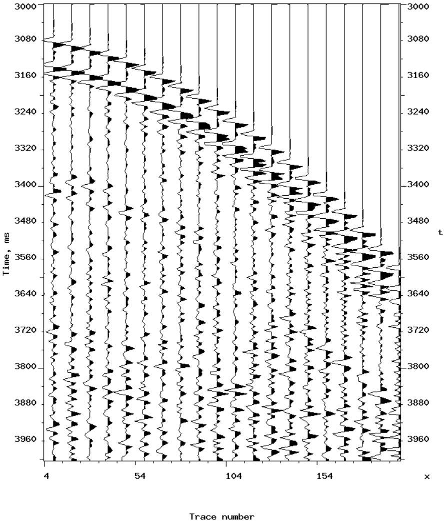
- Toggle between this slide and next to see receiver ghost attenuation
- 2nd event is water bottom primary's receiver ghost and source/receiver ghost
- Think 4th and 5th events are sub water bottom primary's receiver ghost and source/receiver ghost


Trace number
100
200
3.0
3.2
3.4
3.6
3.8
4.0
Time (s)
30
20
10
0
-10
-20
-30
P at 25m (hyd_841_3291_1225_v3_2066.su)


3000
3080
3160
3240
3320
3400
3480
3560
3640
3720
3800
3880
3960
Time, ms
t
4
54
104
154
x
Trace number



R deghosted at 0m (ro5_0408e.su, shot 2066)



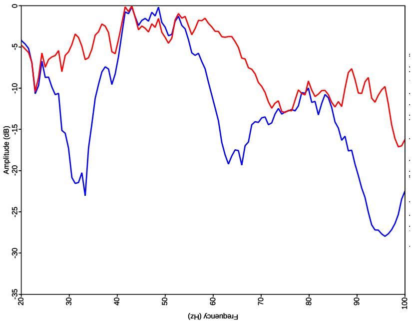


Figure 5: *Field data, deep water Gulf of Mexico: muted hydrophones (blue), receiver side deghosted (red). The receiver notches around 30 Hz, 60 Hz, and 90 Hz have been filled in. Input data courtesy of PGS.*

Source deghosting

- Item 3 in Key takeaways (code requirements)

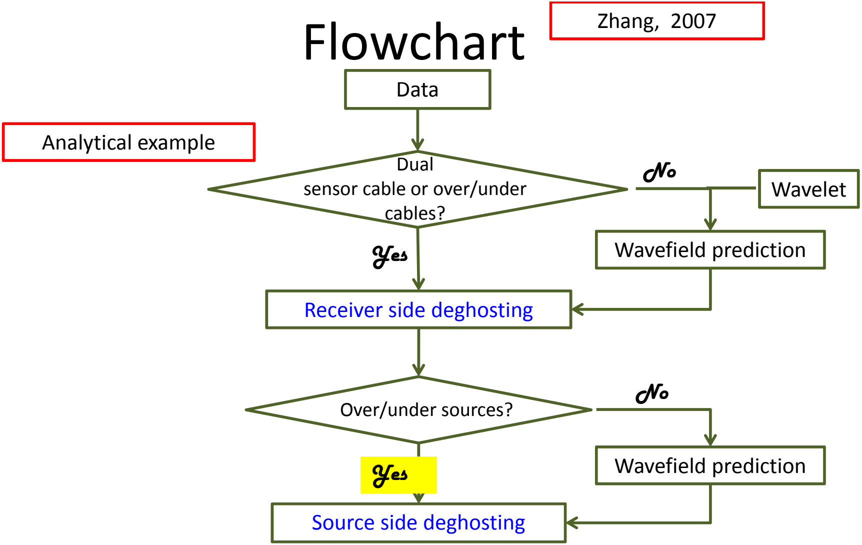


Figure 6: *Green's theorem-derived deghosting can accommodate various configurations of sources and receivers (Wang, 2012, slide 10).*

Source deghosting: Math

If we have over/under sources, source deghosting using

$$P'_{SR} = \int_{sources} dS\, \hat{\mathbf{n}} \cdot [P'_R \nabla G_0^{d+} - G_0^{d+} \nabla P'_R], \qquad (4)$$

where input is output of equation 1 (receiver deghosting), and integration is over measurement surface for a common receiver gather.

Zhang, 2007

Flowchart

Analytical example

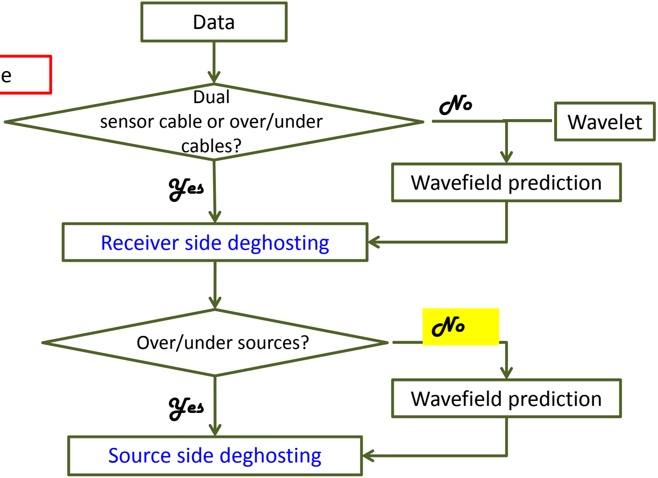


Figure 7: *Green's theorem-derived deghosting can accommodate various configurations of sources and receivers (Wang, 2012, slide 10).*

Source deghosting: Math

Similarly, **if we don't have over/under sources**, source deghost by substituting output of equation 1

$$P'_R = \int_{m.s.} dS\, \hat{\mathbf{n}} \cdot [P \nabla G_0^{d+} - G_0^{d+} \nabla P]$$

into equations 2 and 3 (but without the terms containing the wavelet)

$$P' = \cancel{A G_0^{DD}} + \int_{sources} dS\, \hat{\mathbf{n}} \cdot P'_R \nabla G_0^{DD}$$

$$\frac{\partial P'}{\partial z'_g} = \cancel{A \frac{\partial G_0^{DD}}{\partial z'_g}} + \int_{sources} dS\, \hat{\mathbf{n}} \cdot P'_R \nabla \frac{\partial G_0^{DD}}{\partial z'_g},$$

Source deghosting: Math

and their outputs are the inputs to equation 4

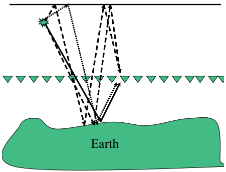
$$P'_{SR} = \int_{sources} dS\, \hat{\mathbf{n}} \cdot [P' \nabla G_0^{d+} - G_0^{d+} \nabla P'].$$

Source deghosting: Math

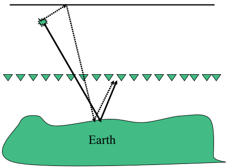
- Equations 2 and 3 allow source deghosting without the need for a wavelet and over/under sources, but $\hat{\mathbf{n}} \cdot \nabla(\partial G_0^{DD}/\partial z_g')(\mathbf{r}, \mathbf{r}_g', \omega)$ uses information at two different depths (measurement surface and free surface) and hence may have stability issues (sensitivity to notches) compared to two measurements at one depth.
- In the interest of time, causal whole space Green's function G_0^{d+} and double Dirichlet Green's function G_0^{DD} moved to Appendix.

Source deghosting

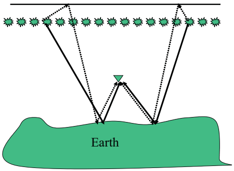
(a) *Input*



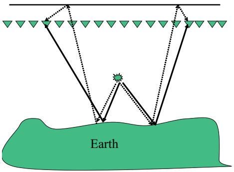
(b) *R deghosted*



(c) *CSG to CRG*



(d) *Exchange coordinates*



Source deghosting

Previous figures are Zhang 2007, Figs. 2.14–2.16

Summary of flowchart: Source deghosting

Over/under sources available?

Yes: S deghost using equation 4

$$P'_{SR} = \int_{sources} dS\, \hat{\mathbf{n}} \cdot [P'_R \nabla G_0^{d+} - G_0^{d+} \nabla P'_R]$$

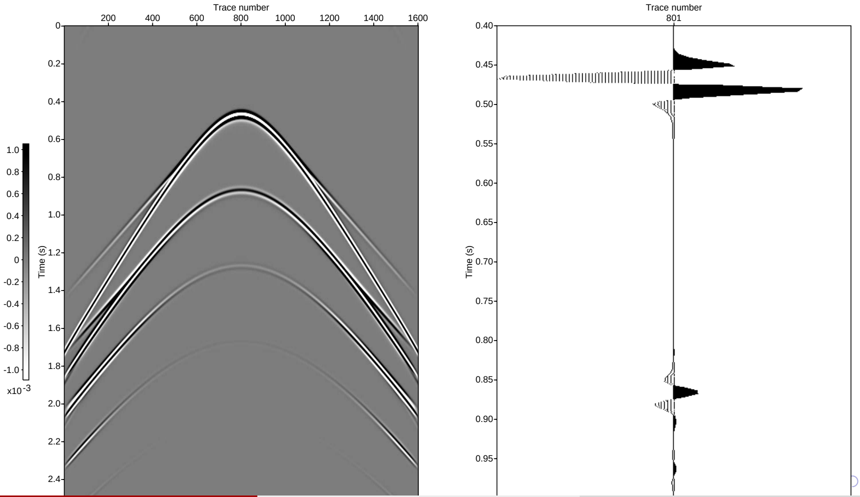
No: S deghost using equations 2, 3, and 4 (setting $A(\omega) = 0$)

$$P' = \cancel{A G_0^{DD}} + \int_{sources} dS\, \hat{\mathbf{n}} \cdot P'_R \nabla G_0^{DD}$$

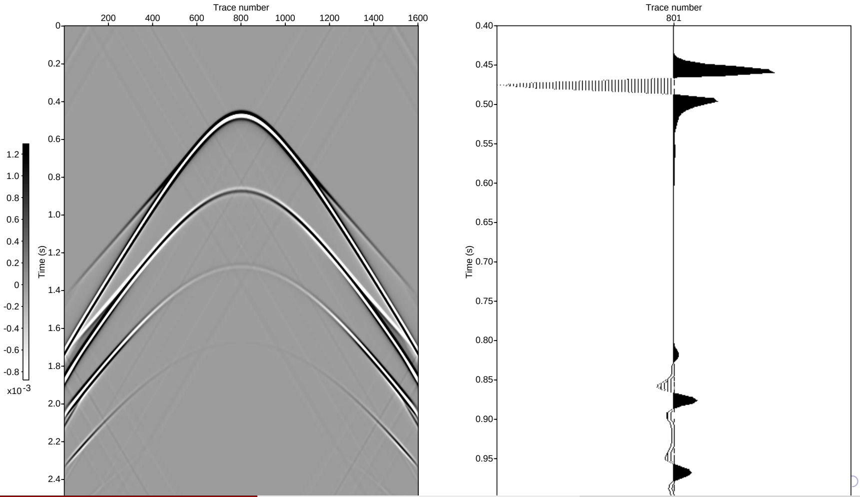
$$\frac{\partial P'}{\partial z'_g} = \cancel{A \frac{\partial G_0^{DD}}{\partial z'_g}} + \int_{sources} dS\, \hat{\mathbf{n}} \cdot P'_R \nabla \frac{\partial G_0^{DD}}{\partial z'_g}$$

Source deghosting: Synthetic data: Overlapping events

Toggle between this slide and next to see source ghost attenuation

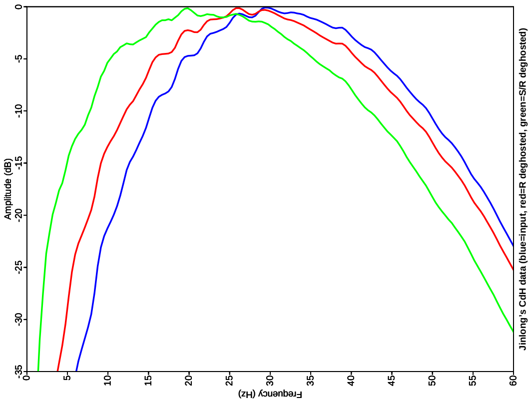


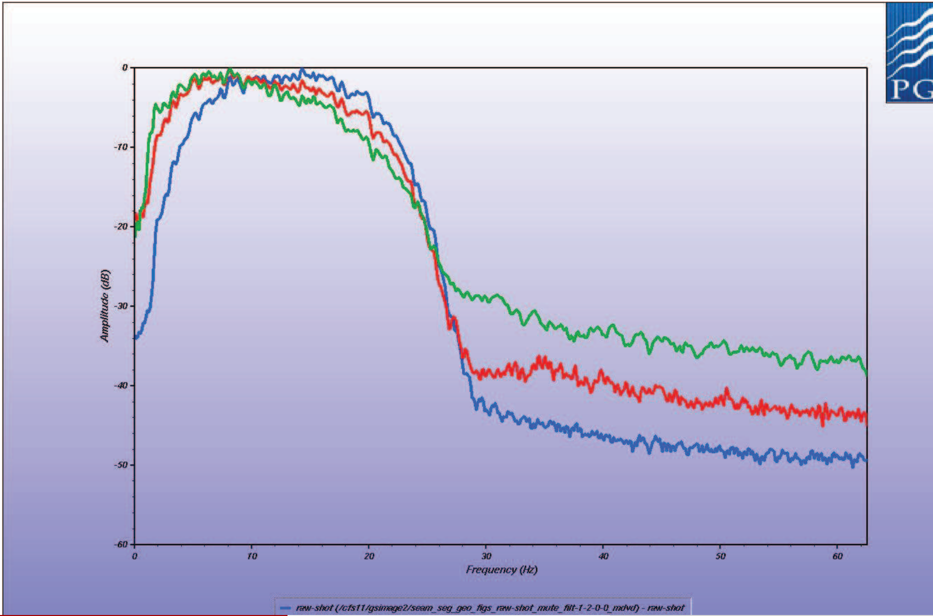
Toggle between this slide and previous to see source ghost attenuation



Deghosting boosts low frequencies

Figure 8: *Jinlong's Cagniard-de Hoop data (blue=input, red=R deghosted, green=S/R deghosted).*




0
-10
-20
-30
-40
-50
-60
Amplitude (dB)
0
10
20
30
40
50
60
Frequency (Hz)
raw-shot (/cfs11/gsimage2/seam_seg_geo_figs_raw-shot_mute_filt-1-2-0-0_mdvd) - raw-shot

Deghosting boosts low frequencies

Figure 9 (previous slide) is SEAM data, shot 131373, frequency spectra: red=P at 17m, blue=receiver deghosted at 10m, green=source and receiver deghosted at 10m. The spectrum uses a window of 201 traces (232-432) by 0.6s (1.4-2.0). The first source notch is at 44Hz which lies above the source frequency range (1–30Hz). Note the shift of the spectrum towards lower frequencies (which may be of interest to FWI).

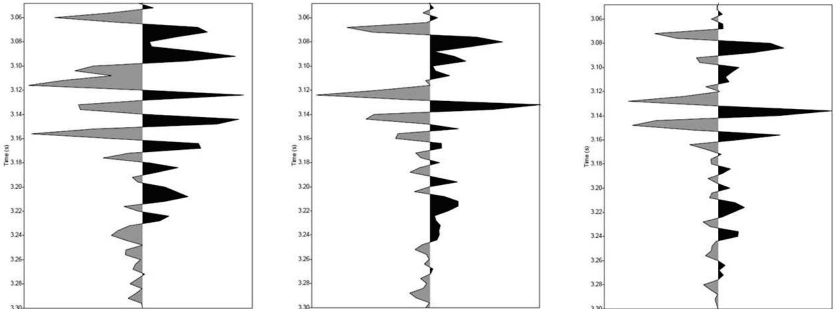


Figure 10: *Field data, deep water Gulf of Mexico: Closeup of trace 5 in hydrophone data (left), R deghosted (center), S/R deghosted (right). Note the gradual recovery of the shape of the wavelet: by receiver deghosting (middle trace) then by both source and receiver deghosting (right trace). Input data courtesy of PGS.*

Green's theorem code

- Item 1 in Key takeaways (code is available on M-OSRP website)
- Given size of typical marine seismic acquisition, deghosting-code efficiency important
- Release 2 runs several times faster than Release 1 because the required Green's functions are pre-computed and stored
- Because the Green's functions are highly local, I found (October 2012) it isn't necessary to integrate over the entire measurement surface

Green's theorem code

- For each shot, SEAM Phase I data has 661 cables $\times$ 661 receivers
- Predicting a receiver-deghosted array requires that the Green's theorem deghosting integral be evaluated over the 661 $\times$ 661 receiver array for each of the 661 $\times$ 661 receivers
- The job was submitted but was expected to run for several weeks

Green's theorem code

- The job was also submitted where the integration was restricted to a radius of $100\Delta x$ or $10\Delta x$ around each predicted receiver (where Δx is the in-line distance between receiver groups)
- The former job ran several days and the latter job ran overnight, and the results look satisfactory
- Comparing Figures 11 and 12 shows the water bottom primary's receiver ghost and source/receiver ghost are attenuated (bottom of the first event)
- Comparing Figure 12 ($10\Delta x$) and Figure 13 ($100\Delta x$) shows latter incrementally better

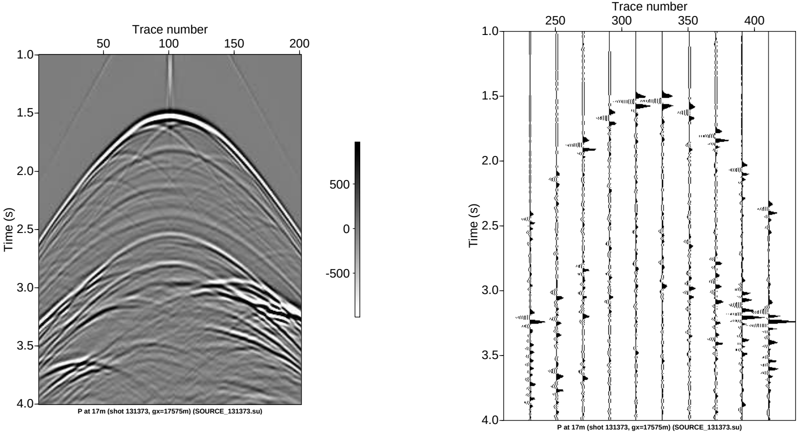


Figure 11: *SEAM Phase I data, shot 131373: P at 17m.*

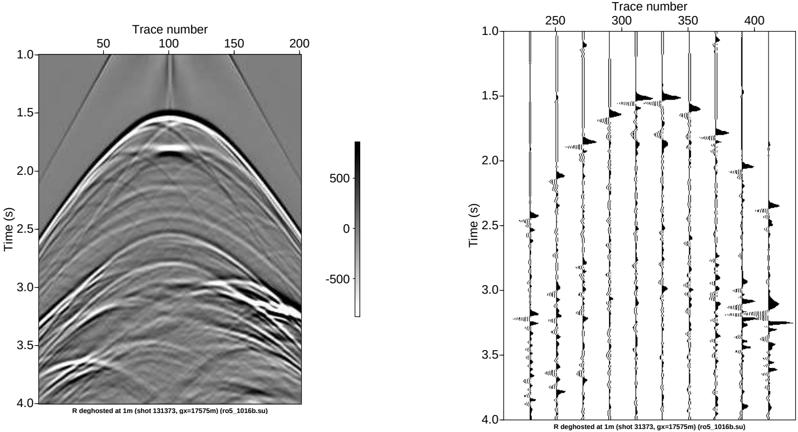


Figure 12: *SEAM Phase I data, shot 131373: receiver deghosted at 1m. The integral was restricted to a radius of* $10\Delta x$ *around each receiver on the measurement surface.*

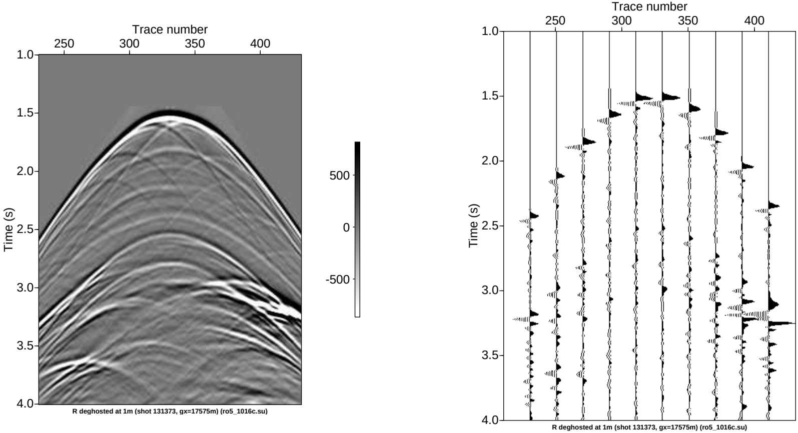


Figure 13: *SEAM Phase I data, shot 131373: receiver deghosted at 1m. The integral was restricted to a radius of* $100\Delta x$ *around each receiver on the measurement surface.*

What's special about integrand that makes it so local?

In 3D, P and G_0^{d+} each fall off like $1/R$, their vertical derivatives fall off even faster
$\Rightarrow$ Integrand falls off faster than $1/R^2$

Summary

1. Green's theorem deghosting code available on M-OSRP website
   — mosrp.uh.edu > Research > Coding Projects > second box labeled "New MOSRP_SU collection of seismic data processing codes based on Seismic Unix."
   — Downloadable package includes documentation and demo script
   — Code handles 2D and 3D data

Summary

2. Code has been tested on deep-water Gulf of Mexico synthetic (SEAM) and field data
3. It works using as input measured data and analytic Green's functions, i.e., it doesn't require
   — Additional information about subsurface or ocean surface
   — Fourier transform over inline or crossline coordinates
   — Integration over entire measurement surface

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Appendix

Analytic form of causal whole-space Green's function G_0^{d+} in $\mathbf{r}, \omega$ domain

- In 2D
  $G_0^{d+}(\mathbf{r}, \mathbf{r}'_g, \omega) = -i/4\, H_0^{(1)}(kR_+)$
  where $H_0^{(1)}$ is zeroth-order Hankel function of first kind, $k = \omega/c_0$, c_0 is wave speed in reference medium (water), and $R_+ = |\mathbf{r} - \mathbf{r}'_g|$.
- In 3D
  $G_0^{d+}(\mathbf{r}, \mathbf{r}'_g, \omega) = -1/(4\pi)\, \exp(ikR_+)/R_+$
- Morse and Feshbach 1953, pp. 810-811

Analytic form of double Dirichlet Green's function G_0^{DD} in $\mathbf{r}, \omega$ domain

In 2D $G_0^{DD}(\mathbf{r}'_g, \mathbf{r}''_g, \omega)$

$$= -\frac{1}{b} \sum_{n=1}^{\infty} \frac{\exp\left(-\sqrt{(n\pi/b)^2 - k^2}\,|x'_g - x''_g|\right)}{\sqrt{(n\pi/b)^2 - k^2}} \times \sin\left(\frac{n\pi}{b} z'_g\right) \sin\left(\frac{n\pi}{b} z''_g\right) \quad (5)$$

where (x''_g, z''_g) are prediction coordinates, (x'_g, z'_g) are receiver coordinates on receiver deghosted cable, free surface at $z'_g = 0$, input cable at $z'_g = b$, and $(n\pi/b)^2 - k^2 > 0$ (Osen et al., 1998; Tan, 1999).

Analytic form of double Dirichlet Green's function G_0^{DD} in $\mathbf{r}, \omega$ domain

In 3D $G_0^{DD}(\mathbf{r}_g'', \mathbf{r}_g', \omega)$

$$= \frac{2\pi i}{b} \sum_{n=1}^{\infty} H_0^{(1)}(\gamma\rho) \sin\left(\frac{n\pi}{b} z_g'\right) \sin\left(\frac{n\pi}{b} z_g''\right)$$

where $\gamma = i\sqrt{(n\pi/b)^2 - k^2}$ and
$\rho = \sqrt{(x_g'' - x_g')^2 + (y_g'' - y_g')^2}$ (Osen et al., 1998).
For numeric evaluation, the Hankel function with imaginary argument is replaced by a hyperbolic Bessel function with real argument (Morse and Feshbach, 1953, page 1323).

Analytic form of double Dirichlet Green's function G_0^{DD} in $\mathbf{r}, \omega$ domain

- The hyperbolic Bessel function (which decays exponentially) and the exponential in equation 5 provide rapid convergence of the infinite series.
- Maximum number of iterations allowed is 100, never bumped into it.

Where does sensitivity to notch matter?

- Receiver notches occur when $kb = n\pi$ (Zhang, 2007, p. 6)
- In equation 5, notches cause numerator $\to 1$ and denominator $\to 0$
  $\Rightarrow$ sensitivity to notch matters just at notch

Synthetic data: Separated events

Parameter	Value
Number of shots	1
Number of channels per shot	801
Number of samples per trace	1500
Time sampling	4ms
Record length	6s
Group interval	6.25m
Shortest offset	0m
Gun depth	30m
Streamer depth	139m & 141m

Synthetic data: Separated events

- Data created using reflectivity code
- Model: free surface, water bottom at 300m, acoustic earth ($c_0 = 2250m/s$, $\rho = 1.667g/cm^3$)
- 3D source, frequency of source: 1-60Hz
- 5% taper applied to each end of cable
- $\partial P/\partial z \simeq [P(141m) - P(139m)]/2m$

Synthetic data: Overlapping events

Parameter	Value
Number of shots	1
Number of channels per shot	1601
Number of samples per trace	625
Time sampling	4ms
Record length	2.5s
Group interval	3m
Shortest offset	0m
Gun depth	7m
Streamer depth	9m & 11m

Synthetic data: Overlapping events

- Data created by Jinlong Yang using Jingfeng Zhang's Cagniard-de Hoop code
- Model: free surface, water bottom at 300m, acoustic earth ($c_0 = 2250 m/s$, $\rho = 1.667 g/cm^3$)
- $\partial P/\partial z \simeq [P(11m) - P(9m)]/2m$

Synthetic data: SEAM Phase I deep-water Gulf of Mexico model

Parameter	Value
Number of shots	9×267
Number of channels per shot	661×661
Number of samples per trace	2001
Time sampling	8ms
Record length	16s
Shot interval	150m
Group interval	30m
Shortest offset	0m

Synthetic data: SEAM Phase I deep-water Gulf of Mexico model

Parameter	Value
Gun depth	15m
Streamer depth	15m & 17m

- 3D source, frequency of source: 1-30Hz
- Distance between towed streamers: 30m
- $\partial P/\partial z \simeq [P(17m) - P(15m)]/2m$

Field data: deep water Gulf of Mexico[1]

Parameter	Value
Number of shots	2451
Number of channels per shot	960
Number of samples per trace	3585
Time sampling	4ms
Record length	14.34s
Shot interval	37.5m
Group interval	12.5m
Shortest offset	112m
Gun depth	9m
Streamer depth	25m

[1] Courtesy of PGS.

Field data: deep water Gulf of Mexico

- 1 dual-sensor towed streamer
- $\partial P/\partial z = i\omega\rho V_z$ where ρ is density of reference medium (seawater)

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