Mission - Oriented Seismic Research Program

A method for the elimination of all first order internal multiples from all reflectors in a 1D medium: theory and examples

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San Antonio, Texas

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Case 1

data

predicted multiple

Solution

Internal multiple attenuator b_3 + adaptive subtraction

Case 2

data

predicted multiple

Solution

Internal multiple elimination

Key Points

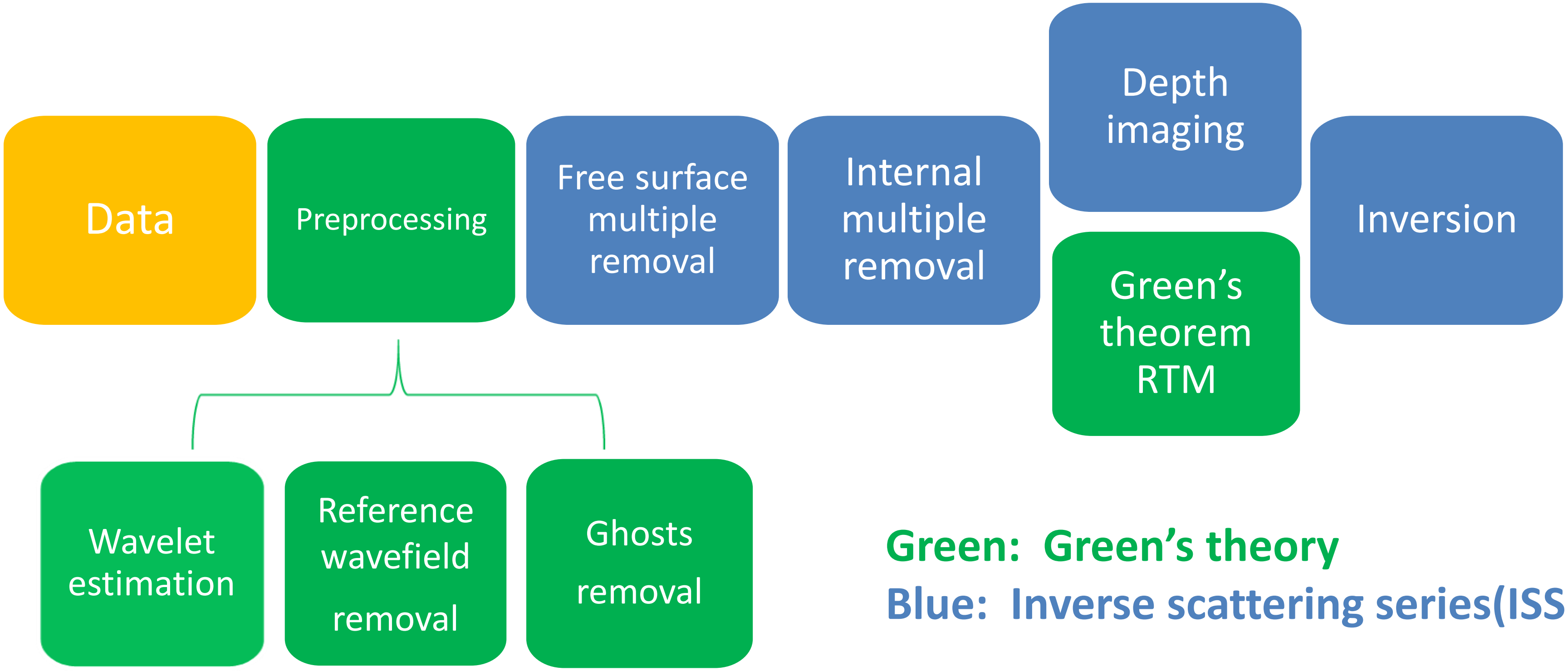
In this presentation, a new method is given to eliminate all first order internal multiples under 1D normal incidence.

This method

1. is derived in a reverse engineering way (not seeking higher order terms within inverse scattering series) to construct an algorithm to eliminate first order internal multiples.

2. achieves the goal directly in terms of data without determining the earth.

Processing Train



Data

Preprocessing

Free surface multiple removal

Internal multiple removal

Depth imaging

Green's theorem RTM

Inversion

Wavelet estimation

Reference wavefield removal

Ghosts removal

Green: Green's theory

Blue: Inverse scattering series(ISS)

Internal multiple removal

Internal multiple removal

Leading order algorithm

Araújo et al(1994),Weglein et al. (1997)

Internal multiple removal

Leading order algorithm

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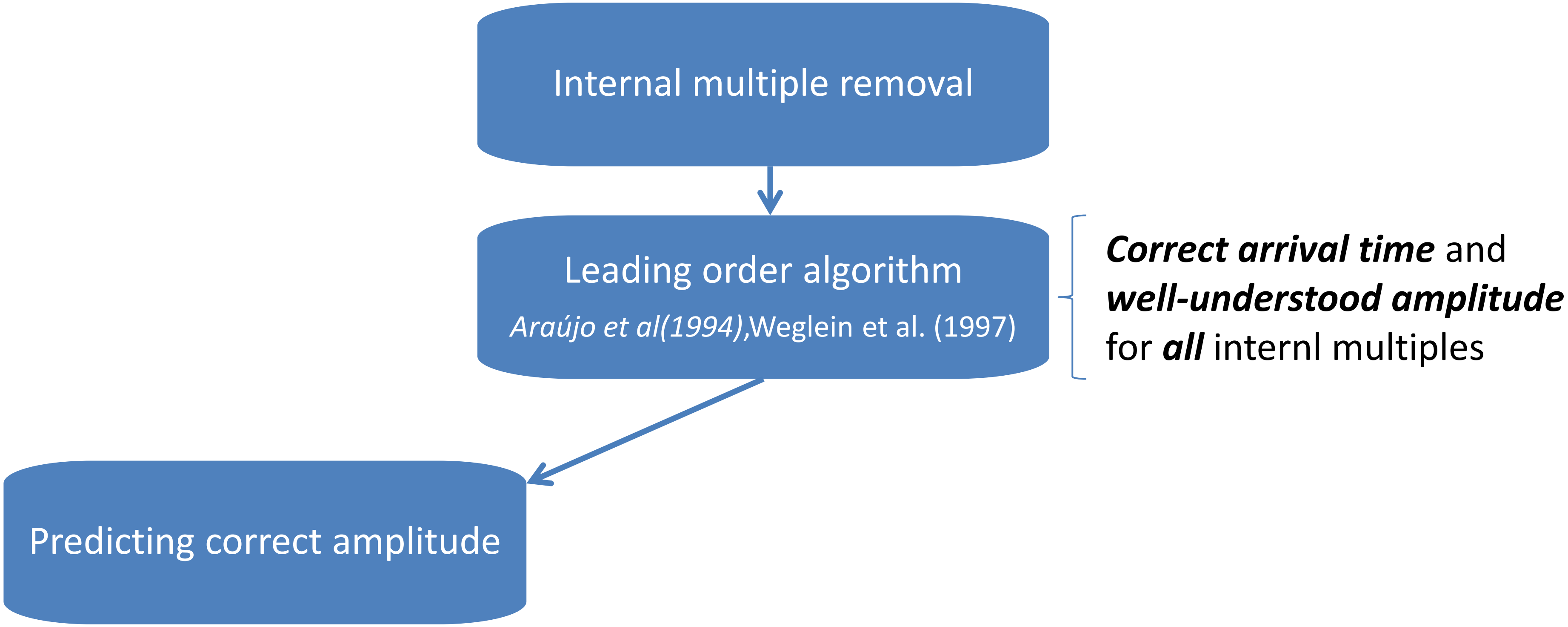
Correct arrival time and

well-understood amplitude

for ***all*** internl multiples



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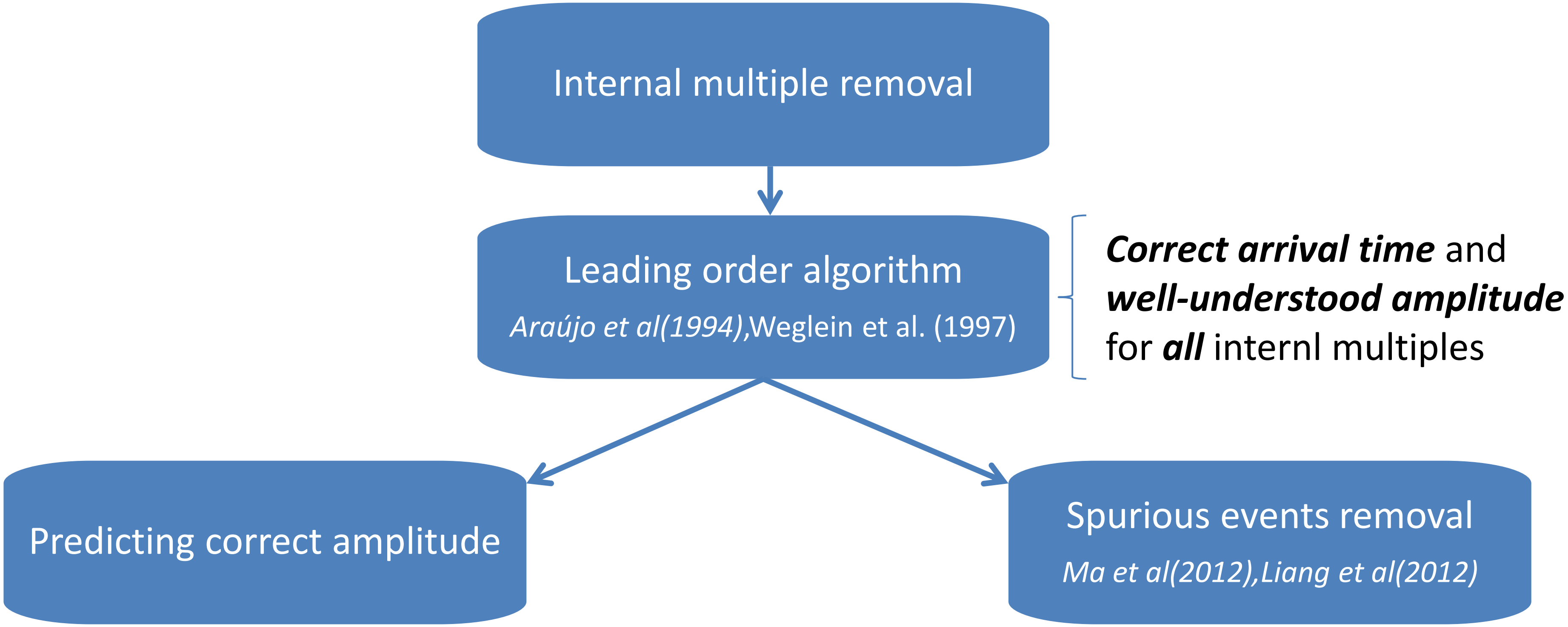

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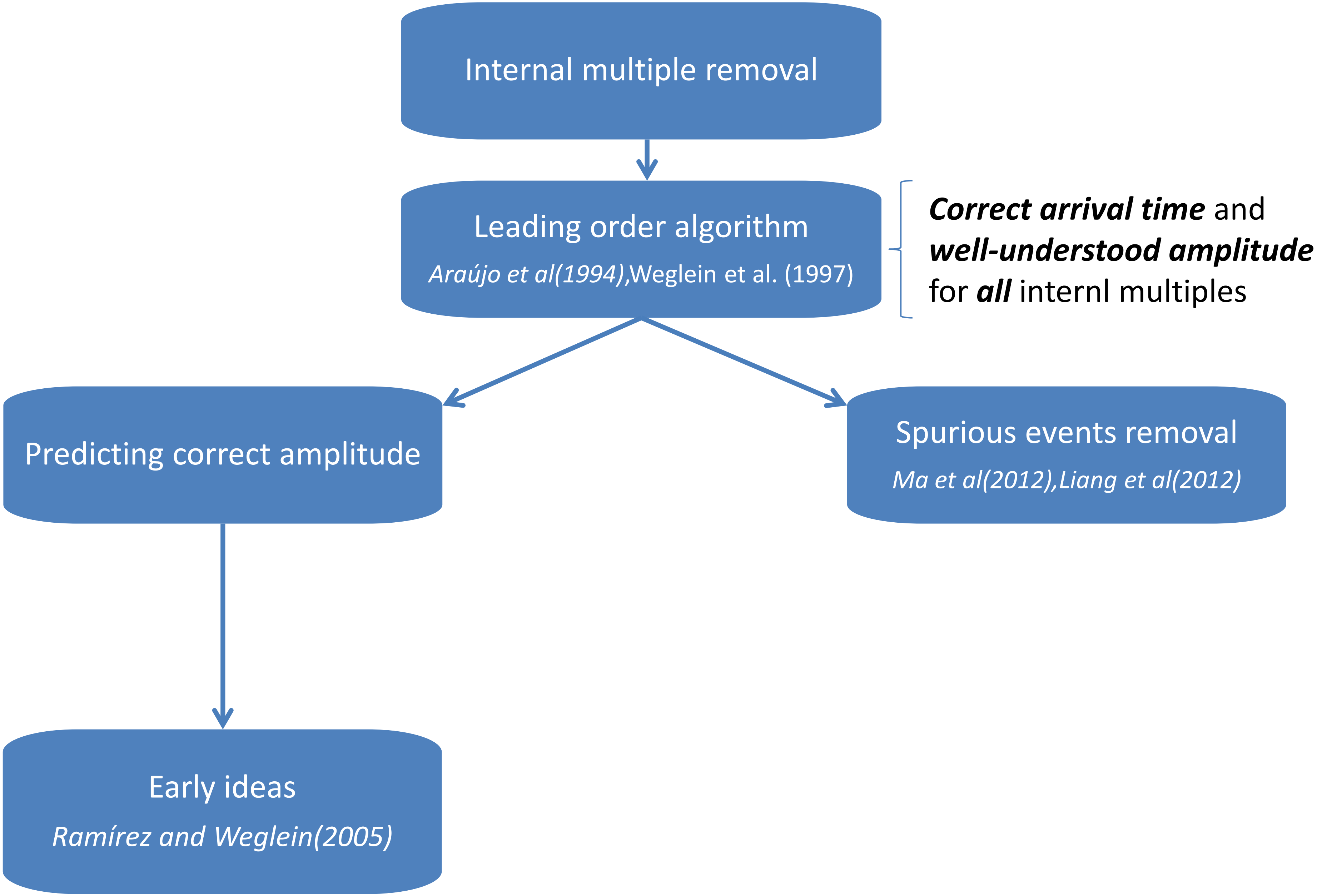
Ma et al(2012),Liang et al(2012)



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Early ideas

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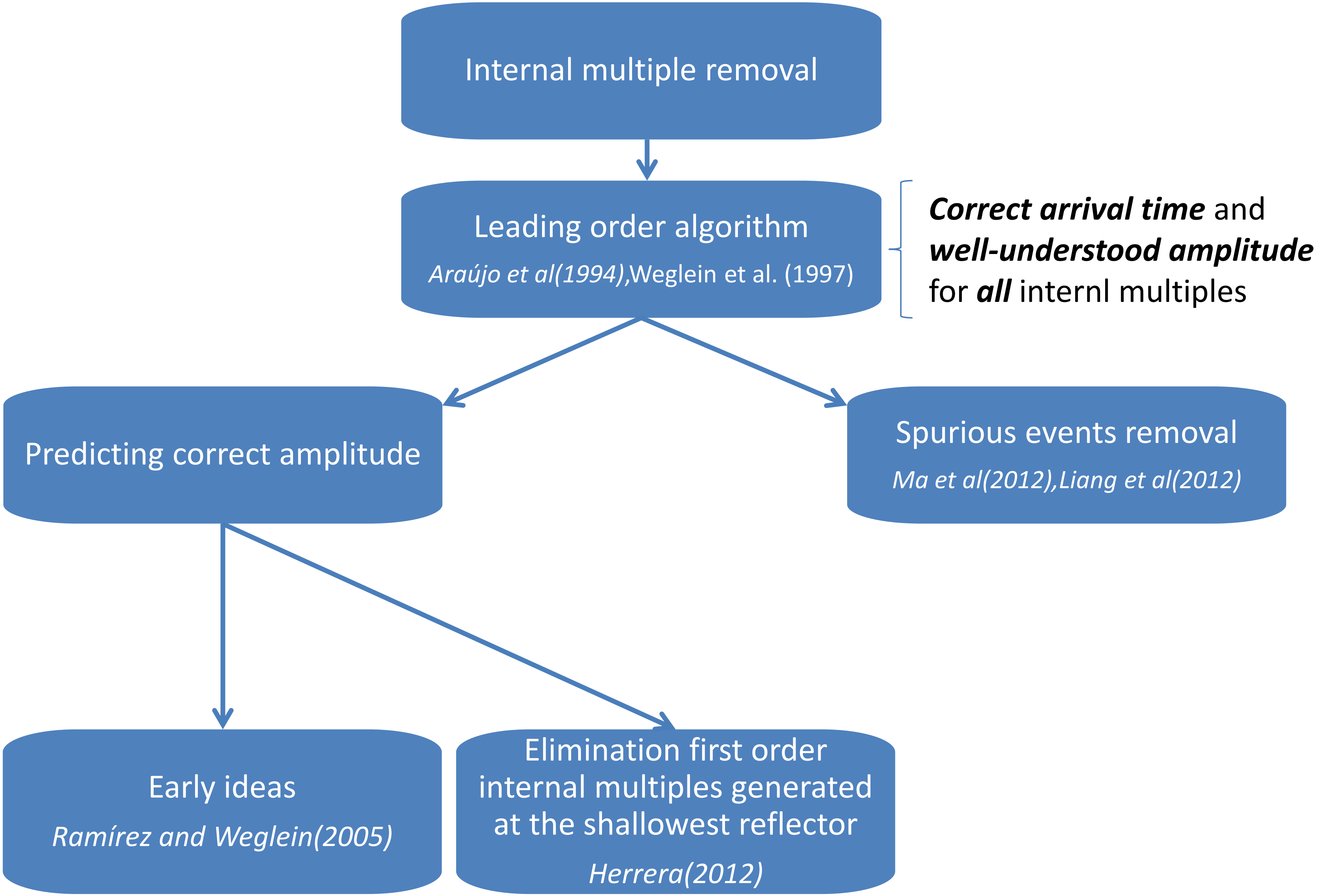

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Elimination first order internal multiples generated at the shallowest reflector

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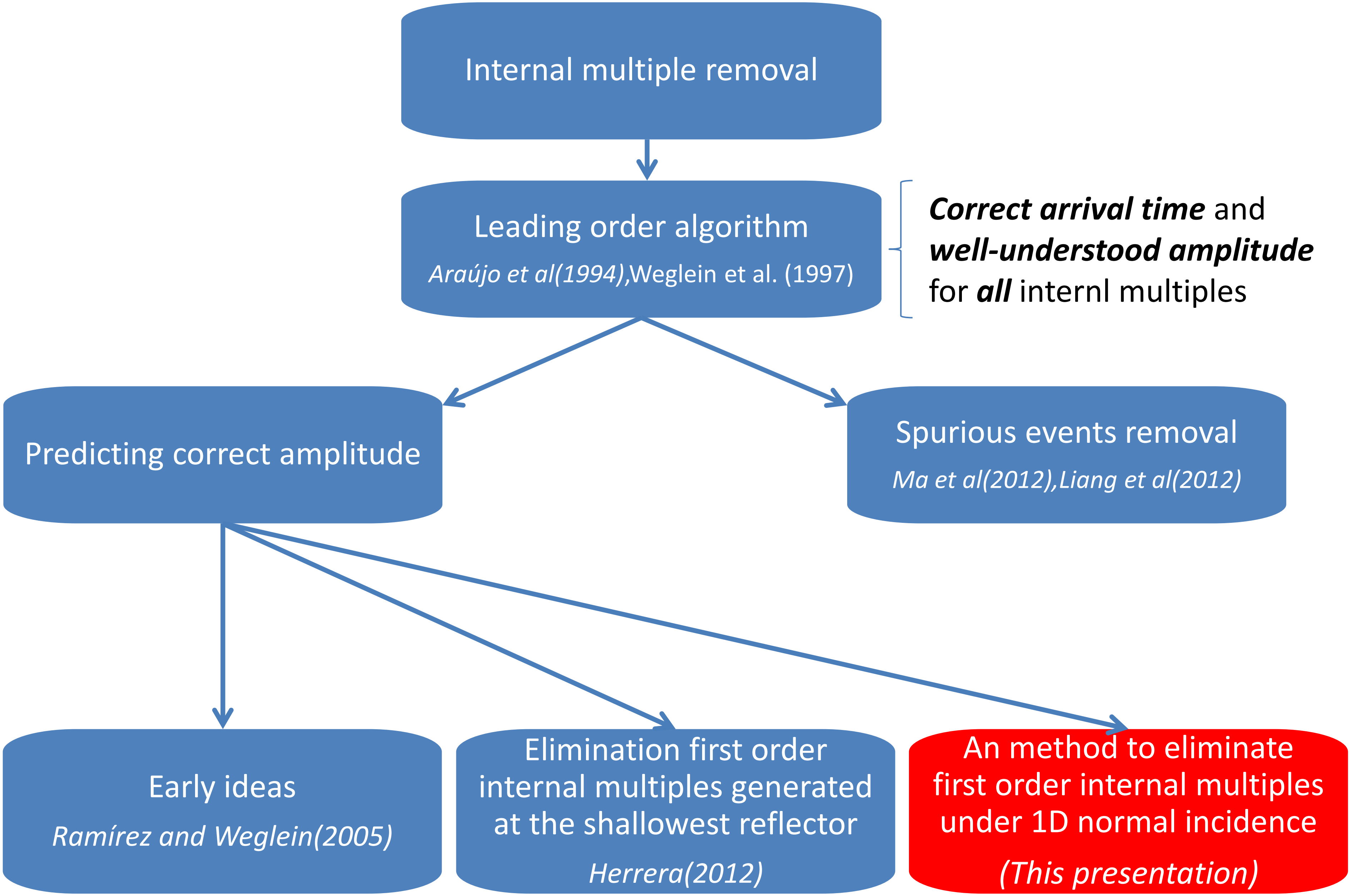

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An method to eliminate first order internal multiples under 1D normal incidence

(This presentation)

Input

data

Goal

predict ***all*** internal multiples with

1)correct amplitude

2)correct time

Input Goal

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2)correct time

Leading order algorithm

data

predict ***first order*** internal multiples with

1)***correct time***

2)***approximate amplitude***

Higher order internal multiples with

correct time and well-understood amplitude

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The method in this presentation

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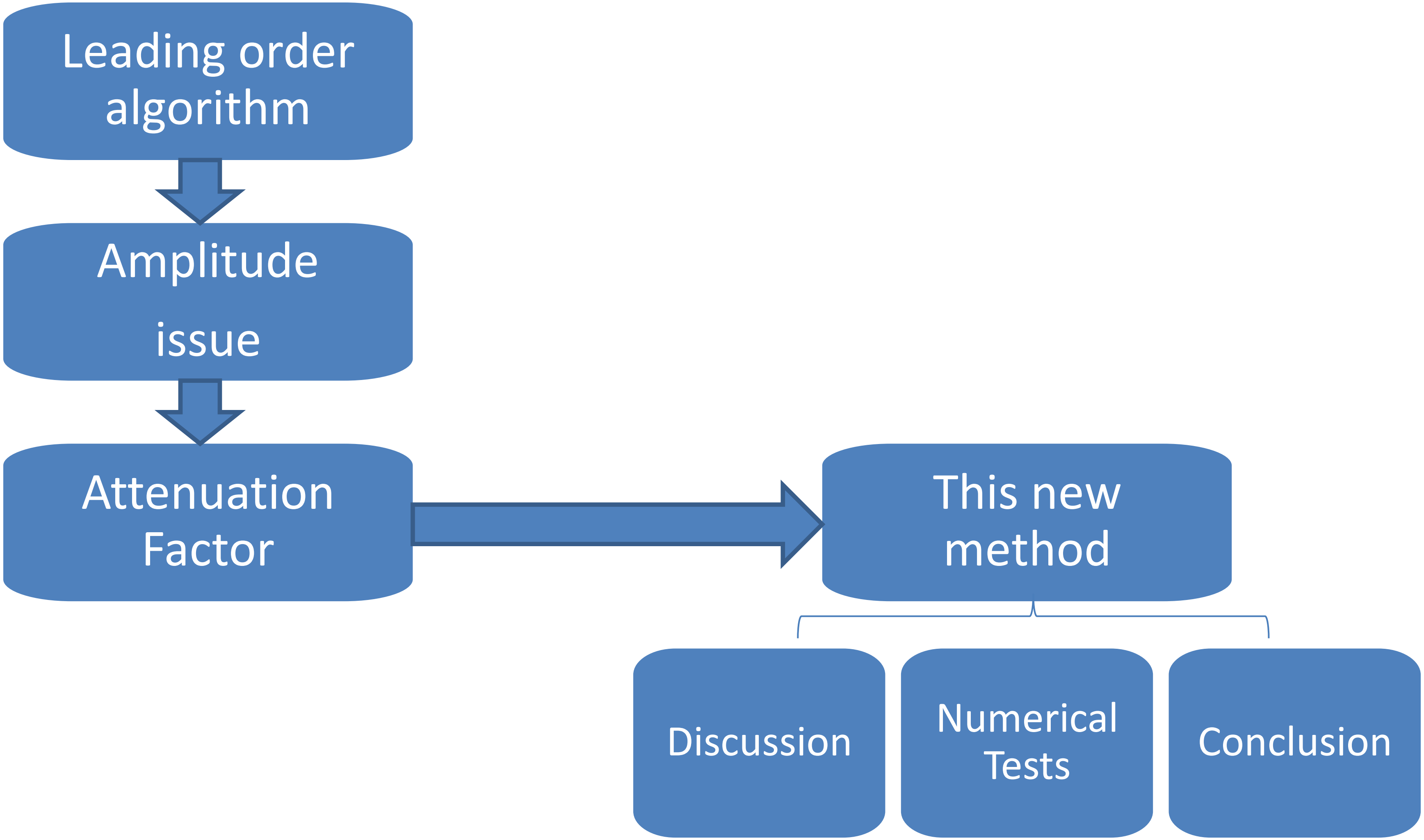
The structure of this presentation



Leading order algorithm

Amplitude issue

Attenuation Factor

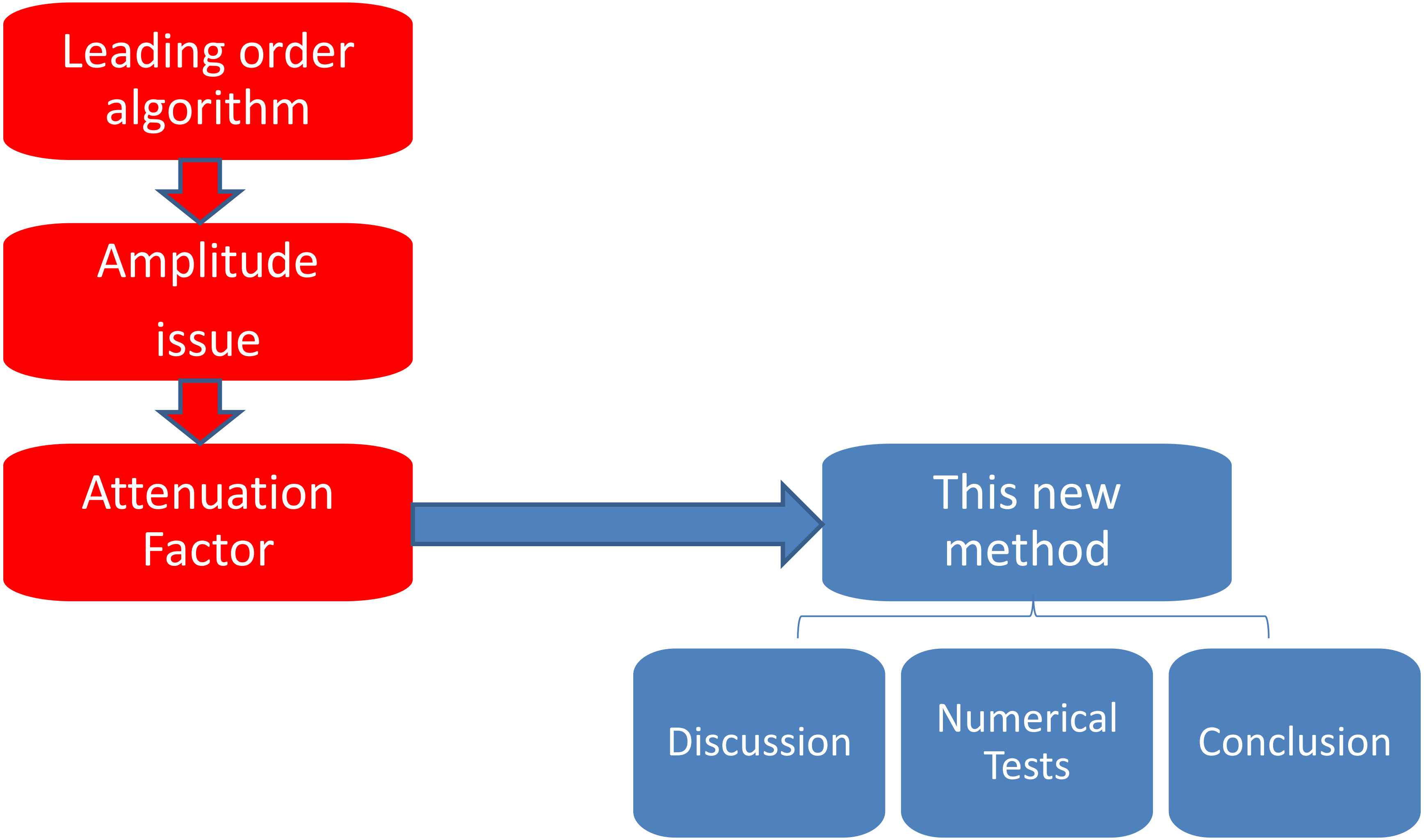
This new method

Discussion

Numerical Tests

Conclusion

The structure of this presentation



Leading order algorithm

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The 1D normal incidence version of the leading order algorithm given by Araújo et al.(1994) and Weglein et al. (1997) is presented as follows:

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \quad (1)$$

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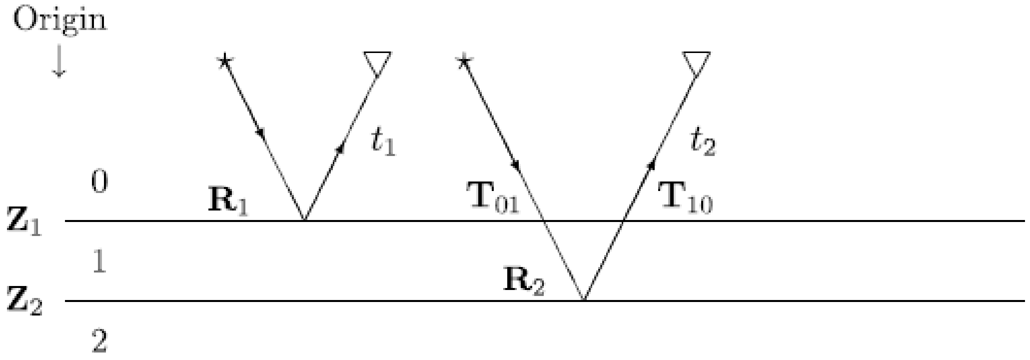
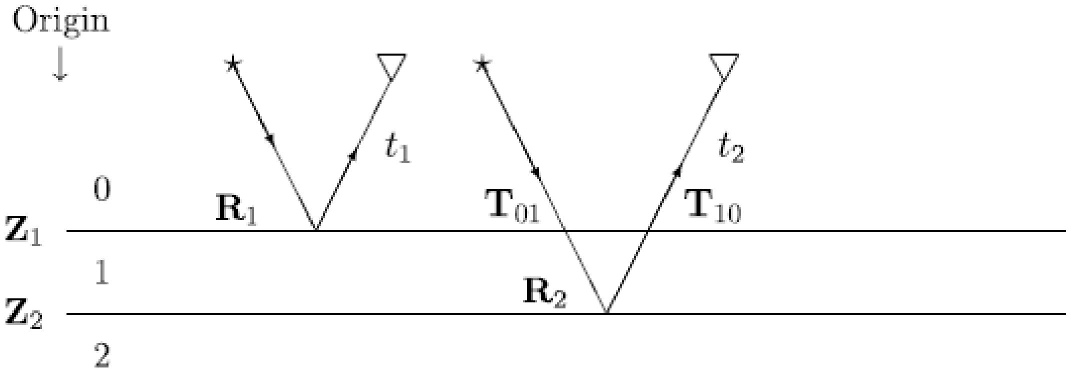
Consider the simplest one-generator model example that can produce an internal multiple given by Weglein et al.(2003)



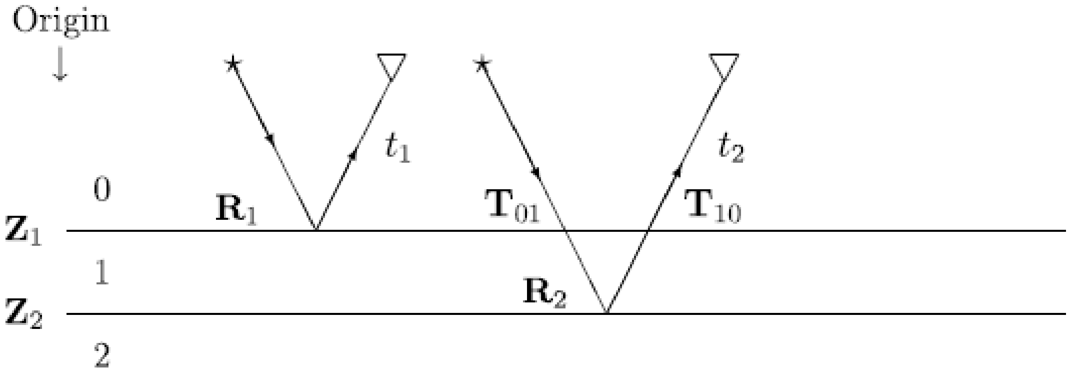
A one dimensional model with two interfaces.



A one dimensional model with two interfaces.

For this model, the reflection data caused by an impulsive incident wave δ(t-z/c) is:

$$D(t) = R_1\delta(t-t_1) + T_{01}R_2T_{10}\delta(t-t_2) + \cdots$$



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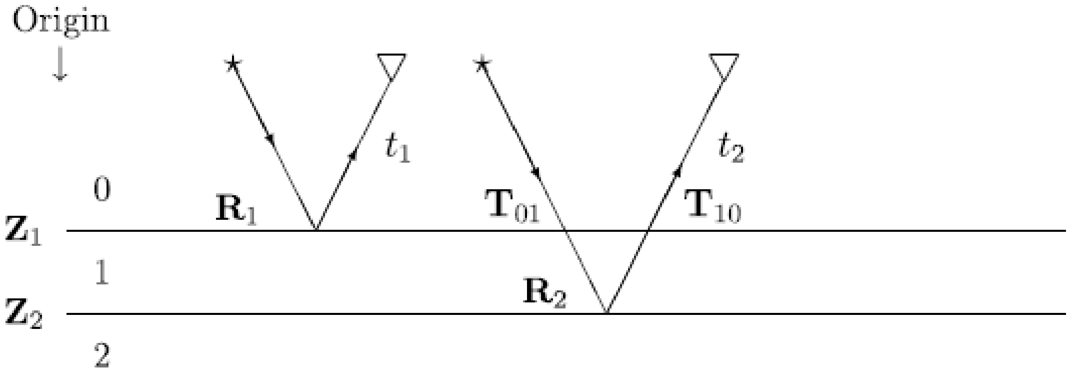

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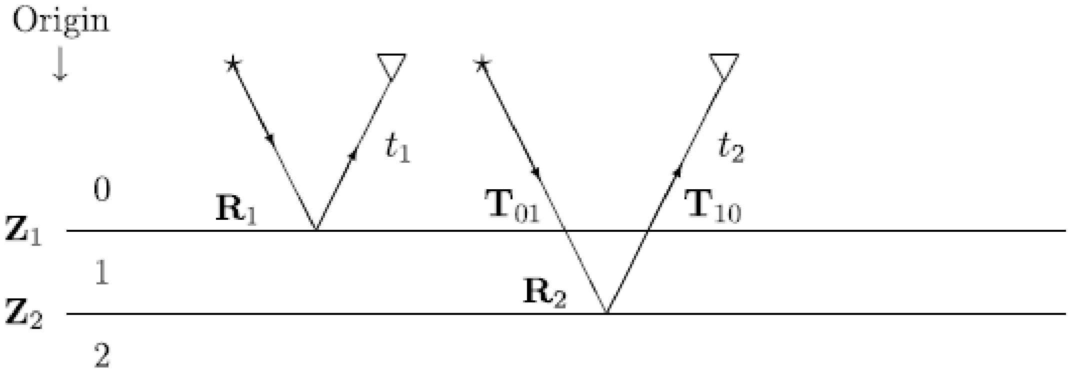

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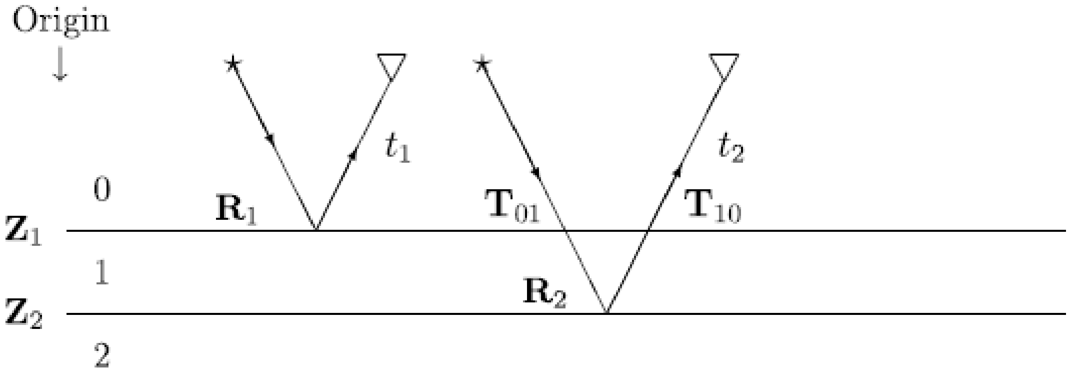
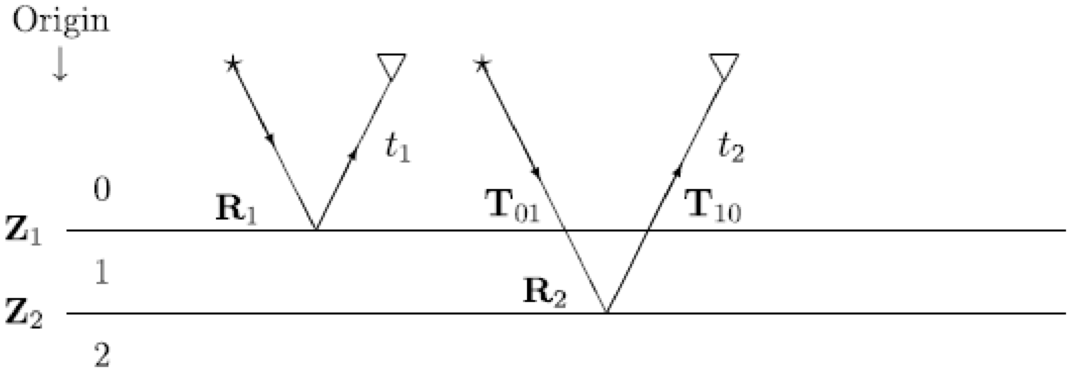
The date is now ready for the internal multiple algorithm.



A one dimensional model with two interfaces.

Substituting $b_1(z)$ into the algorithm, we can derive the prediction (in the time domain):

$$b_3(t) = R_1 R_2^2 T_{01}^2 T_{10}^2 \delta(t - (2t_2 - t_1))$$



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From the example it is easy to compute the actual first order internal multiple precisely:

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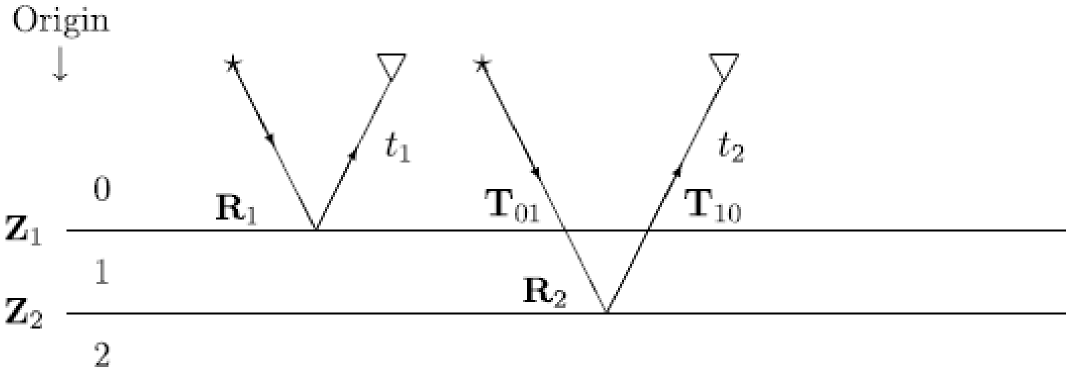

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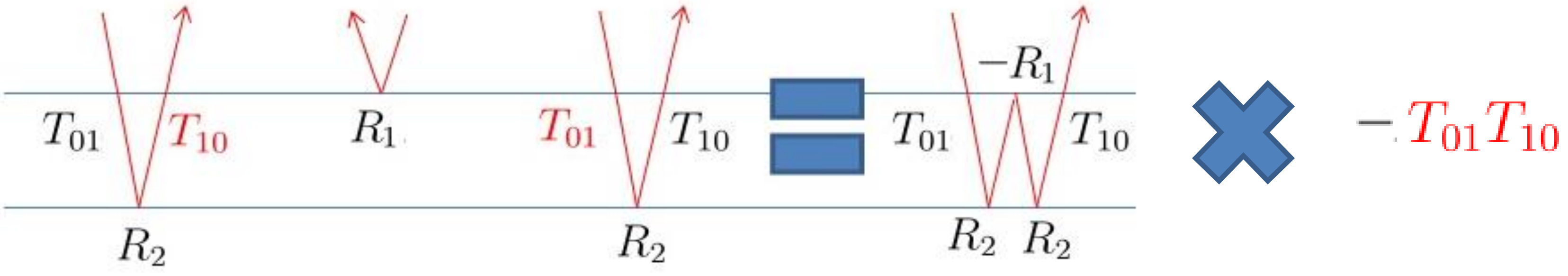
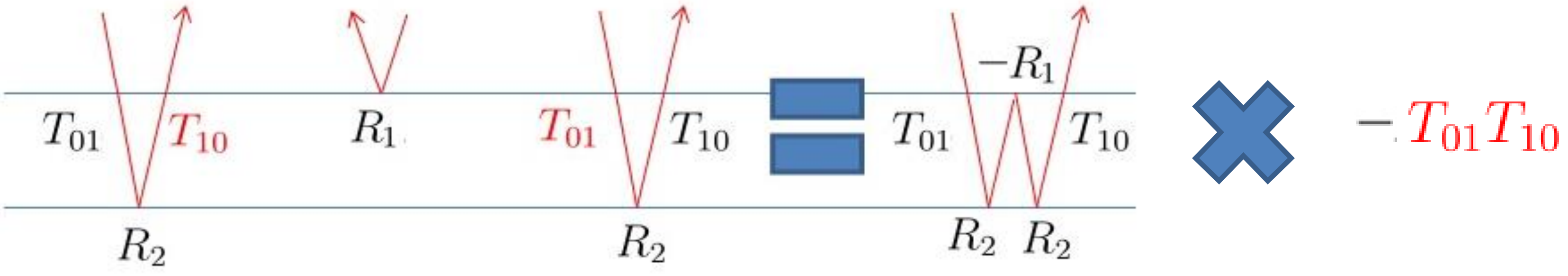
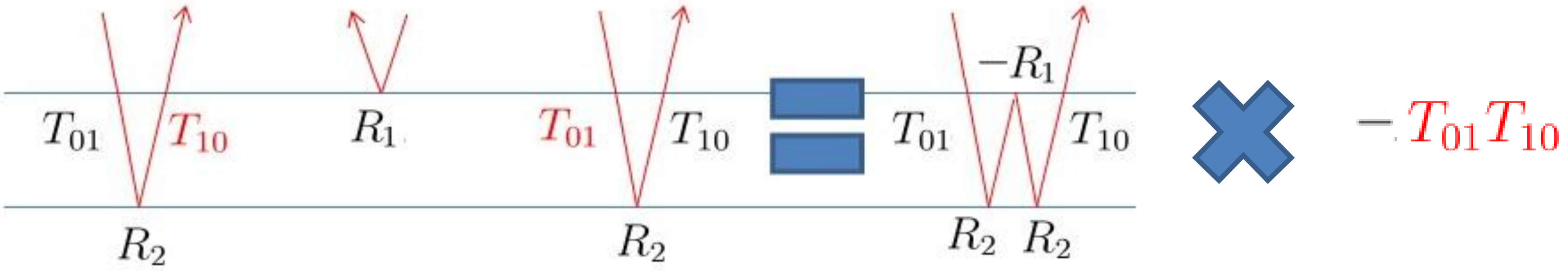
The time prediction is precise, and the amplitude of the prediction has an extra power of **$T_{01}T_{10}$** which is called the **attenuation factor.**


T_{01}
T_{10}
R_2
R_1
T_{01}
T_{10}
R_2
$-R_1$
T_{01}
T_{10}
R_2
R_2
$-T_{01}T_{10}$



$$AF_1 = T_{01}T_{10}$$


T_{01}
T_{10}
R_2
R_1
T_{01}
T_{10}
R_2
T_{01}
$-R_1$
T_{10}
R_2 R_2
$-T_{01}T_{10}$

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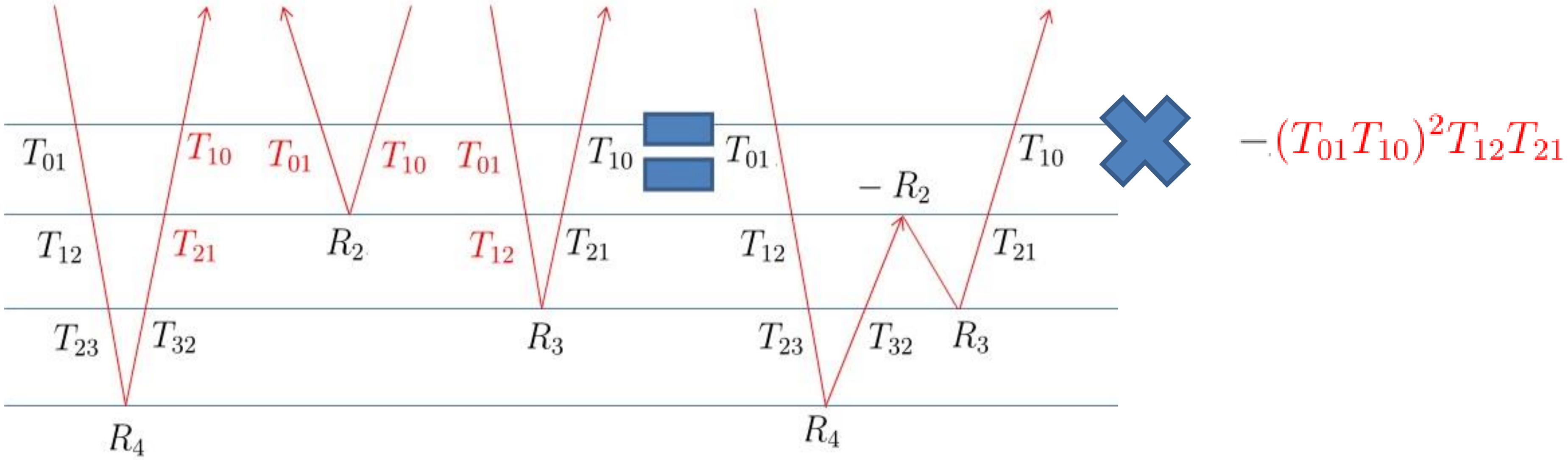
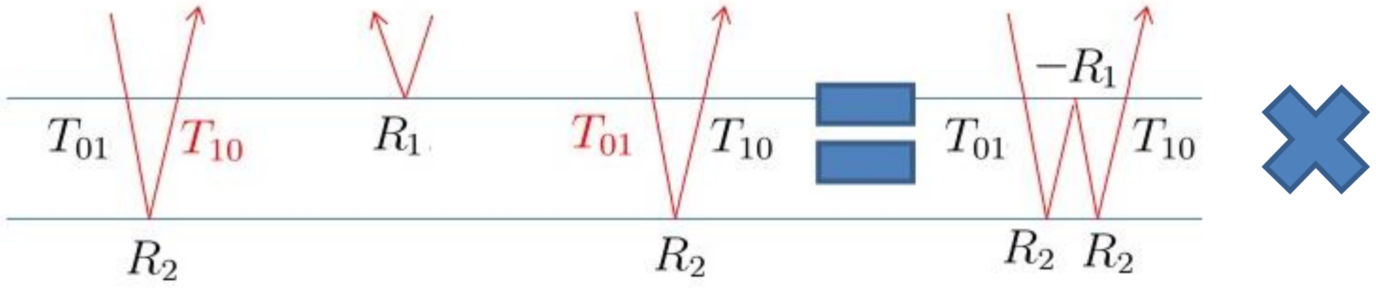

T_{01}
T_{10}
T_{12}
T_{21}
T_{23}
T_{32}
R_4
T_{01}
T_{10}
R_2
T_{01}
T_{10}
T_{12}
T_{21}
R_3
T_{01}
$-R_2$
T_{10}
T_{12}
T_{21}
T_{23}
T_{32}
R_3
R_4
$-(T_{01}T_{10})^2T_{12}T_{21}$



$-T_{01}T_{10}$

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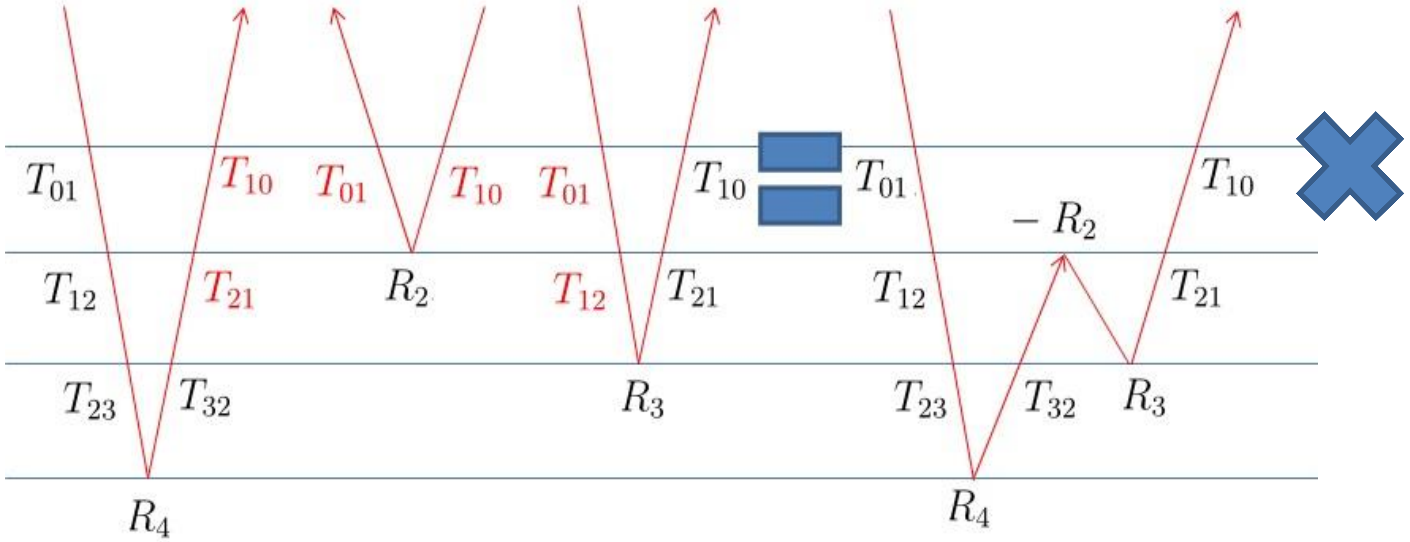

$-(T_{01}T_{10})^2T_{12}T_{21}$

$AF_2 = (T_{01}T_{10})^2T_{12}T_{21}$

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(using transmission coefficients)

$$AF_j = \begin{cases} T_{0,1}T_{1,0} & (j = 1) \\ \prod_{i=1}^{N-1}(T_{i-1,i}^2 T_{i,i-1}^2)T_{j,j-1}T_{j-1,j} & (1 < j < J) \end{cases}$$

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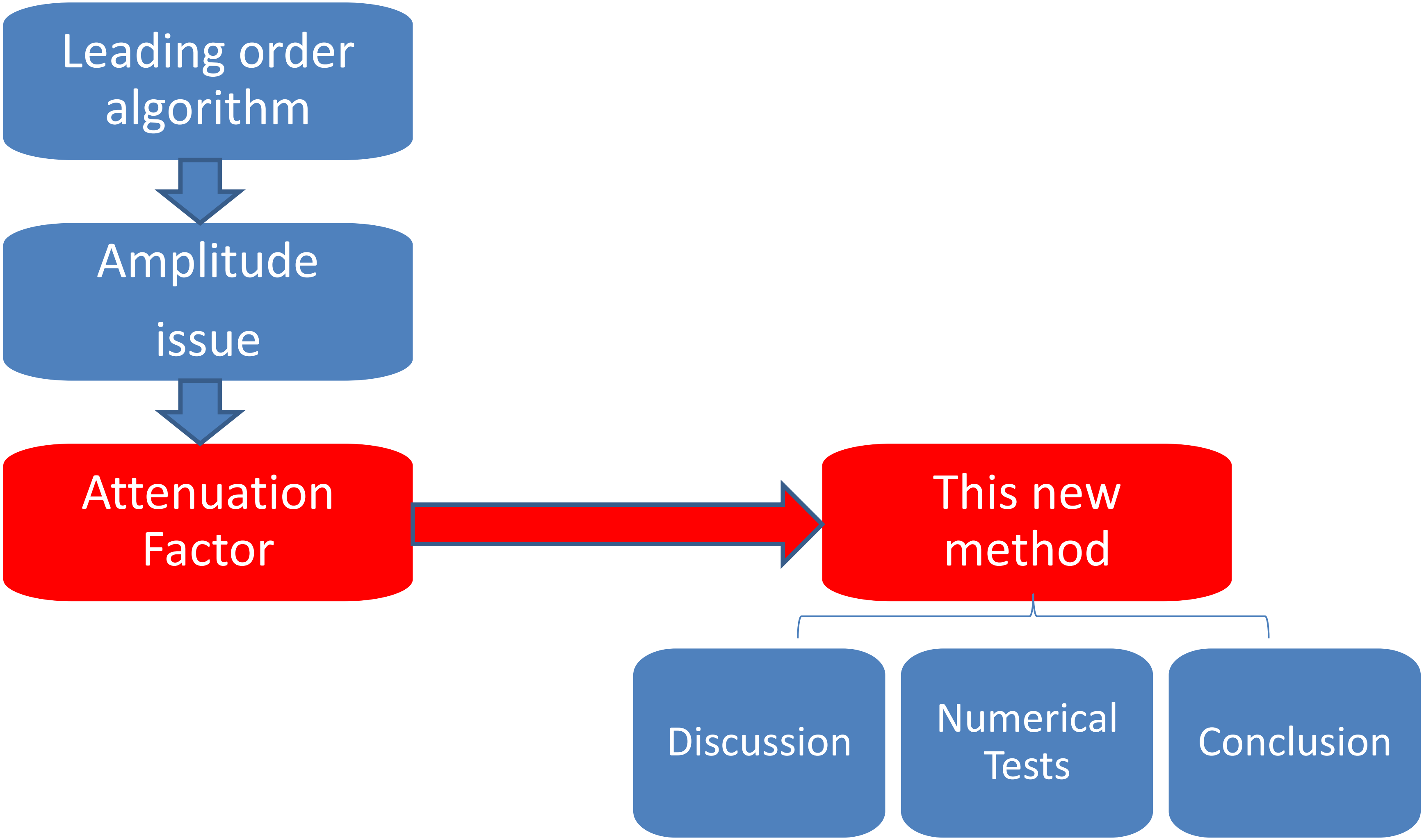
AF_j is the attenuation factor for all first order internal multiples with a downward reflection at the jth reflector.

The structure of this presentation



Leading order algorithm

Amplitude issue

Attenuation Factor

This new method

- Discussion
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- Conclusion

The idea to remove first order internal multiples is to build a new function in the second integral to remove the attenuation factor.

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

$$b_E^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

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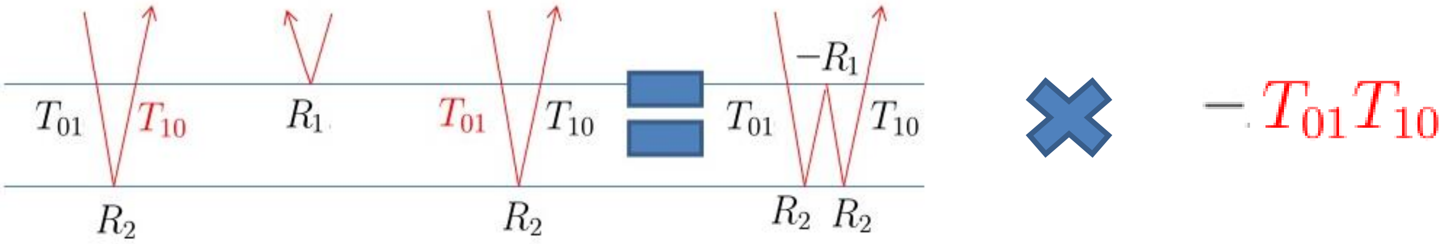

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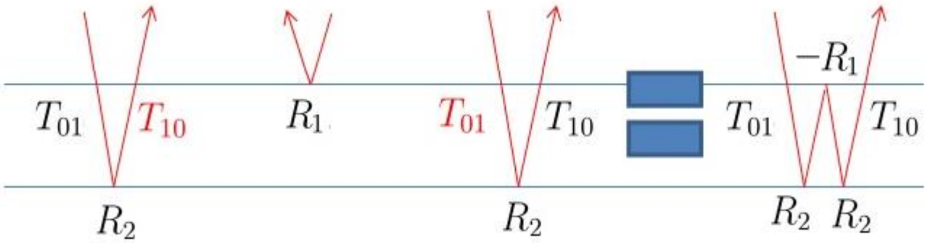
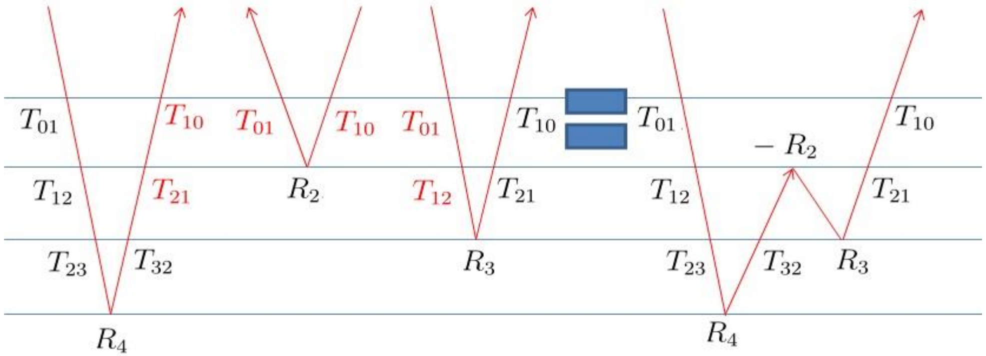


× $-T_{01}T_{10}$



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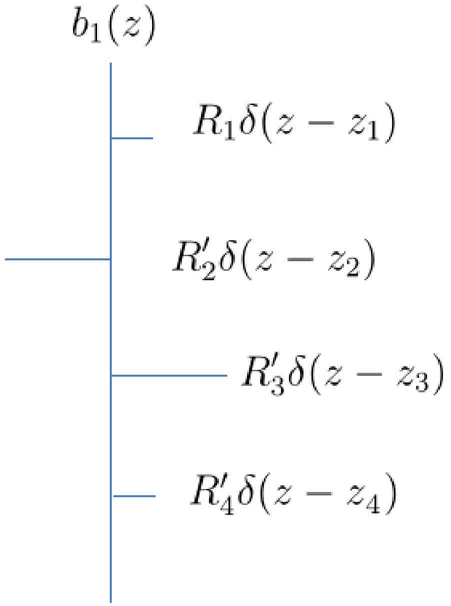

$b_1(z)$

$R_1\delta(z-z_1)$

$R_2'\delta(z-z_2)$

$R_3'\delta(z-z_3)$

$R_4'\delta(z-z_4)$

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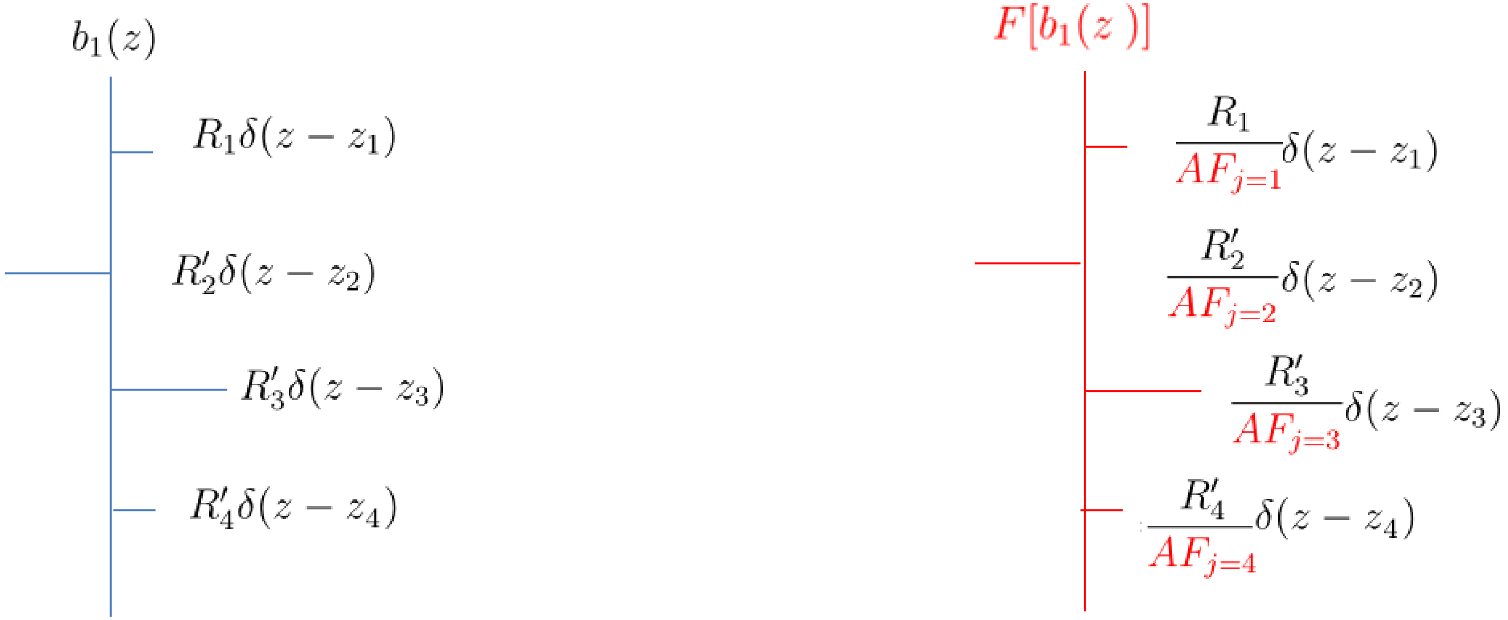

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$R_2'\delta(z-z_2)$

$R_3'\delta(z-z_3)$

$R_4'\delta(z-z_4)$

$F[b_1(z\ )]$

$\frac{R_1}{AF_{j=1}}\delta(z-z_1)$

$\frac{R_2'}{AF_{j=2}}\delta(z-z_2)$

$\frac{R_3'}{AF_{j=3}}\delta(z-z_3)$

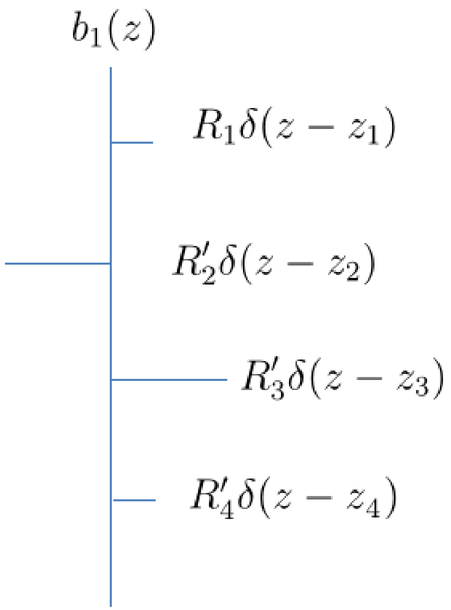
$\frac{R_4'}{AF_{j=4}}\delta(z-z_4)$

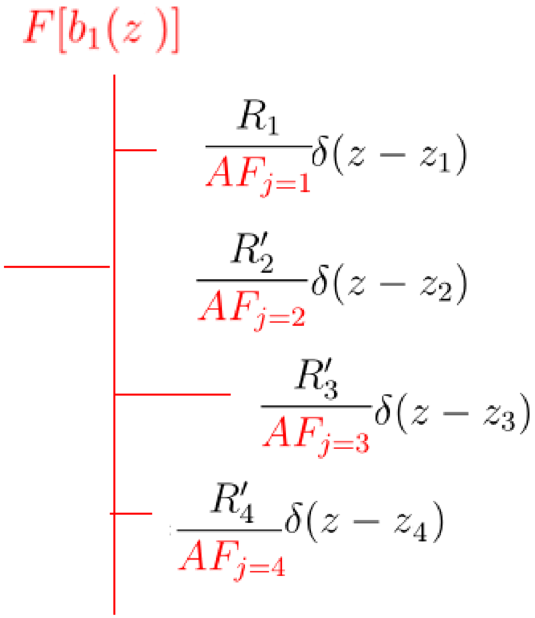
The idea to remove first order internal multiples is to build a new function in the second integral to remove the attenuation factor.

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

$$b_E^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

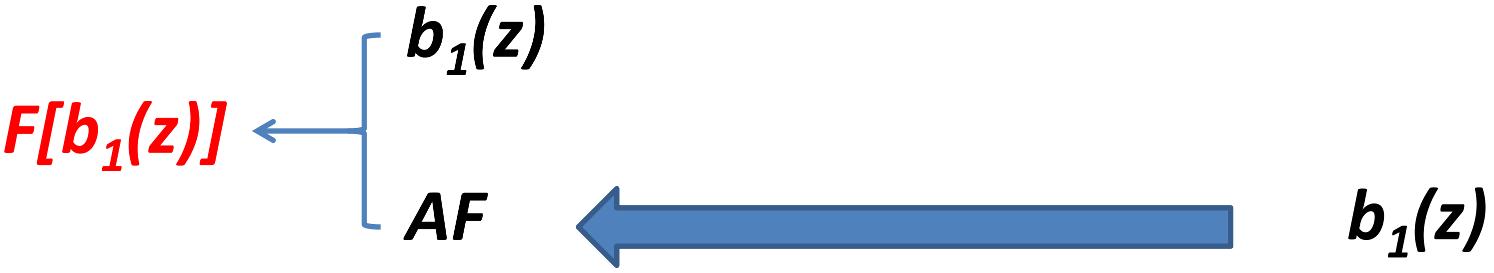
With the definition: $R_i' = (1-R_1^2)(1-R_2^2)\cdots(1-R_{i-2}^2)(1-R_{i-1}^2)\, R_i$





$F[b_1(z)]$

$$F[b_1(z)] \leftarrow \left\{ \begin{array}{l} b_1(z) \\ \\ AF \end{array} \right.$$



$b_1(z)$

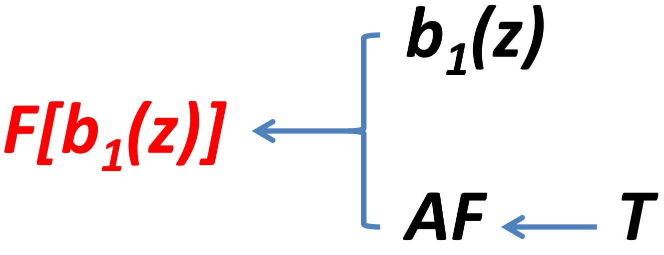
$F[b_1(z)]$

AF

$b_1(z)$

$$F[b_1(z)] \leftarrow \left\{ \begin{array}{l} b_1(z) \\ \\ AF \end{array} \right.$$

$b_1(z)$



$F[b_1(z)]$

$b_1(z)$

AF

T

$b_1(z)$

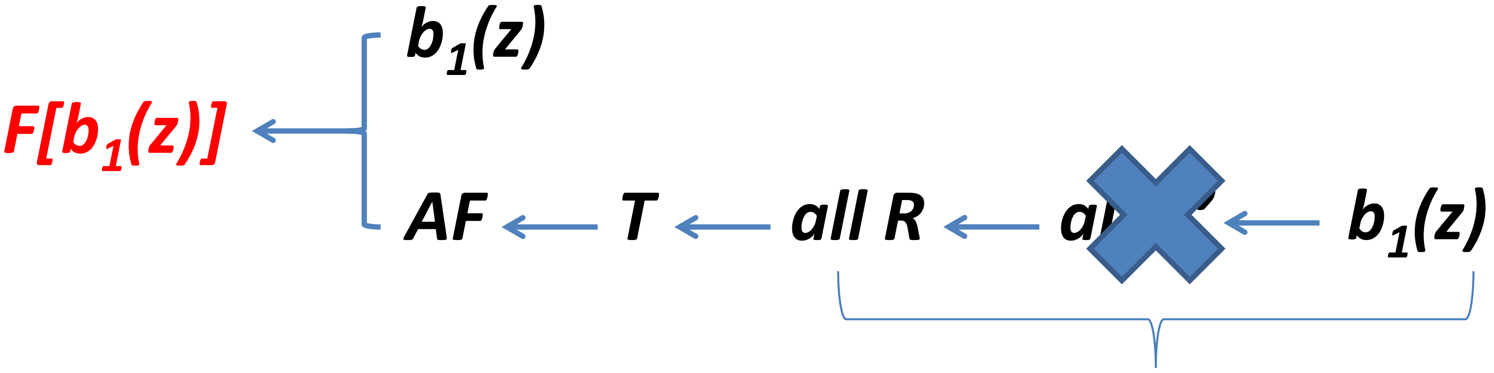
$$F[b_1(z)] \longleftarrow \left\{ \begin{array}{l} b_1(z) \\ AF \longleftarrow T \longleftarrow all\ R \end{array} \right.$$

$b_1(z)$

$$F[b_1(z)] \longleftarrow \left\{ \begin{array}{l} b_1(z) \\ \\ AF \longleftarrow T \longleftarrow \text{all } R \longleftarrow \text{all } R' \qquad b_1(z) \end{array} \right.$$

$$\begin{tikzcd}
 & b_1(z) \arrow[dl] & & & & \\
F[b_1(z)] & & & & & \\
 & AF \arrow[ul] & T \arrow[l] & \text{all } R \arrow[l] & \text{all } R' \arrow[l] & b_1(z) \arrow[l]
\end{tikzcd}$$

$$F[b_1(z)] \leftarrow \left\{\begin{array}{l} b_1(z) \\ \\ AF \leftarrow T \leftarrow \underbrace{\text{all } R \leftarrow \text{all } R' \leftarrow b_1(z)}_{\text{Very complex}} \end{array}\right.$$

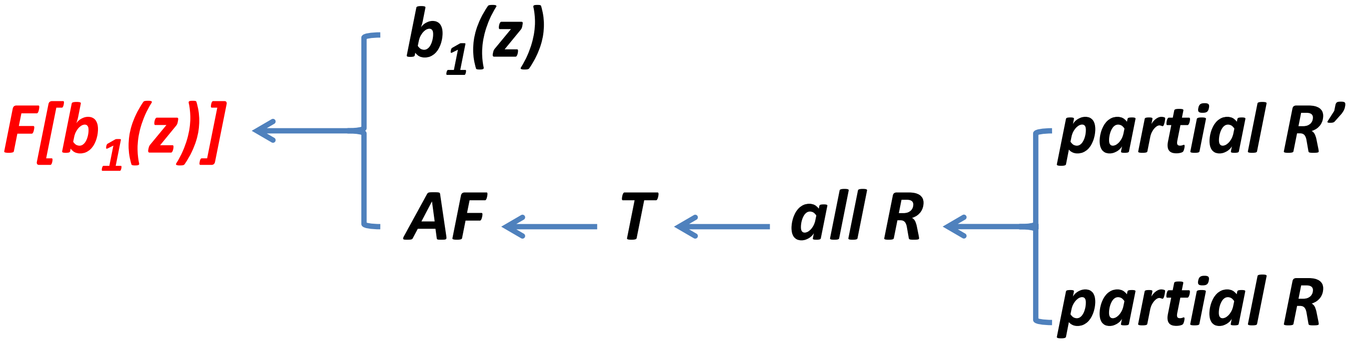


$b_1(z)$

$F[b_1(z)]$

$AF \leftarrow T \leftarrow \text{all } R \leftarrow \text{al} \leftarrow b_1(z)$

Very complex



$b_1(z)$

$F[b_1(z)]$

partial R’

AF ← T ← all R

partial R

$b_1(z)$

$F[b_1(z)]$

AF ← T ← all R ← partial R', partial R ← $b_1(z)$, $g(z)$

$F[b_1(z)]$

$b_1(z)$

AF

T

all R

partial R’

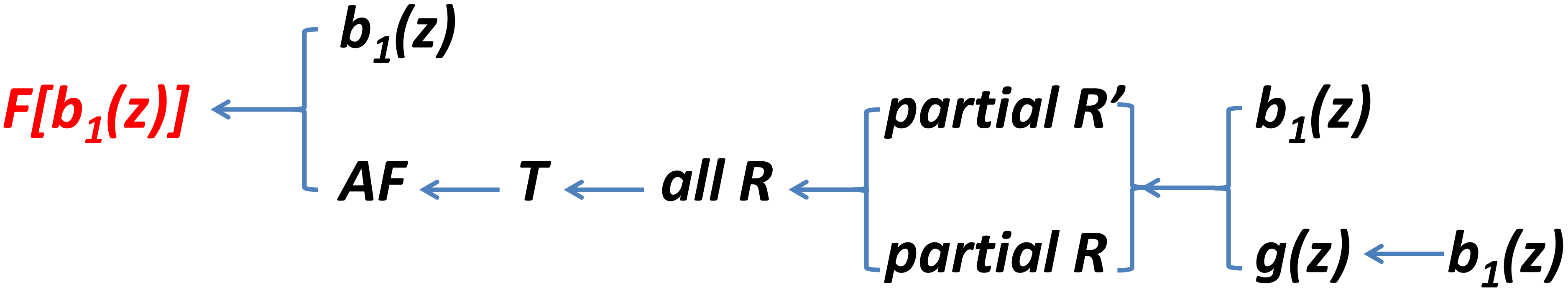
partial R

$b_1(z)$

$g(z)$

g(z) is a new function defined with R as coefficients.

$$g(z) = R_1\delta(z-z_1) + R_2\delta(z-z_2) + R_3\delta(z-z_3) + \cdots + R_n\delta(z-z_n) + \cdots$$



$F[b_1(z)]$

$b_1(z)$

AF ← T ← all R

partial R'

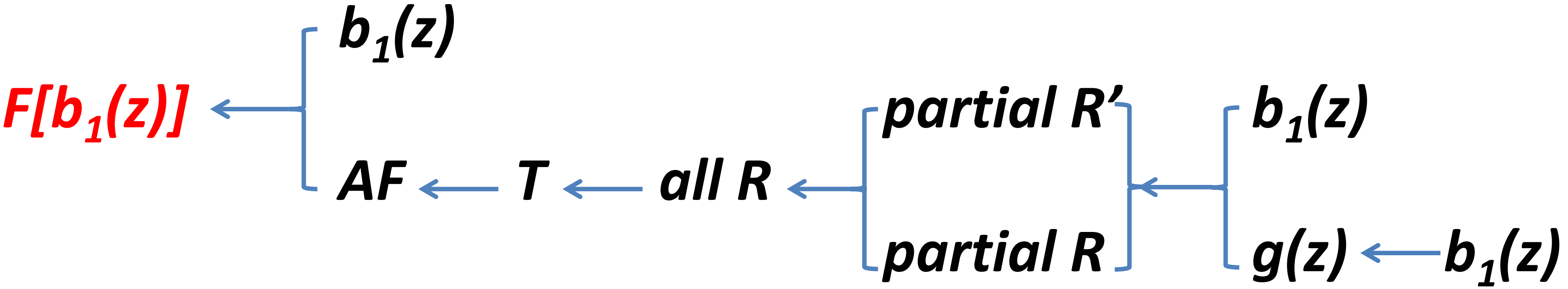
partial R

$b_1(z)$

$g(z)$ ← $b_1(z)$

g(z) is a new function defined with R as coefficients.

$$g(z) = R_1\delta(z - z_1) + R_2\delta(z - z_2) + R_3\delta(z - z_3) + \cdots + R_n\delta(z - z_n) + \cdots$$



$b_1(z)$

$F[b_1(z)]$

partial R'

$b_1(z)$

AF ← T ← all R

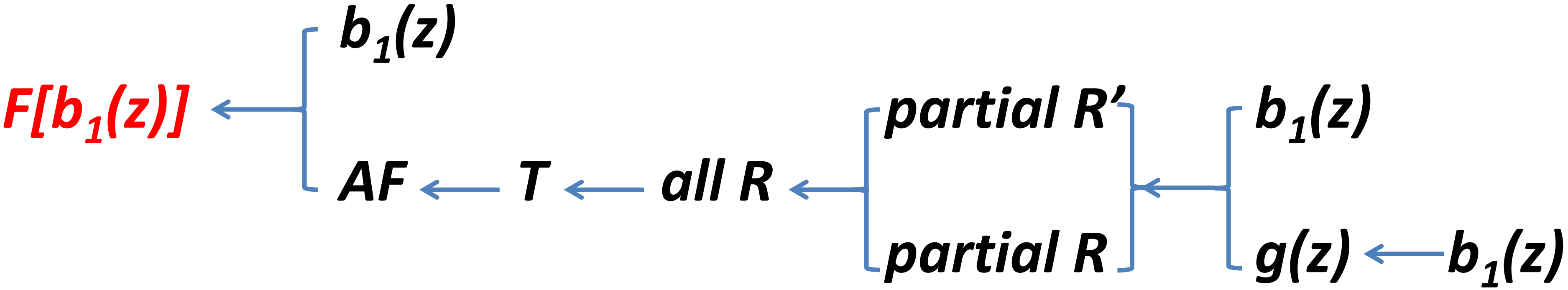
partial R

$g(z) \leftarrow b_1(z)$

$$\boxed{F[b_1(z)] \longleftarrow b_1(z)}$$

g(z) is a new function defined with R as coefficients.

$$g(z) = R_1\delta(z - z_1) + R_2\delta(z - z_2) + R_3\delta(z - z_3) + \cdots + R_n\delta(z - z_n) + \cdots$$



$b_1(z)$

$F[b_1(z)]$

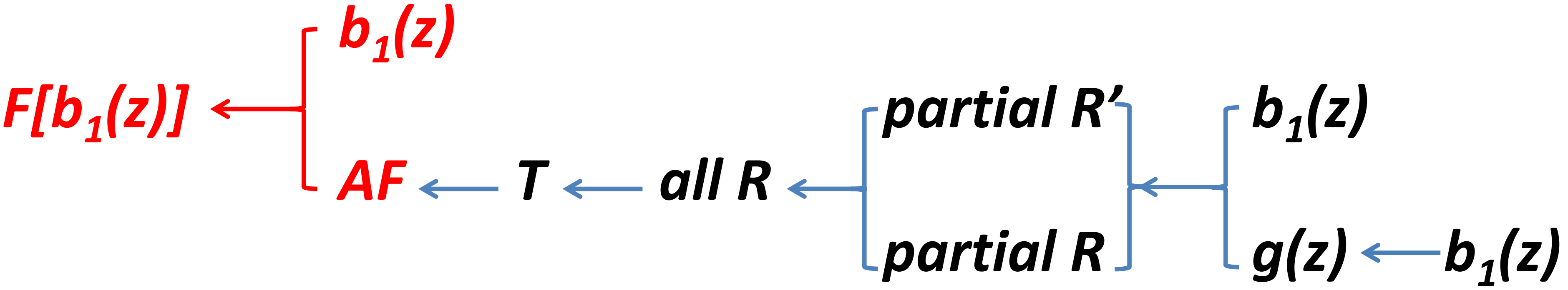
partial R’

$b_1(z)$

AF ← *T* ← *all R*

partial R

$g(z) \leftarrow b_1(z)$



$F[b_1(z)]$

$b_1(z)$

AF

T

all R

partial R'

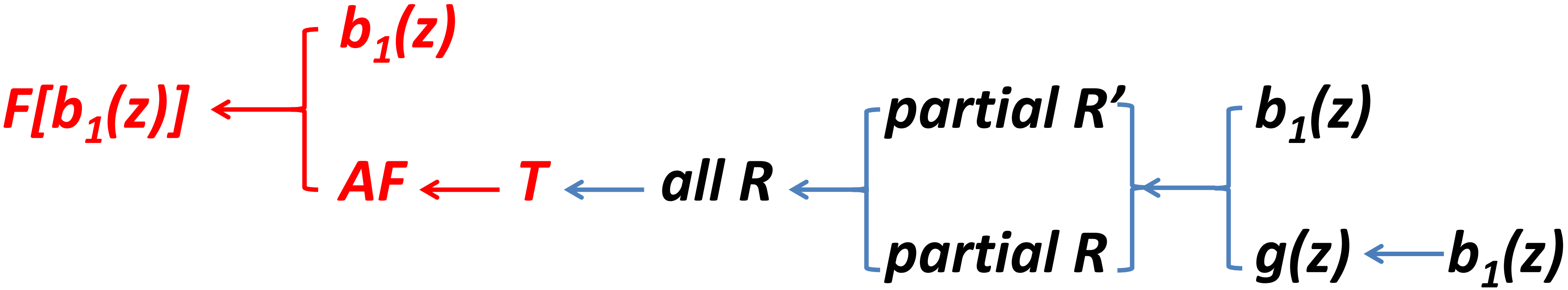
partial R

$b_1(z)$

$g(z)$

$b_1(z)$

The coefficient of the ith term in $F[b_1(z)]$ is: $\frac{R'_i}{AF_i}$

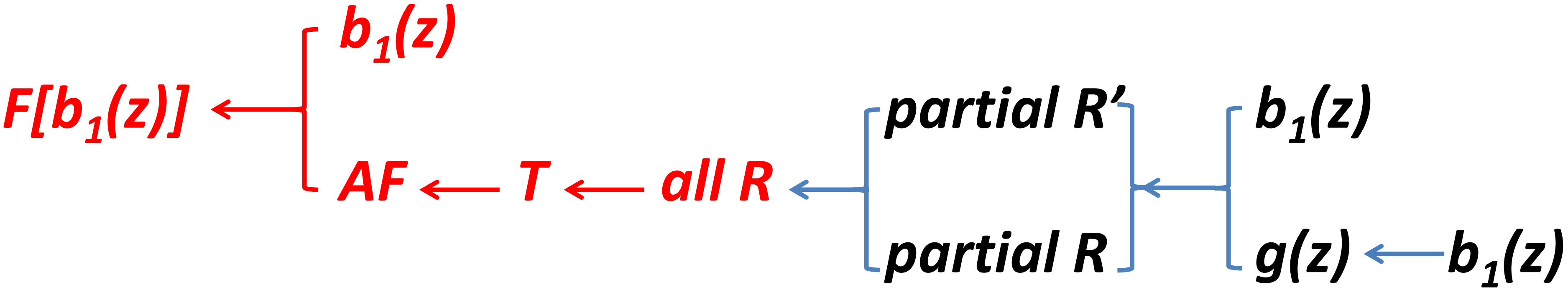


$F[b_1(z)]$ ← $b_1(z)$, AF

AF ← T ← all R ← partial R', partial R ← $b_1(z)$, $g(z)$ ← $b_1(z)$

The coefficient of the ith term in $F[b_1(z)]$ is: $\frac{R'_i}{AF_i}$

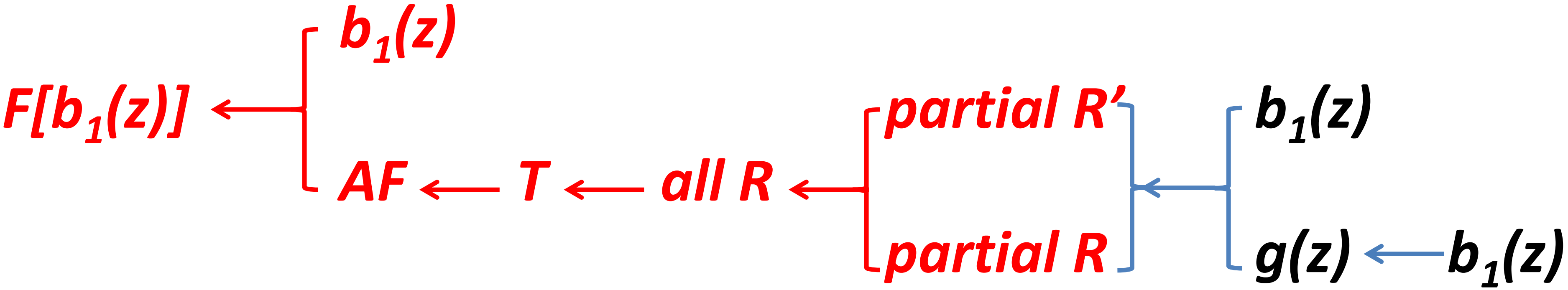
$$\frac{R'_i}{AF_i} = \frac{R'_i}{(T_{01}T_{10})^2(T_{12}T_{21})^2\cdots(T_{i-2,i-1}T_{i-1,i-2})^2(T_{i-1,i}T_{i,i-1})}$$



The coefficient of the ith term in $F[b_1(z)]$ is: $\frac{R'_i}{AF_i}$

$$\frac{R'_i}{AF_i} = \frac{R'_i}{(T_{01}T_{10})^2(T_{12}T_{21})^2\cdots(T_{i-2,i-1}T_{i-1,i-2})^2(T_{i-1,i}T_{i,i-1})}$$

$$= \frac{R'_i}{(1-R_1^2)^2(1-R_2^2)^2\cdots(1-R_{i-1}^2)^2(1-R_i^2)}$$



$F[b_1(z)]$

$b_1(z)$

AF ← T ← all R

partial R'

partial R

$b_1(z)$

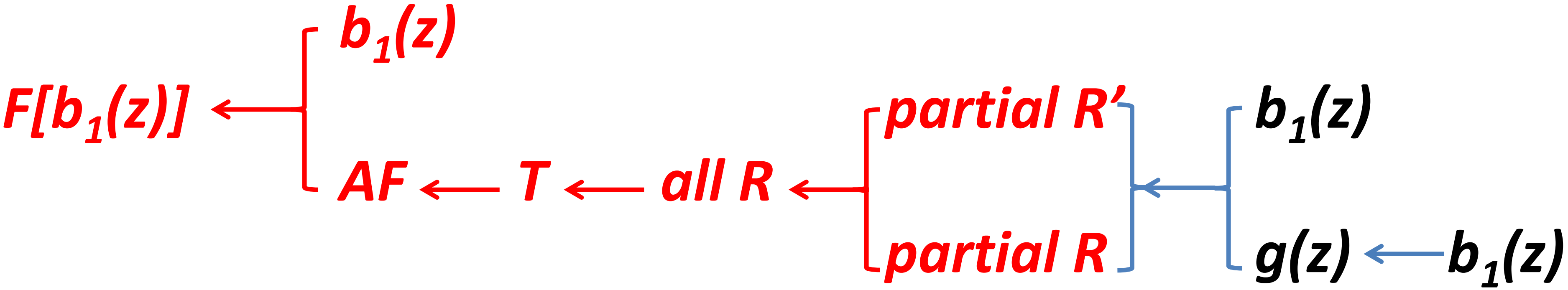
$g(z)$ ← $b_1(z)$

The coefficient of the i^{th} term in $F[b_1(z)]$ is: $\frac{R'_i}{AF_i}$

$$\frac{R'_i}{AF_i} = \frac{R'_i}{(T_{01}T_{10})^2(T_{12}T_{21})^2\cdots(T_{i-2,i-1}T_{i-1,i-2})^2(T_{i-1,i}T_{i,i-1})}$$

$$= \frac{R'_i}{(1-R_1^2)^2(1-R_2^2)^2\cdots(1-R_{i-1}^2)^2(1-R_i^2)}$$

$$= \frac{R'_i}{(1-R_1R_1-R'_2R_2-\cdots-R'_{i-1}R_{i-1})^2(1-R_i^2)}$$



$F[b_1(z)]$ ← $b_1(z)$, AF

AF ← T ← all R ← partial R', partial R

partial R', partial R ← $b_1(z)$, $g(z)$

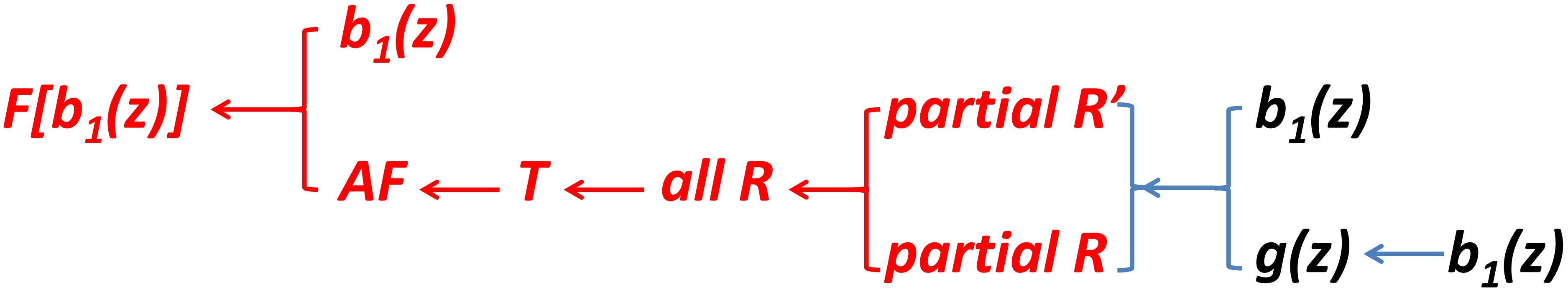
$g(z)$ ← $b_1(z)$

The coefficient of the ith term in $F[b_1(z)]$ is: $\frac{R'_i}{AF_i}$

$$
\begin{aligned}
\frac{R'_i}{AF_i} &= \frac{R'_i}{(T_{01}T_{10})^2(T_{12}T_{21})^2\cdots(T_{i-2,i-1}T_{i-1,i-2})^2(T_{i-1,i}T_{i,i-1})} \\
&= \frac{R'_i}{(1-R_1^2)^2(1-R_2^2)^2\cdots(1-R_{i-1}^2)^2(1-R_i^2)} \\
&= \frac{R'_i}{(1-R_1R_1-R'_2R_2-\cdots-R'_{i-1}R_{i-1})^2(1-R_i^2)}
\end{aligned}
$$

In the derivation, I used the expression:

$$
\begin{aligned}
&(1-R_1^2)(1-R_2^2)\cdots(1-R_{i-2}^2)(1-R_{i-1}^2) \\
=&\,1-R_1R_1-R'_2R_2-\cdots-R'_{i-1}R_{i-1}
\end{aligned}
$$


$F[b_1(z)]$

$b_1(z)$

AF

T

all R

partial R'

partial R

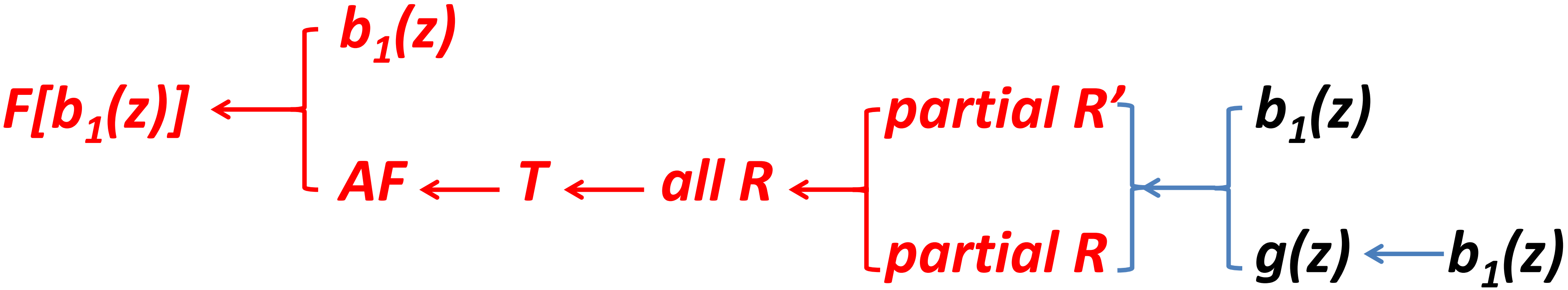
$b_1(z)$

$g(z)$

$b_1(z)$

The coefficient of the i^{th} term in $F[b_1(z)]$ is: $\frac{R'_i}{AF_i}$

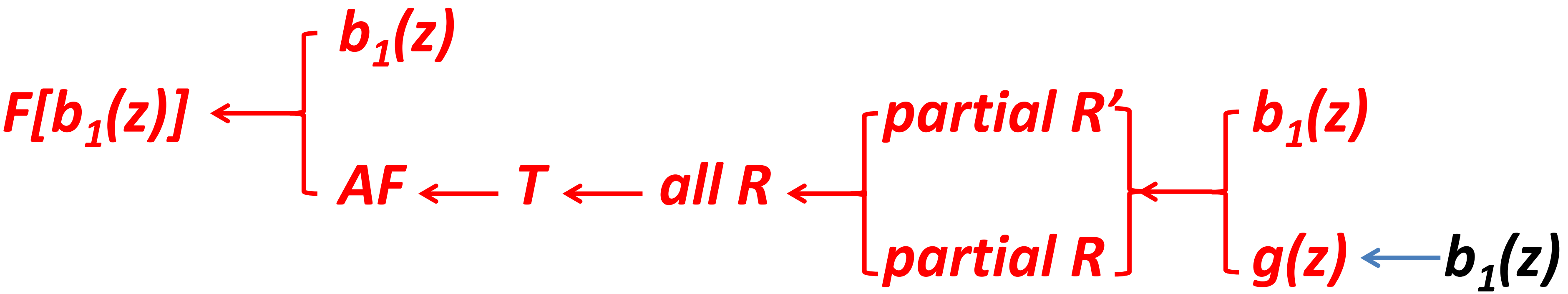
$$\frac{R'_i}{AF_i} = \frac{R'_i}{(1 - R_1R_1 - R'_2R_2 - \cdots - R'_{i-1}R_{i-1})^2(1 - R_i^2)}$$



The coefficient of the i^{th} term in $F[b_1(z)]$ is: $\frac{R'_i}{AF_i}$

$$\frac{R'_i}{AF_i} = \frac{R'_i}{(1 - R_1R_1 - R'_2R_2 - \cdots - R'_{i-1}R_{i-1})^2(1 - R_i^2)}$$

Expressions we will use		The coefficient of the i^{th} term
$(\int_{z-\varepsilon}^{z+\varepsilon} dz' g(z'))^2$	$\longrightarrow$	R_i^2
$\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')$	$\longrightarrow$	$R_1R_1 + R'_2R_2 + \cdots + R'_{i-1}R_{i-1}$



F[$b_1(z)$] ← $b_1(z)$, AF

AF ← T ← all R ← partial R′, partial R

partial R′, partial R ← $b_1(z)$, g(z)

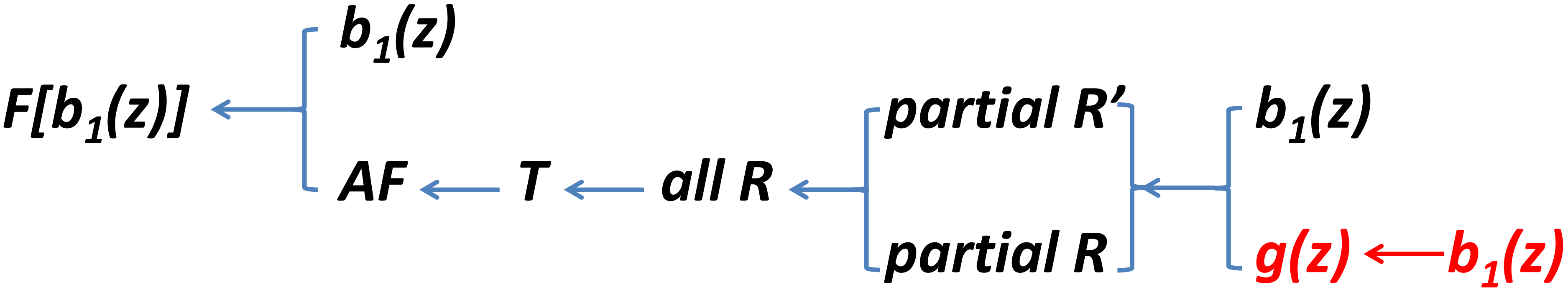
g(z) ← $b_1(z)$

The coefficient of the ith term in F[b_1(z)] is: $\frac{R'_i}{AF_i}$

$$\frac{R'_i}{AF_i} = \frac{R'_i}{(1-R_1R_1-R'_2R_2-\cdots-R'_{i-1}R_{i-1})^2(1-R_i^2)}$$

Expressions we will use		The coefficient of the ith term
$(\int_{z-\varepsilon}^{z+\varepsilon} dz' g(z'))^2$	→	R_i^2
$\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')$	→	$R_1R_1+R'_2R_2+\cdots+R'_{i-1}R_{i-1}$

$$\boxed{F[b_1(z)] = \frac{b_1(z)}{[1-(\int_{z-\varepsilon}^{z+\varepsilon} dz' g(z'))^2][1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')]^2}}$$



$F[b_1(z)]$

$b_1(z)$

AF

T

all R

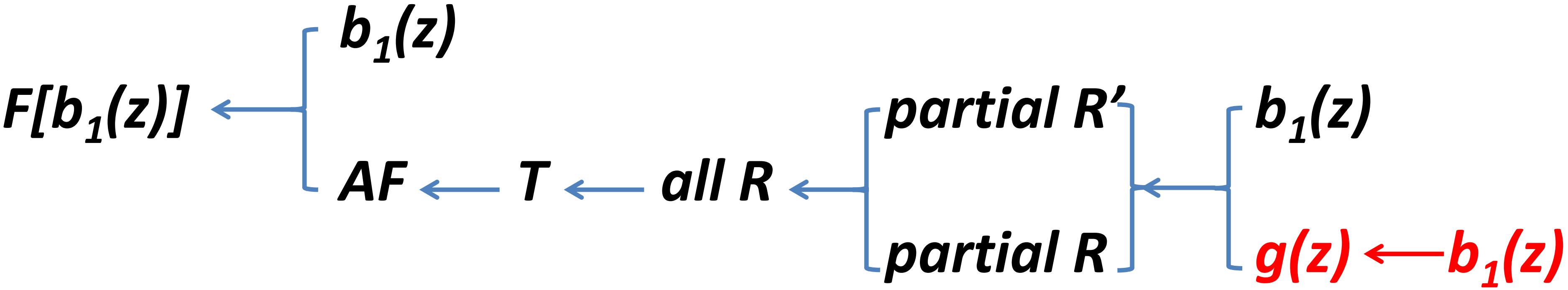
partial R'

partial R

$b_1(z)$

$g(z)$

$b_1(z)$



$F[b_1(z)]$

$b_1(z)$

AF

T

all R

partial R'

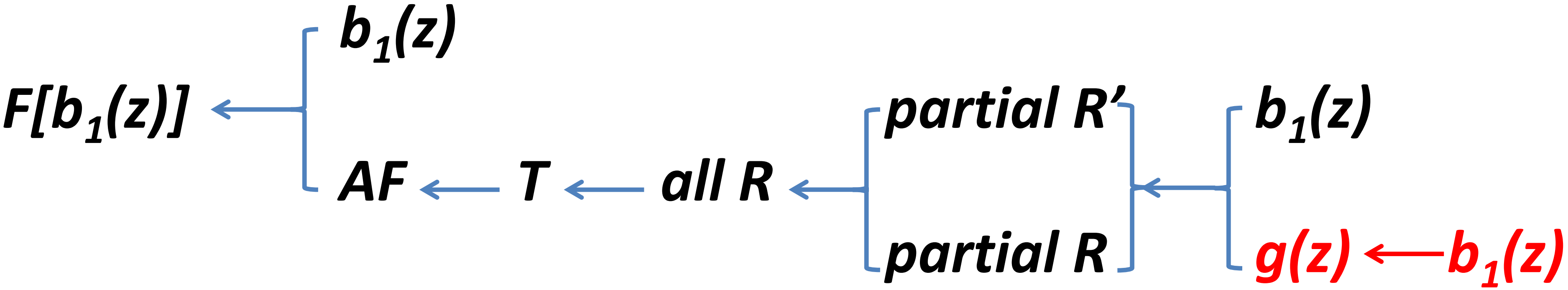
partial R

$b_1(z)$

$g(z)$

$b_1(z)$

The coefficient of the ith term of *g(z) is:* R_i



F[b$_1$(z)]

b$_1$(z)

AF ← ***T*** ← ***all R***

partial R'

partial R

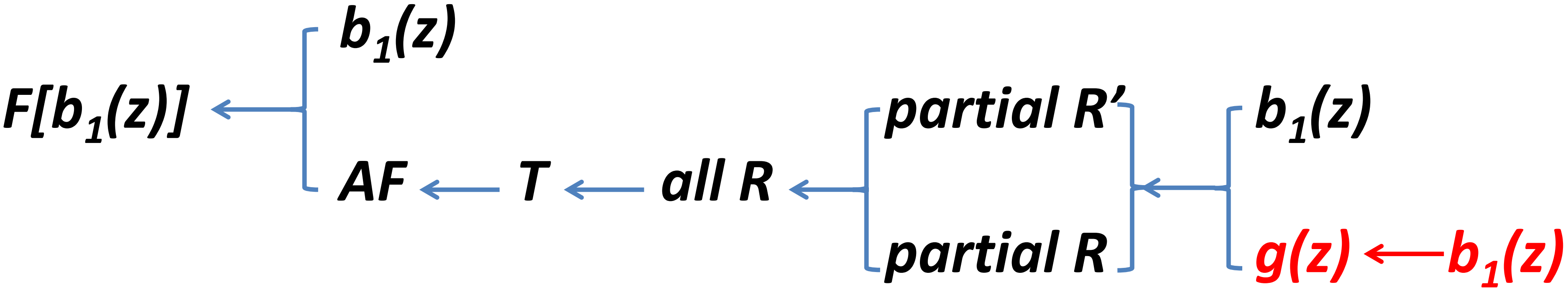
b$_1$(z)

g(z) ← ***b$_1$(z)***

The coefficient of the ith term of *g(z) is:* R_i

$$R_i = \frac{R'_i}{(1-R_1^2)(1-R_2^2)\cdots(1-R_{i-2}^2)(1-R_{i-1}^2)}$$

$$= \frac{R'_i}{1 - R_1R_1 - R'_2R_2 - \cdots - R'_{i-1}R_{i-1}}$$



$F[b_1(z)]$ ← ***$b_1(z)$***, ***AF***

AF ← ***T*** ← ***all R*** ← ***partial R'***, ***partial R***

partial R', ***partial R*** ← ***$b_1(z)$***, ***$g(z)$*** ← ***$b_1(z)$***

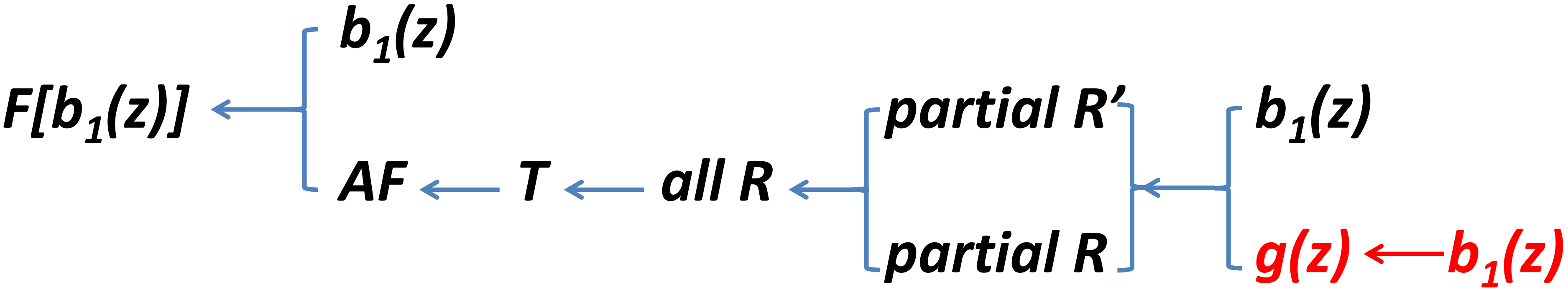
The coefficient of the i^th^ term of *g(z) is:* R_i

$$\begin{aligned} R_i =& \frac{R_i'}{(1-R_1^2)(1-R_2^2)\cdots(1-R_{i-2}^2)(1-R_{i-1}^2)} \\ =& \frac{R_i'}{1-R_1R_1-R_2'R_2-\cdots-R_{i-1}'R_{i-1}} \end{aligned}$$

Expression we will use

The coefficient of the i^th^ term

$$\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'') \longrightarrow R_1R_1 + R_2'R_2 + \cdots + R_{i-1}'R_{i-1}$$



F[b₁(z)] ← **b₁(z)**, **AF** ← **T** ← **all R** ← **partial R'**, **partial R** ← **b₁(z)**, **g(z)** ← **b₁(z)**

The coefficient of the i^th term of *g(z) is:* R_i

$$R_i = \frac{R'_i}{(1-R_1^2)(1-R_2^2)\cdots(1-R_{i-2}^2)(1-R_{i-1}^2)}$$

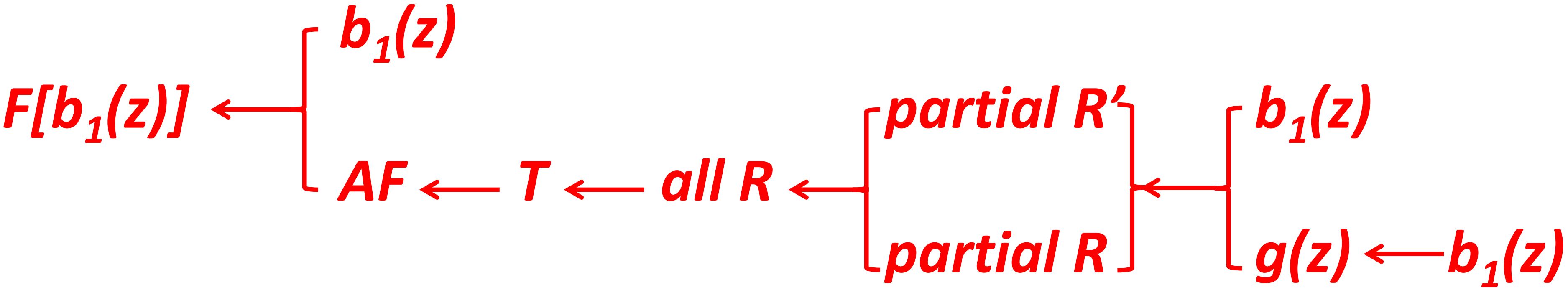
$$= \frac{R'_i}{1 - R_1R_1 - R'_2R_2 - \cdots - R'_{i-1}R_{i-1}}$$

Expression we will use

The coefficient of the i^th term

$$\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'') \longrightarrow R_1R_1 + R'_2R_2 + \cdots + R'_{i-1}R_{i-1}$$

$$\boxed{g(z) = \frac{b_1(z)}{1 - \int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')}}$$



$b_1(z)$

$F[b_1(z)]$

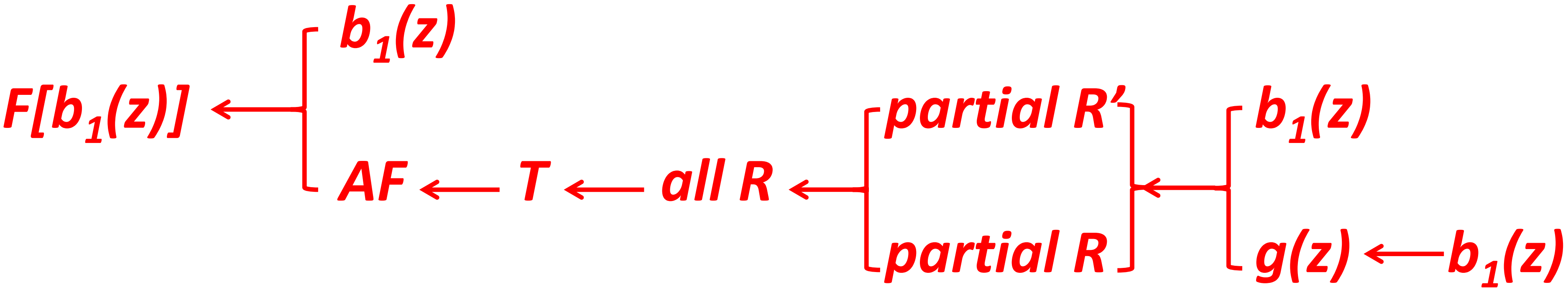
$AF \leftarrow T \leftarrow \text{all } R$

partial R'

partial R

$b_1(z)$

$g(z) \leftarrow b_1(z)$



$b_1(z)$

$F[b_1(z)]$ ←

partial $R’$

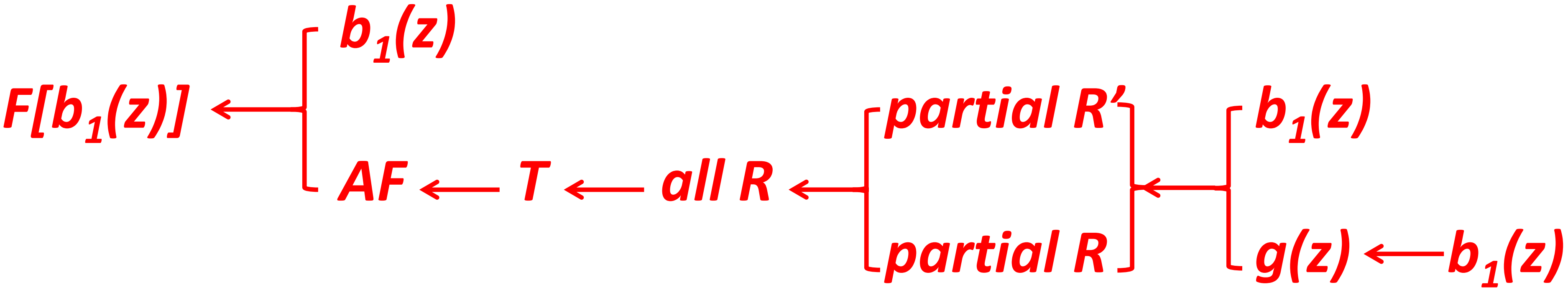
$b_1(z)$

AF ← T ← all R ←

partial R

$g(z)$ ← $b_1(z)$

$$\boxed{F[b_1(z)] \longleftarrow b_1(z)}$$



$$\boxed{F[b_1(z)] \longleftarrow b_1(z)}$$

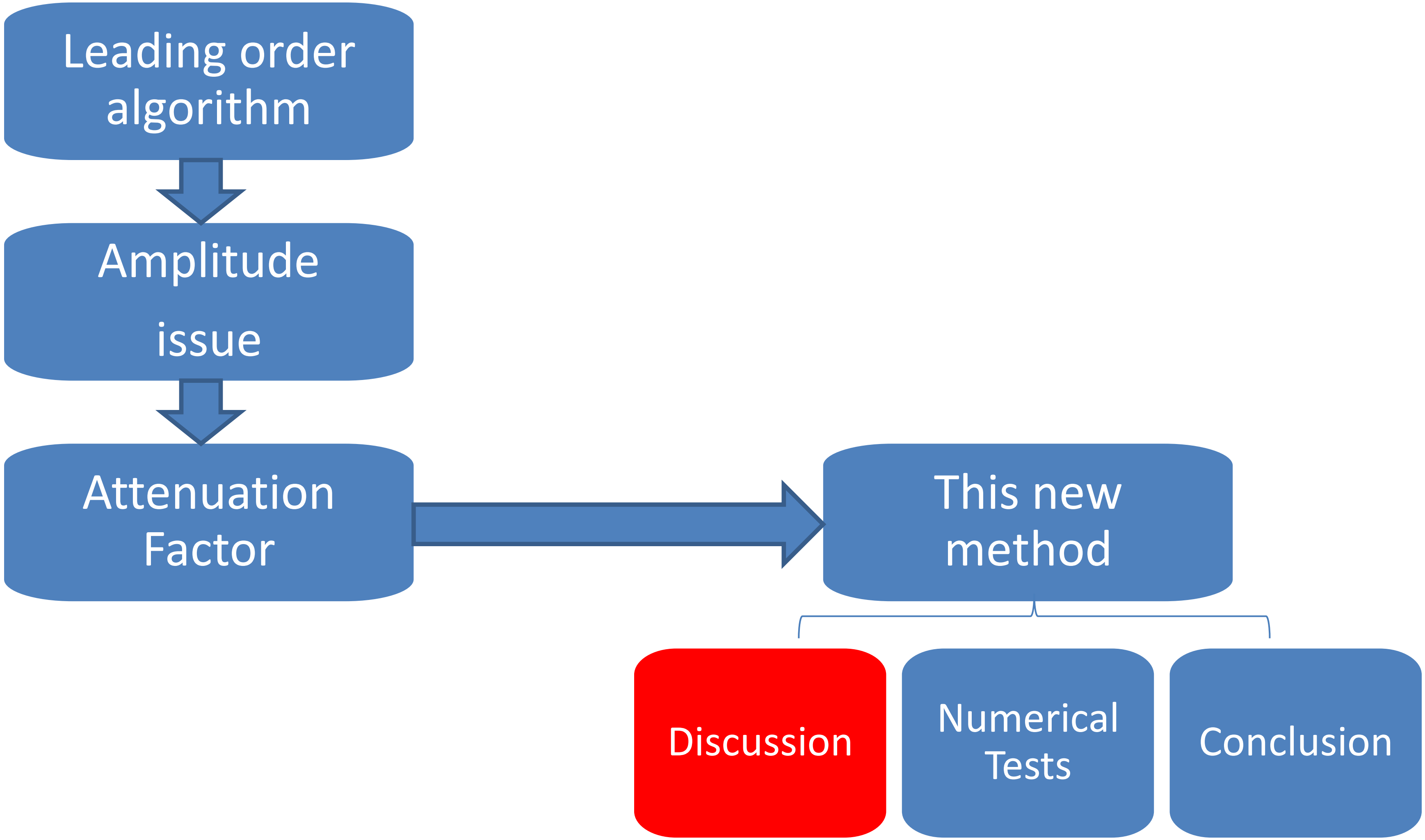
$$b_E^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

$$F[b_1(z)] = \frac{b_1(z)}{[1-(\int_{z-\varepsilon}^{z+\varepsilon} dz' g(z'))^2][1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')]^2} \tag{5}$$

$$g(z) = \frac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(z'')} \tag{6}$$

First type approximation for equation(6)	$g(z)=\dfrac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')}$ $\approx\dfrac{b_1(z)}{1-0}$ $\approx b_1(z)$
Second type approximation for equation(6)	$g(z)=\dfrac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')}$ $\approx\dfrac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''b_1(z'')}$
Third type approximation for equation(6)	$g(z)=\dfrac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''g(z'')}$ $\approx\dfrac{b_1(z)}{1-\int_{-\infty}^{z-\varepsilon}dz'b_1(z')\int_{z'-\varepsilon}^{z'+\varepsilon}dz''\dfrac{b_1(z'')}{1-\int_{-\infty}^{z''-\varepsilon}dz'''b_1(z''')\int_{z'''-\varepsilon}^{z'''+\varepsilon}dz^{(4)}b_1(z^{(4)})}}$

The structure of this presentation



Leading order algorithm

Amplitude issue

Attenuation Factor

This new method

Discussion

Numerical Tests

Conclusion

Discussion

This method considers only primaries as the input.

Discussion

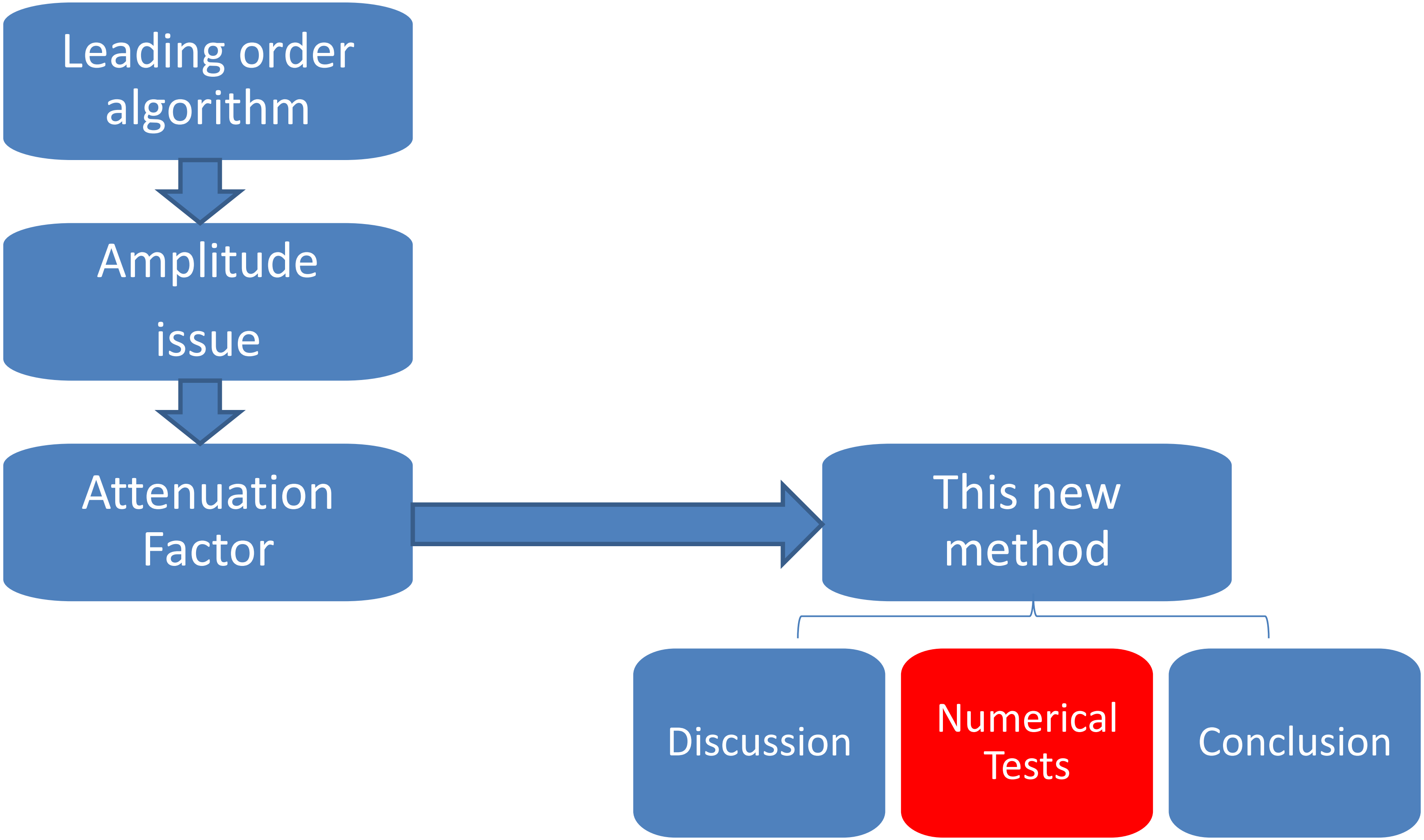
The input data contains both primaries and internal multiples.

<table>
<tr><td>First type approximation</td><td colspan="2">all $IM_{j=1}$: correct
all $IM_{j>1}$: more accurate than internal multiple attenuator</td></tr>
<tr><td>Second type approximation</td><td colspan="2">all $IM_{j=1}$ and $IM_{j=2}$: correct
all $IM_{j>2}$: more accurate than the first type</td></tr>
<tr><td rowspan="2">Third type approximation</td><td>Internal multiples arrive after the 3rd primary</td><td>all $IM_{j=1}$, all $IM_{j=2}$ and all $IM_{j=3}$:correct
all $IM_{j>3}$: more accurate than the second type</td></tr>
<tr><td>Internal multiples arrive before the 3rd primary</td><td>all $IM_{j=1}$ and all $IM_{j=2}$:correct
all $IM_{j>2}$: more accurate than the second type</td></tr>
<tr><td>…</td><td colspan="2">…</td></tr>
</table>

Discussion

To deal with this problem, we can first run the internal multiple attenuation algorithm, then attenuate the amplitude of internal multiples in the data and then run this method using the new data to eliminate all first order internal multiples.

The structure of this presentation



Leading order algorithm

Amplitude

issue

Attenuation Factor

This new method

Discussion

Numerical Tests

Conclusion

In this section we test 3 different equations under 1D normal incidence:

(1)internal multiple attenuator

(2)First type of approximation of the new method.

(3)Second type of approximation of the new method.

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

$$b_E^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'')$$

$$\begin{cases} F[b_1(z)]_{1T} = \dfrac{b_1(z)}{1-(\int_{z-\varepsilon}^{z+\varepsilon} dz' b_1(z'))^2} \\ \\ F[b_1(z)]_{2T} = \dfrac{b_1(z)}{[1-(\frac{\int_{z-\varepsilon}^{z+\varepsilon} dz' b_1(z')}{1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' b_1(z'')})^2][1-\int_{-\infty}^{z-\varepsilon} dz' b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' b_1(z'')]} \end{cases}$$

Model

V=1500m/s ρ=1.0g/cm^3

500m

V=1700m/s ρ=1.8g/cm^3

1700m

V=1700m/s ρ=1.0g/cm^3

2700m

V=3500m/s ρ=4.0g/cm^3

5700m

V=5000m/s ρ=4.0g/cm^3

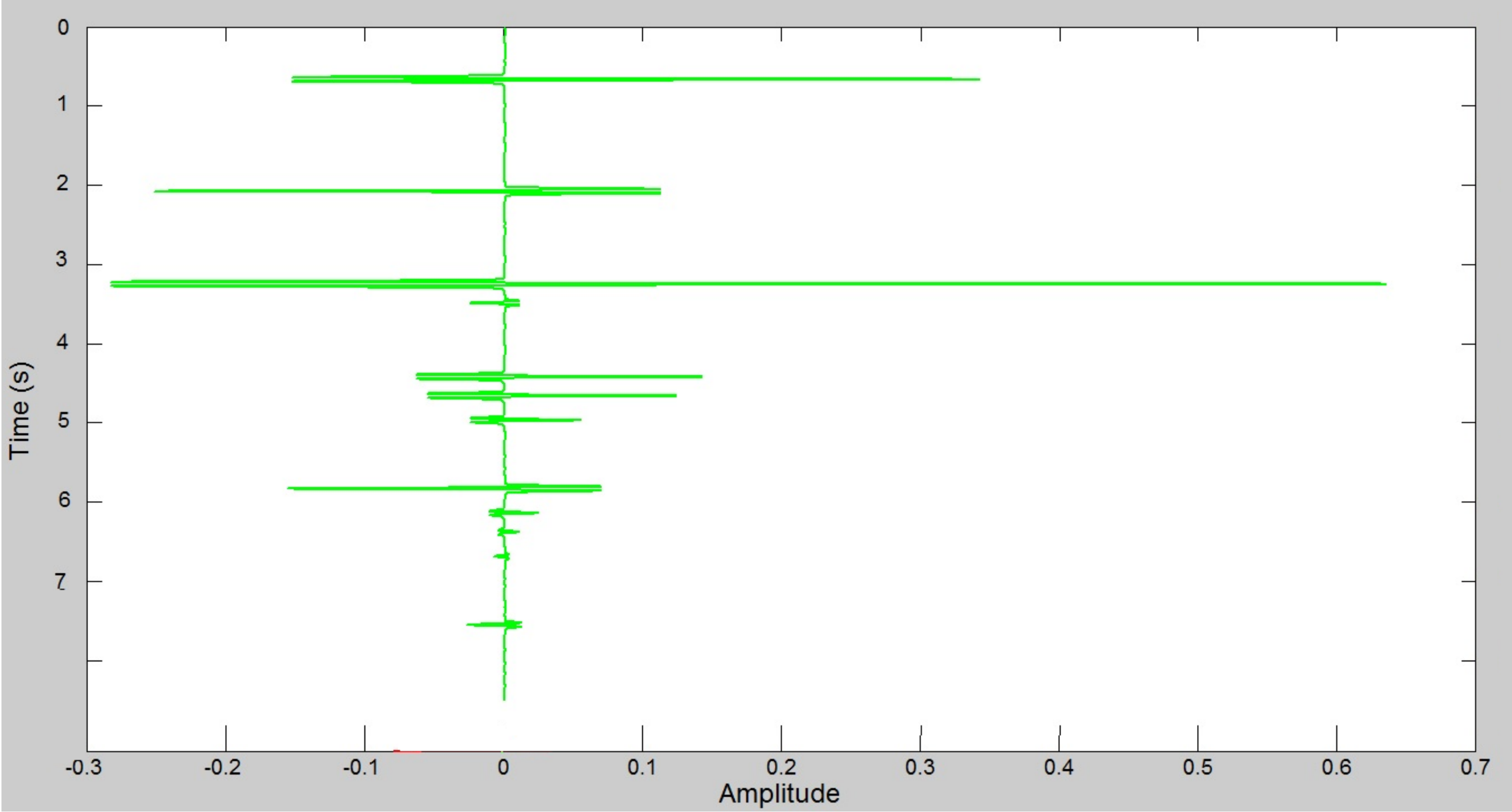
Initial Tests

A. Test for perfect data.
B. Test for bandlimited data.
C. Test for data with white noise.

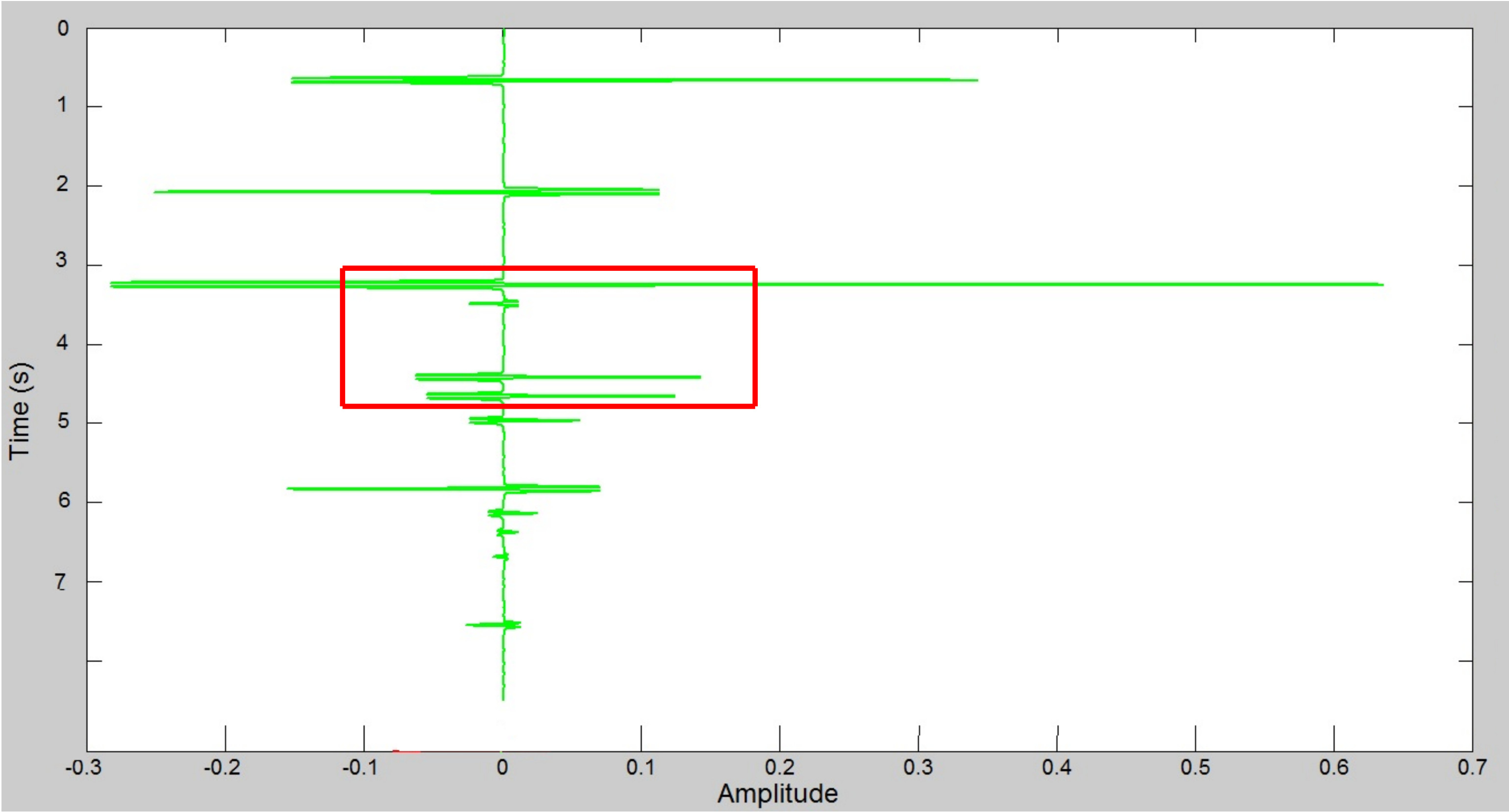
Initial Tests

A. Test for perfect data.
B. Test for bandlimited data.
C. Test for data with white noise.

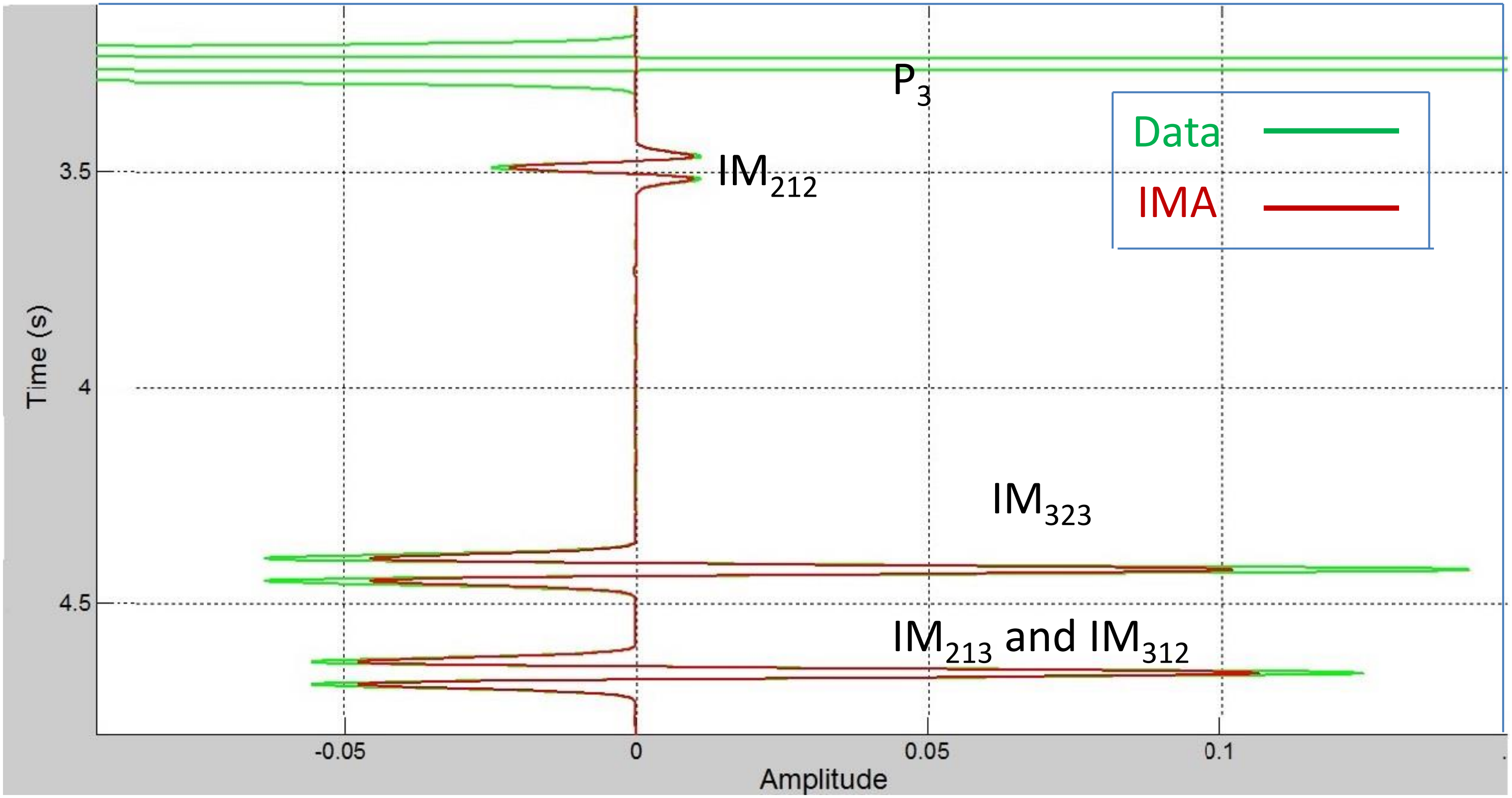
The input data(1D normal incidence)

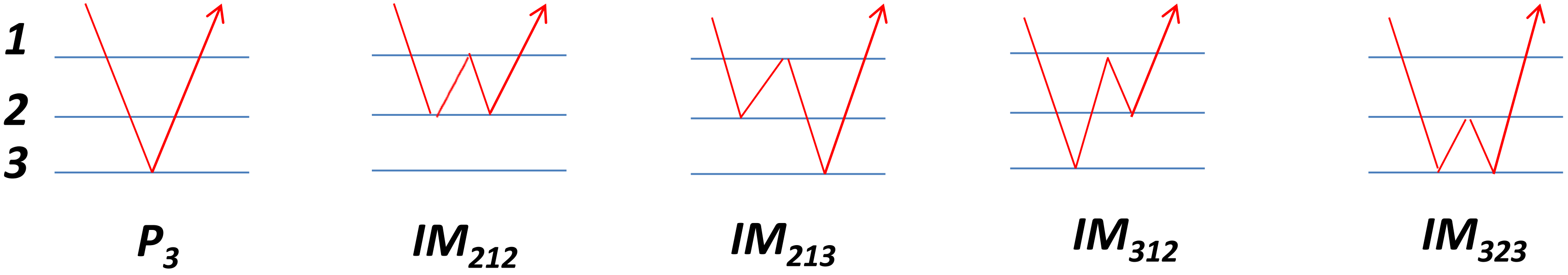


The input data(1D normal incidence)

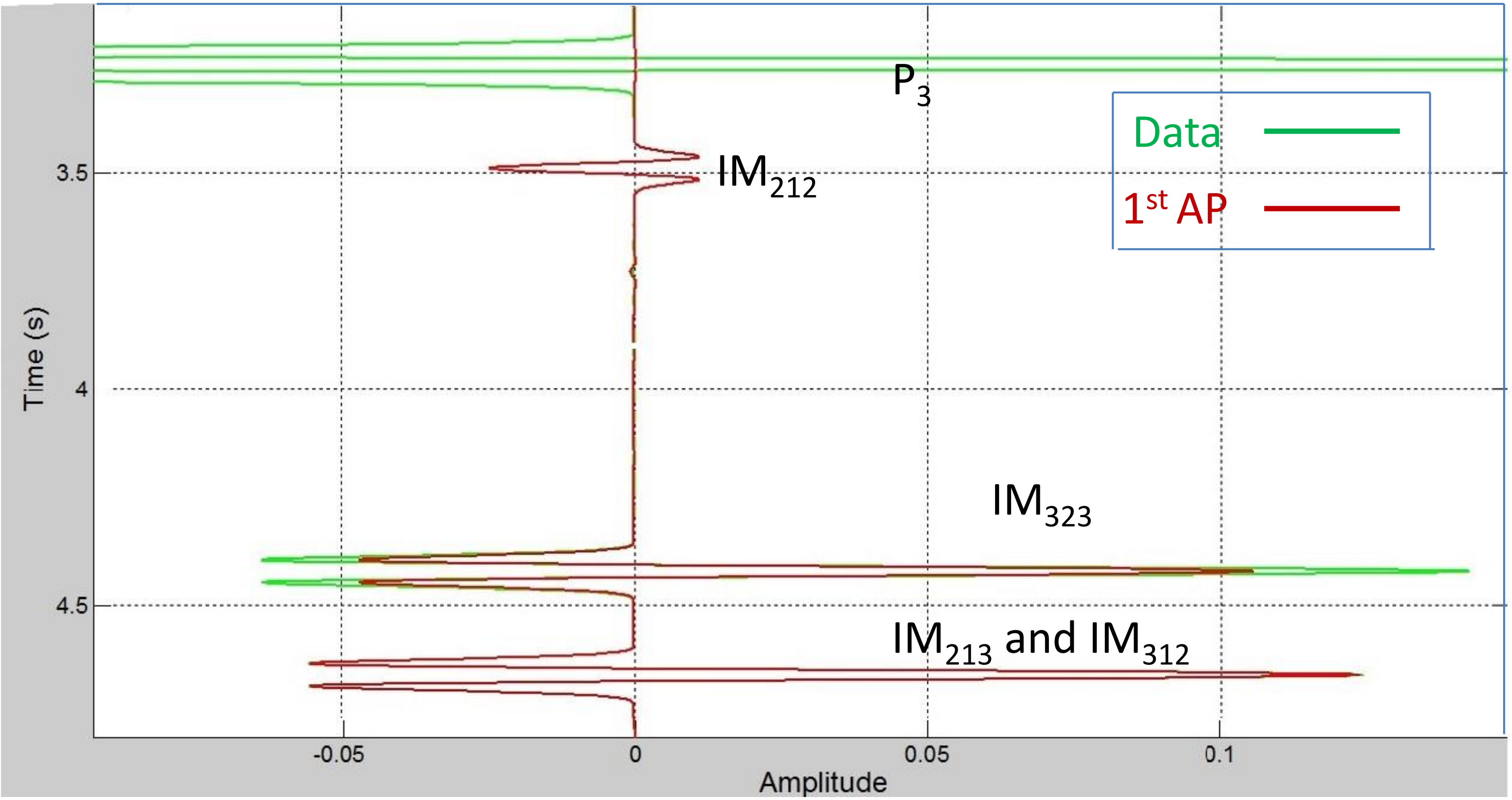


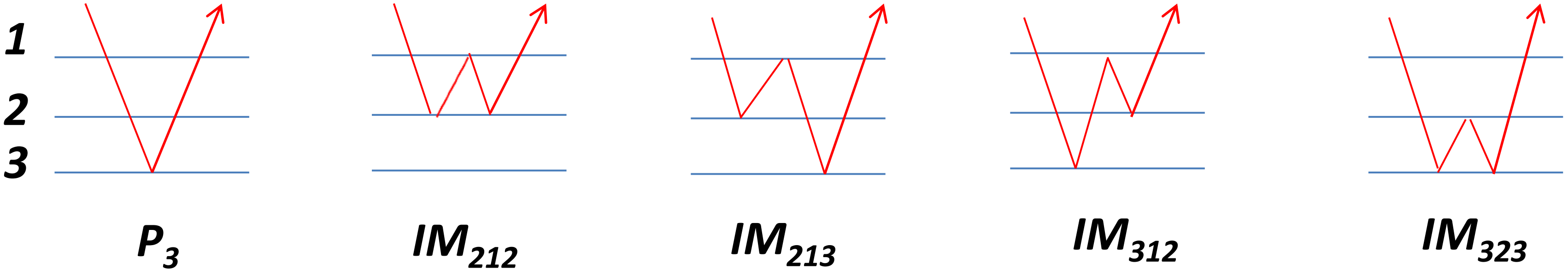
Internal multiple attenuator



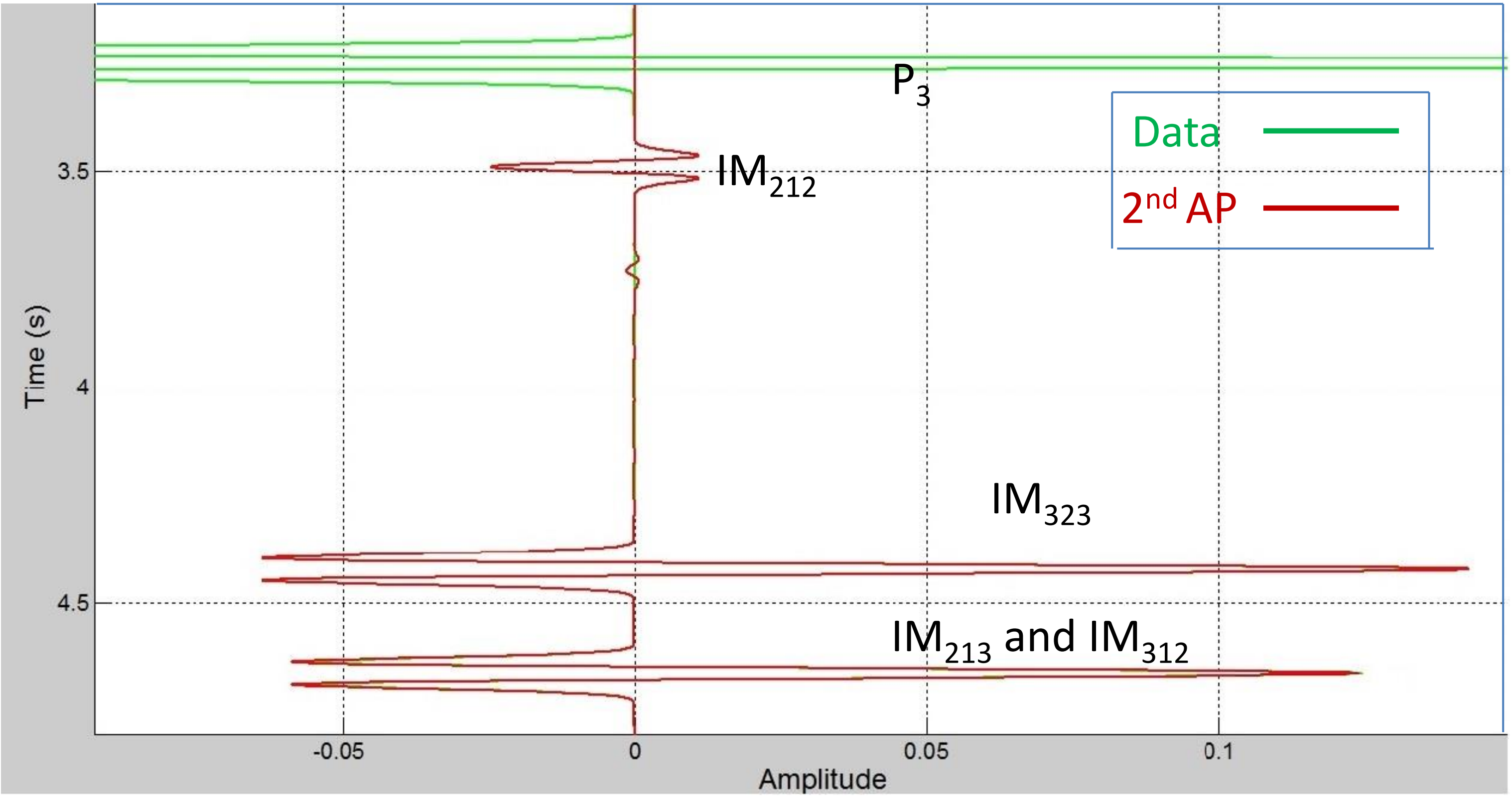


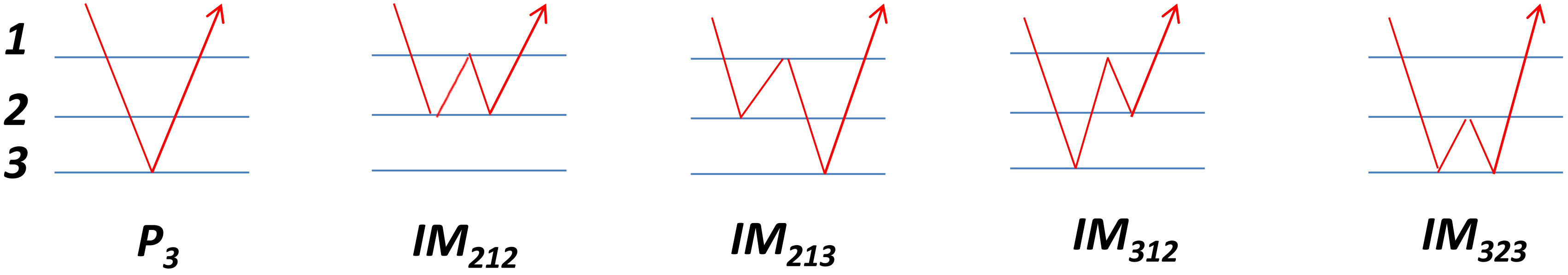
First type of equation approximation





Second type of equation approximation





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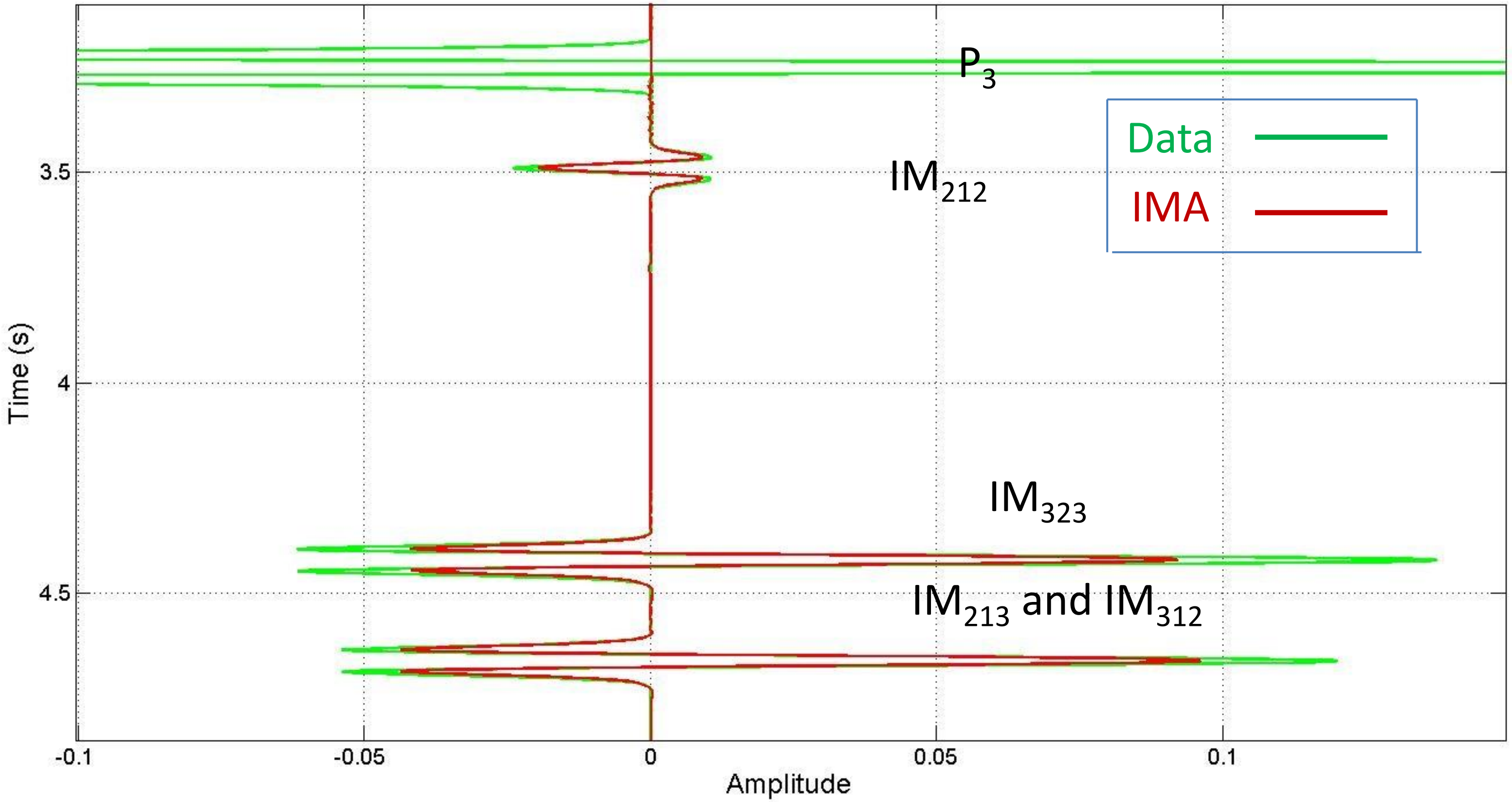
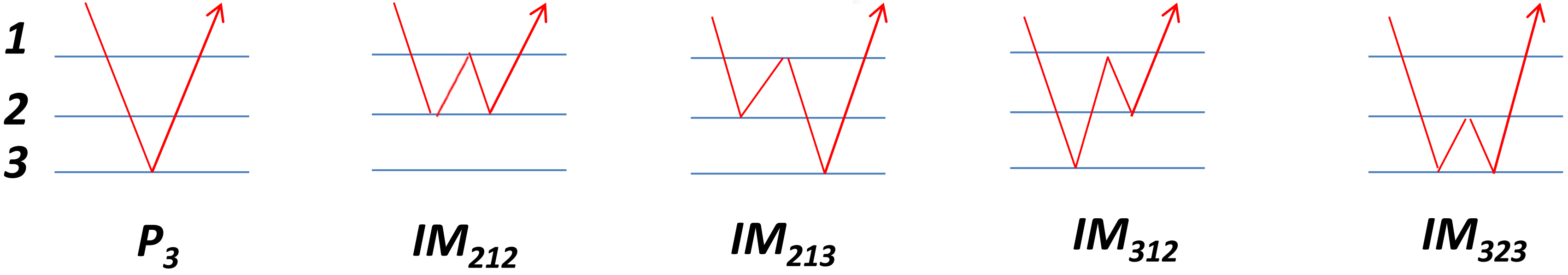
Internal multiple attenuator





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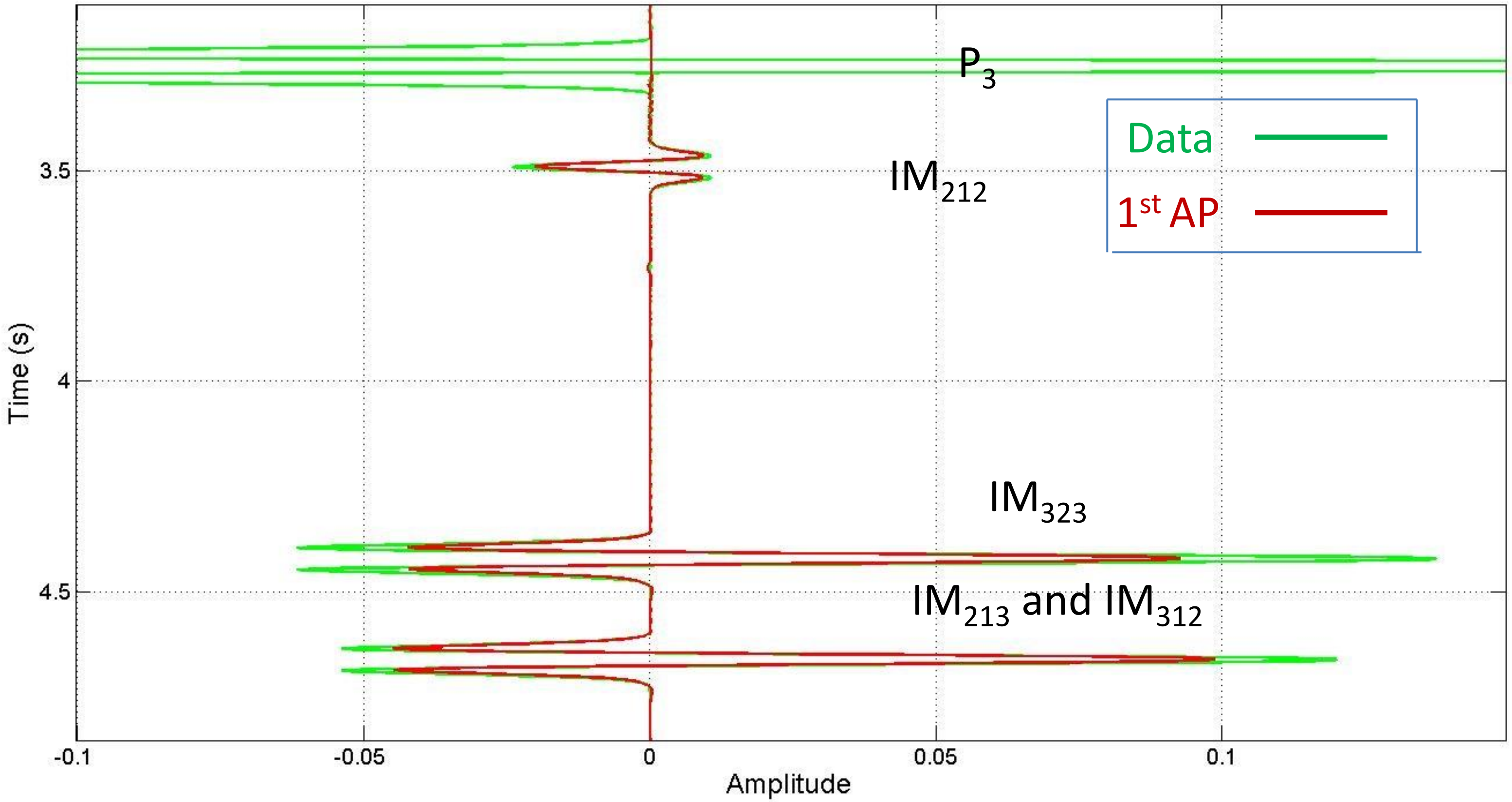
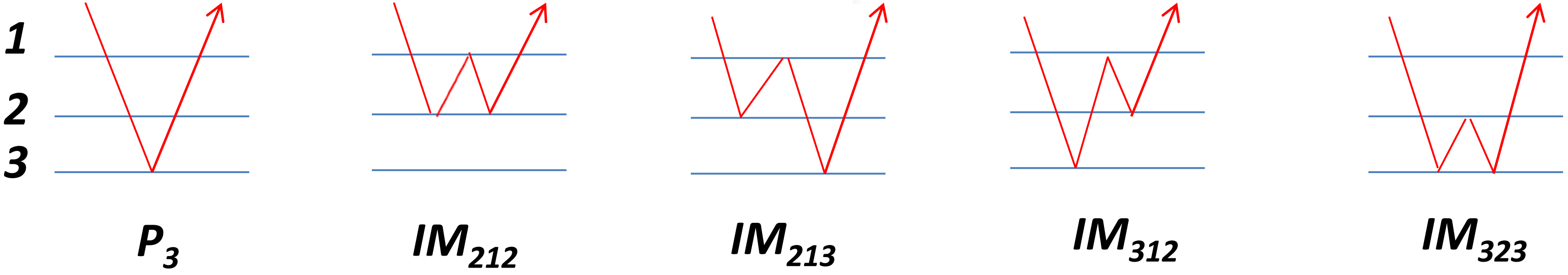
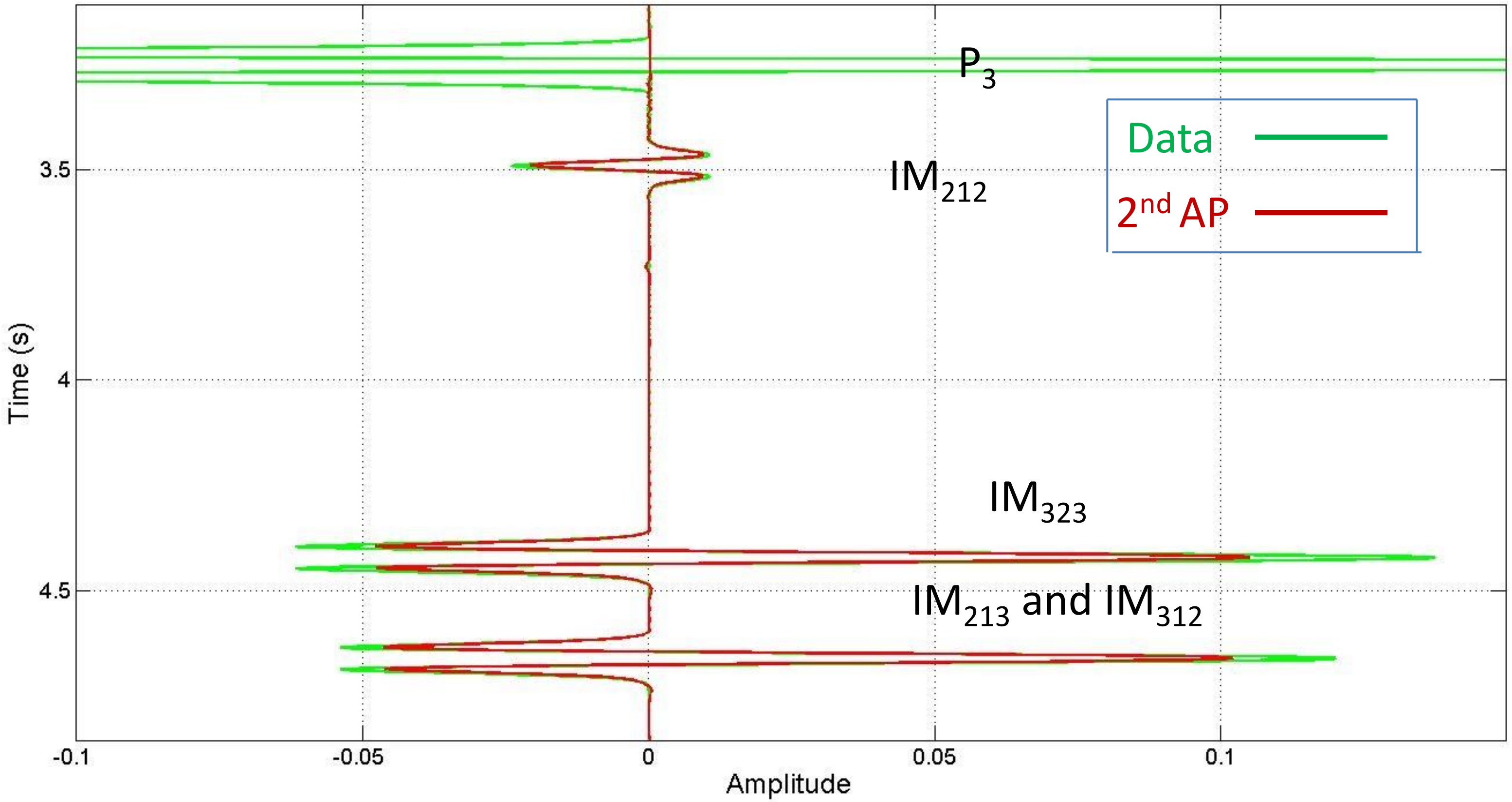
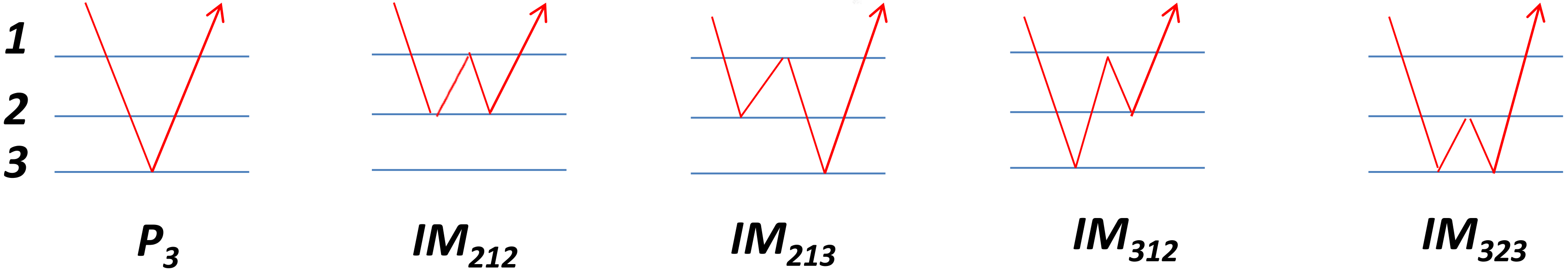




Second type of equation approximation





Initial Tests

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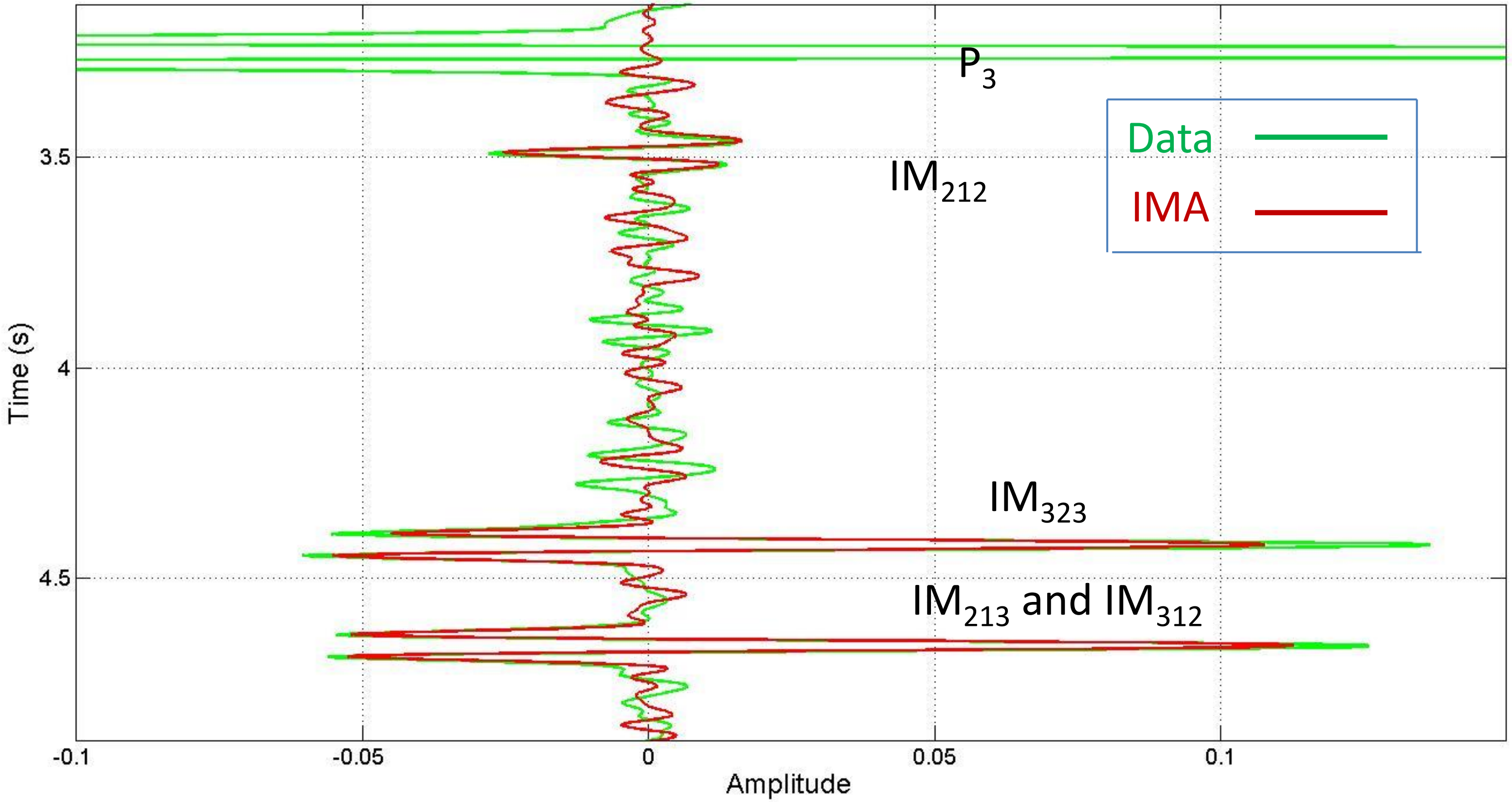
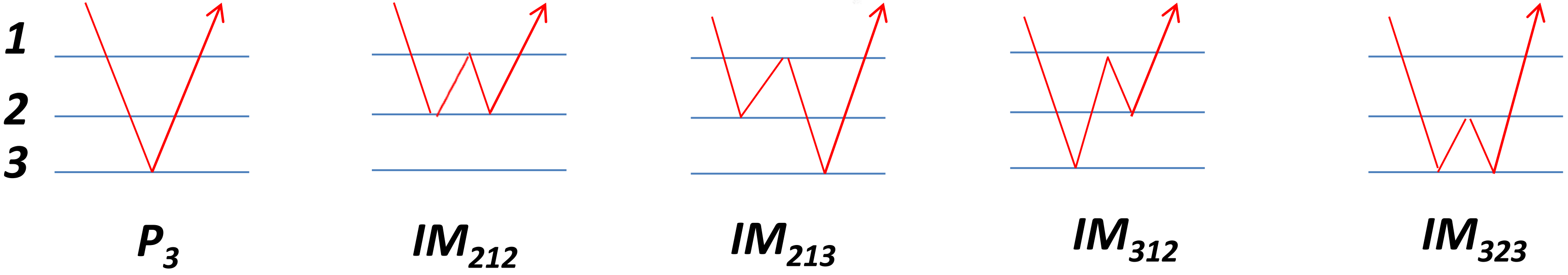
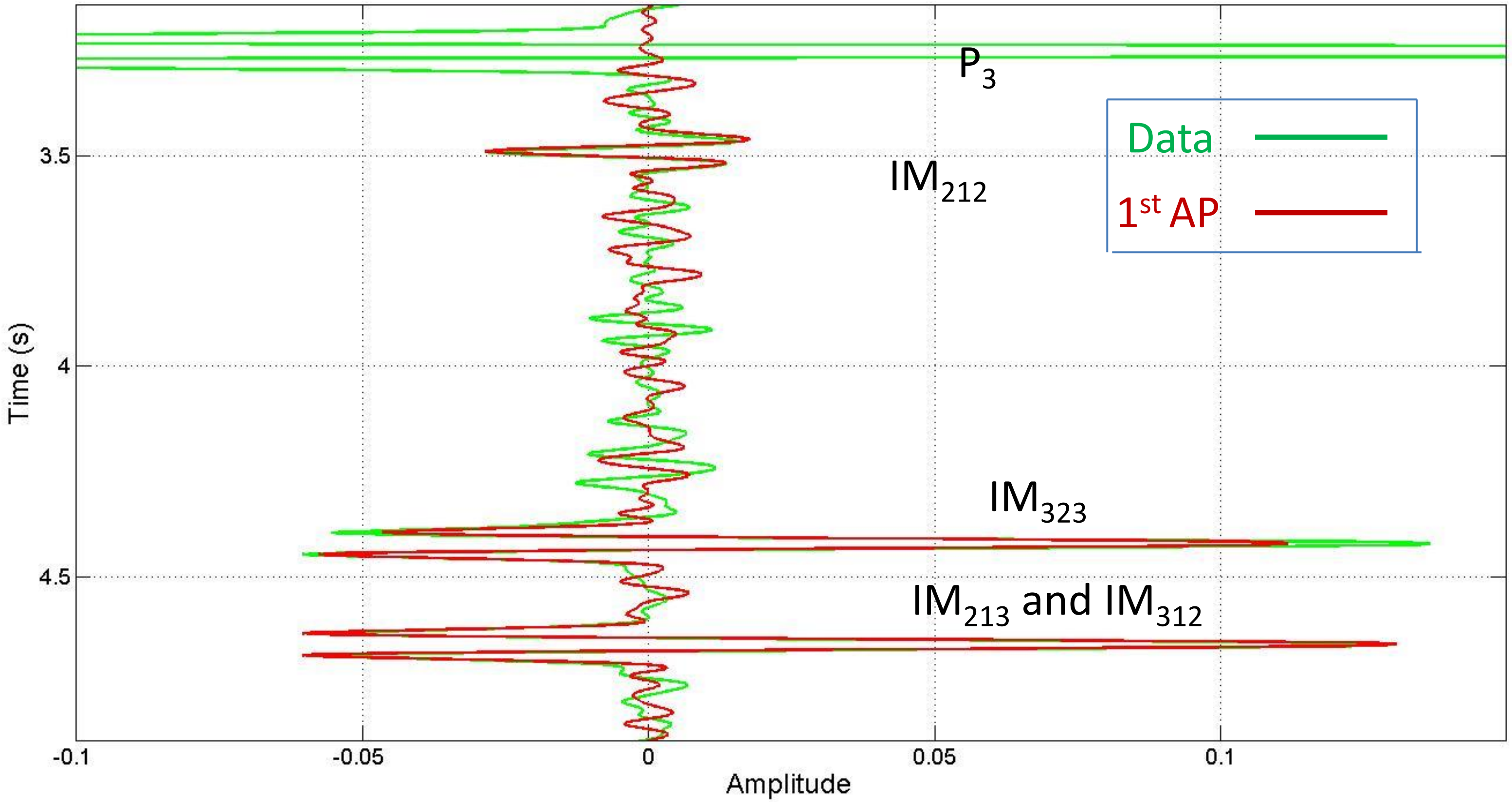
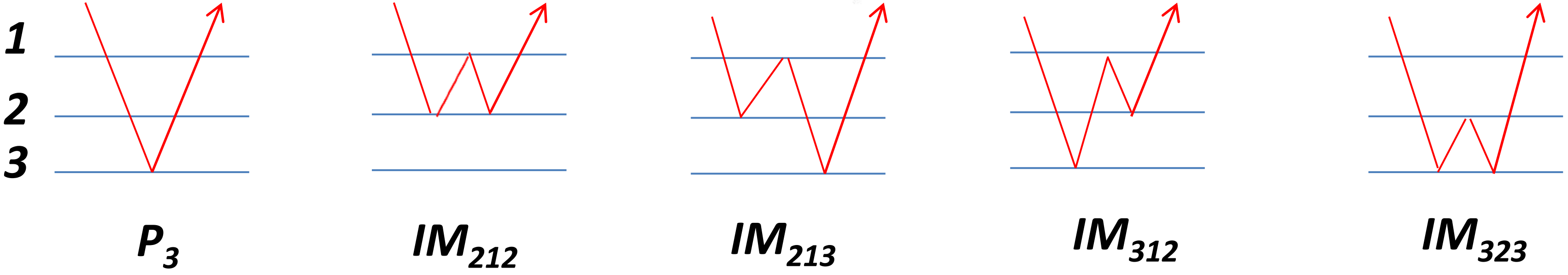
Internal multiple attenuator



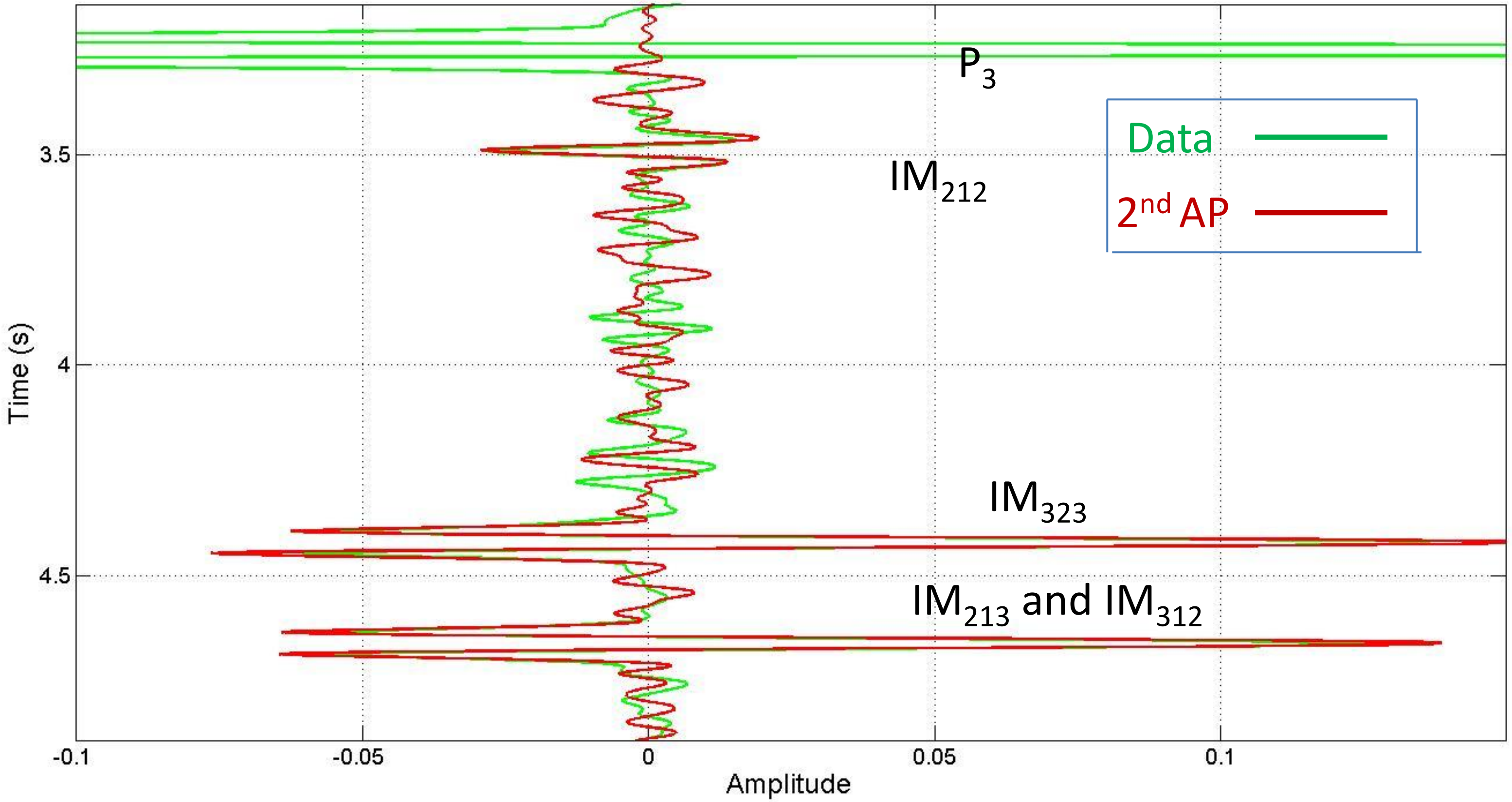


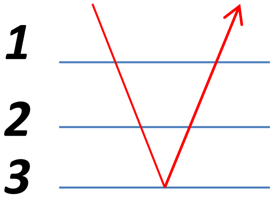
First type of equation approximation



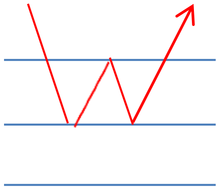


Second type of equation approximation

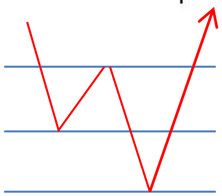




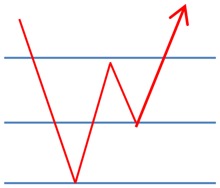
P_3



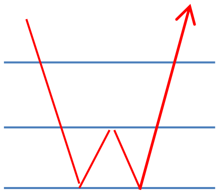
IM_{212}



IM_{213}

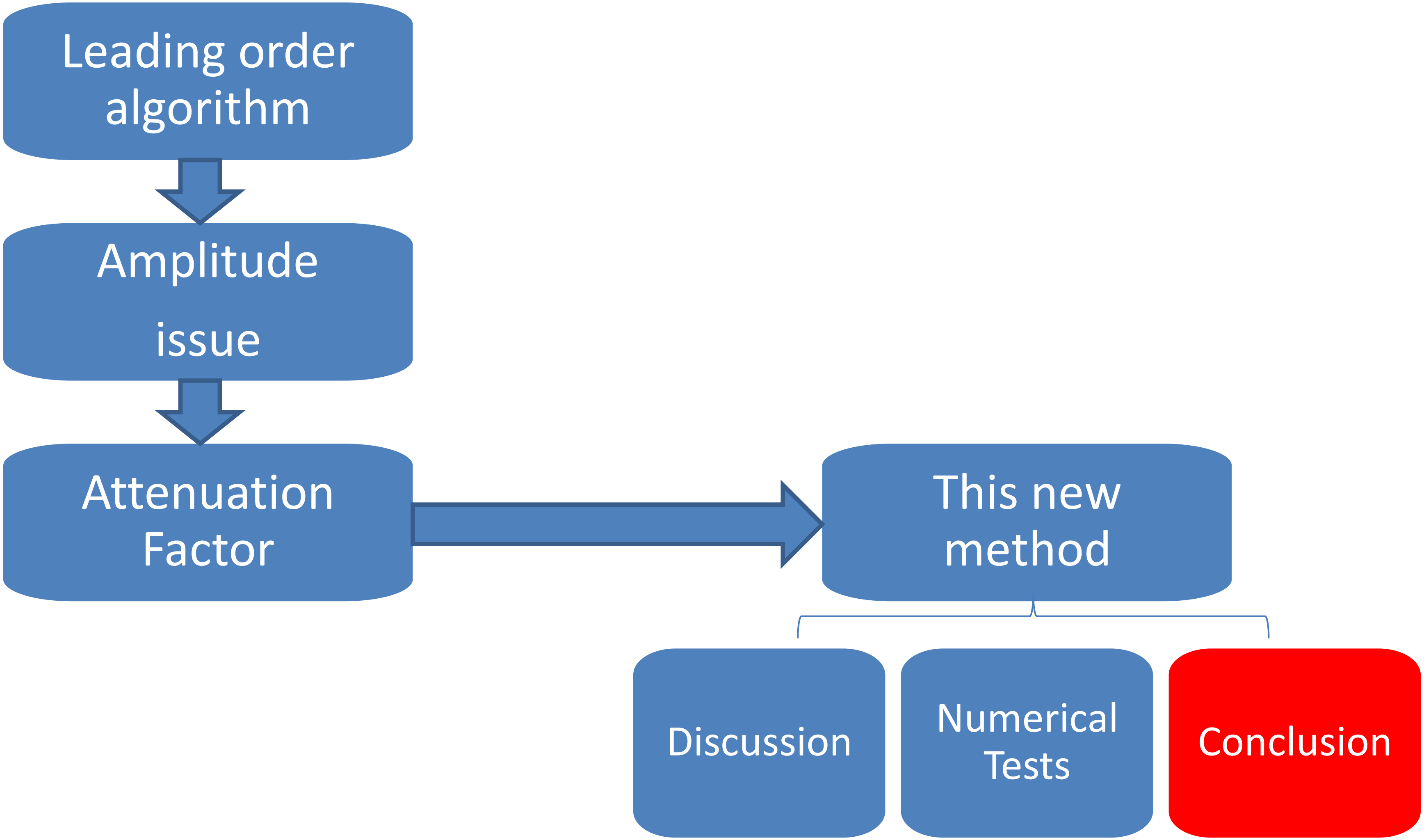


IM_{312}



IM_{323}

The structure of this presentation



Leading order algorithm

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Conclusion

Predicting correct amplitude of internal multiples is an important problem.

1. In this presentation, a new method is given to eliminate all first order internal multiples under 1D normal incidence directly in terms of data without determining the earth.

2. As one of the equations in the method is an integral equation , we can make different types of approximations to it and achieve different levels of delivery by using different orders of approximations.

Conclusion

3. This method considers only primaries in the data ($b_1(z)$).

To address this issue, we can first run the internal multiple attenuation algorithm, then attenuate the amplitude of internal multiples in the data and then run this method using the new data to eliminate all first order internal multiples.

4. From the test we can see, this method is robust to bandwith and noise.

Conclusion

5. This method is a part of a project which is aimed at predicting correct amplitude and time of all internal multiples. This method is a step within seeking this purpose.

The project may be relevant and provide value when primaries and internal multiples interfere with each other in both on-shore and off-shore data.

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