The first wave theory RTM, examples with a layered medium, predicting the source and receiver at depth and then imaging, providing the correct location and reflection amplitude at every depth location, and where the data includes primaries and all internal multiples.

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RTM (Reverse Time Migration) in M-OSRP

- Weglein and Stolt and Mayhan 2011
- PML by Herrera and Weglein.
- We need $D(z_g, z_s)$, $\partial D/\partial z_g$ and $\partial D/\partial z_s$ which can be easily obtained after deghosting.
- Our objective: (1) two-way propagation, (2) complex medium, (3) amplitude.

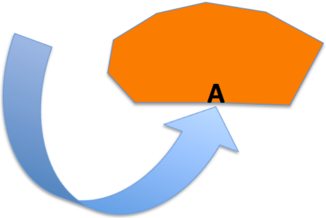
RTM outside M-OSRP

- A cutting edge seismic imaging method first appeared in geophysical literature around 1983.
- The basic and popular idea is to run the finite difference modeling backwards in time.
- Advantage: two- vs one-way propagation.
- Disadvantage: much more time comsuming than one-way procedures.

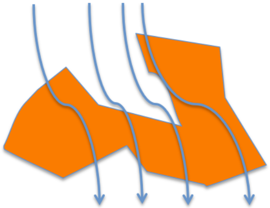
Key contributions

- The wave theory method to calculate G_0^{DN} for arbitrary medium, its finite difference version can be extended to multi-dimension with lateral varying velocity models.
- Incorporating density contribution in the Green's theorem RTM.
- Our two-way method recovered not only the precise location of the subsurface reflector from data include internal multiples, but also its actual amplitude that is precise, clearly defined, and quantatively meaningful.

Asymptotic propagation for simple and complicated geology



Simple geology



Complicated geology

Asymptotic vs wave theory imaging: simple medium

- If the medium is simple enough, asymptotic may be enough for the structural map. The amplitude results, however, may not be sufficient for AVO analysis.

- As demonstrated by numerical example in this presentation, wave theory will give you something in principal to do quantitative interpretation.

Asymptotic vs wave theory imaging: complicated medium

- The industry often prefers wave theory over asymptotic method when we have to get through salt.
- If the medium is complicated, wave theoretical procedure is needed even to achieve an accurate structural map.

Numerical example: Data from a two reflector model

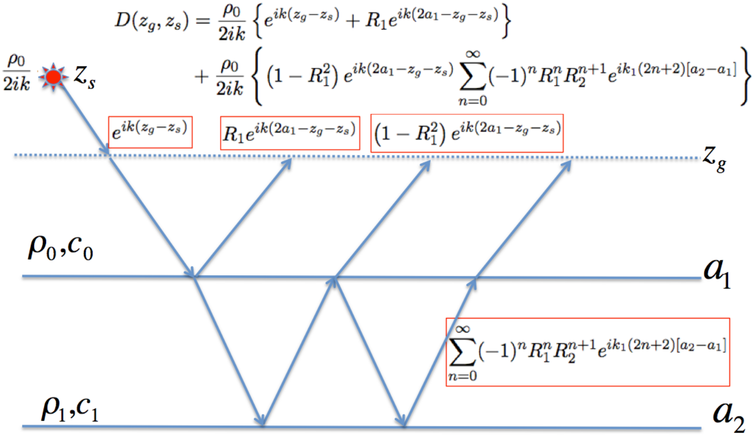


Figure: The input data for a source at z_s, and receiver at z_g, the geological model has two reflectors. We use the following notations: $k = \omega/c_0$, $k_1 = \omega/c_1$, $k_2 = \omega/c_2$. R_1 and R_2 are the reflection coefficients of two reflectors in the model.

Downward continuation above the first reflector

$$E(z,z) = \frac{\rho_0}{2ik}\left\{1 + e^{ik(2a_1-2z)}\left(R_1 + (1-R_1^2)\sum_{n=0}^{\infty}(-1)^n R_1^n R_2^{n+1} e^{ik_1(2n+2)[a_2-a_1]}\right)\right\}$$

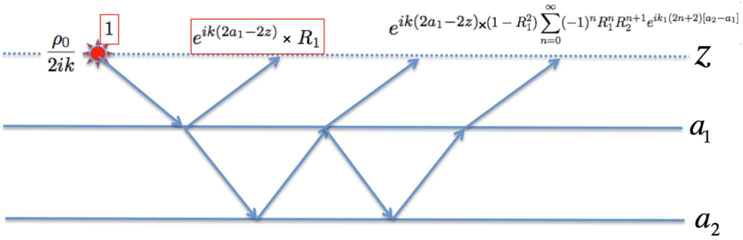


Figure: The downward continuation result for above the first reflector. The history, amplitude and phase of each event in the downward continued result is shown below the formula.

Imaging above the first reflector

Converting the result above to the time domain:

$$E(z,z,t) = -\frac{\rho_0 c_0}{2} \left\{ \begin{array}{l} H(t) + R_1 H\left(t - \tau_1\right) + (1 - R_1^2)\times \\ \sum\limits_{n=0}^{\infty} (-1)^n R_1^n R_2^{n+1} H\left(t - \tau_1 - (n+1)\tau_2\right) \end{array} \right\},$$

where $\tau_1 = \frac{2a_1 - 2z}{c_0}$, $\tau_2 = \frac{2a_2 - 2a_1}{c_1}$. H is the step function. Balancing out the amplitude of the incidence wave (the $\frac{\rho_0 c_0}{-2}$ factor), removing the direct wave $H(t)$, and taking the $t = 0$ imaging condition, we have:

$$\mathcal{D}(z,t) = \begin{cases} 0 & \texttt{if } (z < a_1) \\ R_1 & \texttt{if } (z = a_1) \end{cases}$$

Downward continuation between the first and second reflector

$$E(z,z) = \frac{\rho_1}{2ik_1}\left\{\begin{array}{l} \overset{\text{Event 2}}{R_2 e^{ik_1(2a_2-2z)}} - \overset{\text{Event 6}}{R_1 R_2^2 e^{ik_1(4a_2-2a_1-2z)}} + R_1^2 R_2^3 e^{ik_1(6a_2-4a_1-2z)} + \cdots \\ \\ \underset{\text{Event 1}}{-R_1 e^{ik_1(2z-2a_1)}} + \underset{\text{Event 5}}{R_1^2 R_2 e^{ik_1(2z+2a_2-4a_1)}} - R_1^3 R_2^2 e^{ik_1(2z+4a_2-6z_1)} + \cdots \\ \\ +1 + \underset{\text{Event 3 and 4}}{2\left[-R_1 R_2 e^{ik_1(2a_2-2a_1)}\right]^1} + \underset{\text{Event 7 and 8}}{2\left[-R_1 R_2 e^{ik_1(2a_2-2a_1)}\right]^2} + \cdots \end{array}\right\}$$

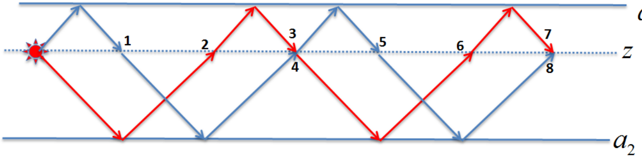


Figure: The downward continuation result between the first and second reflector. The history, amplitude and phase of each event in the downward continued result is shown below the formula.

Imaging between the first and second reflector

Converting the result above to the time domain,

$$E(z,z,t) = -\frac{\rho_1 c_1}{2}\left\{\begin{array}{l} H(t) + 2\sum_{n=1}^{\infty}(-1)^n R_1^n R_2^n H\left(t - \frac{2n(a_2-a_1)}{c_1}\right) \\ + \sum_{n=0}^{\infty}(-1)^{n+1} R_1^{n+1} R_2^n H\left(t - \frac{2z+2na_2-2(n+1)a_1}{c_1}\right) \\ + \sum_{n=0}^{\infty}(-1)^n R_1^n R_2^{n+1} H\left(t - \frac{2(n+1)a_2-2na_1-2z}{c_1}\right) \end{array}\right\}$$

Balancing out the amplitude of the incidence wave (the $\frac{\rho_1 c_1}{c_1}$ fator), removing the direct wave, and taking the $t = 0$ imaging condition, we have:

$$\mathcal{D}(z,t) = \begin{cases} -R_1 & \texttt{if } (z = a_1) \\ 0 & \texttt{if } (a_1 < z < a_2) \\ R_2 & \texttt{if } (z = a_2) \end{cases} \tag{1}$$

Downward continuation below the second reflector

$$E(z,z) = \frac{\rho_2}{2ik_2}\left\{1 \overbrace{- R_2 e^{ik_2(2z-2a_2)}}^{\text{Event 1}} + (1-R_2^2)e^{ik_2(2z-2a_2)} \overbrace{\sum_{n=0}^{\infty}(-1)^{n+1}R_1^{n+1}R_2^{n}e^{ik_1(2n+2)(a_2-a_1)}}^{\text{Event 2,3,4, ...}}\right\}$$

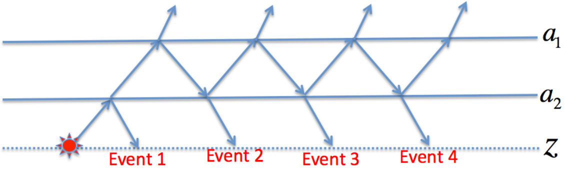


Figure: The downward continuation result below the second reflector. The history, amplitude and phase of each event in the downward continued result is shown below the formula.

Imaging below the second reflector

Convertin the result above to the time domain,

$$E(z,z,t) = -\frac{\rho_2 c_2}{2}\left\{\begin{array}{l} H(t) - R_2 H\left(t-\tau_1\right) + (1-R_2^2)\times \\ \sum_{n=0}^{\infty} H\left(t-\tau_1-(n+1)\tau_2\right) \end{array}\right\},$$

where $\tau_1 = \frac{2z-2a_2}{c_2}$, $\tau_2 = \frac{2a_2-2a_1}{c_1}$.
Balancing out the amplitude of the incidence wave *(the $\frac{\rho_2 c_2}{-2}$ factor), removing the direct wave, and taking the $t=0$ imaging condition, we have:

$$\mathcal{D}(z,t) = \begin{cases} -R_2 & \texttt{if } (z=a_2) \\ 0 & \texttt{if } (a_2<z) \end{cases}$$

Notations

- $G_0^{DN}(z, z', \omega)$ is the Green's function with vanishing Dirichlet and Neumann boundary conditions at the deeper boundary B. $\left(\frac{\partial}{\partial z'} \frac{1}{\rho(z')} \frac{\partial}{\partial z'} + \frac{\omega^2}{\rho(z')c^2(z')}\right) G_0^{DN}(z, z', \omega) = \delta(z - z')$
- z' is the field location in equation defining the Green's function, and is the location of the receiver (A) on the measurement surface in the Green's theorem.
- z is the source location in equation defining the Green's function, and is the depth we want to downward continue the wave field to.
- Before graphical display, a bandlimited wavelet is added by convolution. The wavelet is $i\omega e^{-\omega^2/\beta}$ in the frequency domain or $\frac{1}{2}\sqrt{\frac{\beta}{\pi}} e^{-\beta t^2/4}$ in the time domain, where $\beta = (20\pi)^2$.

The problem

$$\left(\frac{\partial}{\partial z'}\frac{1}{\rho(z')}\frac{\partial}{\partial z'}+\frac{\omega^2}{\rho(z')c^2(z')}\right)P(z',\omega)=0$$
$$\left(\frac{\partial}{\partial z'}\frac{1}{\rho(z')}\frac{\partial}{\partial z'}+\frac{\omega^2}{\rho(z')c^2(z')}\right)G_0(z,z',\omega)=\delta(z-z') \tag{2}$$

We know the value of P and $\partial P/\partial z'$ at the measurement surface $z' = A$, the objective is to predict its value at any depth z in the subsurface.

Green's theorem for downward continuing the receiver

$$P(z,\omega) = \frac{1}{\rho(z')}\left\{P(z',\omega)\frac{\partial G_0(z,z',\omega)}{\partial z'} - G_0(z,z',\omega)\frac{\partial P(z',\omega)}{\partial z'}\right\}\Bigg|_{z'=\mathcal{A}}^{z'=\mathcal{B}},$$

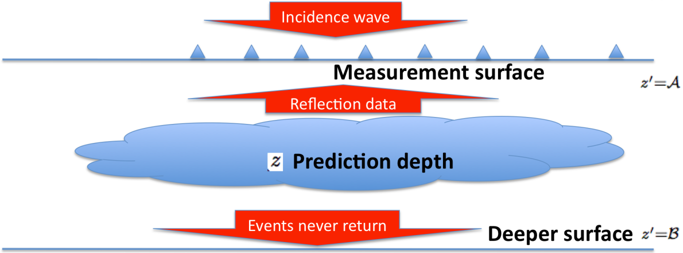


Figure: The Green's theorem predict the wavefield at an arbitrary depth z between the shallower depth A and deeper depth B. If G_0 vanishes at the lower boundary $z' = B$, we call it G_0^{DN}, then the measurement at B is not needed in the calculation.

Green's theorem for downward continuing the source

The aforementioned Green's theorem is derived for downward continuing the wave field in a source free region. How can we use it to downward continue the source as desired in seismic migration?

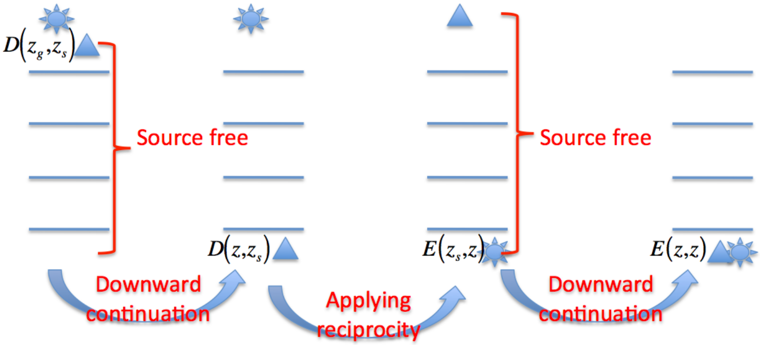


Figure: The scheme to downward continue both the source and receiver to the subsurface using Green's theorem. The imaginary data E is defined by exchanging the source and receiver location of the actual data D, they are equal due to reciprocity.

The double Green's theorem for downward continue both the source and receiver

Similar ideas in applying the double Green's theorem to downward continue both the source and receiver to the subsurface can be found in the "INVERSION WITH A VARIABLE BACKGROUND" section of Clayton and Stolt 1981.

$$E\left(z,z\right)=\rho(z_g)\rho(z_s)\times$$
$$\left\{\begin{array}{l} D\left(z_g,z_s\right)\frac{\partial G_0^{DN}(z,z_g)}{\partial z_g}\frac{\partial G_0^{DN}(z,z_s)}{\partial z_s}-\frac{\partial D(z_g,z_s)}{\partial z_s}\frac{\partial G_0^{DN}(z,z_g)}{\partial z_g}G_0^{DN}\left(z,z_s\right) \\ +\frac{\partial^2 D(z_g,z_s)}{\partial z_g \partial z_s}G_0^{DN}\left(z,z_g\right)G_0^{DN}\left(z,z_s\right)-\frac{\partial D(z_g,z_s)}{\partial z_g}\frac{\partial G_0^{DN}(z,z_s)}{\partial z_s}G_0^{DN}\left(z,z_g\right) \end{array}\right\}$$

Figure: The actual data on the measure surface is denoted as $D(z_g, z_s)$, the downward continued data at subsurface is denoted as $E(z, z)$. z_g, z_s, and z are the receiver depth, source depth, and target location respectively.

The Green's function

For one-way wave propagation the double downward continued data, D is

$$D(\text{at depth}) = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} D \, dS_g \, dS_s \quad , \qquad (5)$$

where D in the integrand $= D(\text{on measurement surface})$, $\partial G_0^{-D}/\partial z_s =$ anticausal Green's function with Dirichlet boundary condition on the measurement surface, $s =$ shot, and $g =$ receiver.

The Green's function

For two-way wave double downward continuation:

$$
\begin{aligned}
D(\text{at depth}) \;&=\; \int_{S_s}\left[\frac{\partial G_0^{DN}}{\partial z_s}\int_{S_g}\left\{\frac{\partial G_0^{DN}}{\partial z_g}D+\frac{\partial D}{\partial z_g}G_0^{DN}\right\}dS_g\right. \\
&+\; \left. G_0^{DN}\frac{\partial}{\partial z_s}\int_{S_g}\left\{\frac{\partial G_0^{DN}}{\partial z_g}D+\frac{\partial D}{\partial z_g}G_0^{DN}\right\}dS_g\right] dS_s \quad (6)
\end{aligned}
$$

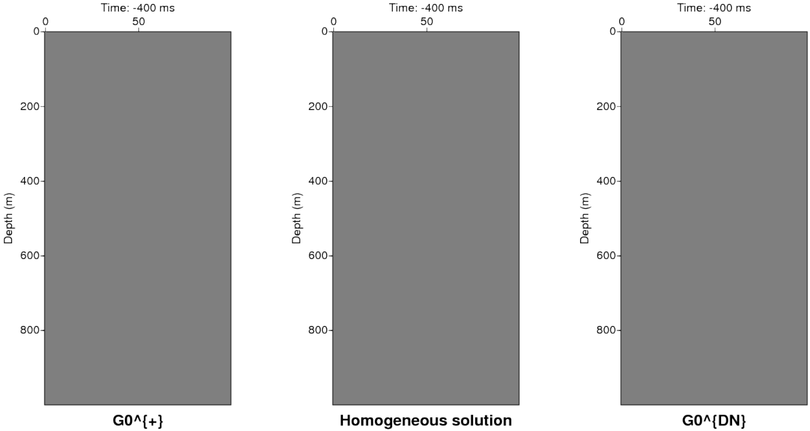
where D in the integrands $= D$(on measurement surface). G_0^{DN} is *neither* causal nor anticausal. G_0^{DN} is not an *anti*causal Green's function; it is not the inverse or adjoint of any physical propagating Green's function. It is the Green's function needed for RTM.

Construction of G_0^{DN}: method 1

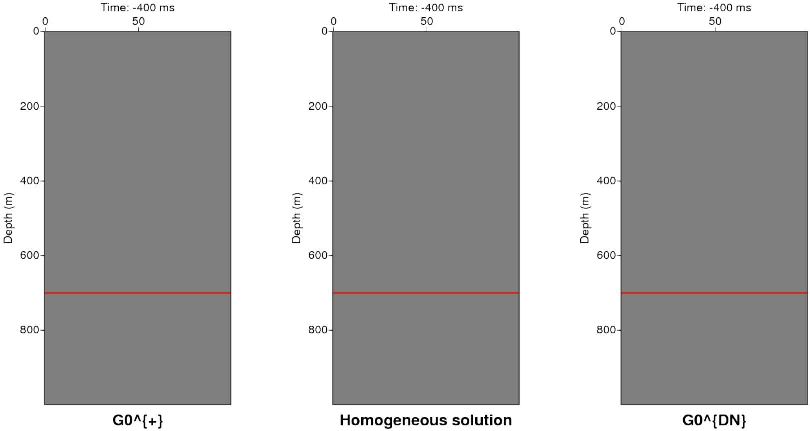
$$\left(\frac{\partial}{\partial z'}\frac{1}{\rho(z')}\frac{\partial}{\partial z'}+\frac{\omega^2}{\rho(z')c^2(z')}\right)G_0^{DN}(z,z',\omega)=\delta(z-z').$$

- First calculate the causal solution G_0^+:
  $$\left(\frac{\partial}{\partial z'}\frac{1}{\rho(z')}\frac{\partial}{\partial z'}+\frac{\omega^2}{\rho(z')c^2(z')}\right)G_0^+(z,z',\omega)=\delta(z-z').$$
- Find a particular solution for the same geological model without source:
  $$\left(\frac{\partial}{\partial z'}\frac{1}{\rho(z')}\frac{\partial}{\partial z'}+\frac{\omega^2}{\rho(z')c^2(z')}\right)\phi(z,z',\omega)=0$$
  such that G_0^+ and ϕ cancel with each other at $z'=B$.
- We have the solution: $G_0^{DN}=G_0^+ + \phi$.
- Since ϕ has 2 degree of freedom, it is always possible to make sure both Dirichlet and Neumann boundary conditions at the deeper boundary are satisfied.
- This approach is complicated, but it offers a construction from two physical components that actually happen.

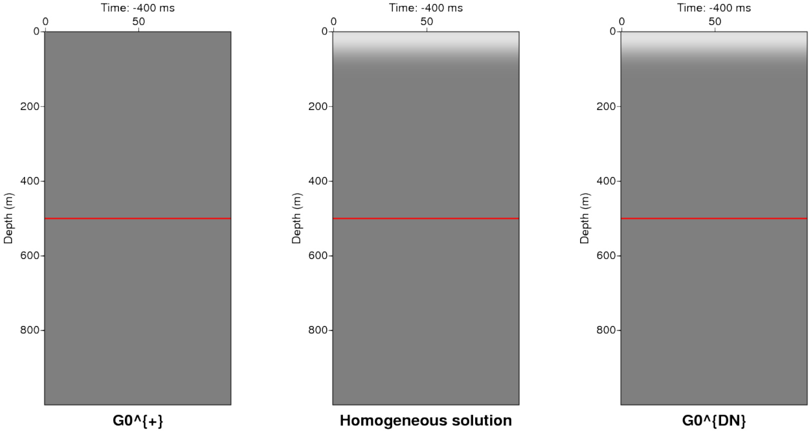
Construction of G_0^{DN} from G_0^{+} and a homogeneous solution



Construction of G_0^{DN} from G_0^{+} and a homogeneous solution

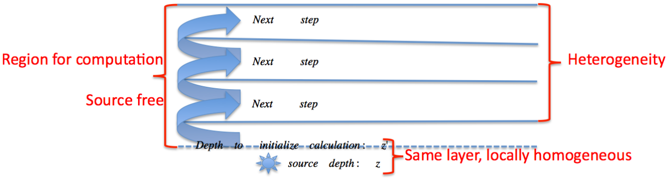


Construction of G_0^{DN} from G_0^{+} and a homogeneous solution



Construction of G_0^{DN}: method 2 - iterative approach

- We will take advantage of independence of $G_0^{DN}(z, z')$ from any heterogeneity beyond the (z', z) interval. This approach is much simpler and much easier to compute for an arbitrary medium, but the solution is less straightforward since it is not physical.
- Calculate $G_0^{DN}(z, z')$ for a location $z' > z$ sufficiently close to z such that they locate in the same layer. In the this case the solution had already been provided by Weglein, Stolt and Mayhan 2011.



Cost to construct G_0^{DN}

- Problem reduces to "solving differential equation with a know boundary conditon" at the lower surface.

- Computationally it is the same as an ordinary forward modeling procedure, for example, finite difference.

From simple to complicated medium: wave theoretical approach

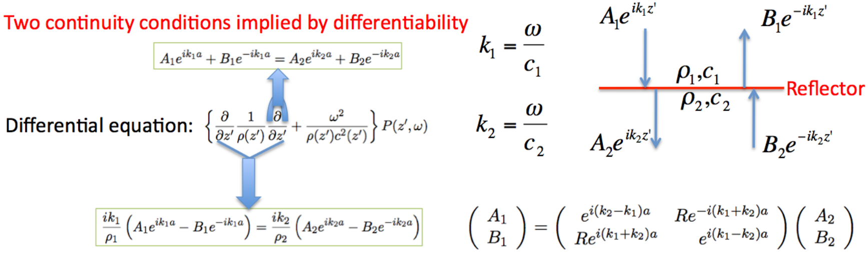


Figure: An iterative scheme to calculate the wave field above a reflector, where $R = (\rho_2 c_2 - \rho_1 c_1)/(\rho_2 c_2 + \rho_1 c_1)$ is the reflection coefficient. A_2 and B_2 can be culculated through a simpler model without the aforementioned reflector.

From simple to complicated medium: wave theoretical approach

- Inside each layer the medium is homogeneous, and the wave field withing can be expressed as $Ae^{ikz'} + Be^{-ikz'}$ where $k = \omega/c$ and c is the velocity in the layer.

- The procedure can be found in a classical geophysics literature, for example, Robinson & Treitel.

- In a typical reflection problem, the incidence strength A_1 is assumed known, and no wave comes up below the reflector, in other words $B_2 = 0$. The objective is to calculate the reflection amplitude B_1 and transmission strength A_2.

- In our case, we assume A_2 and B_2 are known, and the objective is to calculate A_1 and B_1.

From simple to complicated medium: finite difference approach

Differential equation: $$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2(z)}\frac{\partial^2}{\partial t^2}\right)\phi(z,t) = 0.$$

$$\frac{\phi_{m+1,n} + \phi_{m-1,n} - 2\phi_{m,n}}{(\Delta z)^2} - \frac{1}{c^2}\frac{\phi_{m,n+1} + \phi_{m,n-1} - 2\phi_{m,n}}{(\Delta t)^2} = 0,$$

Propagate forward in time:

$$\phi_{m,n+1} = (2 - 2p^2)\phi_{m,n} - \phi_{m,n-1} + p^2 * (\phi_{m+1,n} + \phi_{m-1,n})$$

Propagate upward in space:

$$\phi_{m-1,n} = (2 - 2p^{-2})\phi_{m,n} - \phi_{m+1,n} + p^{-2} * (\phi_{m,n+1} + \phi_{m,n-1})$$

Figure: The deeper solution is known through the property of the Green's function, the Green's function at one step shallower can be calculated through this scheme, where $p = c\Delta t/\Delta z$.

G_0^{DN} analytic and numerical examples: Homogeneous case

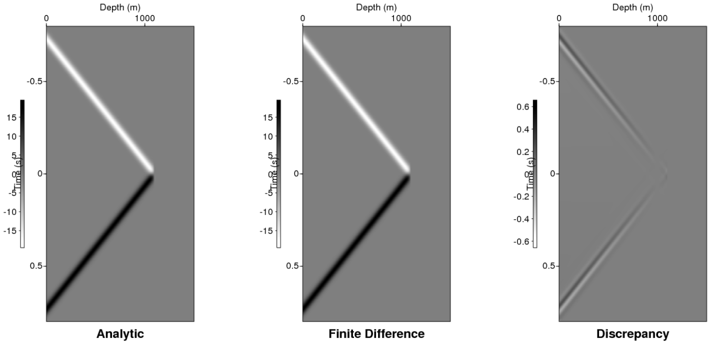


Figure: $G_0^{DN}(z = 1100m, z', t)$ for a homogeneous medium with velocity $1500m/s$. Left: Analytic solution, middle: finite difference result, right: their difference.

G_0^{DN} analytic and numerical examples: one-reflector model

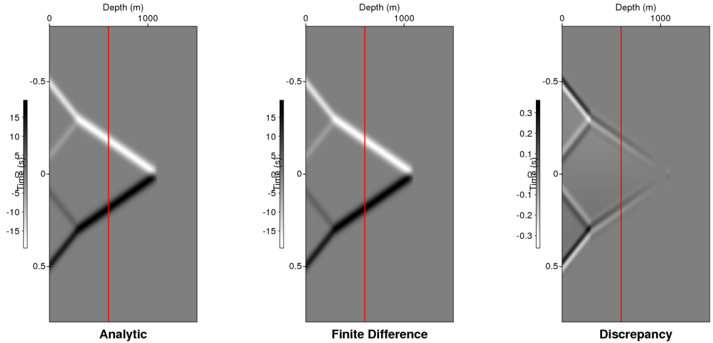


Figure: $G_0^{DN}(z = 1100m, z', t)$ for a medium with one reflector at $z' = 600m$, the velocities above and below the reflector are 1500 and $2700m/s$, respectively. Left: Analytic solution, middle: finite difference result, right: their difference.

G_0^{DN} analytic and numerical examples: two-reflector model

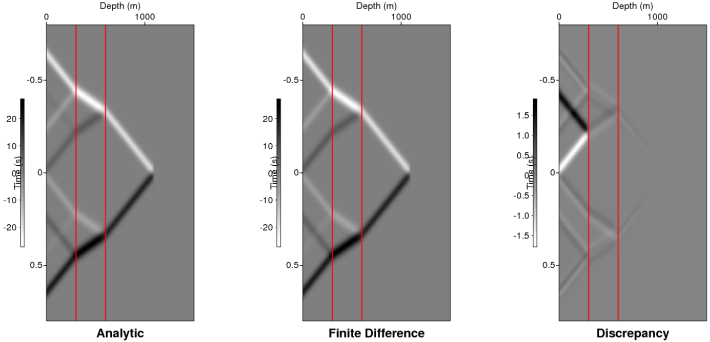


Figure: $G_0^{DN}(z = 1100m, z', t)$ for a medium with two reflectors at deth $300m$ and $600m$, respectively (the velocities from top to bottom are 1500, 2700, and $1500m/s$). Left: Analytic solution, middle: finite difference result, right: their difference.

Conclusions

- The wave theory method to calculate G_0^{DN} for arbitrary medium, its finite difference version can be extended to multi-dimension with lateral varying velocity models.
- Incorporating density contribution in the Green's theorem RTM.
- Our two-way method recovered not only the precise location of the subsurface reflector from data include internal multiples, but also its actual amplitude that is precise, clearly defined, and quantatively meaningful.

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