M-OSRP
Misson - Oriented Seismic Research Program
Solve The Right Problem

Direct Inversion and FWI
A key-note address

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Society of Exploration Geophysicists
The international society of applied geophysics

- Modeling
- Inversion

Direct

Indirect

Direct

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Indirect

Search for x such that

$(ax^2 + bx + c)^2$ is a minimum

Indicators of “indirect”

- model matching
- objective/cost functions
- search algorithm
- iterative linear “inversion”
- necessary and not sufficient conditions, e.g., CIG flatness

There’s a role for direct and indirect methods in practical real world application.

Direct Forward and Direct Inverse

$$L_0 G_0 = \delta \qquad LG = \delta$$

$$V = L_0 - L \quad \psi_s = G - G_0$$

Relationship $$G = G_0 + G_0 V G \qquad (1)$$

An operator identity that follows from

$$L^{-1} = L_0^{-1} + L_0^{-1}(L_0 - L)L^{-1}$$

Modeling $$L \to G \quad L_0, V \to G$$

Direct Forward and Direct Inverse

Relationship $$G = G_0 + G_0 V G \quad (1)$$

Modeling $$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots \quad (2)$$

Form $$G - G_0 = S = ar + ar^2 + \cdots = \frac{ar}{1-r}$$

with $$a = G_0$$ $$r = VG_0$$

$$S = S_1 + S_2 + S_3 + \cdots$$

Direct Forward and Direct Inverse

$$S = S_1 + S_2 + S_3 + \cdots$$

$$S = \frac{ar}{1-r}$$

Solve for *r*

$$r = \frac{S/a}{1 + S/a} = S/a - \left(S/a\right)^2 + \left(S/a\right)^3 + \cdots$$

$$= r_1 + r_2 + r_3 + \cdots$$

Direct Forward and Direct Inverse

$$S = (G - G_0)_{ms} = Data$$

Forward *S* in terms of *V*, inverse *V* in terms of *S*

$$V = V_1 + V_2 + \cdots \qquad (3)$$

where V_n is the portion of *V*, n-th order in the data

(2) is the forward series;

(3) is the inverse series.

- The relationship (2) provides a Geometric forward series rather than a Taylor series.
- In general, a Taylor series doesn’t have an inverse series; however, a Geometric series has an inverse series.
- All conventional current mainstream inversion, including iterative linear inversion and FWI, are based on a Taylor series concept.
- Solving a forward problem in an inverse sense is not the same as solving an inverse problem directly.

- The r_1, r_2, … equations generalize

$$r = S/a - \left(S/a\right)^2 + \left(S/a\right)^3 + \cdots$$
$$= r_1 + r_2 + r_3 + \cdots$$

$$G_0 V_1 G_0 = D$$
$$G_0 V_2 G_0 = -G_0 V_1 G_0 V_1 G_0$$
$$G_0 V_3 G_0 = -G_0 V_1 G_0 V_1 G_0 V_1 G_0 + \ldots$$

(2D) Elastic (isotropic)

$$L\vec{u} = \begin{pmatrix} f_x \\ f_z \end{pmatrix} \qquad \hat{L}\begin{pmatrix} \phi^P \\ \phi^S \end{pmatrix} = \begin{pmatrix} F^P \\ F^S \end{pmatrix}$$

where

$$L = \left[\rho\omega^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \partial_x \gamma \partial_x + \partial_z \mu \partial_z & \partial_x (\gamma - 2\mu) \partial_z + \partial_z \mu \partial_x \\ \partial_z (\gamma - 2\mu) \partial_x + \partial_x \mu \partial_z & \partial_z \gamma \partial_z + \partial_x \mu \partial_x \end{pmatrix} \right]$$

$$L_0 = \left[\rho_0\omega^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \gamma_0 \partial_x^2 + \mu_0 \partial_z^2 & (\gamma_0 - \mu_0) \partial_x \partial_z \\ (\gamma_0 - \mu_0) \partial_x \partial_z & \mu_0 \partial_x^2 + \gamma_0 \partial_z^2 \end{pmatrix} \right]$$

$$V \equiv L_0 - L$$

$$= \begin{bmatrix} a_\rho \omega^2 + \alpha_0^2 \partial_x a_\gamma \partial_x + \beta_0^2 \partial_z a_\mu \partial_z & \partial_x (\alpha_0^2 a_\gamma - 2\beta_0^2 a_\mu) \partial_z + \beta_0^2 \partial_z a_\mu \partial_x \\ \partial_z (\alpha_0^2 a_\gamma - 2\beta_0^2 a_\mu) \partial_x + \beta_0^2 \partial_x a_\mu \partial_z & a_\rho \omega^2 + \alpha_0^2 \partial_z a_\gamma \partial_z + \beta_0^2 \partial_x a_\mu \partial_x \end{bmatrix}$$

$$a_\rho \equiv \frac{\rho}{\rho_0} - 1, a_\gamma \equiv \frac{\gamma}{\gamma_0} - 1, a_\mu \equiv \frac{\mu}{\mu_0} - 1$$

The forward problem

$$\hat{G} - \hat{G}_0 = \hat{G}_0 \hat{V} \hat{G} = \hat{G}_0 \hat{V} \hat{G}_0 + \hat{G}_0 \hat{V} \hat{G}_0 \hat{V} \hat{G}_0 + \cdots \quad (4)$$

$$\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix} = \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}$$

$$+ \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}$$

$$\begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}$$

$$+ \cdots$$

The inverse problem (solving for *r* in terms of S)

$$\hat{V}^{PP} = \hat{V}_1^{PP} + \hat{V}_2^{PP} + \hat{V}_3^{PP} + \cdots$$

$$\hat{V}^{PS} = \hat{V}_1^{PS} + \hat{V}_2^{PS} + \hat{V}_3^{PS} + \cdots$$

$$\hat{V}^{SP} = \hat{V}_1^{SP} + \hat{V}_2^{SP} + \hat{V}_3^{SP} + \cdots \quad (5)$$

$$\hat{V}^{SS} = \hat{V}_1^{SS} + \hat{V}_2^{SS} + \hat{V}_3^{SS} + \cdots$$

$$\hat{V}^{PP} = -\nabla^2 a_\gamma - \frac{\omega^2}{\alpha_0^2}(a_\rho \partial_x^2 + \partial_z a_\rho \partial_z)\frac{1}{\nabla^2} - \left[-2\partial_z^2 a_{mu} \partial_x^2 - 2\partial_x^2 a_{mu} \partial_z^2 + 4\partial_x^2 \partial_z a_{mu} \partial_z\right]\frac{1}{\nabla^2}$$

$$\hat{V}^{PS} = \frac{\alpha_0^2}{\beta_0^2}\left[\frac{\omega^2}{\alpha_0^2}(\partial_x a_\rho \partial_z - \partial_z a_\rho \partial_x) + 2\partial_x \partial_z a_{mu}(\partial_z^2 - \partial_x^2) - 2(\partial_z^2 - \partial_x^2)a_{mu} \partial_z \partial_x\right]\frac{1}{\nabla^2}$$

$$\hat{V}^{SP} = -\left[\frac{\omega^2}{\alpha_0^2}(\partial_x a_\rho \partial_z - \partial_z a_\rho \partial_x) + 2\partial_x \partial_z a_{mu}(\partial_z^2 - \partial_x^2) - 2(\partial_z^2 - \partial_x^2)a_{mu} \partial_z \partial_x\right]\frac{1}{\nabla^2}$$

$$\hat{V}^{SS} = -\frac{\alpha_0^2}{\beta_0^2}\left[\frac{\omega^2}{\alpha_0^2}(a_\rho \partial_x^2 + \partial_z a_\rho \partial_z) + (\partial_z^2 - \partial_x^2)a_{mu}(\partial_z^2 - \partial_x^2) + 4\partial_x \partial_z a_{mu} \partial_x \partial_z\right]\frac{1}{\nabla^2}$$

$$a_{mu} = \frac{\mu - \mu_0}{\gamma_0} = \frac{\beta_0^2}{\alpha_0^2} a_\mu$$

$$\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix} = \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_2^{PP} & \hat{V}_2^{PS} \\ \hat{V}_2^{SP} & \hat{V}_2^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}$$

$$= - \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix} \begin{pmatrix} \hat{V}_1^{PP} & \hat{V}_1^{PS} \\ \hat{V}_1^{SP} & \hat{V}_1^{SS} \end{pmatrix} \begin{pmatrix} \hat{G}_0^P & 0 \\ 0 & \hat{G}_0^S \end{pmatrix}$$

$$\vdots$$

Hence, for $\hat{V}^{PP} = \hat{V}_1^{PP} + \hat{V}_2^{PP} + \hat{V}_3^{PP} + \cdots$ any one of the four matrix elements of V requires $\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix}$

- $\hat{D}^{PP}$ can be determined independently

  in terms of $\begin{pmatrix} \hat{V}^{PP} & \hat{V}^{PS} \\ \hat{V}^{SP} & \hat{V}^{SS} \end{pmatrix}$

- but $\hat{V}^{PP}$ or $\hat{V}^{PS}, \hat{V}^{SP}, \hat{V}^{SS}$ requires $\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix}$

- That's what the general relationship

  $G = G_0 + G_0 V G$ requires.

- A direct non-linear solution

  order by order in the data matrix $\begin{pmatrix} \hat{D}^{PP} & \hat{D}^{PS} \\ \hat{D}^{SP} & \hat{D}^{SS} \end{pmatrix}$

- ISS is not iterative linear inversion.
- Iterative linear starts with $G_0 V_1 G_0 = D$ (7)

  solves for V_1, changes the reference medium, finds a new L_0 and G_0 (and require generalized inversions of noisy bandlimited data dependent operators).
- To find V_1' , $G_0' V_1' G_0' = D' = (G - G_0')_{ms}$

$$L_0' = L_0 - V_1$$

$$L_0' G_0' = \delta$$

- The problem is much more serious than a different approach to solve $G_0 V_1 G_0 = D$ (7) for V_1 .
- If (7) is our <u>entire</u> basic theory, you can mistakenly think that $\hat{D}^{PP} = \hat{G}_0^P \hat{V}_1^{PP} \hat{G}_0^P$

  is sufficient to update $\hat{D}^{PP'} = \hat{G}_0^{P'} \hat{V}_1^{PP'} \hat{G}_0^{P'}$
- That step loses contact with and violates

$$G = G_0 + G_0 V G$$

  for the elastic wave equation.

1. That is, it violates the basic relationship between changes in a medium, ***V*** and changes in the wavefield, $G\text{-}G_0$ for the simplest elastic model.
2. This direct inverse method gives you a platform for FWI and communicates when a “FWI” method should work in principle.
3. Iteratively inverting multi-component data has the correct data but doesn’t corresponds to a direct inverse algorithm.
4. To honor $G=G_0+G_0VG$, you need both the data and the algorithm that direct inverse prescribes.

A.B. Weglein and Jinlong Yang

A first comparison of the inverse scattering series and iterative linear inverse for parameter estimation, M-OSRP (2015), SEG Abstract (2015).

There's a role for direct and indirect methods in practical real world application.

4D Application

Mission - Oriented Seismic Research Program

Discrimination between pressure and fluid saturation using direct non-linear inversion method: *an application to time-lapse seismic data*

Haiyan Zhang, Arthur B. Weglein, Robert Keys, Douglas Foster and Simon Shaw

M-OSRP Annual Meeting
University of Houston
May 10 –12, 2006

Statement of the problem

- **Distinguishing pressure changes from reservoir fluid changes is difficult with conventional seismic time-lapse attributes.**
- **Pressure changes or fluid changes?**
  - **shear modulus** ***sensitive to*** **pressure changes**
  - **Vp/Vs** ***sensitive to*** **fluid changes**
- **A direct non-linear inversion method may be useful for accomplishing this goal.**

Introduction of the method

Inverse scattering series	Time-lapse seismic monitoring
Reference medium L_0	**Initial reservoir condition**
Actual medium L	**Current reservoir condition**
Earth property changes in space $V=L_0-L$	**Reservoir property changes in time**
Reference wave field G_0	**Baseline survey**
Actual wave field G	**Monitor survey**
Scattered wave field $D=G-G_0$	**Monitor-Baseline**

Conclusion

- **Comparing the first and second order algorithms in estimating shear modulus and Vp/Vs contrasts.**
  - **Second order direct inverse was able to distinguish pressure changes from fluid changes.**

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