Elastic Green's theorem preprocessing for on-shore internal multiple attenuation: theory and initial synthetic data tests

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SUMMARY

Prerequisites are important for the Inverse Scattering Series (ISS) multiple removal method, that assumes reference wave field has been removed and the source wavelet has been deconvolved. This paper derives the elastic Green's theorem reference wave prediction algorithm, which extends the off-shore acoustic to the on-shore elastic wave field separation, in preparation for on-shore ISS internal multiple attenuation.

INTRODUCTION

Weglein (2013) proposes a three-pronged strategy to respond to the current pressing challenges in removing multiples: (1) develop the ISS prerequisites for predicting the reference wave field and producing deghosted data that are direct and do not require subsurface information; (2) develop internal multiple elimination algorithms from the ISS; (3) develop a replacement for the energy-minimization criteria for adaptive subtraction. For part (1), Green's theorem preprocessing has documented effectiveness for off-shore plays (e.g., Weglein et al., 2002; Zhang, 2007; Mayhan et al., 2012; Mayhan and Weglein, 2013; Tang et al., 2013; Yang et al., 2013).

For on-shore plays, because of their complex structures with lateral variation, as well as significant ground roll, there are more fundamental issues and challenges for resolving the near surface problem. Among these issues and challenges, identification and removal of the reference wave is one pressing and essential topic. Scattering theory separates the real world into two parts: the reference medium, whose property is known, plus a perturbation. The wave that travels in the reference medium is called the reference wave, which does not experience the earth that we are interested in (Weglein et al., 2003; Tang and Weglein, 2014). Especially for on-shore, the ground roll as the main energy of the reference wave can obscure the reflections. In addition, the reference wave contains the source signature information, which is important and will be used for deconvolution before the subsequent ISS multiple removal. Therefore, it's an important step to identify and remove the reference wave field on land.

Matson (1997) provides the ISS free surface multiple elimination and internal multiple attenuation algorithms in PS space, i.e., by using potentials, rather than displacements. He assumes the reference wave has been removed by using linear mute; however, the linear mute may harm/destroy useful information, especially when the reference wave and the scattering wave are seriously interfering with each other. Weglein and Secrest (1990) propose the reference wave prediction method for the elastic media based on Green's theorem, and derive the wavelet estimation algorithm in displacement space. When the medium is assumed to consist of a homogeneous elastic whole space, Jiang et al. (2013) test the algorithm.

In order to simulate the land acquisition situation, we choose two homogeneous half spaces as the reference medium, an acoustic half-space over an elastic half-space. We locate the source in the acoustic medium and receivers in the elastic medium. The perturbation will be in the lower elastic half-space. By using Green's second identity, we derive the algorithm to separate the reference wave and scattering wave in PS space. In this paper, the algorithms are derived in both the space-frequency domain and the wavenumber-frequency domain. The wavelet can be estimated from the predicted reference wave. Numerical tests are shown to evaluate the accuracy of the algorithm for predicting the source wavelet for this acoustic over elastic half-space problem, that models the on-shore play acquisition. The results are positive and encouraging.

BACKGROUND FOR 2D ELASTIC MEDIUM

We are deriving the wave field separation method for on-shore application and we start with the elastic formulation. For convenience, the basis is changed from $\mathbf{u} = \begin{pmatrix} u_x \\ u_z \end{pmatrix}$ to $\Phi = \begin{pmatrix} \phi^P \\ \phi^S \end{pmatrix}$. $\mathbf{u}$ has x and z components; whereas Φ has potential components for P wave and S wave.

In PS space, the basic wave equations (Weglein and Stolt, 1995; Zhang, 2006) are

$$\hat{L}\Phi = \mathbf{F}, \tag{1}$$
$$\hat{L}\hat{G} = \delta, \tag{2}$$
$$\hat{L}_0\Phi_0 = \mathbf{F}, \tag{3}$$
$$\hat{L}_0\hat{G}_0 = \delta, \tag{4}$$

where $\hat{L}$ and $\hat{L}_0$ are the differential operators that describe the wave propagation in the actual and the reference media, respectively. $\mathbf{F}$ is the source term. $\hat{G}$ and $\hat{G}_0$ are the corresponding Green's function operators for the actual and reference media.

For a homogeneous medium,

$$\hat{L}_0 = \begin{pmatrix} \nabla^2 + \frac{\omega^2}{\alpha_0^2} & \\ & \nabla^2 + \frac{\omega^2}{\beta_0^2} \end{pmatrix} = \begin{pmatrix} \hat{L}_0^P & \\ & \hat{L}_0^S \end{pmatrix}, \tag{5}$$

and

$$\hat{G}_0 = \begin{pmatrix} \hat{G}_0^P & \\ & \hat{G}_0^S \end{pmatrix}. \tag{6}$$

Equations 5 and 6 are diagonal. However, in an actual inhomogeneous medium, $\hat{G}$ is no longer a diagonal matrix, but has a form

$$\hat{G} = \begin{pmatrix} \hat{G}^{PP} & \hat{G}^{PS} \\ \hat{G}^{SP} & \hat{G}^{SS} \end{pmatrix}. \tag{7}$$

For the superscripts, the right one represents the wave type of source side, whereas the left one represents the wave type of receiver side.

DOI http://dx.doi.org/10.1190/segam2014-1513.1

GREEN'S THEOREM WAVE FIELD SEPARATION ALGORITHM IN PS SPACE

Problem Description

Transforming the elastic wave equations from the displacement space to the PS space, we have

$$\hat{L}\Phi = \mathbf{F},$$
$$\hat{L}_0\hat{G}_0 = \delta,$$
$$\hat{L} = \hat{L}_0 - \hat{V}. \tag{8}$$

The basic form of these equations is the same as in the acoustic case. Based on the successful applications of Green's theorem wave field separation in the acoustic case (e.g., Zhang, 2007; Mayhan et al., 2012), it's feasible to apply Green's theorem wave field separation algorithm to the elastic medium in a similar way. The reference medium ($\hat{L}_0$) can be chosen for different objectives. To separate the reference and the scattering wave, the reference medium should be chosen equal to the actual medium above the measurement surface.

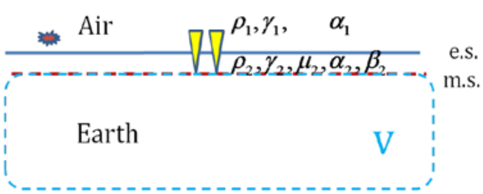

Figure 1: On-shore acquisition

For land acquisition, we assume that the source is located slightly above the earth's surface (e.s.), and the geophone is in the earth but close to the earth's surface shown as Fig.1. Actually, because of weathered layer and tundra, the property of near surface can be complicated, with lateral varying densities and velocities. For this initial study, we assume that the medium, which is below the earth's surface and above measurement surface (m.s.), is homogeneous. The reference wave can be predicted in any point inside the volume in Fig.1 by using Green's theorem.

Reference Wave Field Prediction in PS Space

In Fig.2, source ($\mathbf{r}_s$) is above the earth's surface, i.e., the boundary, receiver ($\mathbf{r}'$) is on the measurement surface, and the prediction location is represented by $\mathbf{r}$. The reference medium is chosen as a discontinuous two-half-space medium, above the boundary is homogeneous air, below is homogeneous elastic. A hemispherical surface integral upper bounded by the measurement surface will separate the total wave Φ into the reference wave Φ_0 and the scattering wave Φ_S. The prediction in the volume is the reference wave Φ_0 as shown in Fig.2.

The elastic Green's theorem algorithm in the space-frequency domain for the reference wave prediction in the volume is

$$\Phi_0(\mathbf{r},\mathbf{r}_s) = \oint \left(\Phi(\mathbf{r}',\mathbf{r}_s)\cdot\nabla'\hat{G}_0(\mathbf{r}',\mathbf{r}) - \nabla'\Phi(\mathbf{r}',\mathbf{r}_s)\cdot\hat{G}_0(\mathbf{r}',\mathbf{r})\right)\cdot\hat{n}dS', \tag{9}$$

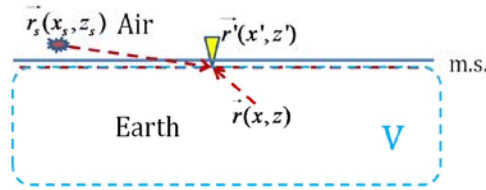

Figure 2: Volume enclosed (blue dashed line) for reference wave field prediction at $\mathbf{r}$ in the volume and $\mathbf{r}$ is under the measurement surface that is represented by $\mathbf{r}'$.

where $\Phi_0(\mathbf{r},\mathbf{r}_s) = \begin{pmatrix} \Phi_0^P(\mathbf{r},\mathbf{r}_s) \\ \Phi_0^S(\mathbf{r},\mathbf{r}_s) \end{pmatrix}, \Phi(\mathbf{r},\mathbf{r}_s) = \begin{pmatrix} \Phi^P(\mathbf{r},\mathbf{r}_s) \\ \Phi^S(\mathbf{r},\mathbf{r}_s) \end{pmatrix},$ and Green's function for the reference medium in the $(\mathbf{r},\omega)$ domain is

$$\begin{aligned}
&\hat{G}_0(\mathbf{r}',\mathbf{r}) \\
&= \begin{pmatrix} \hat{G}_0^P(\mathbf{r}',\mathbf{r}) + \hat{G}_0^{PP}(\mathbf{r}',\mathbf{r}) & \hat{G}_0^{PS}(\mathbf{r}',\mathbf{r}) \\ \hat{G}_0^{SP}(\mathbf{r}',\mathbf{r}) & \hat{G}_0^S(\mathbf{r}',\mathbf{r}) + \hat{G}_0^{SS}(\mathbf{r}',\mathbf{r}) \end{pmatrix} \\
&= \frac{1}{2\pi}\int e^{ik_x(x'-x)}dk_x \\
&\left(\begin{pmatrix} \frac{e^{i\nu_2|z'-z|}}{2i\nu_2} & 0 \\ 0 & \frac{e^{i\eta_2|z'-z|}}{2i\eta_2} \end{pmatrix} + \begin{pmatrix} \acute{P}\grave{P}\frac{e^{i\nu_2 z}e^{i\nu_2 z'}}{2i\nu_2} & \acute{S}\grave{P}\frac{e^{i\eta_2 z}e^{i\nu_2 z'}}{2i\eta_2} \\ \acute{P}\grave{S}\frac{e^{i\nu_2 z}e^{i\eta_2 z'}}{2i\nu_2} & \acute{S}\grave{S}\frac{e^{i\eta_2 z}e^{i\eta_2 z'}}{2i\eta_2} \end{pmatrix} \right),
\end{aligned} \tag{10}$$

where $\acute{P}\grave{P}$, $\acute{P}\grave{S}$, $\acute{S}\grave{P}$, $\acute{S}\grave{S}$ represent the reflection coefficients along the boundary, respectively, and

$$\nu_2 = \begin{cases} \sqrt{k_{\alpha_2}^2 - k_x^2} & if \quad k_x < k_{\alpha_2} \\ i\sqrt{k_x^2 - k_{\alpha_2}^2} & if \quad k_x > k_{\alpha_2} \end{cases} \qquad k_{\alpha_2} = \frac{\omega}{\alpha_2} \quad ,$$

$$\eta_2 = \begin{cases} \sqrt{k_{\beta_2}^2 - k_x^2} & if \quad k_x < k_{\beta_2} \\ i\sqrt{k_x^2 - k_{\beta_2}^2} & if \quad k_x > k_{\beta_2} \end{cases} \qquad k_{\beta_2} = \frac{\omega}{\beta_2} \quad .$$

Since both Φ and $\hat{G}_0$ in the integral are tensors, the symbol '$\cdot$' represents a tensor product.

If the measurement surface is horizontal, $\hat{n} = (0,-1)$, and it represents the outward normal vector directs upward. The algorithm can be simplified as:

$$\Phi_0(\mathbf{r},\mathbf{r}_s) = -\int \left(\Phi(\mathbf{r}',\mathbf{r}_s)\cdot\partial z'\hat{G}_0(\mathbf{r}',\mathbf{r}) - \partial z'\Phi(\mathbf{r}',\mathbf{r}_s)\cdot\hat{G}_0(\mathbf{r}',\mathbf{r})\right)dx'. \tag{11}$$

Using the reciprocity of Green's function and Fourier transforming over x, the algorithm in the wavenumber-frequency domain for the reference wave prediction in the volume can be obtained:

$$\tilde{\Phi}_0(k_x,z,\mathbf{r}_s) = -\tilde{\Phi}(k_x,z,\mathbf{r}_s)\cdot\partial z'\tilde{\hat{G}}_0^T(k_x,z,\mathbf{r}') + \partial z'\tilde{\Phi}(k_x,z,\mathbf{r}_s)\cdot\tilde{\hat{G}}_0^T(k_x,z,\mathbf{r}'), \tag{12}$$

where $\tilde{\hat{G}}_0^T$ is the transverse of $\tilde{\hat{G}}_0$.

With the separated reference wave from the total wave, the wavelet $A(\omega)$ can be estimated. Since

$$\begin{pmatrix} \Phi_0^P(\mathbf{r},\mathbf{r}_s,\omega) \\ \Phi_0^S(\mathbf{r},\mathbf{r}_s,\omega) \end{pmatrix} = \begin{pmatrix} A(\omega)\hat{G}_0^{PP}(\mathbf{r},\mathbf{r}_s,\omega) \\ A(\omega)\hat{G}_0^{SP}(\mathbf{r},\mathbf{r}_s,\omega) \end{pmatrix}, \tag{13}$$

DOI http://dx.doi.org/10.1190/segam2014-1513.1

$A(\omega)$ can be obtained by either

$$A(\omega) = \frac{\Phi_0^P(\mathbf{r},\mathbf{r}_s,\omega)}{\hat{G}_0^{PP}(\mathbf{r},\mathbf{r}_s,\omega)} \quad \text{or} \quad A(\omega) = \frac{\Phi_0^S(\mathbf{r},\mathbf{r}_s,\omega)}{\hat{G}_0^{SP}(\mathbf{r},\mathbf{r}_s,\omega)}, \tag{14}$$

where

$$\begin{aligned} \hat{G}_0^{PP}(\mathbf{r},\mathbf{r}_s,\omega) &= \frac{1}{2\pi}\int \grave{P}\grave{P}\frac{e^{-i\nu_1 z_s}e^{i\nu_2 z}}{2i\nu_1}e^{ik_x(x-x_s)}dk_x, \\ \hat{G}_0^{SP}(\mathbf{r},\mathbf{r}_s,\omega) &= \frac{1}{2\pi}\int \grave{P}\grave{S}\frac{e^{-i\nu_1 z_s}e^{i\eta_2 z}}{2i\nu_1}e^{ik_x(x-x_s)}dk_x, \end{aligned} \tag{15}$$

and $\nu_1 = \begin{cases} \sqrt{k_{\alpha_1}^2 - k_x^2} & if \quad k_x < k_{\alpha_1} \\ i\sqrt{k_x^2 - k_{\alpha_1}^2} & if \quad k_x > k_{\alpha_1} \end{cases} \qquad k_{\alpha_1} = \frac{\omega}{\alpha_1}$,

and $\grave{P}\grave{P}$, $\grave{P}\grave{S}$ represent the transmission coefficients along the boundary, respectively.

NUMERICAL EVALUATION

Since the methodology in this paper chooses the reference medium above the earth's surface to be acoustic, either fluid (water) or air can be chosen as the medium above the earth's surface. Those two cases would correspond to ocean bottom and onshore applications, respectively. In this section, two models are chosen to evaluate Green's theorem wave field separation algorithm, one is water/elastic, and the other is air/elastic.

Water/Elastic Model

A water/elastic model is first selected to examine the accuracy of the algorithm. The parameters are listed in Table 1. The water/elastic boundary is at depth 0m, the source's depth is -5m, and the measurement's depth is 0m (on the boundary) but coupled with the lower elastic. The trace interval is 2m.

Layer Number	P Velocity (m/s)	S Velocity (m/s)	Density (kg/m^3)
1	1500	0	1000
2	2250	1200	2000

Table 1: The water/elastic model parameters

Since there is no perturbation from earth in this model, the reference medium is the same as the actual one. Therefore, if the prediction point in the elastic medium is close to depth 0m, the predicted reference wave should be the same as the total wave. This will serve as a criteria later to test the algorithm.

P wave is produced by source in the water, and transmitted P and S waves will be collected by the receivers in the elastic medium. The synthetic data are generated by multiplying a wavelet with the analytical forms of Green's function in the frequency domain (equation 13), shown in Fig.3(a) for PP and Fig.4(a) for SP. The most significant energy of the total wave is surface waves since source and receivers are very close to the boundary. The direct waves have relatively weaker energy.

The predicted reference P waves (PP0) and S waves (SP0) are listed in Fig.3(b) and Fig.4(b), respectively, and as anticipated are similar to the input data. Traces with offset 400m are extracted for further comparison, as shown in Fig.3(c) and Fig.4(c). The prediction results match well with input data, which confirm the effectiveness of our wave field separation algorithm.

After obtaining the reference wave, the wavelet can be estimated by using equation 14. The results of comparison between the actual wavelet (red line in Fig.3(d)) and the wavelet estimated from PP0 at offset 400m (blue line in Fig.3(d)), and the actual wavelet (red line in Fig.4(d)) and the wavelet estimated from SP0 at offset 400m (blue line in Fig.4(d)) further confirm the accuracy of the wavelet estimation algorithm.

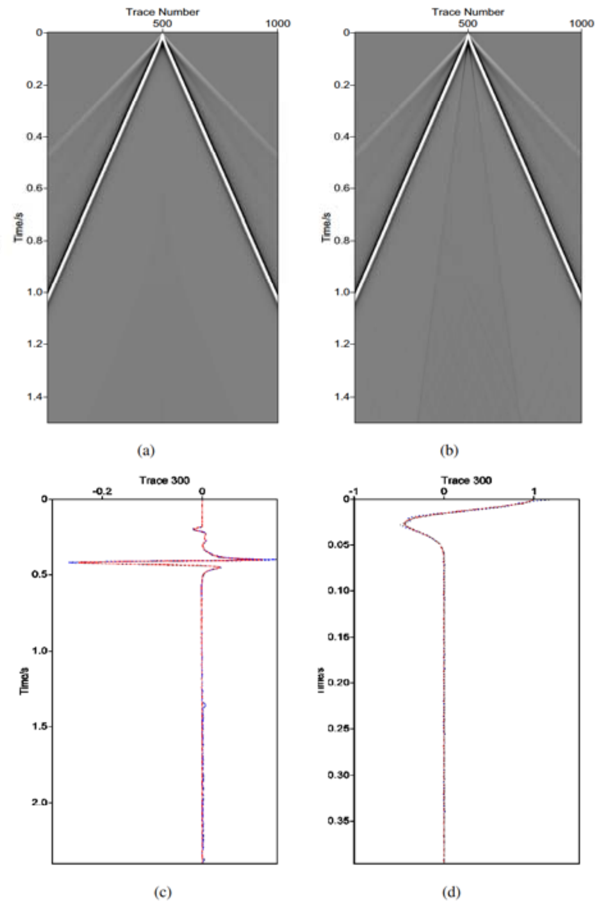

Figure 3: Numerical result for water/elastic model. (a): input data PP; (b): predicted reference wave PP0 at depth 0m; (c): traces with offset 400m, red line for PP, blue line for PP0; (d): actual wavelet (red line) and wavelet estimated from PP0 at offset 400m (blue line). Figures in (a) and (b) are in the same scale.

Air/Elastic Model

An air/elastic model is selected to examine the accuracy of the algorithm. The parameters are listed in Table 2. The depth of source is 0m, and we arrange the boundary as belonging to the upper half-space of air; whereas the depth of receiver is 5m and it is inside the elastic half-space. The prediction depth is chosen to be 25m. The result will confirm the theory that

DOI http://dx.doi.org/10.1190/segam2014-1513.1

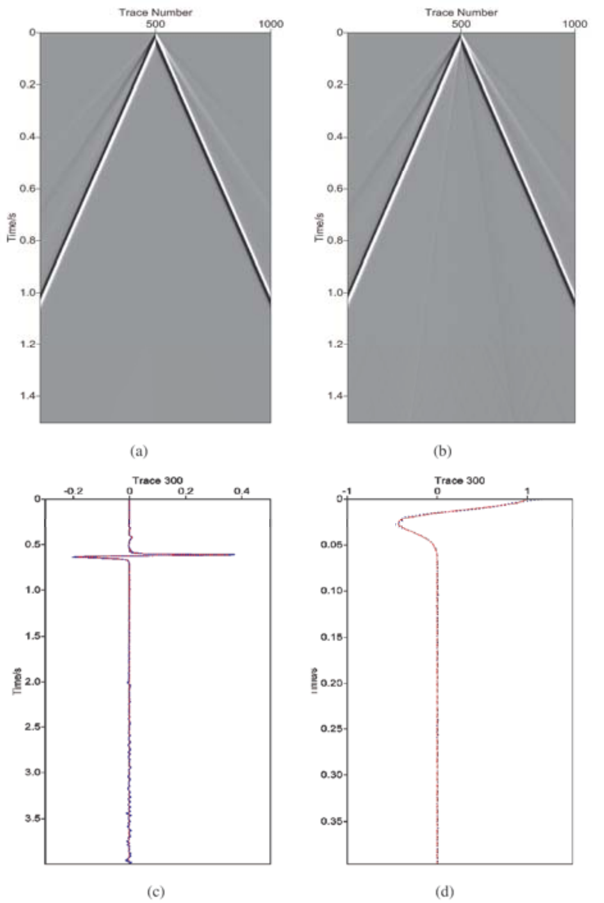

Figure 4: Numerical result for water/elastic model. (a): input data SP; (b): predicted reference wave SP0 at depth 0m; (c): traces with offset 400m, red line for SP, blue line for SP0; (d): actual wavelet (red line) and wavelet estimated from SP0 at offset 400m (blue line). Figures in (a) and (b) are in the same scale.

Green's theorem algorithm can predict the reference wave at any point in the volume, and the volume is upper bounded by the measurement surface. The trace interval is 2m.

Layer Number	P Velocity (m/s)	S Velocity (m/s)	Density (kg/m^3)
1	340	0	3
2	2250	1200	2000

Table 2: The air/elastic model parameters

The synthetic input data PP and SP are similarly generated as in the first case, as shown in Fig.5(a) and Fig.5(d). The predicted reference waves are shown in Fig.5(b) for PP0 and Fig.5(e) for SP0. In order to confirm the result, the analytical data (by multiplying the original wavelet with the analytical forms of Green's function in the frequency domain) for a receiver at depth 25m are also listed in Fig.5(c) and Fig.5(f).

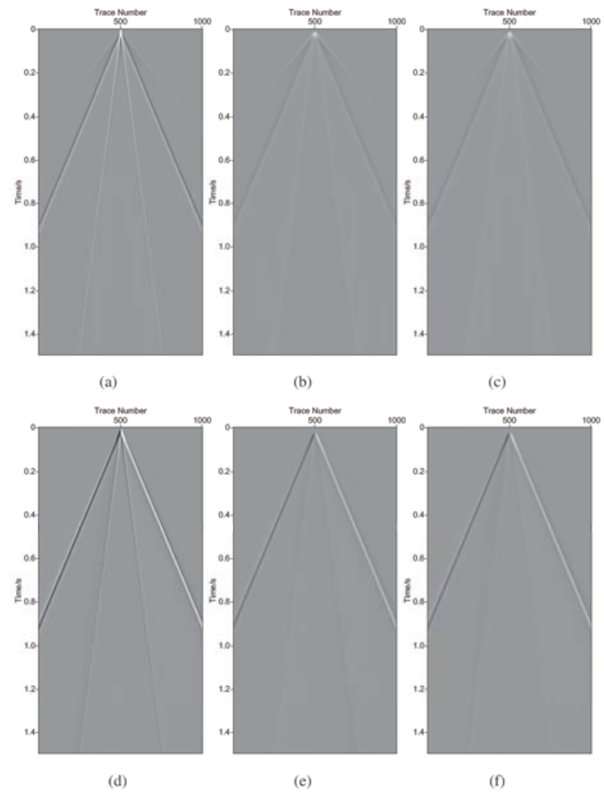

Figure 5: Numerical result for air/elastic model. (a): input data PP; (b): predicted reference wave PP0 at depth 25m; (c): analytical reference wave PP0 at depth 25m; (d): input data SP; (e): predicted reference wave SP0 at depth 25m; (f): analytical reference wave SP0 at depth 25m. All the figures are in the same scale.

CONCLUSION AND FUTURE PLAN

From the theoretical derivation and numerical tests in this paper, we understand that it's possible to apply the Green's theorem wave field separation algorithm on land. For on-shore application, the reference medium consists of two half spaces: one acoustic and the other elastic. This will provide a possible way to remove ground roll, which has the majority of the energy of the reference wave for on-shore acquisition. In order to apply Green's theorem to remove ground roll for practical complicated land acquisition data, a modified reference model and further research are required.

ACKNOWLEDGEMENTS

We are grateful to all M-OSRP sponsors for encouragement and support in this research.

http://dx.doi.org/10.1190/segam2014-1513.1

EDITED REFERENCES

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