Green's theorem derived methods for preprocessing seismic data when the pressure P and its normal derivative are measured

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Summary

We discuss deghosting of marine seismic data using Green's theorem. Deghosting is put into context in the complete M-OSRP processing chain, Green's theorem derived theory is presented, and an algorithm implementing the theory is discussed. The algorithm has been tested on field data and several kinds of synthetic data with positive and encouraging results.

Introduction

The inverse scattering series (ISS) can perform certain tasks (e.g., free surface multiple elimination) without a priori estimates of the spatial distribution of velocity. The Mission-Oriented Seismic Research Program (M-OSRP) has generated algorithms to accomplish seismic data processing goals based on the ISS (free surface multiple elimination, internal multiple removal, depth imaging, nonlinear direct AVO, and Q compensation) and Green's theorem (deghosting, source signature estimation, and data reconstruction). While the ISS is independent of subsurface velocity (and in fact of all subsurface properties), it is data dependent and makes certain assumptions about its input data. Weglein et al. (2003) describe how every ISS isolated task subseries requires (1) the removal of the reference wavefield, (2) an estimate of the source signature and radiation pattern, and (3) source and receiver deghosting and how the ISS has a nonlinear dependence on these preprocessing steps. Therefore, the Green's theorem deghosting methods are critically important to the success of the inverse series methods since they may be used to bring seismic data in line with the assumptions of inverse scattering. The fact that the ISS is nonlinear places a higher bar on preprocessing. An error in the input to a linear process creates a linear error in its output, but the same error in ISS input creates linear, quadratic, cubic, etc. errors in its output.

A brief aside on terminology. The total wavefield P consists of the reference wavefield P_0 (which doesn't experience the earth) and the scattered wavefield P_s (which does experience the earth). Ghosts begin their propagation moving upward from the source (source ghosts) or end their propagation moving downward to the receiver (receiver ghosts) or both (source/receiver ghosts) and have at least one upward reflection from the earth. Free surface multiples have at least one downward reflection from the free surface (air-water interface) and at least one upward reflection from the earth. Internal multiples have no downward reflections from the free surface, more than one upward reflection from the earth, and at least one downward reflection from inside the earth. An nth order internal multiple has n downward reflections from the earth. Primaries have only one upward reflection from the earth.

The freedom of choosing a convenient reference medium means Green's theorem offers a flexible framework for defining a number of useful algorithms — wavefield separation (reference and scattered), wavelet estimation, ghost removal, and two way wavefield continuation (RTM). Green's theorem methods are exact (fully consistent with the wave equation), multidimensional, make no assumptions about the earth, and work in the $(\omega, \mathbf{r})$ data space (and hence are simple to apply to irregularly spaced data). Therefore, Green's theorem preprocessing methods and ISS isolated task subseries are fully consistent.

This paper is focused on the deghosting prerequisite. Deghosting is important because (1) it is a prerequisite for many processing algorithms including data driven multiple elimination (ISS free surface multiples and internal multiples and SRME) and imaging (wavefield continuation often assumes one way waves), and (2) removing the downward component of the field enhances seismic resolution and boosts the low frequencies. Deghosting has benefit for both traditional seismic processing as well as providing an important role in all ISS based isolated task processing. For a discussion of the ISS see Liu et al. (2011).

Theory

The ISS is based on perturbation theory, and Green's theorem based preprocessing utilizes perturbation theory. A reference medium (and its associated Green's function) is chosen to facilitate solving the problem at hand, and the perturbation is the real world properties minus the reference medium. Within that framework, Green's theorem based preprocessing is remarkably wide ranging. For example, Fig. 1 shows the configuration chosen for Green's theorem deghosting. Choosing a reference medium consisting of a whole space of water, a hemispherical surface of integration bounded below by the measurement surface, and the prediction/observation point inside the surface of integration gives deghosted data $P^{deghosted}$. A different choice of a reference medium (a half space of air and a half space of water and the prediction/observation point outside/inside the surface of integration) gives wavefield separation in which the total wavefield P is separated into the reference wavefield P_0 and scattered wavefield P_s. It should be noted that several processing algorithms for multiple elimination (including the ISS) assume deghosting has been performed on the data and that an accurate estimate of the source wavelet is available.

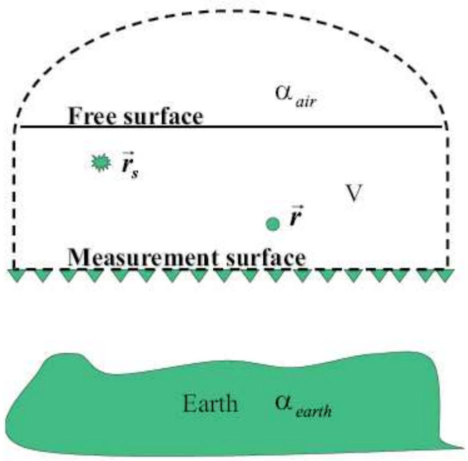

Fig. 1: *Configuration for deghosting using Green's theorem (Zhang, 2007). α_{air} and α_{earth} are perturbations, the differences between the actual medium (air, water, earth) and reference medium (water).*

Deghosting (both receiver and source side) is based on Zhang and Weglein (2005), Zhang and Weglein (2006), and Zhang (2007). The theory assumes measurement of the pressure wavefield P and its normal derivative $\partial P/\partial n \equiv \nabla P(\mathbf{r}', \mathbf{r}_s, \omega) \cdot \hat{n}$ where $\mathbf{r}'$ is the measurement point, $\mathbf{r}_s$ is the location of the air gun array, and $\hat{n}$ is the normal to the measurement surface. The reference medium is a whole space of water (where a causal, analytic solution exists for the acoustic wave equation) and the prediction point is between the free surface and the measurement surface, i.e., inside the volume V bounded by the closed surface of integration consisting of the measurement surface and dashed line in Fig. 1.

Using the above configuration and Green's theorem gives the key equation

$$P^{deghosted}(\mathbf{r}, \mathbf{r}_s, \omega) = \oint_S dS\, \hat{n} \cdot \tag{1}$$
$$[P(\mathbf{r}', \mathbf{r}_s, \omega) \nabla' G_0^d(\mathbf{r}, \mathbf{r}', \omega) - G_0^d(\mathbf{r}, \mathbf{r}', \omega) \nabla' P(\mathbf{r}', \mathbf{r}_s, \omega)],$$

(Zhang, 2007, Eq. (2.23)) where $\mathbf{r}$ is the prediction point, $\mathbf{r}_s$ is the location of the air gun array, S is the closed surface consisting of the measurement surface and dashed line in Fig. 1, $\hat{n}$ is the normal to S (pointing away from the enclosed volume V), $\mathbf{r}'$ is the measurement point, and G_0^d is a whole space Green's function. Extending the radius of the hemisphere to infinity, invoking the Sommerfeld radiation condition, and assuming a horizontal measurement surface, the integral over the closed surface becomes an integral over the measurement surface:

$$P^{deghosted}(\mathbf{r}, \mathbf{r}_s, \omega) = \int_{m.s.} dS \tag{2}$$
$$[P(\mathbf{r}', \mathbf{r}_s, \omega) \frac{\partial}{\partial z'} G_0^d(\mathbf{r}, \mathbf{r}', \omega) - G_0^d(\mathbf{r}, \mathbf{r}', \omega) \frac{\partial}{\partial z'} P(\mathbf{r}', \mathbf{r}_s, \omega)]$$

(Zhang, 2007, Eq. (2.24)). In 3D $G_0^d(\mathbf{r}, \mathbf{r}', \omega) = -1/(4\pi)\exp(ikR_+)/R_+$ where $k = \omega/c_0$, c_0 is the speed of sound in the reference medium, and $R_+ = |\mathbf{r} - \mathbf{r}'|$. In 2D $G_0^d(\mathbf{r}, \mathbf{r}', \omega) = -i/4\, H_0^{(1)}(kR_+)$ where $H_0^{(1)}$ is the zeroth order Hankel function of the first kind (Morse and Feshbach, 1953, pp. 810-811).

The implementation of the above theory is done in a straightforward manner. The Green's theorem algorithm computes the surface integral in Eq. (2). The method requires two wavefields as input, the pressure measurements P and their normal derivatives $\partial P/\partial z'$. The latter requires dual sensor cables or dual streamer cables. Source side deghosting is straightforward and amounts to applying reciprocity to exchange sources and receivers. Our illustrations will focus on receiver side deghosting.

Example: Flat layer model

The left panel of Fig. 2 shows synthetic data (produced using ray tracing in a flat layer model) designed so that deghosting is easy to demonstrate. The depth of the receivers is chosen such that primaries and ghosts appear as distinct seismic events. The right panel of Fig. 2 shows Green's theorem output using Eq. (2; note the primary's receiver side ghost at 0.45s and the free surface multiple's receiver side ghost at 0.85s are suppressed. Fig. 3 shows the spectra of the input data (blue) and receiver side deghosted output (red). As expected, the receiver side deghosted data fills in notches related to receiver ghosts.

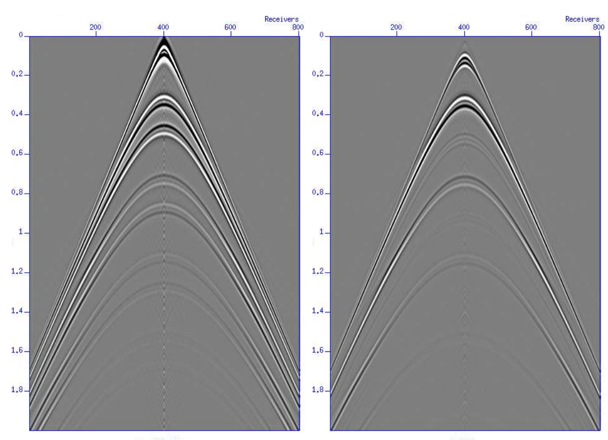

Fig. 2: *Flat layer model (source at 30m, cable at 140m, water bottom at 300m): input data at 110m (left), receiver side deghosted at 100m (right).*

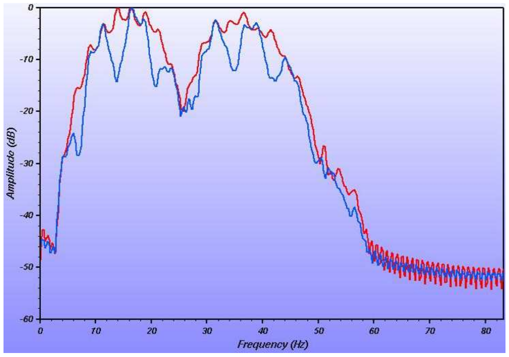

Fig. 3: *Flat layer model: muted input data (blue), receiver side deghosted (red). The receiver notches (at intervals of 5.4Hz) have been filled in; the notch at 25Hz is a source notch.*

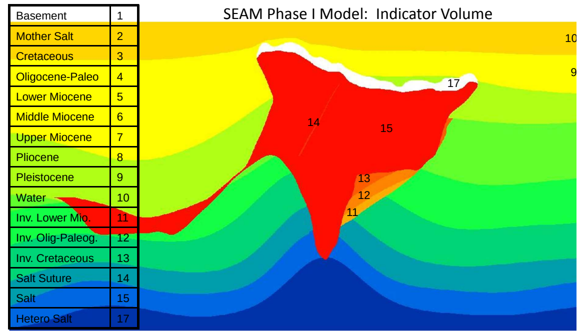

Fig. 4: *SEAM deepwater Gulf of Mexico model: inline section from the middle of the model. Figure courtesy of SEAM.*

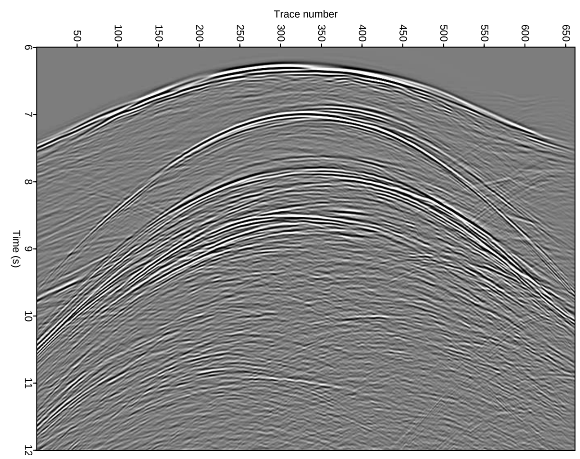

Fig. 5: *SEAM data, shot 130305 (located at sx=16975m, sy=20000m, sz=15m near center of shot grid).*

Example: SEAM application

We applied Green's theorem to the SEAM dataset generated based on a deepwater Gulf of Mexico earth model (Fig. 4) (Society of Exploration Geophysicists, 2011). We used the special SEAM classic dataset modeled to simulate dual sensor acquisition by recording the pressure wavefield at two different depths, 15 and 17m respectively. This dual sensor data consisted of nine sail lines for an equivalent wide azimuth towed streamer survey. The source interval is 150m by 150m while the receiver interval is 30m in both inline and crossline directions. Fig. 5 displays a typical shot gather from the SEAM model. Given the low frequency of the data (less than 30Hz) and the source and receiver depths of 15m and 17m, the ghost reflections are not as separable as in the previous flat layer model. In this situation, successful deghosting would correspond to a change in the wavelet shape. Fig. 6 shows SEAM input (a window of Fig. 5) and receiver side deghosted output computed by the Green's theorem approach. In the right panel of Fig. 6, note the collapsed wavelet. In Fig. 7, note the increased amplitude in lower frequencies and decreased amplitude in higher frequencies, i.e., the shift of the amplitude spectrum towards low frequencies. This is supported by the integral formulation in Eq. (2) which acts like a low pass filter.

Example: Field data

We also applied the deghosting approach to a field survey from the deep water Gulf of Mexico. The data were acquired using dual sensor streamers comprised of hydrophones and vertical geophones. The left panel in Fig. 8 shows a close up on an input shot record while the right panel displays the same traces after receiver side deghosting. Note the collapsed wavelet in the output image. This is also demonstrated in Fig. 9 that compares the amplitude spectra before and after deghosting. As expected, the deghosting solution successfully removed the notches from the spectrum that are associated with the receiver ghost.

Conclusions

We have implemented deghosting based on Green's theorem and have tested the algorithm on field data and several kinds of synthetic data. Testing to date has shown the algorithm works as expected.

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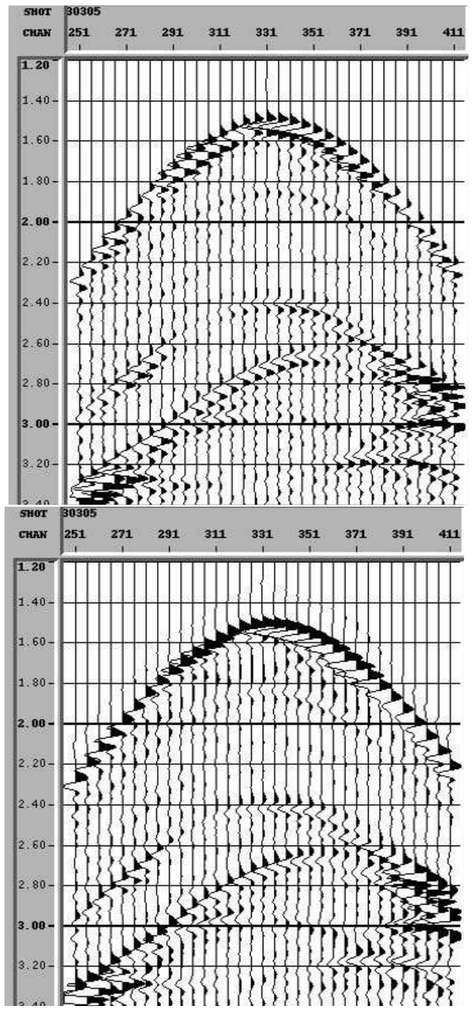

Fig. 6: *SEAM data, shot 130305: recorded data (top), receiver side deghosted at free surface (bottom). Note the collapsed wavelet in the bottom panel.*

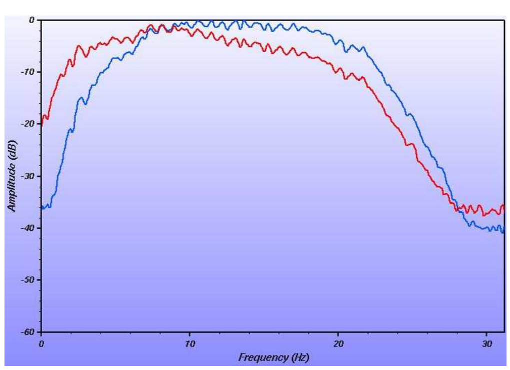

Fig. 7: *SEAM data: shot 130305: recorded data (blue), receiver side deghosted (red). Note the shift of the spectrum towards lower frequencies (first receiver notch is at 50 Hz).*

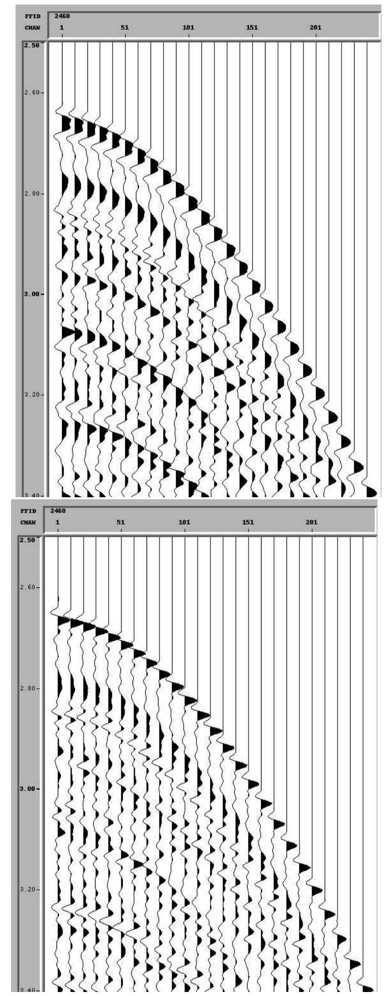

Fig. 8: *Field data: hydrophones (top), receiver side deghosted at free surface (bottom). Note the collapsed wavelet in the bottom panel. Input data courtesy of PGS.*

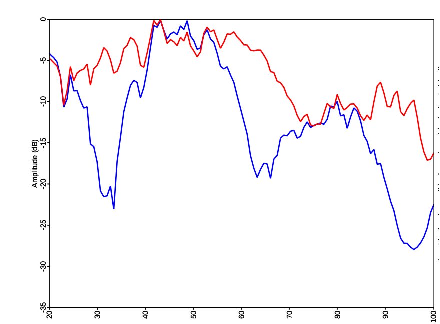

Fig. 9: *Field data: muted hydrophones (blue), receiver side deghosted (red). The receiver notches around 30 Hz, 60 Hz, and 90 Hz have been filled in. Input data courtesy of PGS.*

EDITED REFERENCES

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