

The inverse scattering series depth imaging algorithms: development, tests and progress towards field data application

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SUMMARY

The status of inverse scattering series (ISS) direct depth imaging algorithmic development and strategy is reviewed, and the recent progress towards a field data ready algorithm and capability is described. The progress to report towards field data test readiness relates to two distinct issues: (1) addressing the need for an adequate earth model type in the ISS depth imaging algorithms, where use of both event amplitude and phase in the ISS is required for a direct velocity independent structure-only map, and (2) the practical necessity of addressing certain data limitations, in particular band-limited data. This overview spans several sub-projects within the M-OSRP imaging project, and hence, only a sampling of results are presented here, and details are in reports and cited papers. Our goal in this paper is to provide some sense of issues that were faced and overcome, in this fundamental directed research project and why we feel that launching the first in a series of field data tests is now indicated.

INTRODUCTION / BACKGROUND

The inverse scattering series (ISS) allows for all processing objectives (including removing multiples and depth imaging) to be achieved directly and without subsurface information. It is essential to understand the significance of the term 'direct' in 'direct depth imaging'. Given an accurate velocity model, all current leading-edge imaging methods (e.g., Kirchhoff, FK, Beam and RTM) are able to directly output the depth (actually the spatial configuration) of reflectors. In the same 'direct' sense, ISS imaging algorithms can directly output the spatial configuration without the velocity model. It is the only method with that capability.

All currently used direct imaging and indirect imaging concepts believe that depth and velocity are inextricably linked.

What about 'indirect methods' for depth imaging? Indirect imaging methods (e.g., flat common image gathers, differential moveout, CFP, CRS and path integral approaches) seek to satisfy a property or condition that an image with an accurate velocity would satisfy. Those properties are necessary conditions, but not sufficient, and hence satisfying the indirect proxy for an adequate velocity model is not equivalent to knowing the velocity and direct depth imaging. Most importantly, they fervently believe that a direct depth imaging method would require and demand a velocity model, and that there is absolutely no way around it, and that depth and velocity are inextricably connected. That thinking is clear, and 100% correct within the framework of current imaging concepts.

However, that thinking is limited and challenged within the new and superseding framework for imaging provided by the ISS.

The ISS subseries for direct depth imaging communicates that depth and velocity are not inextricably linked.

ISS provides a new superseding theory that views the current velocity-depth relationship and framework as a special limiting case, as quantum mechanics and relativity view classical physics as a limiting and special case, within a new comprehensive and broader platform and framework. The new broader framework for imaging reduces to current imaging thinking when your velocity model is adequate, and most amazingly it determines on its own whether for any particular data set, or portion of your data set, whether the new framework is needed, or whether the current imaging framework will suffice. The new imaging framework determines if its services are called upon, and only if indicated, and then activates the new framework terms and calls them into action.

All current leading edge migration methods, such as, beam, Kirchhoff and RTM, are linear. The ISS direct depth imaging without the velocity algorithm is a non-linear relationship between data and the wavefield at depth.

WE OFTEN HEAR "HOW CAN DEPTH IMAGING WITHOUT VELOCITY BE POSSIBLE, LET ALONE TRUE? AND IF IT'S TRUE, THEN I STILL DON'T BELIEVE IT!"

If one can answer how the ISS acts in removing free surface and internal multiples without subsurface information, then the answer to the depth imaging question is exactly the same logic and response. Free surface and internal multiple removal 'understanding' using ISS was incomprehensible and counterintuitive and was equally resisted when first introduced in the early 1990's. Today the imaging subseries is early, embryonic and soon to have its first field data test, in 2010. It's one and the same starting point and one logic, from one set of equations for multiple removal and imaging, see e.g., Weglein et al. (2003).

INVERSE SCATTERING SERIES DIRECT DEPTH IMAGING

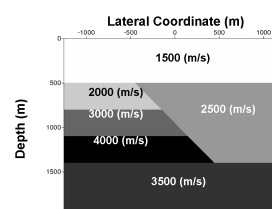


Figure 1: The geological model used to test the imaging algorithm in Fig. (2) and Fig. (3).

Model type and ISS imaging subseries:

Direct depth imaging without velocity

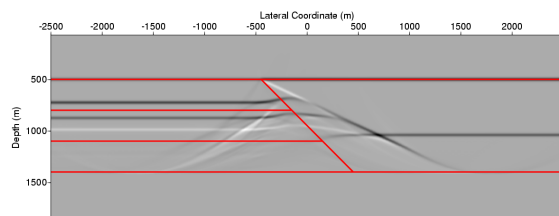


Figure 2: The linear image (water speed migration). The fault is vertically and horizontally mislocated. The red lines show the actual location of each reflector (Liu and Weglein, 2010).

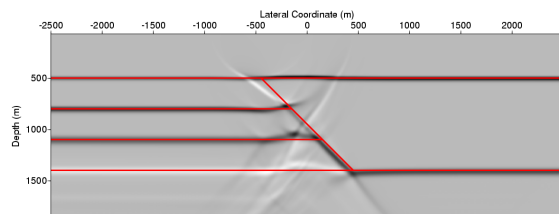


Figure 3: The HOIS inverse scattering imaging result produced the actual spatial location (shown by red lines) of the fault using a single constant water velocity migration as the input to HOIS (Liu and Weglein, 2010).

The earlier free surface and internal multiple developments were first examined and tested in simple acoustic models before being extended and ultimately applied in their current model-type independent forms. A model-type independent algorithm doesn't change when you assume the earth is e.g., acoustic, elastic or inelastic. The M-OSRP fundamental research projects on depth imaging, target identification and Q compensation each pass through the same stages that we went through in the earlier work on multiples. At some point, we decide that the new formalism has passed tests with sufficient synthetic data model realism to warrant a field data test. At this moment, we have not developed a model-type independent inverse scattering series depth imaging algorithm. Therefore, in the steps leading to field data testing within the ISS imaging project it is reasonable to ask what specific type of model data success would point to the possibility of a successful test with field data.

In seismic exploration, we want a minimally complicated model adequate to achieve our predictive purposes, but not too simple to be unrealistic, and misleading or harmful, nor too complicated to be more than necessary to reach E&P goals.

ISS imaging in a velocity only varying earth

Among the basic assumptions behind all traditional and currently applied seismic depth imaging methods are: (1) the ability to determine an adequate velocity model, and (2) that an imaging algorithm is available to adequately back-propagate waves through that medium, where only the velocity is changing. If the velocity configuration is achievable, and your goal is a structure map, then thinking about (and then worrying about) additional subsurface properties such as e.g., density is completely unnecessary and a more complicated model than

needed for the goal of structure determination.

Since the goal of our M-OSRP imaging project is to use the inverse scattering series for locating structure (without concern for the amplitude of the image), and without knowing *any* subsurface properties, it was reasonable to begin with an earth model that only allowed variation in velocity, and where the velocity would be unknown, and never would be estimated or determined. In that thinking, no other physical property other than velocity needs to be considered. Our inverse scattering imaging subseries with different degrees of imaging capture and capability (Weglein et al., 2000, 2002, 2003), LOIS (Shaw et al., 2003a,b; Innanen, 2003), HOIS (Liu, 2006) followed that path and reasoning. LOIS and HOIS refer to leading and higher order imaging subseries, respectively. The thinking went as follows: 'if having the velocity was adequate to determine depth, then assuming you didn't know the velocity would be adequate for an inverse scattering series method to directly determine structure without subsurface information'. Tests in 2008-2009 with Fang Liu's multi-D ISS HOIS imaging algorithm (Liu, 2006) were successful and encouraging on complex synthetics corresponding to fault shadow and presalt challenges, with large velocity contrasts, and without knowing (or determining) the velocity model. In Figures 1, 2 and 3, we show a fault shadow zone model, the single water speed FK Stolt migration, and F. Liu's closed form HOIS image. Figure 3 takes 30% more time than the single water speed FK migration of Figure 2. In other words the ISS depth imaging result is very fast, essentially the cost of a single water speed FK migration. These effective and efficient results produced an announced plan in 2008-2009 to test Fang Liu's HOIS algorithm on 3D marine field data. What happened?

1. ISS imaging depends on non-linear communication between reflections from different reflectors.
2. Density changes can often by themselves cause seismic reflections.
3. In the nonlinear ISS communication with reflections that correspond to only density changes have no bearing on a structure only ISS objective and those reflections need to be excluded from ISS imaging conversations and algorithms.

ISS imaging in a velocity and density varying earth

In the LOIS and HOIS imaging theories, the velocity was the only property that varied and was unknown, and hence the model could not accommodate density only reflections. The need to eliminate reflections due to only density changes required a generalization of the velocity only imaging theories. That extension allowed ISS imaging methods in velocity and density varying media in what is called the 'imaging conjecture' (Weglein (2008), Page 1~8). The imaging conjecture uses the strength of a multi-parameter inverse series to exclude density only variations from the multiplicative conversations of event amplitudes and phases, towards a structure only map, while avoiding the new and daunting issues of multi-parameter inverse of seeking several separate images, one for each parameter, with new complications due, e.g., to linear inverse

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leakage, that don't exist in the earlier one parameter velocity only varying earth model and experience of LOIS and HOIS. A single collective image of a reflectivity like quantity is output as part of the imaging conjecture. Zhang et al. (2007) confirmed the conjecture for the elastic case. Initial synthetic tests of the conjecture were carried out with distinct front end and imaging issues that need to be examined, tested and analyzed. That analysis and testing began in Annual Report (2008) in separate reports of Li et al. (2008) and Jiang et al. (2008). The imaging conjecture has a multi-parameter front end that excludes density only reflections, and outputs reflectivity, sitting on top of a Fang Liu type of HOIS imaging algorithm engine.

In the three examples in this section, the earth model is one dimensional and the data is prestack PP data. $D(x, t)$ is the shot record and the water speed depth image is $D(z, \theta)$, where z is depth and $\theta = \sin^{-1} \left(\frac{k_x}{\omega/c_0} \right)$, where k_x , ω are the conjugates of x and t , respectively, and c_0 is the constant water speed reference velocity.

The development for this new velocity and density varying imaging theory progresses from simple 1D to complex multi-D tests, as in the earlier LOIS, HOIS velocity only varying world; a sampling of 1D tests are shown in this paper. There is only an imaging contribution required of the ISS if the actual velocity varies and the input velocity is assumed to be constant. We show in Figure 4 an elastic model example where the V_p and V_s and density all vary. In Figure 5 and 6 we show the reflectivity imaged with reference velocity and the ISS imaging conjecture for reflectivity, respectively. There is no imaging contribution for ISS when only density varies across the first reflector and the ISS result agrees with the constant velocity image (Li and Weglein, 2010; Liang et al., 2010).

An unexpected AVO by-product resulted from the imaging conjecture, with its automatically outputting a reflectivity like output as a function of angle at the correct depth and a flat common image gather, and without a velocity model, and without a trim mean filter to force the flat common image gather. The latter reflectivity like CIG flat output is an intermediate result before AVO analysis. In practice, the zero crossing defining type 1 and type 2 AVO targets can be lost by the combination of velocity analysis and "ironing". This opportunity and tool developed from the imaging conjecture, for automatic flat common image gathers of a reflectivity like output without the need for ironing, and without ironing away polarity reversals. This followed a suggestion by Doug Foster to examine this imaging conjecture output for type 1 and type 2 AVO targets and application. Early tests of that possibility are encouraging and an elastic test is presented below (Li and Weglein, 2010). The effectiveness of the conjecture has been extended to include ISS imaging terms beyond that initial imaging capability within a multi-parameter acoustic world (Wang et al., 2010). It will be extended to a multi-parameter elastic earth for field data application. The danger of using an inadequate model (e.g., acoustic model for an elastic earth data) for inversion purposes or ISS imaging has been studied by Liang et al. (2010). There are serious and significant implications for having a mismatch between the model of your data, and the model assumed in your processing algorithms, in both current conventional pro-

cessing and for so-called "full wave field inversion", as well as the new platform ISS imaging and inversion algorithms (see Figure 8). Our strategy for field data application recognizes that issue, using the minimally acceptable earth model for amplitude analysis, the isotropic elastic model. The plan is now for the imaging conjecture to be used for our first field data imaging test, and we expect that first set of tests this year.

In order to remove density only changing reflections and include only reflections corresponding to where p wave velocity and/or s wave velocity have changed, in ISS data conversations that relate to locating only structure, requires a linear data conversation front end that combines data at different angles. Issues that arise from band-limited data, and the need to exclude the latter density only changing reflections, with different band-limited data distortions at different angles, were addressed in a subtle and sophisticated regularization scheme developed for this purpose by Liu and Weglein (2010). The latter band-limited data regularization contribution is a significant conceptual and practical advance that is essential to maintain ISS efficacy on field data, and will allow the ISS imaging algorithms to reach their potential.

• Model

$$\begin{array}{l} \rho_0 = 2.27 \text{ g/cm}^3 \quad v_p = 2030 \text{ m/s} \quad v_s = 1020 \text{ m/s} \\ \hline \rho_1 = 2.32 \text{ g/cm}^3 \quad v_p = 2133.6 \text{ m/s} \quad v_s = 1122.88 \text{ m/s} \\ \hline \rho_2 = 2.24 \text{ g/cm}^3 \quad v_p = 2743.2 \text{ m/s} \quad v_s = 1828.8 \text{ m/s} \end{array}$$

$Z_1=5$
 $Z_2=1$

Figure 4: Elastic model with a type I AVO at the second reflector, and zero crossing in reflection coefficient happens at $\theta = 28.45^\circ$.

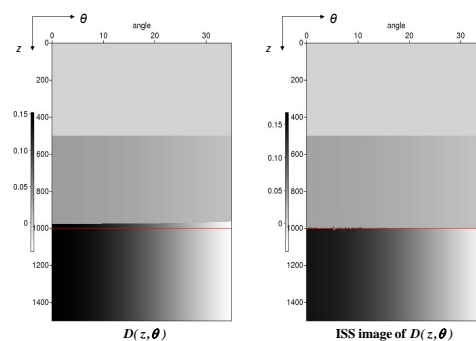


Figure 5: Imaging results: the figure on the left shows data $D(z, \theta)$ imaged with reference P wave velocity; the figure on the right shows the result of inverse scattering series imaging algorithm.

To improve upon the imaging capture capability represented by the imaging conjecture, (arranged by including more imaging terms from the ISS), Wang et al. (2010) developed a beyond conjecture imaging algorithm. The added value of the beyond conjecture contribution will appear when the difference between the actual velocity model and the reference is

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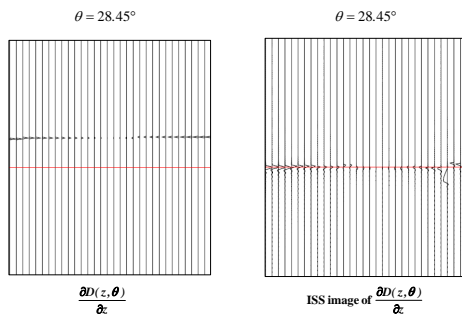


Figure 6: The picture on the left shows $\frac{\partial D(z, \theta)}{\partial v}$ imaged with reference P wave velocity; the picture on the right shows the corresponding inverse scattering series imaging result. Again, we only enlarge the part at second reflector that has zero crossing information. After inverse scattering imaging algorithm, the zero crossing information is preserved at depth.

large. In Figure 7, we see FK constant velocity migration, conjecture imaging and beyond conjecture imaging results. Note that the latter improves the location of the second reflector and the range of angles where the image is reliable. This will be extended to elastic media before field data imaging application/test.

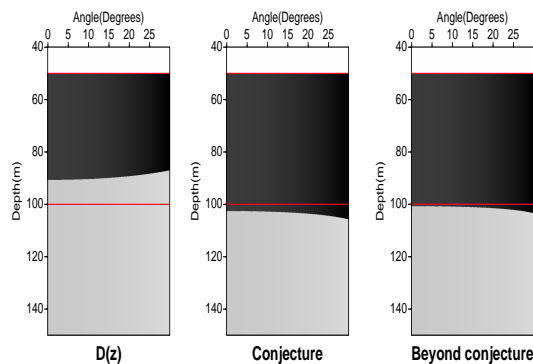


Figure 7: The comparison of 'the conjecture' and 'beyond conjecture' using a three-layer model with large contrasts in properties: $v_0 = 1500 \text{ m/s}$, $\rho_0 = 1.0 \text{ g/cm}^3$; $v_1 = 1850 \text{ m/s}$, $\rho_1 = 1.1 \text{ g/cm}^3$; $v_2 = 1350 \text{ m/s}$, $\rho_2 = 1.2 \text{ g/cm}^3$. The left one is the imaging result of $D(z, \theta)$ using FK constant velocity migration; the middle one is the result of 'the conjecture'; the right one is the result of 'beyond conjecture' imaging capture.

Liang et al. (2010) have studied the issue of model type match and model type mismatch between the data and the ISS imaging and parameter estimation algorithms.

In figure 8, Liang et al. (2010) show that when the earth model is elastic, the elastic imaging algorithm has good capability, but the acoustic conjecture imaging algorithm does not. The acoustic conjecture imaging treats the data as though only the density and P wave velocity varied in the medium, whereas the elastic conjecture allows the P and S velocities and density to all vary. Liang et al. (2010) illustrate how, in general and for ISS imaging and inversion applications, it's important to

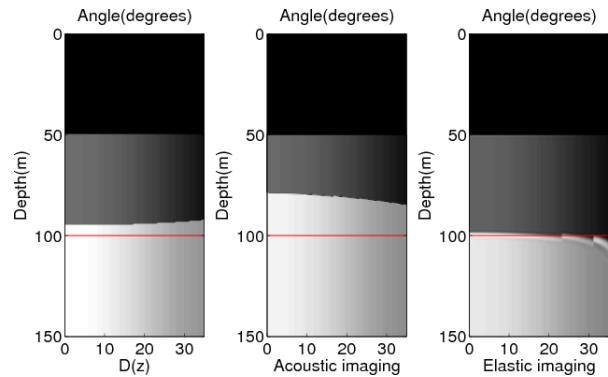


Figure 8: Imaging comparison: the left figure is the result of constant velocity migration, the middle one is acoustic conjecture imaging, and the right one is the result of elastic conjecture imaging. The parameters for the elastic data model are: $\rho_0 = 2.3$, $\rho_1 = 2.4$, $\rho_2 = 2.5$; $v_{p0} = 2700$, $v_{p1} = 3000$, $v_{p2} = 3500$; $v_{s0} = 1500$, $v_{s1} = 1800$, $v_{s2} = 2000$; the depths of reflectors are 50m and 100m; the red line in the figure indicates the exact location of the deeper reflector.

match the processing algorithm's model type to the model that generated the data.

CONCLUSIONS

We have significant conceptual and practical progress to report in the campaign to bring the inverse scattering series direct velocity independent depth imaging algorithms towards field data application. This paper presented an overview of the status of different initiatives within the ISS imaging project and plans for field data tests. The initial objective of the ISS imaging project was a depth accurate structure map. However, while the need to include velocity and density in the theory set us back from a planned field data test in 2009, the resulting extended ISS imaging theory has delivered an AVO by-product beyond the ambition of the initial structure only imaging objective. We are currently soliciting field data examples from our sponsors.

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EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2010 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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