Using Green's theorem to satisfy data requirements of multiple removal methods: The impact of acquisition design

Lin Tang, James D. Mayhan, Jinlong Yang, and Arthur B. Weglein, M-OSRP, University of Houston*

SUMMARY

The freedom of choosing a convenient reference medium means Green's theorem offers a flexible framework for deriving a number of useful algorithms. Methods that can be derived from Green's theorem include: separation of reference and scattered wavefields ($P = P_0 + P_s$), wavelet estimation, and ghost removal. Green's theorem preprocessing methods are fully consistent with the inverse scattering series (ISS) isolated task subseries processing because Green's theorem wave separation methods are multidimensional and make no assumptions about the earth. The ISS multiple removal algorithms require their input data to be deghosted and to have an estimate of the wavelet. We discuss the effect of acquisition design on Green's theorem for predicting P_0 and P_s and for deghosting.

INTRODUCTION

Preprocessing of seismic data, including removal of reference waves, wavelet estimation, and removal of ghosts, is very important in seismic data processing. The reference wave does not experience reflection from the earth, which is our ultimate objective, so it should be removed before subsequent analysis. Seismic data are affected by both the acquisition signature and the properties of the earth. Thus, we need to identify and remove the wavelet's contribution from the seismic data (Weglein and Secrest (1990)). Deghosting will remove the down-going wave from the scattered wave and will enhance the low-frequency content of the data (Mayhan et al., 2011, 2012; Mayhan and Weglein, 2013). These are the prerequisites of the following steps of multiple removal and depth imaging in the Inverse Scattering Series (ISS) algorithm (Weglein et al. (2003)). All three of these processing steps can be achieved by using Green's theorem. In Weglein and Secrest (1990), wave separation and wavelet estimation by using Green's theorem are discussed. By performing an integral along the measurement surface, we can predict the reference wave or the scattered wave, depending on the choice of observation point. Green's theorem can work in multiple dimensions and is especially effective in the case of interfering events, compared with other methods such as simply muting the direct wave from the data, which can lead to the loss of long offset wave information. By the way, the thing we call a wavelet is the factor that separates what we are measuring P_0 vs. a Green's function G_0 and hence includes the source signature and the instrument response, that we could call the acquisition wavelet. In the methods described below, the factor $A(\omega)$ is actually the acquisition wavelet.

In this paper we focus on preparing the data for the subsequent multiple removal steps, including removing reference wave, estimating wavelet and deghosting. The effect of acquisition design on wave separation when using an over/under cable is discussed. The necessity of deghosting for free surface multiple removal will be shown.

THEORY

In scattering theory, we treat the actual medium as a combination of an unperturbed medium, called the reference medium, and a perturbation. Correspondingly, the total measured wavefield P is the summation of the reference wave P_0 and the scattered wave P_s. P_0 does not experience the earth, which is our interest, thus we need to remove it before further processing and analysis. In the marine environment, for the purpose of separating P_0 and P_s, we choose as the reference medium a half-space of water plus a half space of air. Overlaying the reference medium are two sources, the air guns and the earth, which create the measured wavefield P, where $P = P_0 + P_s$. Combining the wave equations for P_0 and the corresponding Green's function G_0 in the reference medium and Green's second identity, we can have the equations for Green's theorem wave separation. When choosing the observation point below the measurement surface, we have the reference wave

$$P_0(\mathbf{r},\mathbf{r_s},\omega) = \int_{m.s.} dS\hat{\mathbf{n}}\cdot \left[P(\mathbf{r}',\mathbf{r_s},\omega)\nabla' G_0(\mathbf{r}',\mathbf{r},\omega) - G_0(\mathbf{r}',\mathbf{r},\omega)\nabla' P(\mathbf{r}',\mathbf{r_s},\omega)\right]. \tag{1}$$

In addition, using the information of reference wave P_0, we can solve for the wavelet

$$A(\omega) = \frac{P_0(\mathbf{r},\mathbf{r_s},\omega)}{G_0(\mathbf{r},\mathbf{r_s},\omega)}. \tag{2}$$

When choosing the observation above the cable, Green's theorem will give us the scattered wave P_s,

$$P_s(\mathbf{r},\mathbf{r_s},\omega) = \int_{m.s.} dS\hat{\mathbf{n}}\cdot \left[P(\mathbf{r}',\mathbf{r_s},\omega)\nabla' G_0(\mathbf{r}',\mathbf{r},\omega) - G_0(\mathbf{r}',\mathbf{r},\omega)\nabla' P(\mathbf{r}',\mathbf{r_s},\omega)\right]. \tag{3}$$

From the above equations, we can see that given the wavefield P and its normal derivative P_n on the measurement surface, we can easily calculate the reference wave P_0 and the scattered wave P_s, depending on the observation point we choose. In other words, the reference wave and the scattered wave are separated by using Green's theorem.

Green's theorem receiver deghosting is carried out via

$$P_R'(\mathbf{r}_g',\mathbf{r}_s,\omega) = \int_{\text{m.s.}} dS\,\hat{\mathbf{n}}\cdot [P(\mathbf{r},\mathbf{r}_s,\omega)\nabla G_0^+(\mathbf{r},\mathbf{r}_g',\omega) - G_0^+(\mathbf{r},\mathbf{r}_g',\omega)\nabla P(\mathbf{r},\mathbf{r}_s,\omega)], \tag{4}$$

DOI http://dx.doi.org/10.1190/segam2013-1083.1

where $P(\mathbf{r},\mathbf{r}_s,\omega)$ is the measured pressure wavefield, $G_0^+(\mathbf{r},\mathbf{r}'_g,\omega)$ is a whole space, causal Green's function, $\mathbf{r}_s$ is the source location, $\mathbf{r}$ is a receiver location, $\mathbf{r}'_g$ is the prediction point, and the integration is over the measurement surface for a common shot gather (Weglein et al., 2002; Zhang and Weglein, 2005, 2006; Zhang, 2007). The input (measurements of P and its normal derivative) requires over/under cables. Similarly, Green's theorem source deghosting uses

$$P'_{SR}(\mathbf{r}'_g,\mathbf{r}'_s,\omega) = \int_{\text{sources}} dS\,\hat{\mathbf{n}}\cdot \left[P'_R(\mathbf{r}'_g,\mathbf{r},\omega)\nabla G_0^+(\mathbf{r},\mathbf{r}'_s,\omega) - G_0^+(\mathbf{r},\mathbf{r}'_s,\omega)\nabla P'_R(\mathbf{r}'_g,\mathbf{r},\omega)\right], \quad (5)$$

where the input (P'_R and its normal derivative) requires over/under sources, and the integration is for a common receiver gather. If we have a single cable measuring P and we can estimate the isotropic wavelet $A(\omega)$, we can receiver deghost by first computing

$$P'(\mathbf{r}'_g,\mathbf{r}_s,\omega) = A(\omega)G_0^{DD}(\mathbf{r}'_g,\mathbf{r}_s,\omega) + \int_{\text{m.s.}} dS\,\hat{\mathbf{n}}\cdot P(\mathbf{r},\mathbf{r}_s,\omega)\nabla G_0^{DD}(\mathbf{r},\mathbf{r}'_g,\omega) \quad (6)$$

$$\frac{\partial P'}{\partial z'_g}(\mathbf{r}'_g,\mathbf{r}_s,\omega) = A(\omega)\frac{\partial G_0^{DD}}{\partial z'_g}(\mathbf{r}'_g,\mathbf{r}_s,\omega) + \int_{\text{m.s.}} dS\,\hat{\mathbf{n}}\cdot P(\mathbf{r},\mathbf{r}_s,\omega)\nabla \frac{\partial G_0^{DD}}{\partial z'_g}(\mathbf{r},\mathbf{r}'_g,\omega). \quad (7)$$

Equation 7 is the derivative of equation 6, and G_0^{DD} is a "double Dirichlet" Green's function constructed to vanish on both the free surface and measurement surface (Osen et al., 1998; Tan, 1999). In this case, the outputs of equations 6 and 7 are the inputs to equation 4. Similarly, in the absence of over/under sources, source deghosting is accomplished by substituting the output of equation 4 into equations 6 and 7 (but without the terms containing the wavelet), then their outputs become the inputs to equation 5.

THE EFFECT OF ACQUISITION DESIGN

Green's theorem requires the wavefield P and its normal derivative P_n on the measurement surface as the input. In marine exploration, an over/under cable has been used to obtain data at two depths. Here we study some practical issues when performing Green's theorem for wave separation using over/under cable acquisition.

The depth difference between the cables

Since the wavefield P is the recorded data, the normal derivative needs to be calculated in the case of a geophone in the marine environment. When using an over/under cable, an easy way to calculate the normal derivative is to subtract the data of the upper cable from the data of the lower cable and then divide by their depth difference, i.e.,

$$\frac{dP(\frac{z_1+z_2}{2})}{dz} = \frac{P(z_2)-P(z_1)}{z_2-z_1}. \quad (8)$$

As the above equation shows, the normal derivative of P is at the depth $(z_1+z_2)/2$, rather than at z_1 or z_2, where wavefield P is measured. This mismatch may affect the wave separation results.

In our synthetic tests using the reflectivity method, we first used a 1D acoustic model with the source at 5m and two cables, one at a depth of 45m and one at 50m. (The cables were placed unrealistically deep to better illustrate the results.) Thus the two cables are separated by 5m. Using Green's theorem, the scattered wave P_s is predicted at 20m, and P_0 is predicted at 80m, as shown in Figure 1. Next, we reduced the depth difference between the two cables to 1m (one cable at 49m, the other at 50m), and in that case Green's theorem gives the predicted P_s at 20m and P_0 at 80m as shown in Figure 2. (The cables were placed unrealistically close again to better illustrate the results.) From these two results, we can clearly see that when the depth difference is 5m, as in Figure 1, there are several residuals left in both cases of P_0 and P_s, while in Figure 2, the predicted results are clean. This indicates that reducing the difference in cable depths can significantly increase the accuracy of wave separation results, since the depth of P_n now better matches with the depth of P in the Green's theorem integral.

The depth of the predicted wave

Other factors may affect the estimated results. The actual experiment shows that the choice of the predicted cable depth can change the quality of the result. Figure 3 shows the choice of different depths when predicting the scattered wave P_s. Here we define the depth difference between the predicted cable and the measurement surface as $\triangle z$. We also define the interval between traces as $\triangle x$. As we can see, the predicted result has many residuals when $\triangle z$ is very small compared with $\triangle x$. Only when $\triangle z$ is at least half of $\triangle x$ are the predicted results acceptable. Likewise, Figure 4 shows the predicted results of P_0 at different depths. We again got the similar conclusion that only when the depth difference between the predicted cable and the actual cable is larger than 1/2 of the interval between traces, does the predicted direct wave have few residuals.

DEGHOSTING SEAM DATA

We applied Green's theorem to the SEAM data set generated based on a deepwater Gulf of Mexico earth model (The SEG Advanced Modeling Corporation (SEAM), 2011). We used the special SEAM classic data set modeled to simulate dual sensor acquisition by recording the pressure wavefield at two different depths, 15 and 17m respectively. This dual sensor data consisted of nine sail lines for an equivalent wide azimuth towed streamer survey. Given the low frequency of the data (less than 30Hz) and the source and receiver depths of 15m and 17m, the ghost reflections overlap/interfere with non-ghost events, and successful deghosting would correspond to a change in the wavelet shape. The result is shown in Figure 5. In the right panel, we see there is no source notch to fill; the first source notch is at 44Hz which lies above the source frequency range (1–30Hz).

DOI http://dx.doi.org/10.1190/segam2013-1083.1

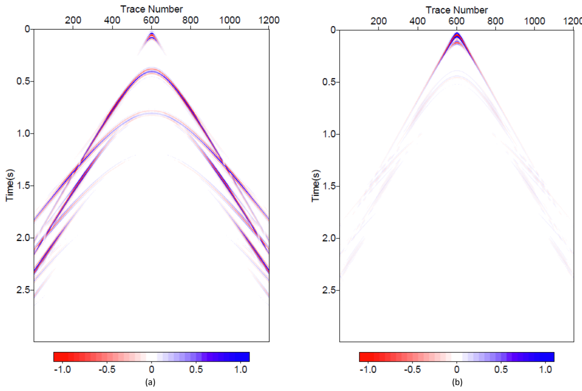

Figure 1: *Using an over/under cable with a 5m depth difference. (a) P_s predicted at 20m, (b) P_0 predicted at 80m.*

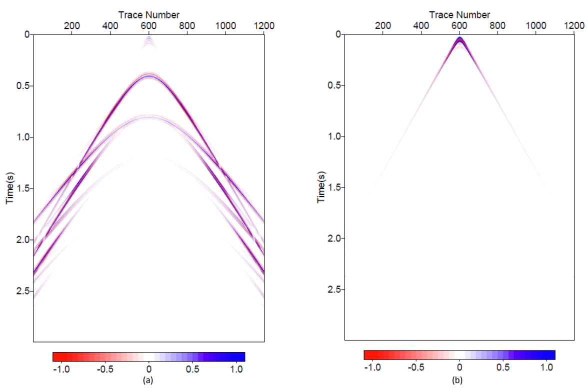

Figure 2: *Using an over/under cable with a 1m depth difference. (a) P_s predicted at 20m, (b) P_0 predicted at 80m.*

FREE SURFACE MULTIPLE REMOVAL

ISS free-surface multiple elimination method has the ability to predict accurately the phase and amplitude of multiples if its pre-requisites (wavelet and deghosted data) are satisfied. Figures 6(a) and 6(b) are the input data with and without ghosts, respectively. Inputting them into ISS free-surface multiple elimination algorithm, Figures 6(c) and 6(d) are their corresponding free-surface multiple predictions. After subtracting from the input data, Figures 6(e) and 6(f) show the results after free-surface multiple removal. If the input data are not deghosted, ISS free-surface multiple removal method can predict the exact phase but only approximate amplitude of multiples. After deghosting, we can see that all free-surface multiples are predicted exactly and through a simple subtraction; they are all well eliminated and most importantly primaries are not touched, as shown in Figure 6(f).

CONCLUSIONS

The ISS multiple removal algorithms require their input data to be deghosted and to have an estimate of the wavelet, each of which can be accomplished with distinct forms of Green's the-

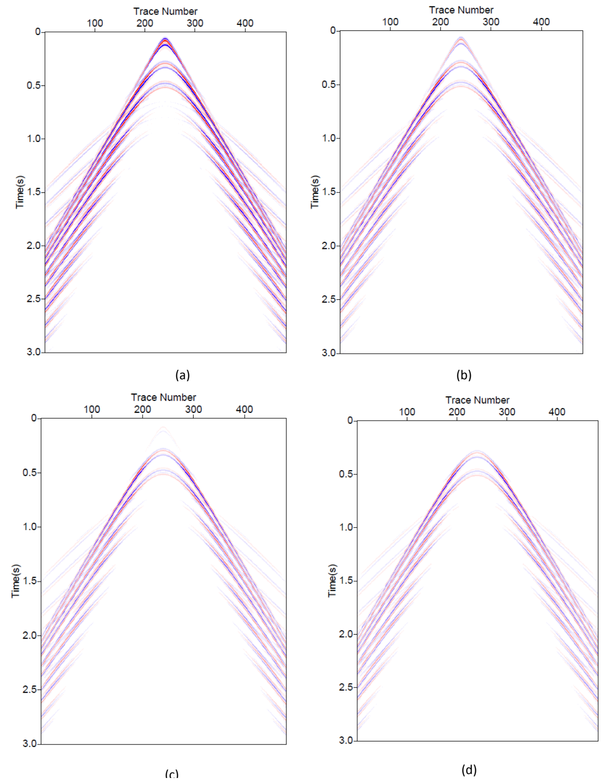

Figure 3: *Predicted P_s when: (a) $\triangle z$ =1/8 $\triangle x$, (b) $\triangle z$ =1/4 $\triangle x$, (c) 1$\triangle z$ =1/2 $\triangle x$, and (d)$\triangle z$ = $\triangle x$.*

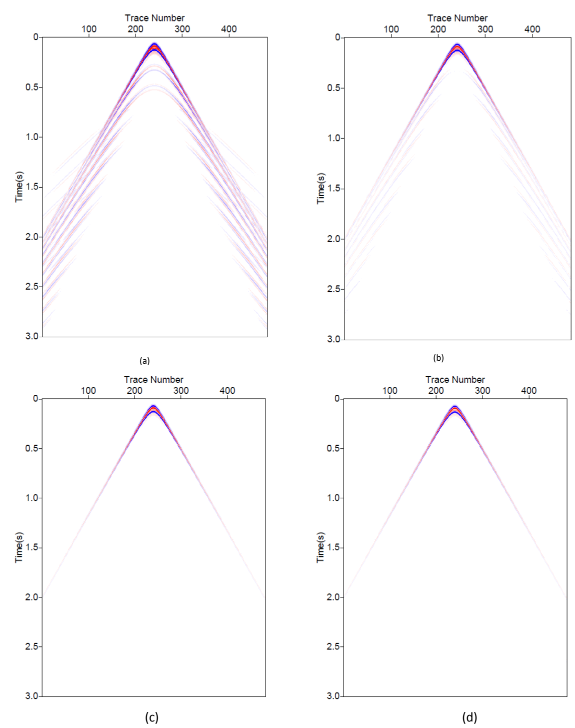

Figure 4: *Predicted P_0 when: (a) $\triangle z$ =1/8 $\triangle x$, (b) $\triangle z$ =1/4 $\triangle x$, (c) 1$\triangle z$ =1/2 $\triangle x$, and (d)$\triangle z$ = $\triangle x$.*

DOI http://dx.doi.org/10.1190/segam2013-1083.1

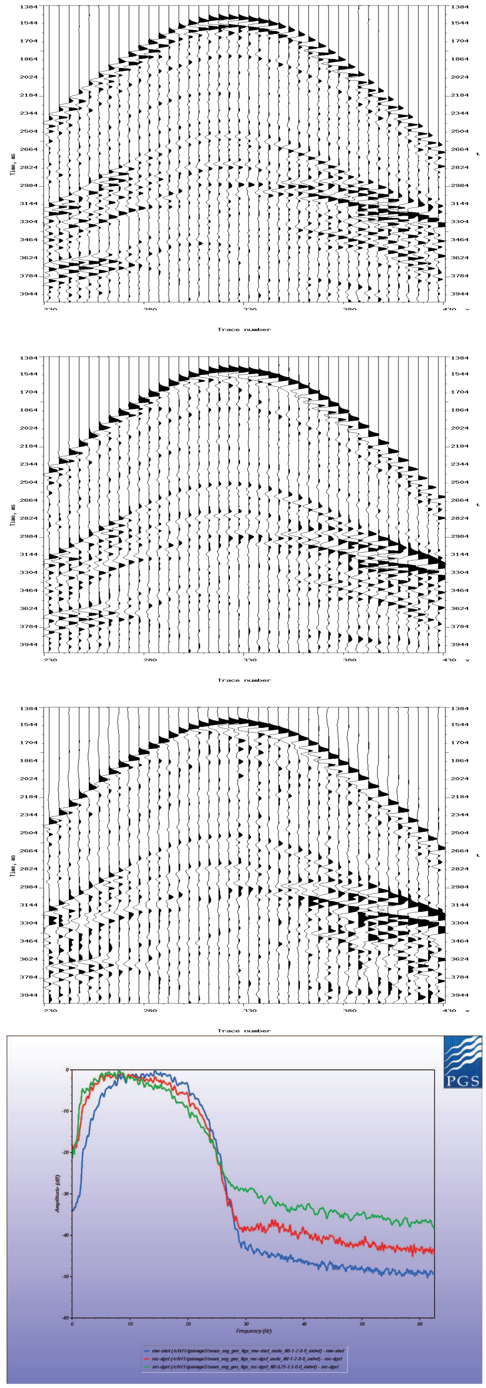

Figure 5: *SEAM data, shot 131373: recorded data at 17m (top panel), receiver deghosted at 10m (second panel), source and receiver deghosted at 10m (third panel). Note the collapsed wavelets in the second and third panels. Frequency spectra (bottom panel): red=P at 17m, blue=receiver deghosted at 10m, green=source and receiver deghosted at 10m. The spectrum uses a window of 201 traces (232-432) by 0.6s (1.4-2.0). The first source notch is at 44Hz which lies above the source frequency range (1–30Hz). Note the shift of the spectrum towards lower frequencies (which may be of interest to FWI).*

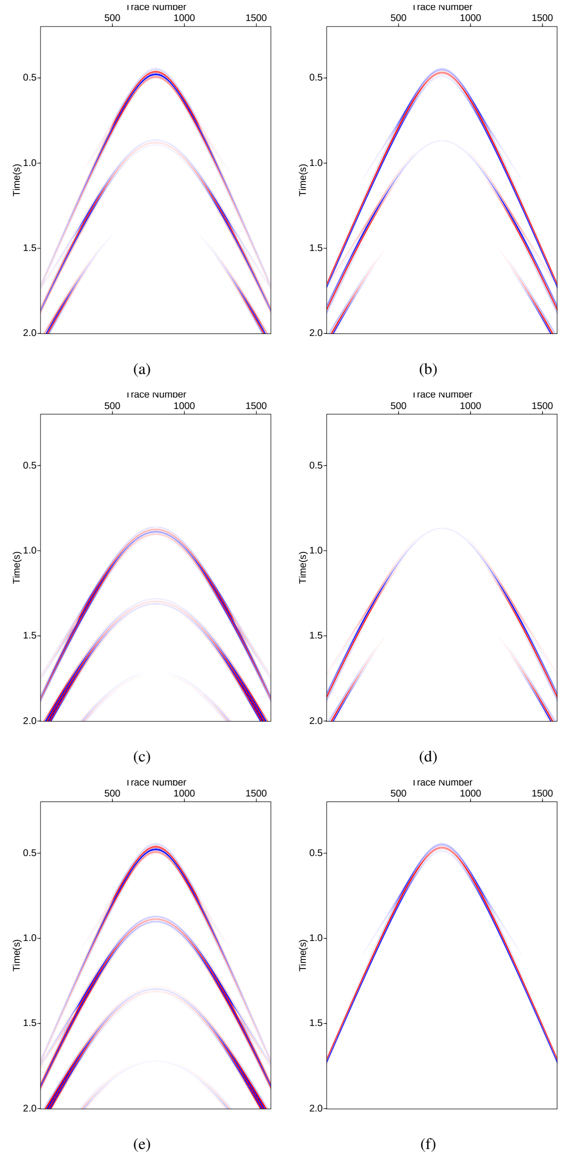

Figure 6: (a)&(b) Input data with and without ghosts; (c)&(d) corresponding free-surface multiple prediction; (e)&(f) After free-surface multiple removal through a simple subtraction.

orem. The effects of (1) the difference in the depth between the over cable and the under cable, and (2) the choice of the location of the predicted reference wave or scattered wave relative to the cable are studied. The tests show that to get useful wave separation results, the depth difference between the two cables should be quite small, and we choose to predict the wave at least 1/2 $\triangle x$ from the cable. The importance of deghosting before free surface multiple removal is also shown.

ACKNOWLEDGMENTS

We thank the M-OSRP sponsors for their support of this research. The second author also thanks PGS for permission to use SEAM data and Nizar Chemingui (PGS) for internships.

DOI http://dx.doi.org/10.1190/segam2013-1083.1

http://dx.doi.org/10.1190/segam2013-1083.1

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2013 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Mayhan, J. D., P. Terenghi, A. B. Weglein, and N. Chemingui, 2011, Green's theorem derived methods for preprocessing seismic data when the pressure P and its normal derivative are measured: 81st Annual International Meeting, SEG, Expanded Abstracts, 2722–2726.

Mayhan, J. D., and A. B. Weglein, 2013, First application of Green's theorem-derived source and receiver deghosting on deep-water Gulf of Mexico synthetic (SEAM) and field data: Geophysics, **78**, no. 2, WA77–WA89.

Mayhan, J. D., A. B. Weglein, and P. Terenghi, 2012, First application of Green's theorem derived source and receiver deghosting on deep water Gulf of Mexico synthetic (SEAM) and field data: 82nd Annual International Meeting, SEG, Expanded Abstracts, 1–5.

Osen, A., B. G. Secrest, and L. Amundsen, 1998, Wavelet estimation from marine pressure measurements: Geophysics, **63**, 2108–2119.

SEG Advanced Modeling Corporation (SEAM), 2011, The SEG advanced model: Technical Report, SEG, (http://www.seg.org/resources/research/seam).

Tan, T. H., 1999, Wavelet spectrum estimation: Geophysics, **64**, 1836–1846.

Weglein, A. B., F. V. Ararujo, P. M. Carvalho, R. H. Stolt, K. H. Matson, R. T. Coates, D. Corrigan, D. J. Foster, S. A. Shaw, and H. Zhang, 2003, Inverse scattering series and seismic exploration: Inverse Problems, **19**, R27–R83.

Weglein, A. B., and B. G. Secrest, 1990, Wavelet estimation for a multidimensional acoustic earth model: Geophysics, **55**, 902–913.

Weglein, A. B., S. A. Shaw, K. H. Matson, J. L. Sheiman, R. H. Stolt, T. H. Tan, A. Osen, G. P. Correa, K. A. Innanen, Z. Guo, and J. Zhang, 2002, New approaches to deghosting towed-streamer and ocean-bottom pressure measurements: 72nd Annual International Meeting, SEG, Expanded Abstracts, 1016–1019.

Zhang, J., 2007, Wave theory based data preparation for inverse scattering multiple removal, depth imaging and parameter estimation: Analysis and numerical tests of Green's theorem deghosting theory: Ph.D. thesis, University of Houston.

Zhang, J., and A. B. Weglein, 2005, Extinction theorem deghosting method using towed streamer pressure data: Analysis of the receiver array effect on deghosting and subsequent free surface multiple removal: 75th Annual International Meeting, SEG, Expanded Abstracts, **24**, 2095–2098.

Zhang, J., and A. B. Weglein, 2006, Application of extinction theorem deghosting method on ocean bottom data: 76th Annual International Meeting, SEG, Expanded Abstracts, **25**, 2674–2678.