Internal Multiple Removal in Offshore Brazil Seismic Data Using the Inverse Scattering Series

Master Thesis

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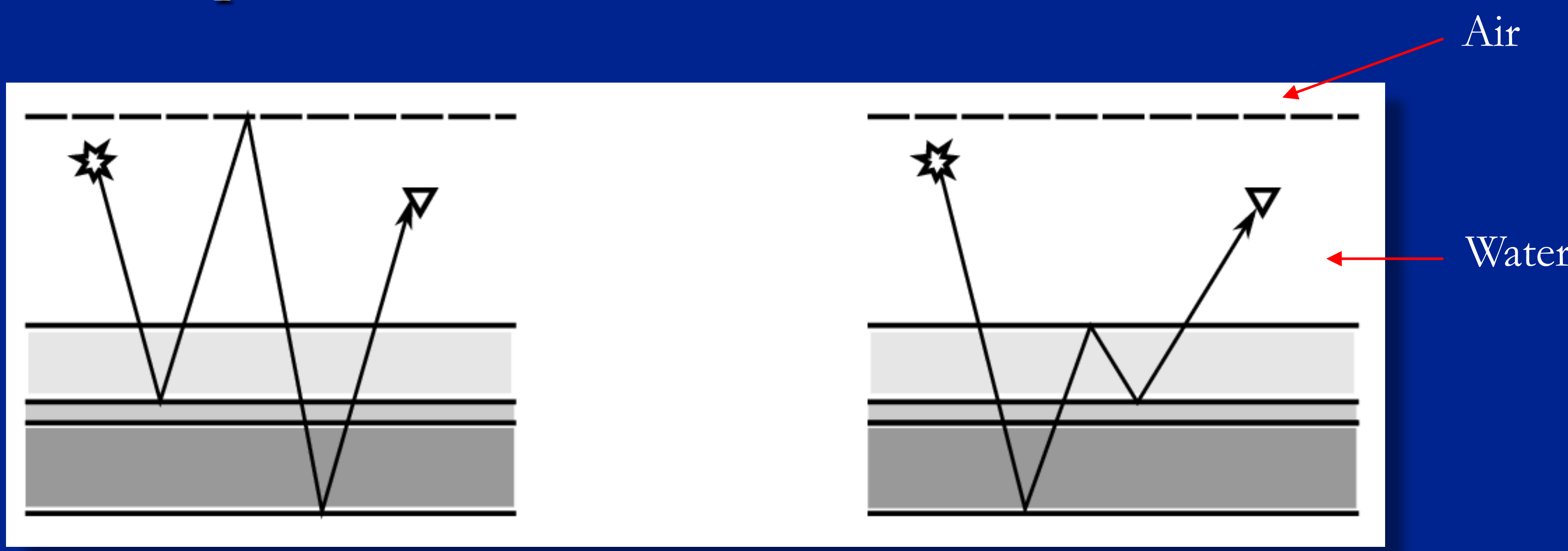
Outline

- Introduction and problem
- Objectives
- Theory
- Data processing
- Multiple attenuation
- Conclusion

Introduction and problem

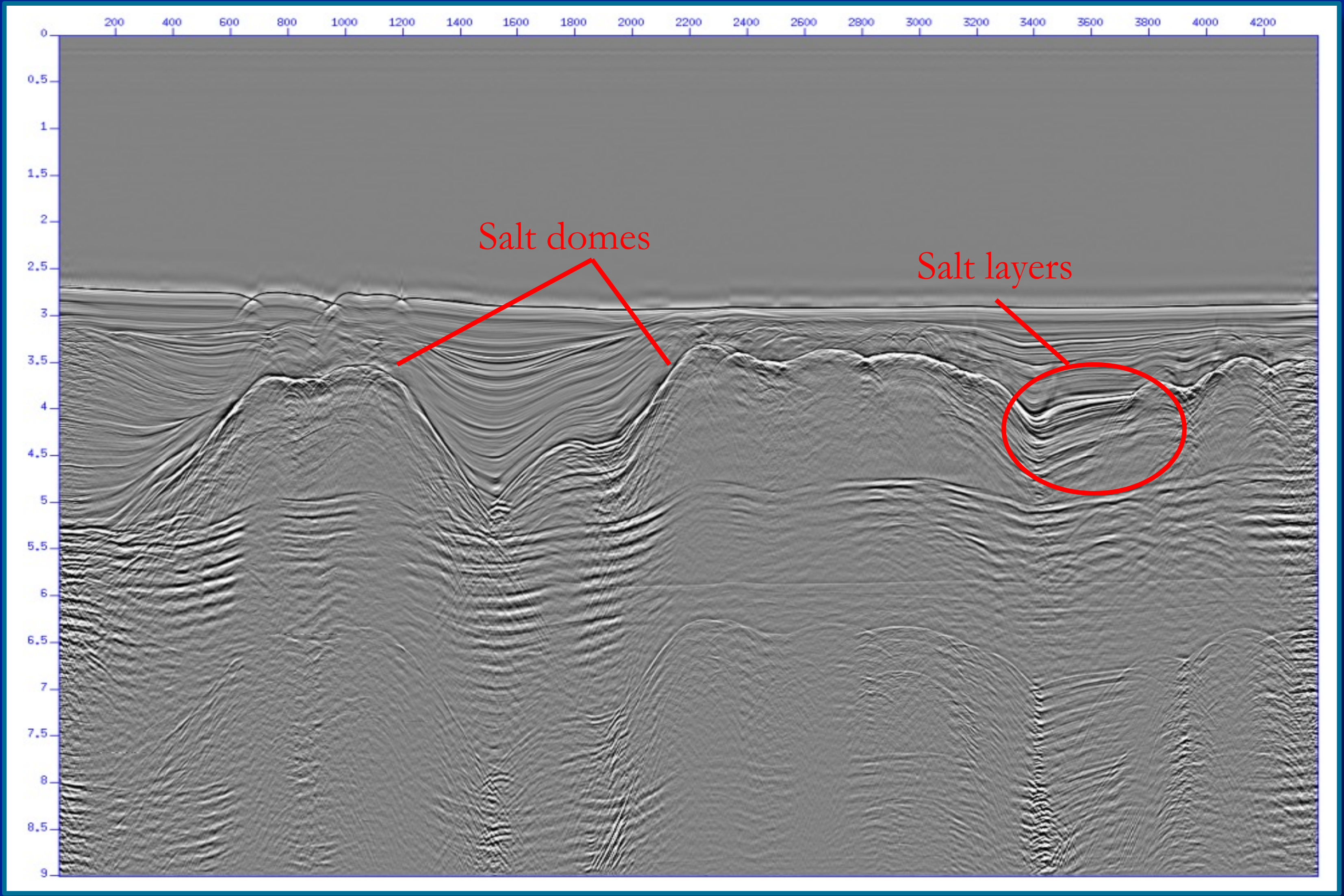
- Multiples in seismic data



- Only primaries are used for imaging and inversion
- There are various methods to remove or attenuate multiples

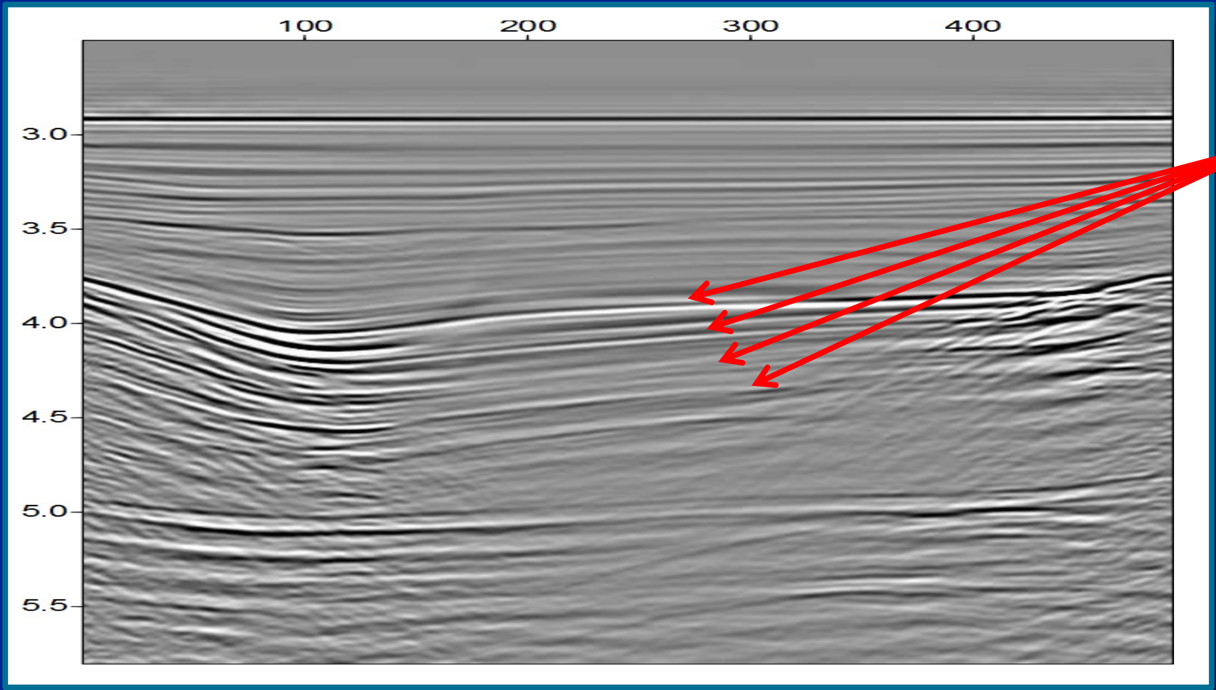
Introduction and problem

- Petrobras dataset stack example (all multiples shown)



Introduction and problem

- Multiple removal challenges
  - Deal with complex structure
  - Effective removal at short offsets
  - Generators for internal multiples cannot be mapped



Several internal multiple generators

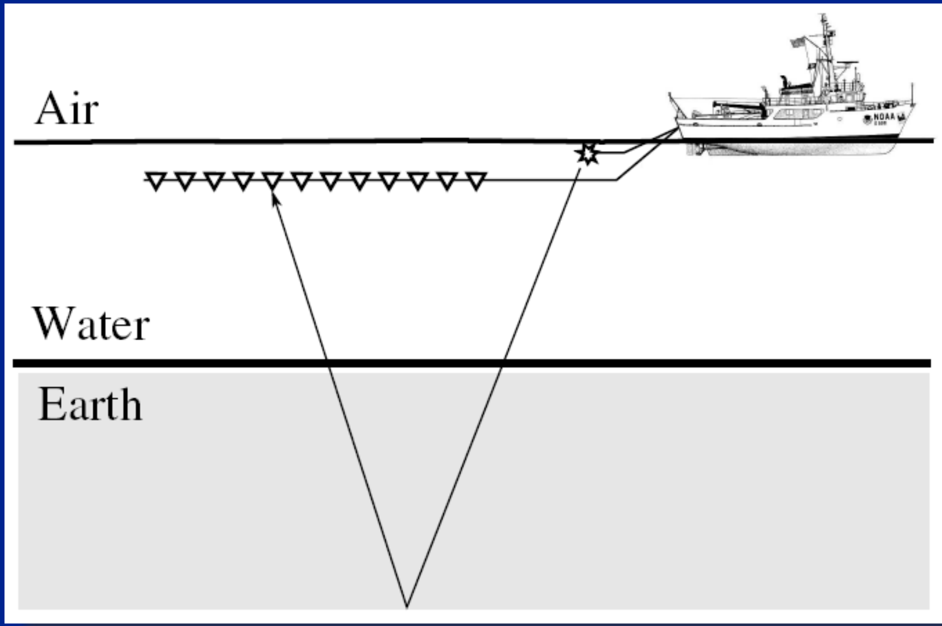
- New method with fewer assumptions is needed

Objectives

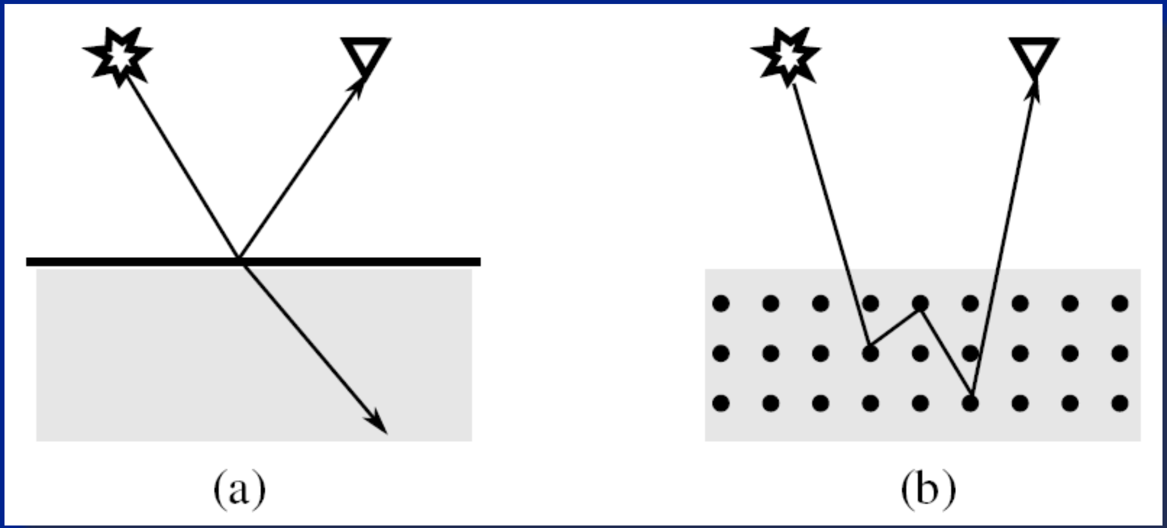
- Review theory and applications of ISS methods
- Reprocess a 2D Petrobras seismic line extracted from 3D to the requirements of ISS multiple attenuation
- Apply ISS multiple attenuation methods to the data
  - ISS methods are direct data driven and do not require any *a priori* information
  - ISS methods work on data with complex structure, on all offsets, and no reflector mapping is necessary
  - Any improvement in multiple attenuation will be relevant

Theory

- Marine seismic experiment:

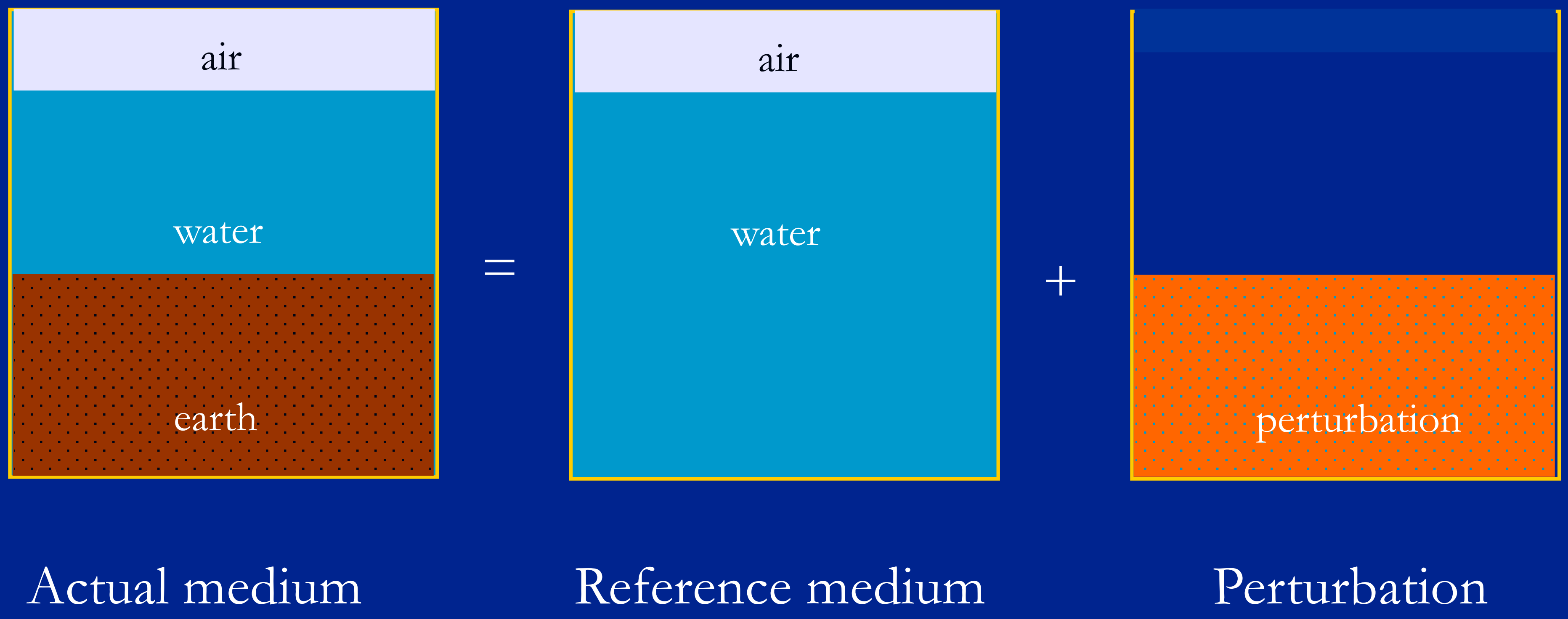


- Scattering theory view:



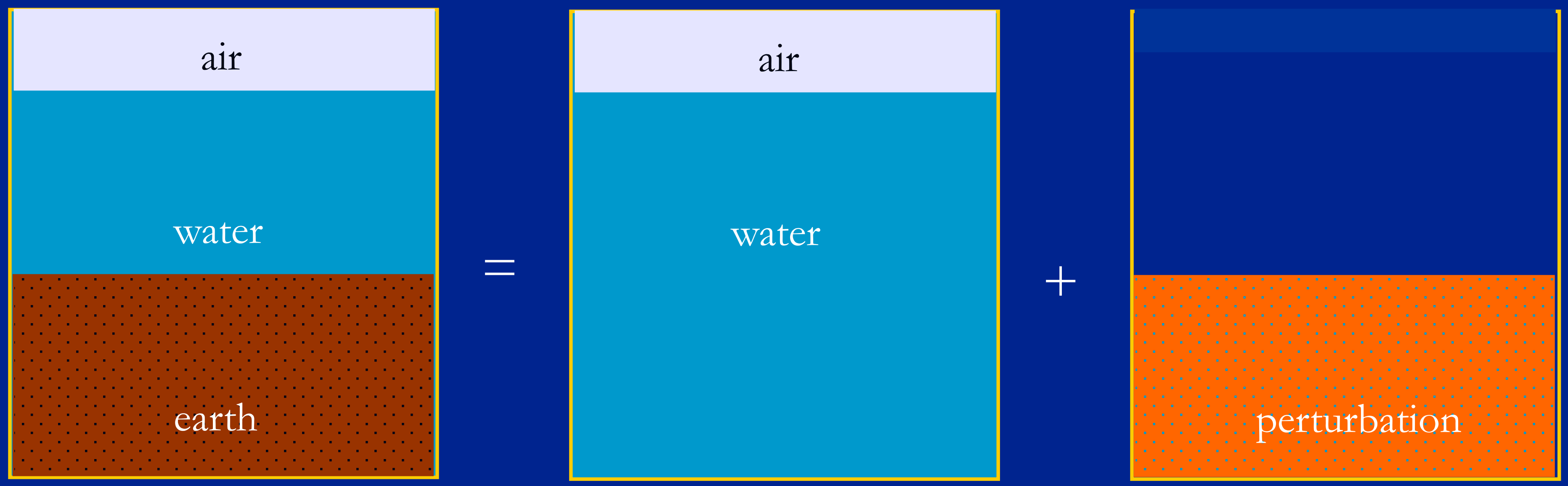
Theory

- Reference medium and perturbation (marine case)



Theory

- Reference medium and perturbation (marine case)



Actual medium | Reference medium | Perturbation

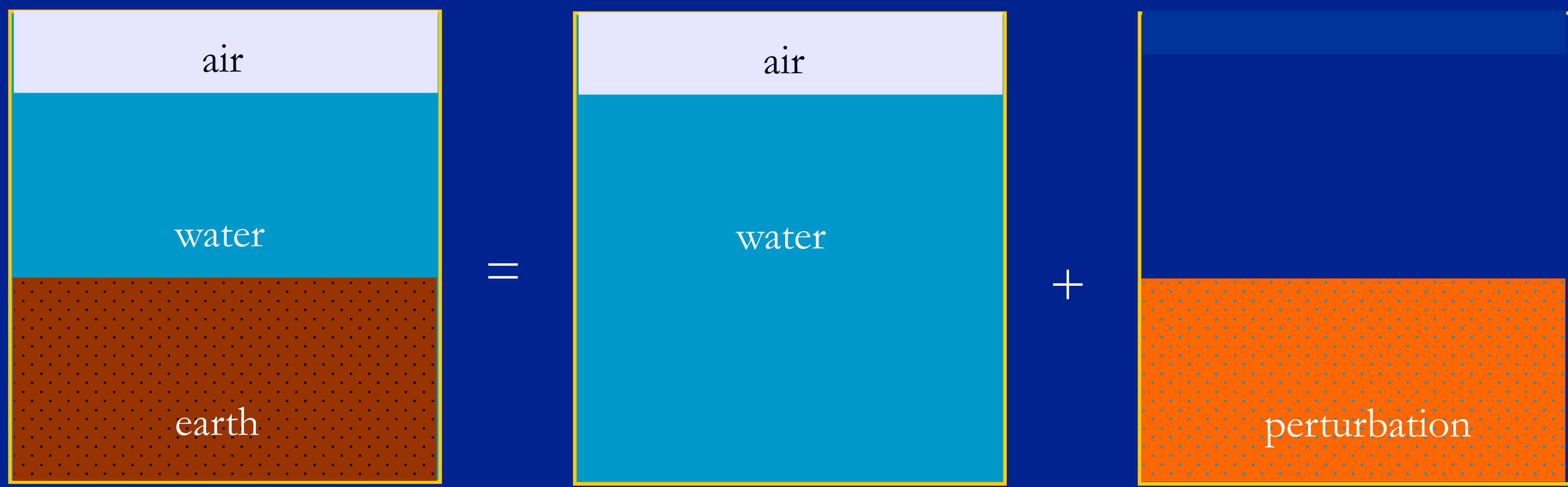
$$\mathcal{L}\mathcal{G} = I$$

$\mathcal{L}$= Linear operator describing the actual medium

$\mathcal{G}$= Green's function describing the actual wavefield

Theory

- Reference medium and perturbation (marine case)



Actual medium | Reference medium | Perturbation

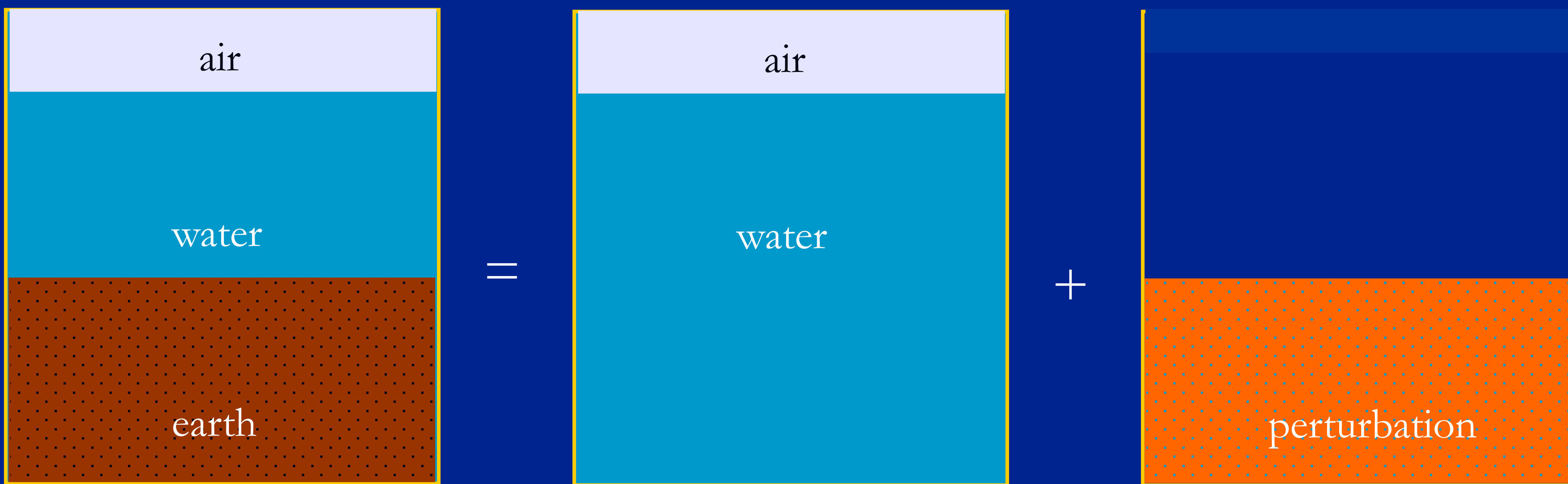
$$\mathcal{L}_0 \mathcal{G}_0 = I$$

$\mathcal{L}_0$ = Linear operator describing the reference medium

$\mathcal{G}_0$ = Green's function describing the reference wavefield

Theory

- Reference medium and perturbation (marine case)



Actual medium | Reference medium | Perturbation

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0 \mathcal{V} \mathcal{G}$$ (Lippmann-Schwinger Equation)

$\mathcal{V} = \mathcal{L}_0 - \mathcal{L} =$ Perturbation operator

Theory

- Equation describing the actual medium: $\mathcal{L}\mathcal{G} = I$

  $\mathcal{L}$= Linear operator describing the actual medium

  $\mathcal{G}$= Green's function describing the actual wavefield

- Equation describing the reference medium: $\mathcal{L}_0\mathcal{G}_0 = I$

  $\mathcal{L}_0$=Linear operator describing the reference medium

  $\mathcal{G}_0$=Green's function describing the reference wavefield

- Relation between actual and reference medium

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G} \qquad \text{(Lippmann-Schwinger Equation)}$$

$\mathcal{V} = \mathcal{L}_0 - \mathcal{L} =$ Perturbation operator

Theory

- Forward scattering series:

$$\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G}_0 + \mathcal{G}_0\mathcal{V}\mathcal{G}_0\mathcal{V}\mathcal{G}_0 + \dots$$

  - Solves for the actual wavefield as a function of the reference medium and subsurface properties
  - The series is non-linear. Truncation at the first order term leads to the Born linear approximation
  - There is a direct relation between the traditional seismic reflection view and the scattering approach
  - However, the goal of seismic is to solve for subsurface properties (inverse problem)

Theory

- Inverse Scattering Series (ISS):

$$
\begin{aligned}
D &= (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M \\
0 &= (\mathcal{G}_0\mathcal{V}_2\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M \\
0 &= (\mathcal{G}_0\mathcal{V}_3\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_2\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_2\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M \\
&\vdots
\end{aligned}
$$

$D=$ actual field measured on a surface (seismic data)

$\mathcal{V}_j=$ Portion of perturbator operator that is of order j in the data (D)

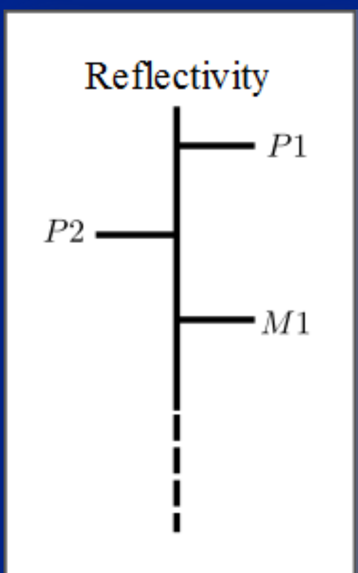
- Solves for the perturbation operator as a function of the reference wavefield and the actual field measured on a surface (seismic data D)
- Direct and non-linear solution to the inverse problem

Theory

- Inverse Scattering Series and mapping of terms

$$
\begin{aligned}
D &= (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M \\
0 &= (\mathcal{G}_0\mathcal{V}_2\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M \\
0 &= (\mathcal{G}_0\mathcal{V}_3\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_2\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_2\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M \\
&\vdots
\end{aligned}
$$

- Mapping between the scattering series and the traditional seismic problem



Map I

Inverse series

$$(\mathcal{G}_0\mathcal{V}\mathcal{G}_0)_M = (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0\mathcal{V}_1\mathcal{G}_0)_M + \dots$$

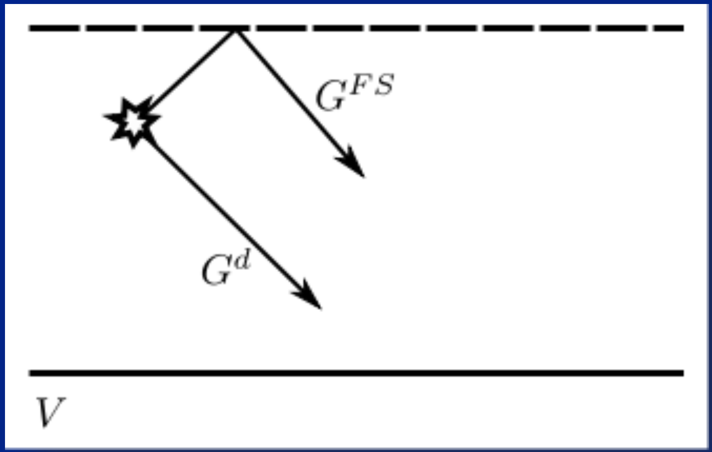
Forward series

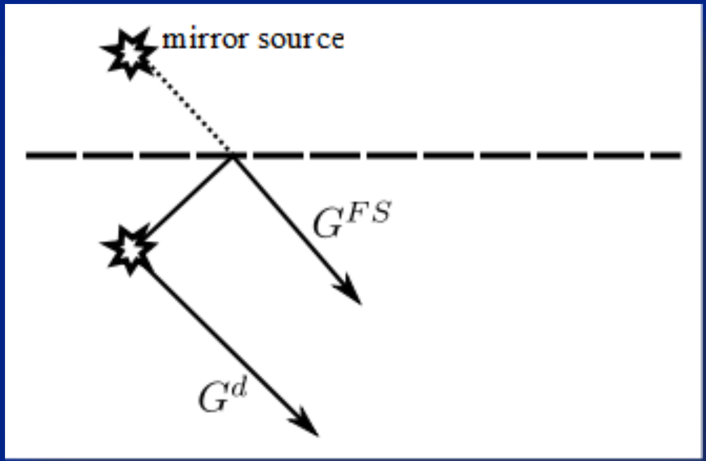
$$(P_s)_M = (\mathcal{G}_0\mathcal{V}\mathcal{G}_0)_M + (\mathcal{G}_0\mathcal{V}\mathcal{G}_0\mathcal{V}\mathcal{G}_0)_M + \dots$$

Map II

Theory

- Reference Green's function
  - Reference medium = air and water separated by a free surface





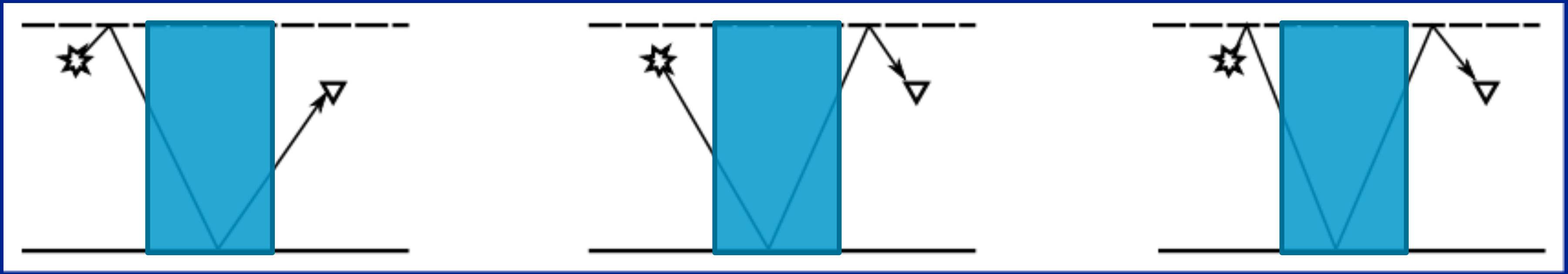
$G_0 = G_0^d + G_0^{FS}$ (Direct Green's function and free surface Green's function)

$$G_0(x_F, z_F, x_I, z_I; \omega) = \frac{1}{2\pi} \int dk_x \frac{e^{ik_x(x_F - x_I)} \left(e^{iq|z_F - z_I|} - e^{iq|z_F + z_I|}\right)}{-2iq}$$

$$q = \operatorname{sng}(\omega)\sqrt{k_0^2 - k_x^2} \qquad k_0 = \frac{\omega}{c_0}$$

Theory

- Deghosting
  - Ghost: event that starts and/or ends its propagation with a reflection at the free surface



General series term: $[G_0 (\ldots) G_0]_M$ $\quad G_0 = G_0^d + G_0^{FS}$

Deghosting operation: $(G_0^d V_i G_0^{FS} V_j G_0^d V_k G_0^d)_M$

Multiply both sides by: $\dfrac{G_0^d}{G_0} = \dfrac{G_0^d}{G_0^d + G_0^{FS}}$ (projected on receiver and source sides)

Deghosted data: $\tilde{D} = \dfrac{D}{(1 - e^{2iq_g \epsilon_g})(1 - e^{2iq_s \epsilon_s})}$

Theory

- ISS algorithm
  - Identify task-specific terms in the entire series
    - e.g.: identify the terms responsible for free surface multiple removal
  - Employ the subseries to perform the specific task (data processing)
    - e.g.: apply free surface multiple removal subseries to the dataset and obtain a processed dataset with multiples removed
  - Restart the problem using the processed data as new input
    - e.g.: the internal multiple attenuation uses the data with free surface multiples removed as input

Theory

- Free surface removal subseries
  - Inverse scattering subseries terms

$$D'_1 = \tilde{D} = (G_0^d V_1 G_0^d)_M$$

$$(G_0^d V_2 G_0^d)_M = -(G_0^d V_1 G_0 V_1 G_0^d)_M$$

$$(G_0^d V_3 G_0^d)_M = -(G_0^d V_1 G_0 V_1 G_0 V_1 G_0^d)_M - (G_0^d V_1 G_0 V_2 G_0^d)_M - (G_0^d V_2 G_0 V_1 G_0^d)_M$$

$$\vdots$$

$$G_0 = G_0^d + G_0^{FS}$$

Types of terms:

$$\text{Type 1:} \quad (G_0^d V_i G_0^{FS} V_j G_0^{FS} V_k G_0^d)_M,$$

$$\text{Type 2:} \quad (G_0^d V_i G_0^{FS} V_j G_0^d V_k G_0^d)_M,$$

$$\text{Type 3:} \quad (G_0^d V_i G_0^d V_j G_0^d V_k G_0^d)_M.$$

We select type 1 (the only task is to create free surface multiples):

$$(G_0^d V_i G_0^{FS} V_j G_0^{FS} V_k G_0^d)_M$$

Theory

- ISS algorithm
  - Identify task-specific terms in the entire series
    - e.g.: identify the terms responsible for free surface multiple removal
  - Employ the subseries to perform the specific task (data processing)
    - e.g.: apply free surface multiple removal subseries to the dataset and obtain a processed dataset with multiples removed
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Theory

- Free surface removal subseries
  - Computation of the first term

$$D_1'(k_g,\epsilon_g,k_s,\epsilon_s;\omega) = \int dr_1 dr_2 G_0^d(k_g,\epsilon_g,r_1;\omega) V_1(r_1,r_2;\omega) G_0^d(r_2,k_s,\epsilon_s;\omega)$$

$$G_0^d(x_2,z_2,k_s,\epsilon_s;\omega) = \frac{1}{2\pi}\frac{e^{ik_s x_2}}{-2iq_s}e^{iq_s(z_2-\epsilon_s)}$$

$$G_0^d(k_g,\epsilon_g,x_1,z_1;\omega) = \frac{1}{2\pi}\frac{e^{-ik_g x_1}}{-2iq_g}e^{iq_g(z_1-\epsilon_g)}$$

$$D_1' = \frac{1}{(2\pi)^2}\frac{e^{-iq_g\epsilon_g}e^{-iq_s\epsilon_s}}{-4q_s q_g}\int dr_1 dr_2 e^{-ik_g x_1}e^{iq_g z_1}V_1(x_1,z_1,x_2,z_2)e^{ik_s x_2}e^{iq_s z_2}$$

$$D_1'(k_g,\epsilon_g,k_s,\epsilon_s;\omega) = \frac{1}{(2\pi)^2}\frac{e^{-iq_g\epsilon_g}e^{-iq_s\epsilon_s}}{-4q_s q_g}V_1(k_g,-q_g,k_s,q_s)$$

$$q_i = \mathrm{sgn}(\omega)\sqrt{k_0^2-k_i^2} = \mathrm{sgn}(\omega)\sqrt{\left(\frac{\omega}{c_0}\right)^2-k_i^2}$$

Theory

- Free surface removal subseries
  - Computation of the second term

$$(G_0^d V_2 G_0^d)_M = -(G_0^d V_1 G_0^{FS} V_1 G_0^d)_M$$

$$\frac{1}{(2\pi)^2}\frac{e^{-iq_g\epsilon_g}e^{-iq_s\epsilon_s}}{-4q_sq_g}V_2(k_g,-q_g,k_s,q_s) = -\int dr_1dr_2dr_3dr_4G_0^d(k_g,\epsilon_g,x_1,z_1)V_1(r_1,r_2)$$
$$\times G_0^{FS}(r_2,r_3)V_1(r_3,r_4)G_0^d(x_4,z_4,k_s,\epsilon_s)$$

$$V_2(k_g,-q_g,k_s,q_s) = \frac{1}{2\pi}\int dk'\,\frac{V_1(k_g,-q_g,k',q')V_1(k',-q',k_s,q_s)}{-2iq'}$$

From first ISS equation: $D_1'(k_g,\epsilon_g,k_s,\epsilon_s;\omega) = \frac{1}{(2\pi)^2}\frac{e^{-iq_g\epsilon_g}e^{-iq_s\epsilon_s}}{-4q_sq_g}V_1(k_g,-q_g,k_s,q_s)$

$$D_2'(k_g,-q_g,k_s,q_s) = -\pi i\int dk_1q_1e^{iq_1(\epsilon_s+\epsilon_g)}D_1'(k_g,-q_g,k_1,q_1)D_1'(k_1,-q_1,k_s,q_s)$$

Theory

- Free surface removal subseries
  - Recurrence relations

$$D'_n(k_g,-q_g,k_s,q_s) = -\pi i \int dk' q' e^{iq'(\epsilon_s+\epsilon_g)} D'_1(k_g,-q_g,k',q') D'_{n-1}(k',-q',k_s,q_s)$$

$$n = 2, 3, \ldots$$

  - Free surface removed data

$$D'(k_g,-q_g,k_s,q_s) = \sum_{n=1}^{\infty} D'_n(k_g,-q_g,k_s,q_s)$$

  - In practice: subtract multiple prediction from original data

Theory

- ISS algorithm
  - Identify task-specific terms in the entire series
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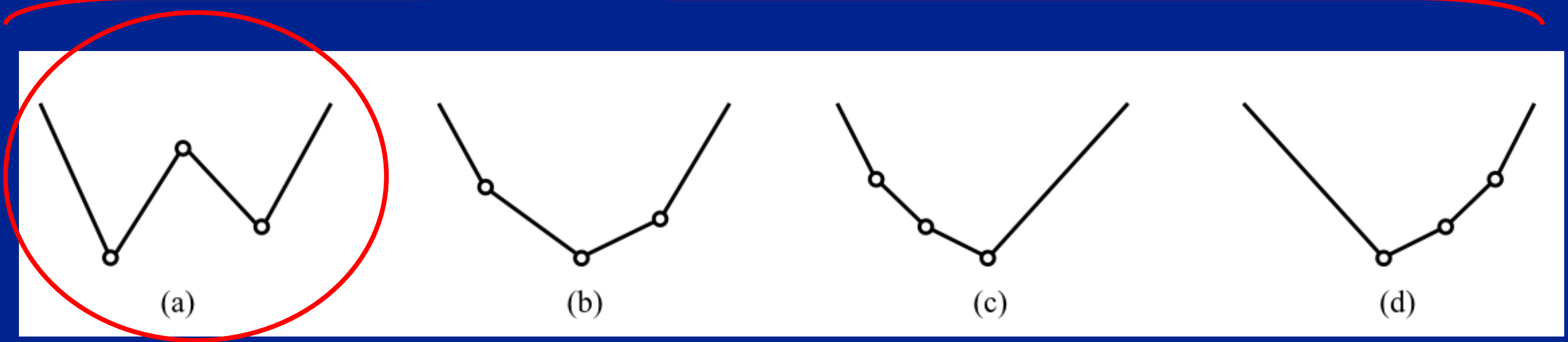
- Internal multiple attenuation subseries
  - Inverse scattering subseries terms

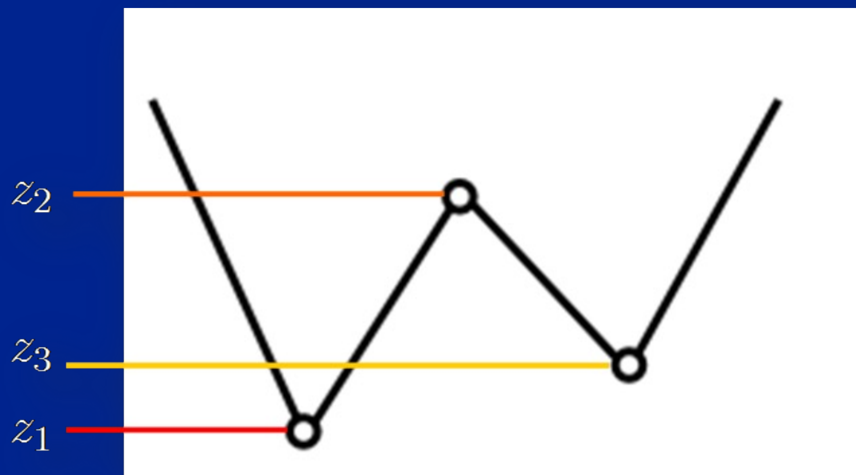
$$D_1' = (G_0^d V_1 G_0^d)_M$$
$$(G_0^d V_2 G_0^d)_M = -(G_0^d V_1 G_0^d V_1 G_0^d)_M$$
$$(G_0^d V_3 G_0^d)_M = -(G_0^d V_1 G_0^d V_1 G_0^d V_1 G_0^d)_M - (G_0^d V_1 G_0^d V_2 G_0^d)_M - (G_0^d V_2 G_0^d V_1 G_0^d)_M$$
$$\vdots$$

  - The internal multiple terms are more difficult to map



Theory

- Internal multiple attenuation subseries
  - Inverse scattering subseries terms



$$V_3^{(1,1)} = -\frac{1}{(2\pi)^2}\iint dk_1 dk_2 \int dz_1 \frac{V_1(k_g, z_1, k_1)}{2iq_1} e^{i(q_g+q_1)z_1} \times$$
$$\times \int_{-\infty}^{z_1} dz_2 V_1(k_1, z_2, k_2) e^{-i(q_1+q_2)z_2} \int_{z_2}^{\infty} dz_3 \frac{V_1(k_2, z_3, k_s)}{2iq_2} e^{i(q_2+q_s)z_3}$$

Theory

- Internal multiple attenuation subseries
  - Inverse scattering subseries terms

$$b_1(k_g, q_g + q_s, k_s) = (2iq_s) D_1(k_g, \epsilon_g, k_s, \epsilon_s; \omega)$$

From previous slide:

$$V_3^{(1,1)} = -\frac{1}{(2\pi)^2} \iint dk_1 dk_2 \int dz_1 \frac{V_1(k_g, z_1, k_1)}{2iq_1} e^{i(q_g+q_1)z_1} \times$$
$$\times \int_{-\infty}^{z_1} dz_2 V_1(k_1, z_2, k_2) e^{-i(q_1+q_2)z_2} \int_{z_2}^{\infty} dz_3 \frac{V_1(k_2, z_3, k_s)}{2iq_2} e^{i(q_2+q_s)z_3}$$

Leading first order multiple attenuation term:

$$b_3(k_g, k_s, q_g + q_s) = \frac{1}{(2\pi)^2} \iint dk_1 e^{-iq_1(\epsilon_g - \epsilon_s)} dk_2 e^{iq_2(\epsilon_g - \epsilon_s)} \int dz_1 b_1(k_g, z_1, k_1) e^{i(q_g+q_1)z_1}$$
$$\times \int_{-\infty}^{z_1} dz_3 b_1(k_1, z_3, k_2) e^{-i(q_1+q_2)z_3} \int_{z_3}^{\infty} dz_5 b_1(k_2, z_5, k_s) e^{i(q_2+q_s)z_5}$$

Theory

- Internal multiple attenuation subseries
  - Recursion relations

$$b_{2n+1}(k_g, k_s, q_g + q_s) = \frac{1}{(2\pi)^{2n}} \int dk_1 e^{-iq_1(\epsilon_g - \epsilon_s)} \times \int dz_1 e^{i(q_g+q_1)z_1} b_1(k_g, z_1, k_1) A_{2n+1}(k_1, k_s, z_1)$$

$$A_3(k_1, k_s, z_1) = \int dk_2 e^{iq_2(\epsilon_g - \epsilon_s)} \int_{-\infty}^{z_1} dz_3 e^{-i(q_1+q_2)z_3} b_1(k_1, z_3, k_2) \times \int_{z_3}^{\infty} dz_5 e^{i(q_2+q_s)z_5} b_1(k_2, z_5, k_s)$$

$$A_{2n+1}(k_1, k_s, z_1) = \int dk_2 e^{iq_2(\epsilon_g - \epsilon_s)} \int_{-\infty}^{z_1} dz_3 e^{-i(q_1+q_2)z_3} b_1(k_1, z_3, k_2) \times \int dk_3 e^{-iq_3(\epsilon_g - \epsilon_s)} \int_{z_3}^{\infty} dz_5 e^{i(q_2+q_3)z_5} b_1(k_2, z_5, k_3) A_{2n-1}(k_3, k_s, z_5)$$

Theory

- Internal multiple attenuation subseries
  - Internal multiple attenuated data

$$D^{IM}(k_g,k_s,\omega) = (-2iq_s)^{-1}\sum_{n=0}^{\infty} b_{2n+1}(k_g,k_s,q_g+q_s)$$

$$\begin{aligned} b_{2n+1}(k_g,k_s,q_g+q_s) &= \frac{1}{(2\pi)^{2n}}\int dk_1 e^{-iq_1(\epsilon_g-\epsilon_s)} \\ &\times \int dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,z_1,k_1) A_{2n+1}(k_1,k_s,z_1) \end{aligned}$$

  - In practice: subtract multiple prediction from original data

Application to real data

- Seismic survey parameters

Parameter	Value
Number of shots	1751
Number of channels per shot	239
Number of samples per trace	2251
Time sampling	0.004s
Record length	9.0s
Shot interval	50m
Group interval	25m
Shortest offset	160m
Gun depth	8.5m
Streamer depth	9.5m

Table 6.1: Petrobras field data parameters.

Data processing

- Pre-processing
  - Deghosting
    - Applied in Petrobras America
    - Reliable and efficient method
  - Processing geometry
    - Coordinates from header changed for fictitious coordinates
    - Processing lattice is a horizontal line of station points
    - Offsets regularized (25m)

Data processing

- Pre-processing
  - Regularization and interpolation

Seismic survey parameters:

Parameter	Value
Number of shots	~~1751~~ 3501
Number of channels per shot	~~239~~ 245
Number of samples per trace	2251
Time sampling	0.004s
Record length	9.0s
Shot interval	~~50m~~ 25m
Group interval	25m
Shortest offset	~~160m~~ Zero offset
Gun depth	8.5m
Streamer depth	9.5m

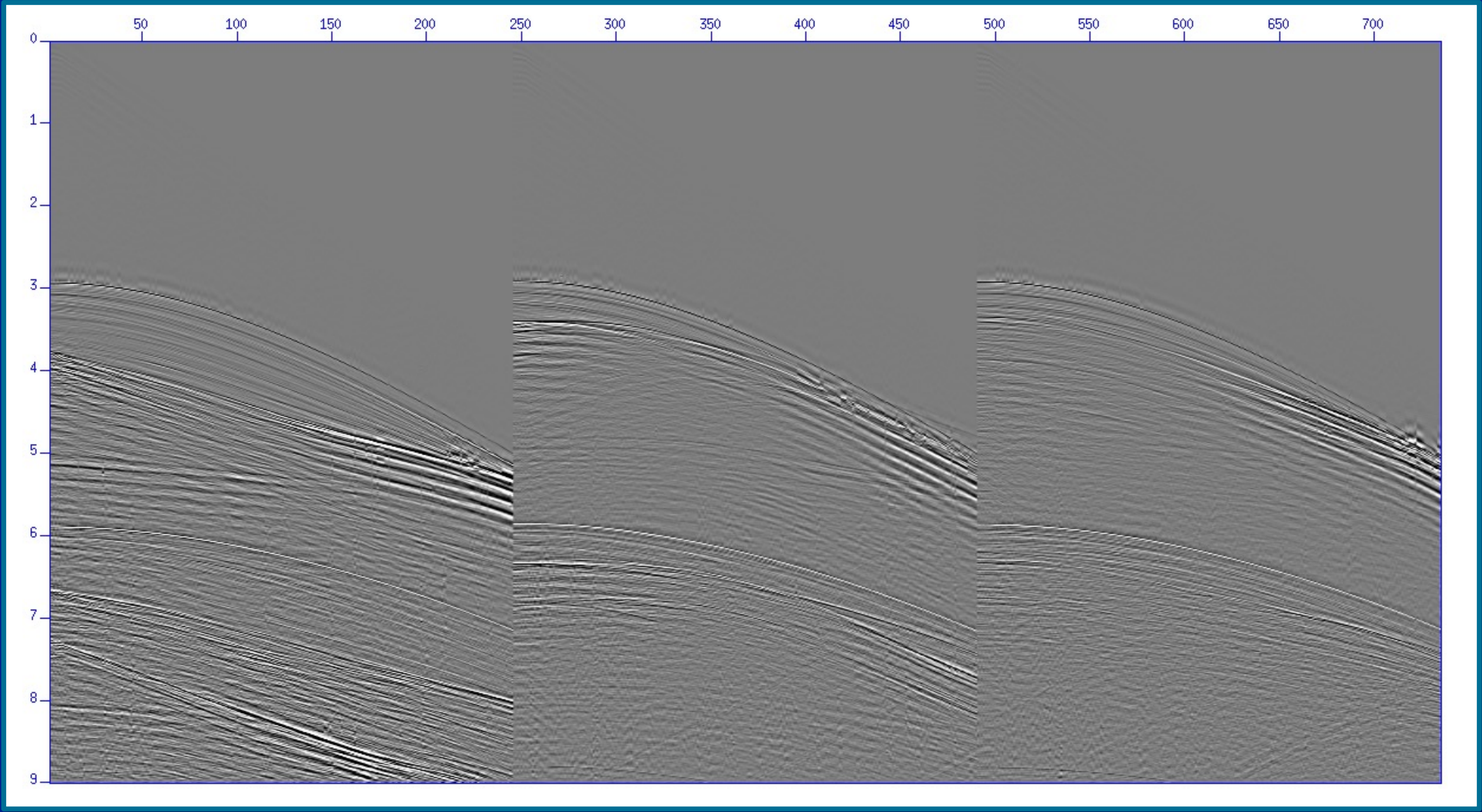
Table 6.1: Petrobras field data parameters.

ISS works with shots and receivers at every station point of the processing lattice
Shot interpolation required!

Data processing

- Pre-processing
  - Source receiver reciprocity

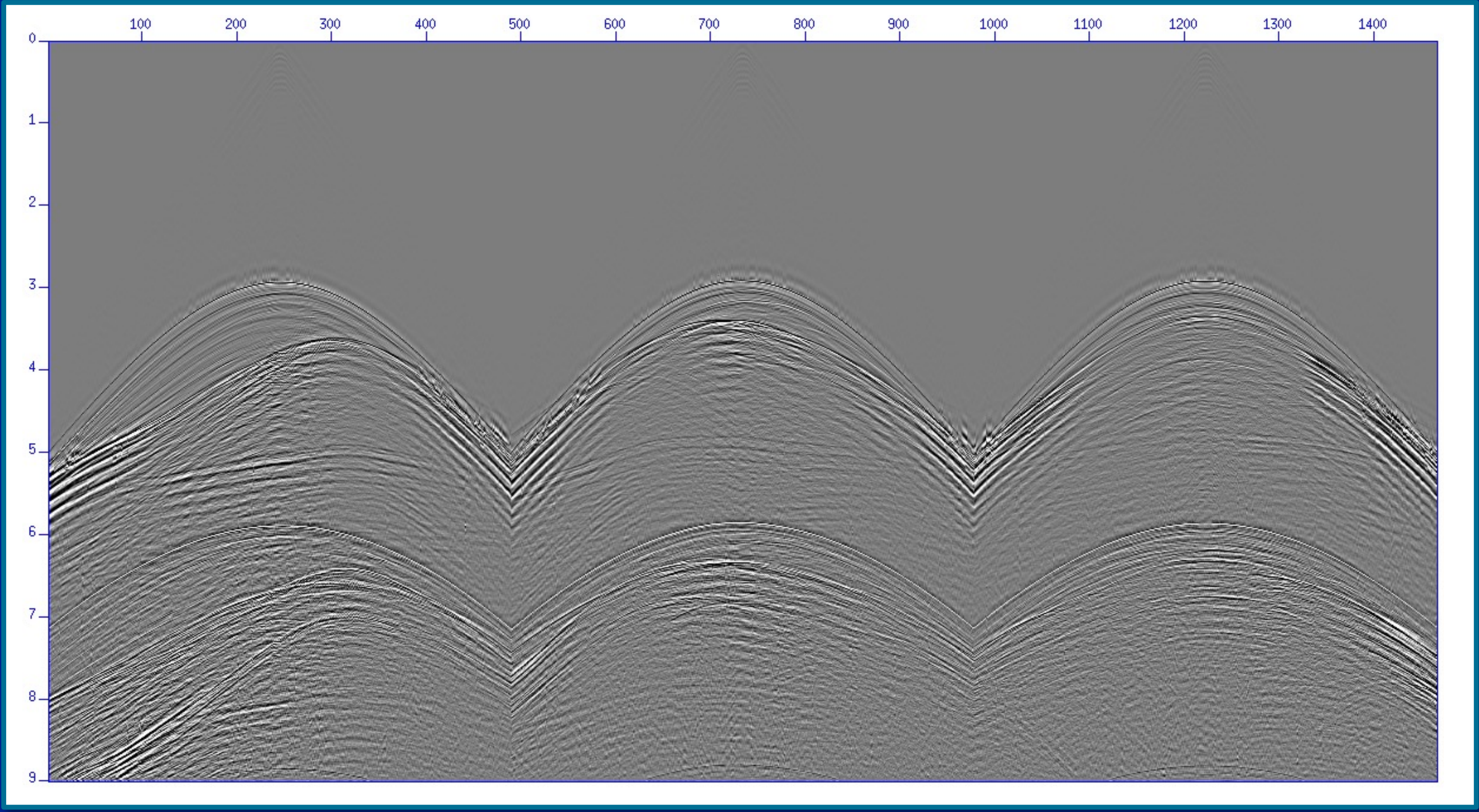
Used to obtain complete split-spread shots



Data processing

- Pre-processing
  - Source receiver reciprocity

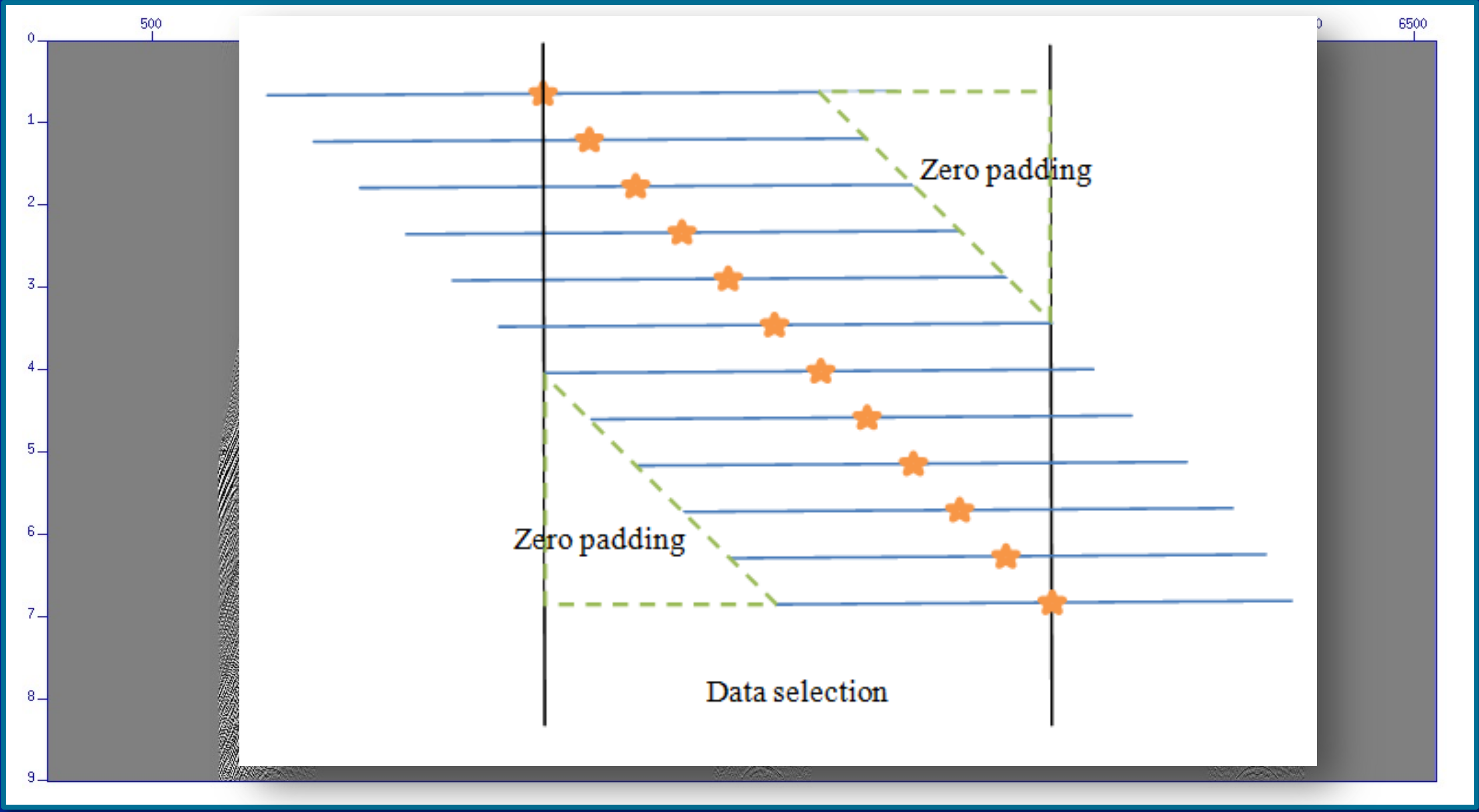
Used to obtain complete split-spread shots



Data processing

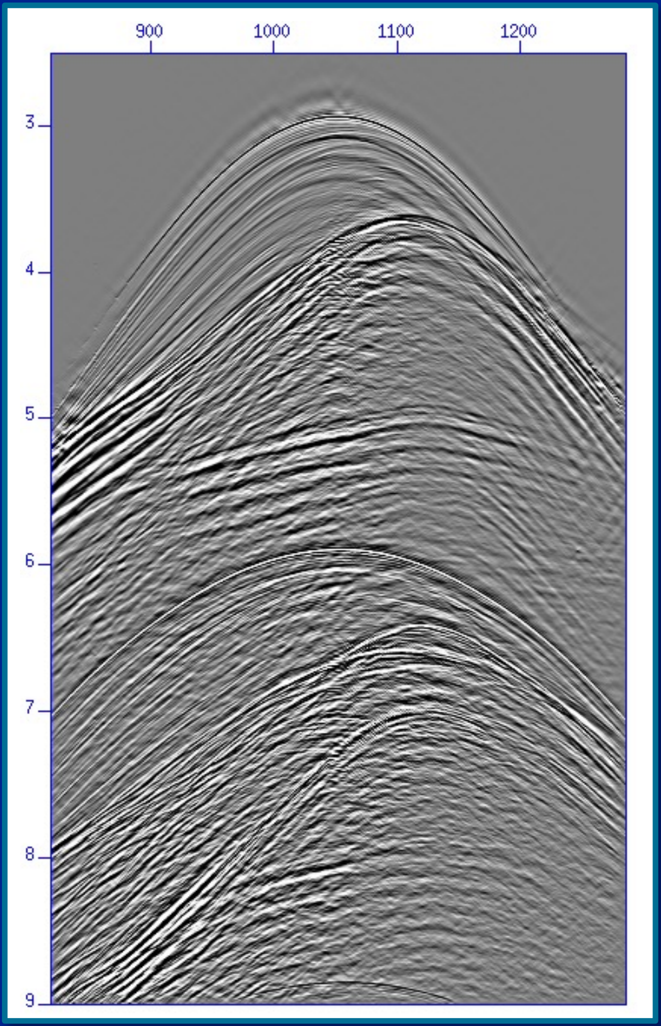
- Pre-processing
  - Trace padding and tapering

Create receiver points at every station to each shot



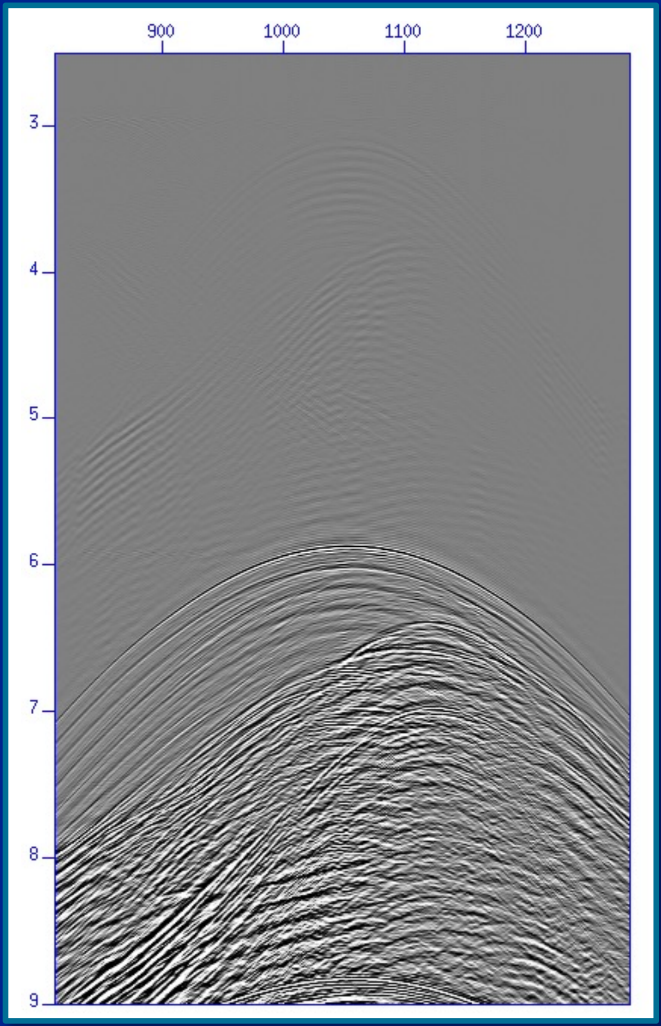
Multiple attenuation

- Free surface multiple attenuation
  - Multiple prediction



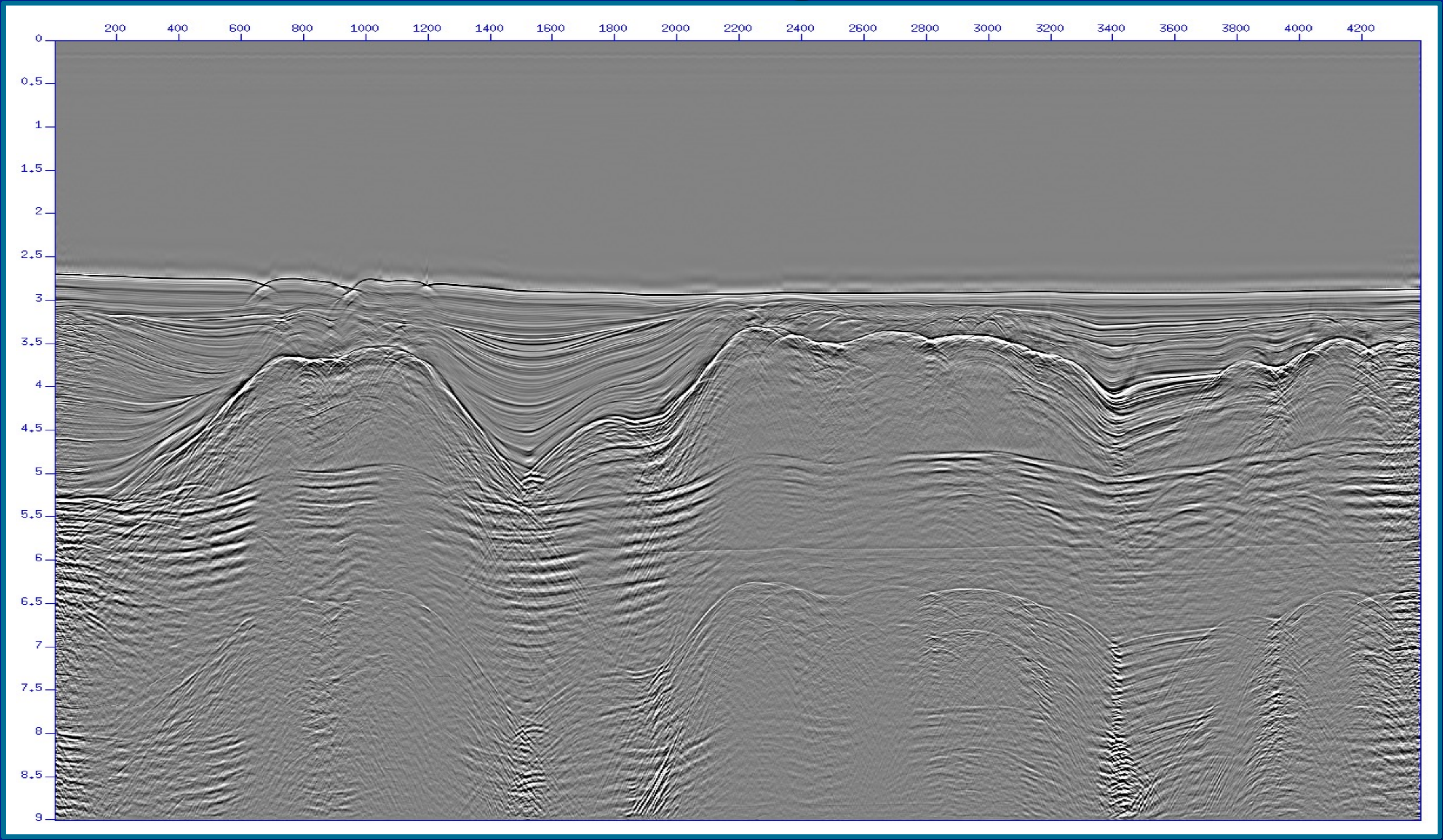
Shot gather

Corresponding multiple prediction



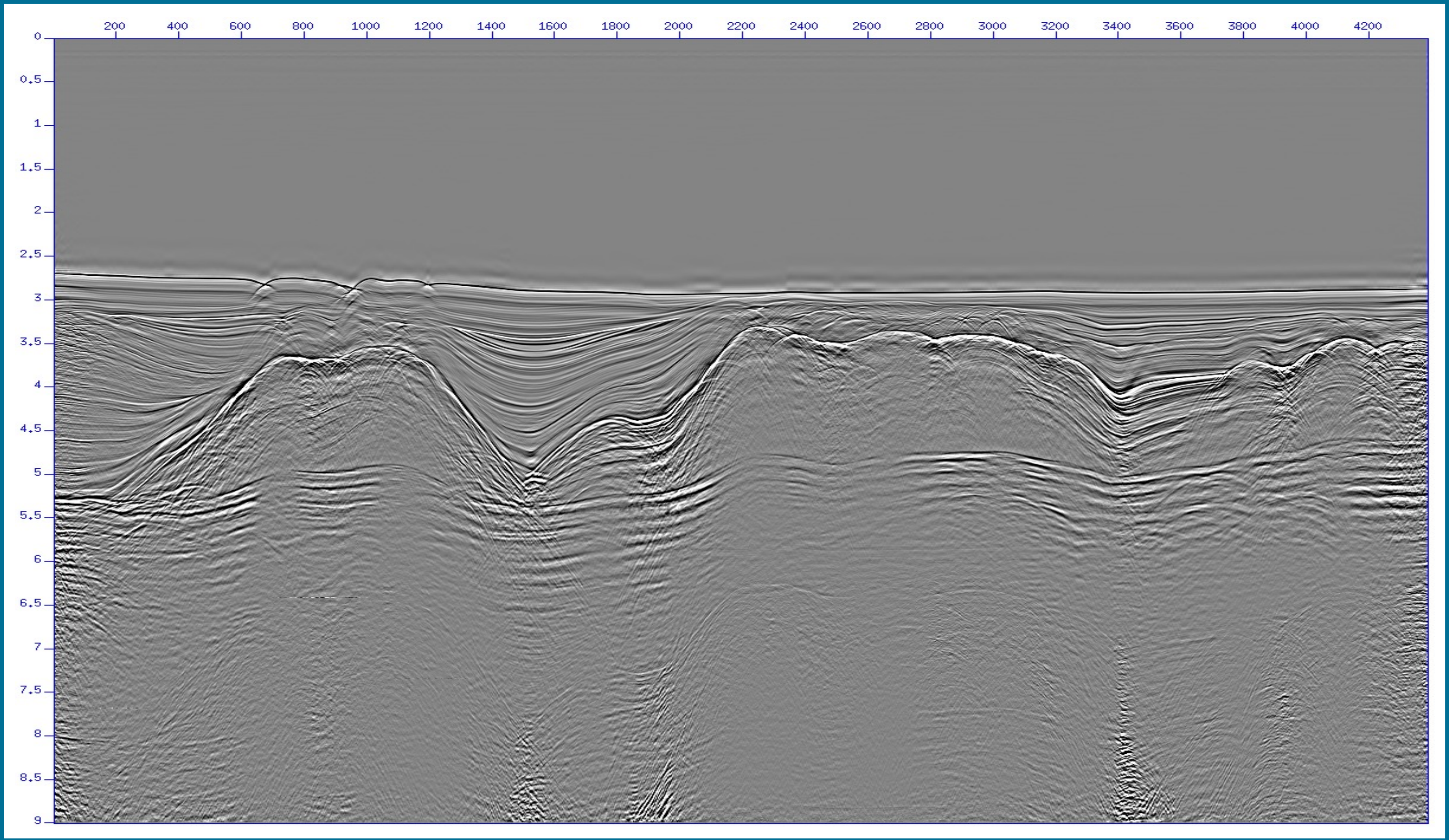
Multiple attenuation

- Free surface multiple attenuation
  - Stack before free surface multiple removal



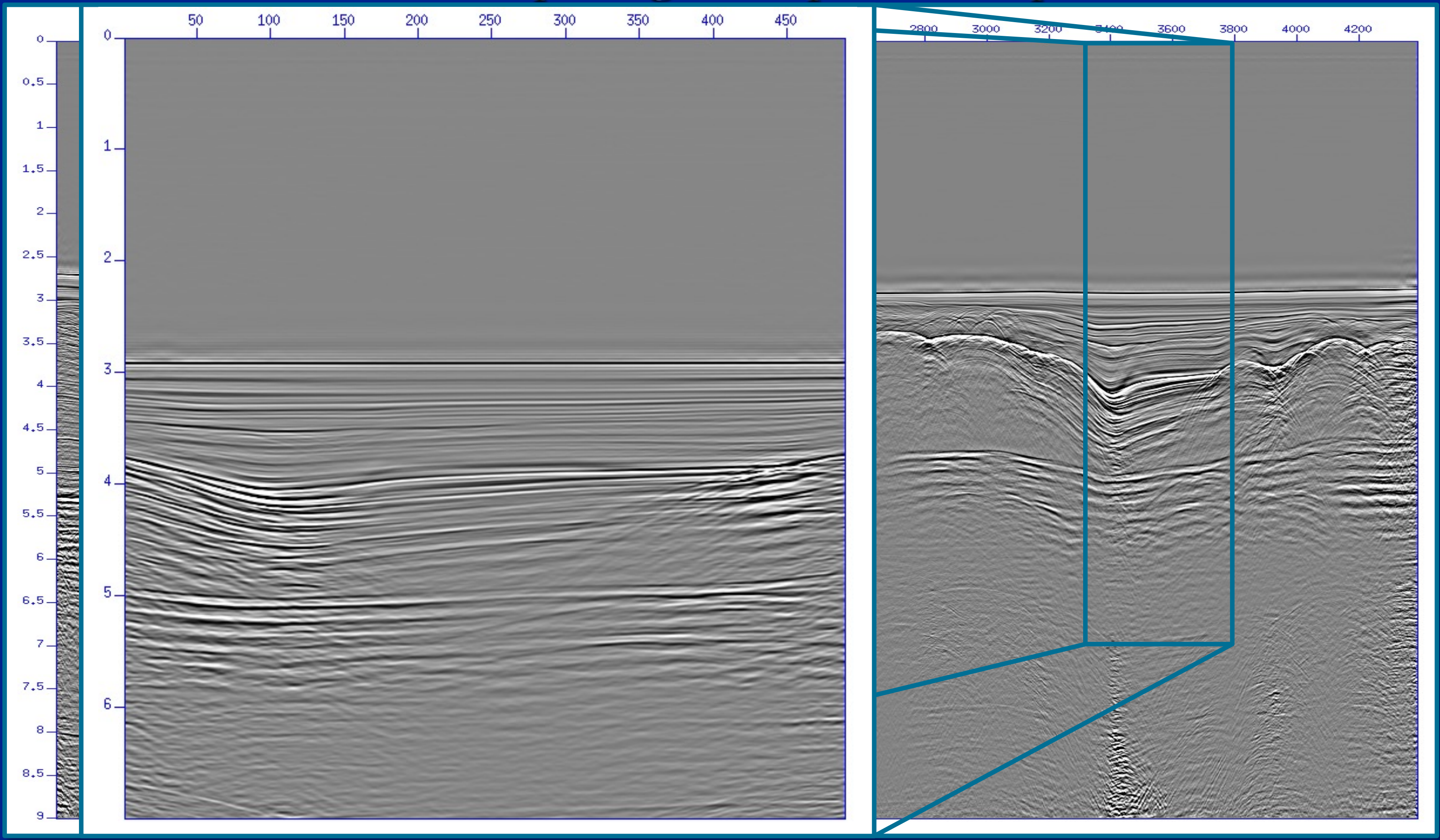
Multiple attenuation

- Free surface multiple attenuation
  - Stack after free surface multiple removal



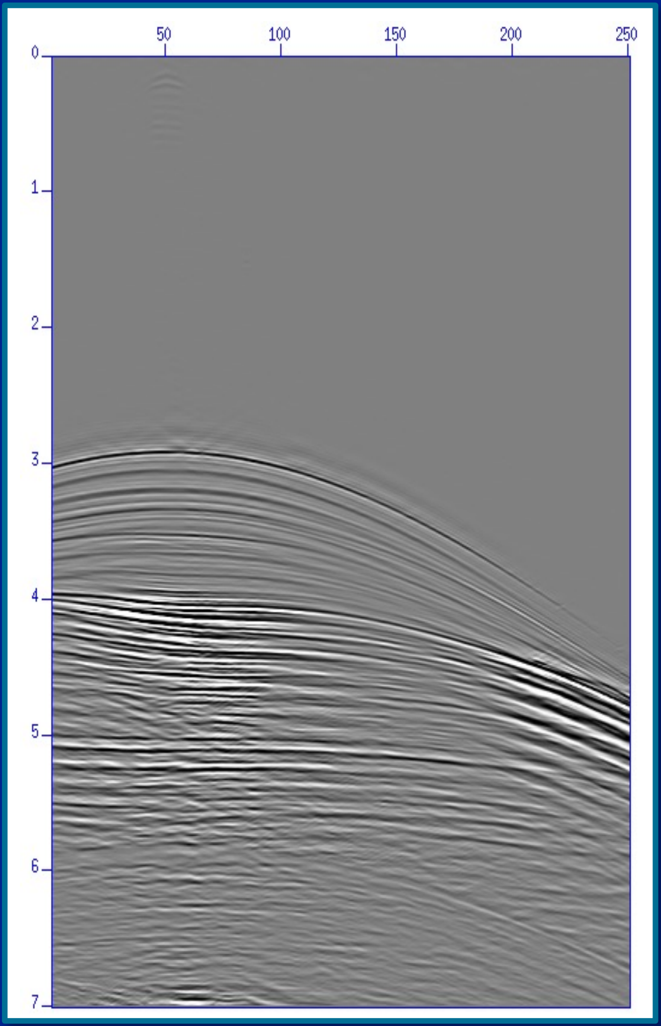
Multiple attenuation

- Internal multiple attenuation
  - The internal multiple high computer cost process



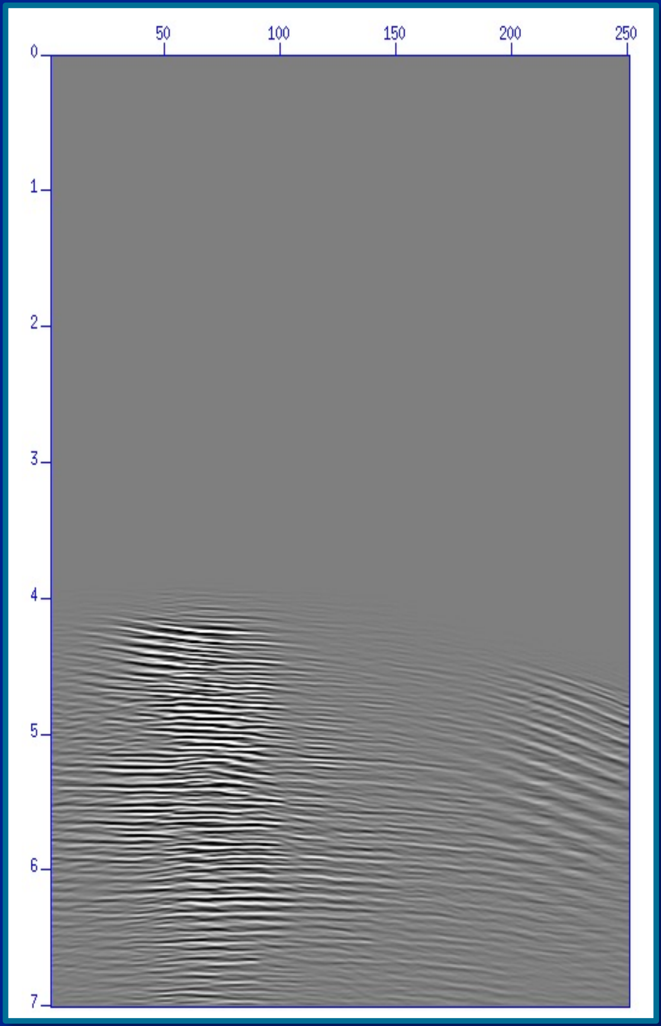
Multiple attenuation

- Internal multiple attenuation
  - Multiple prediction



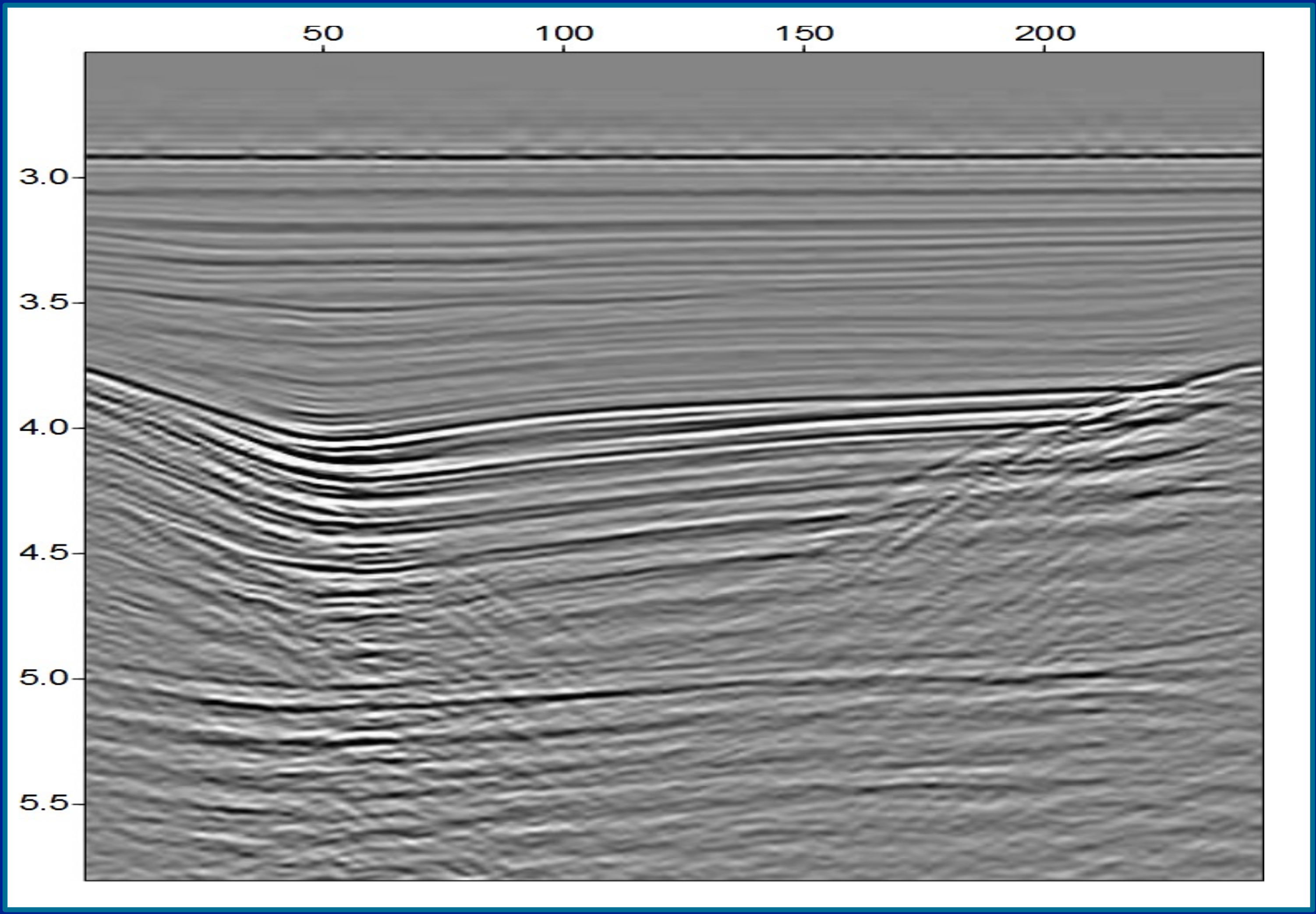
Shot gather

Corresponding multiple prediction



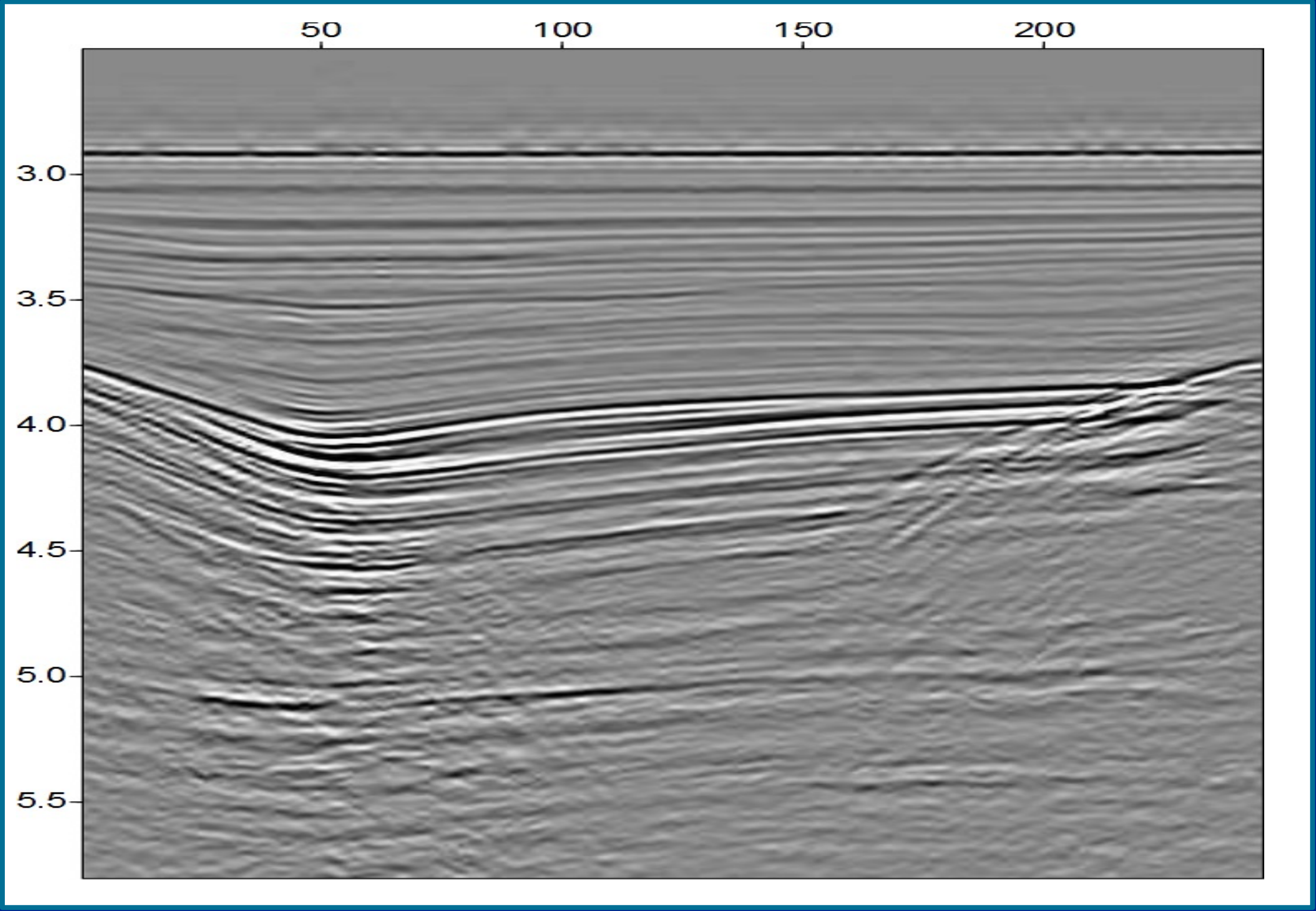
Multiple attenuation

- Internal multiple attenuation results
  - Common offset sections



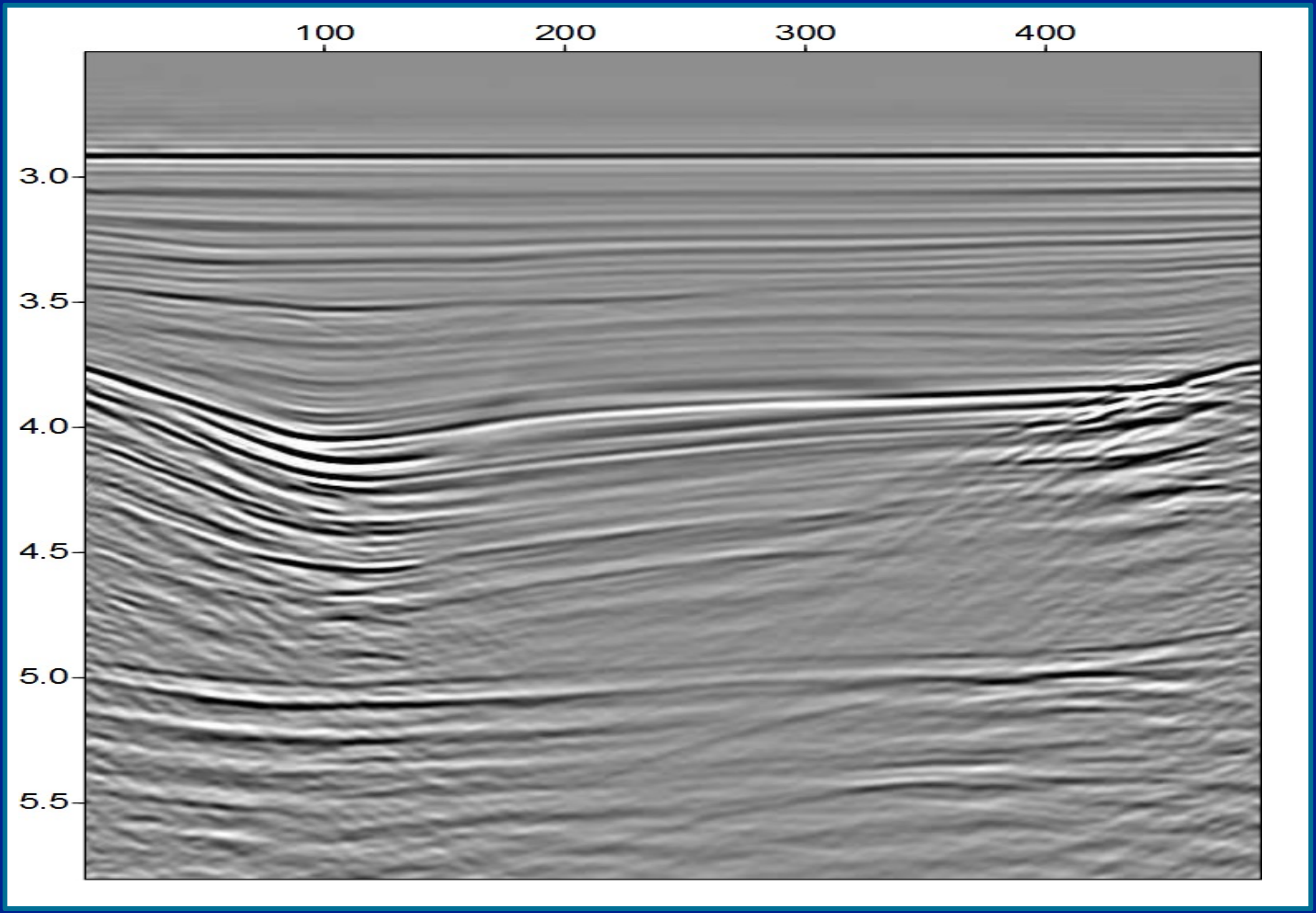
Multiple attenuation

- Internal multiple attenuation results
  - Common offset sections



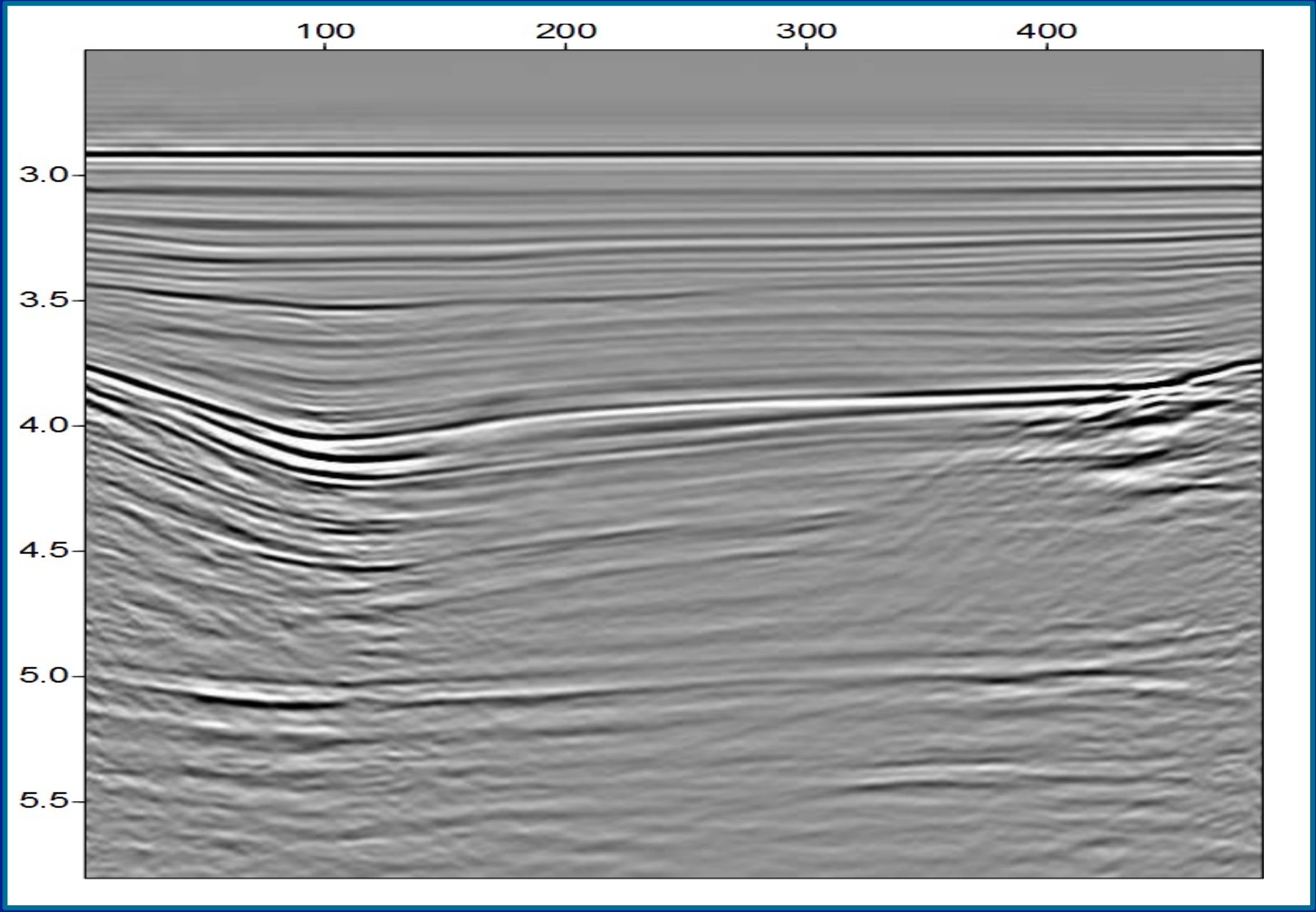
Multiple attenuation

- Internal multiple attenuation results
  - Stacked sections



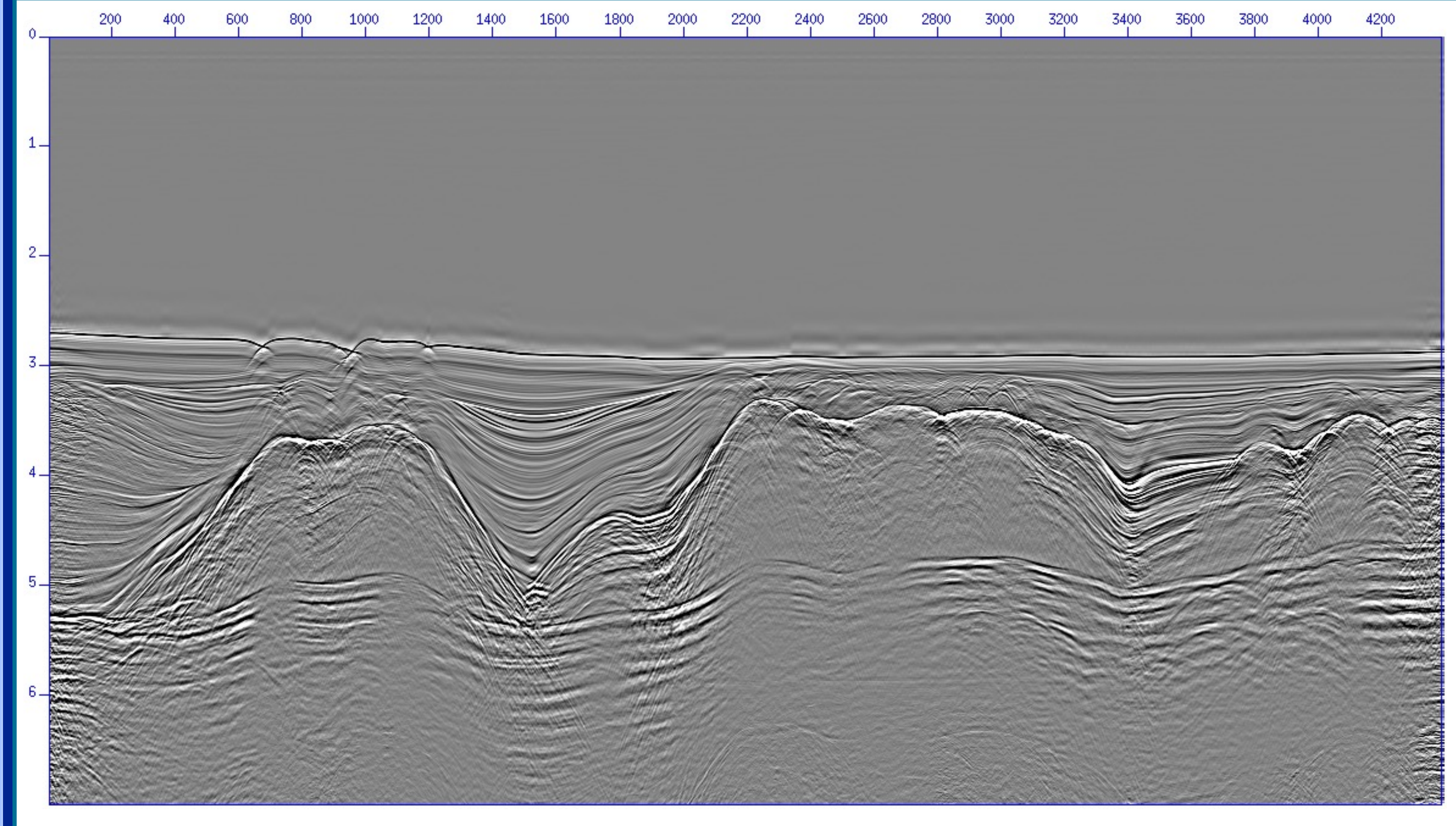
Multiple attenuation

- Internal multiple attenuation results
  - Stacked sections



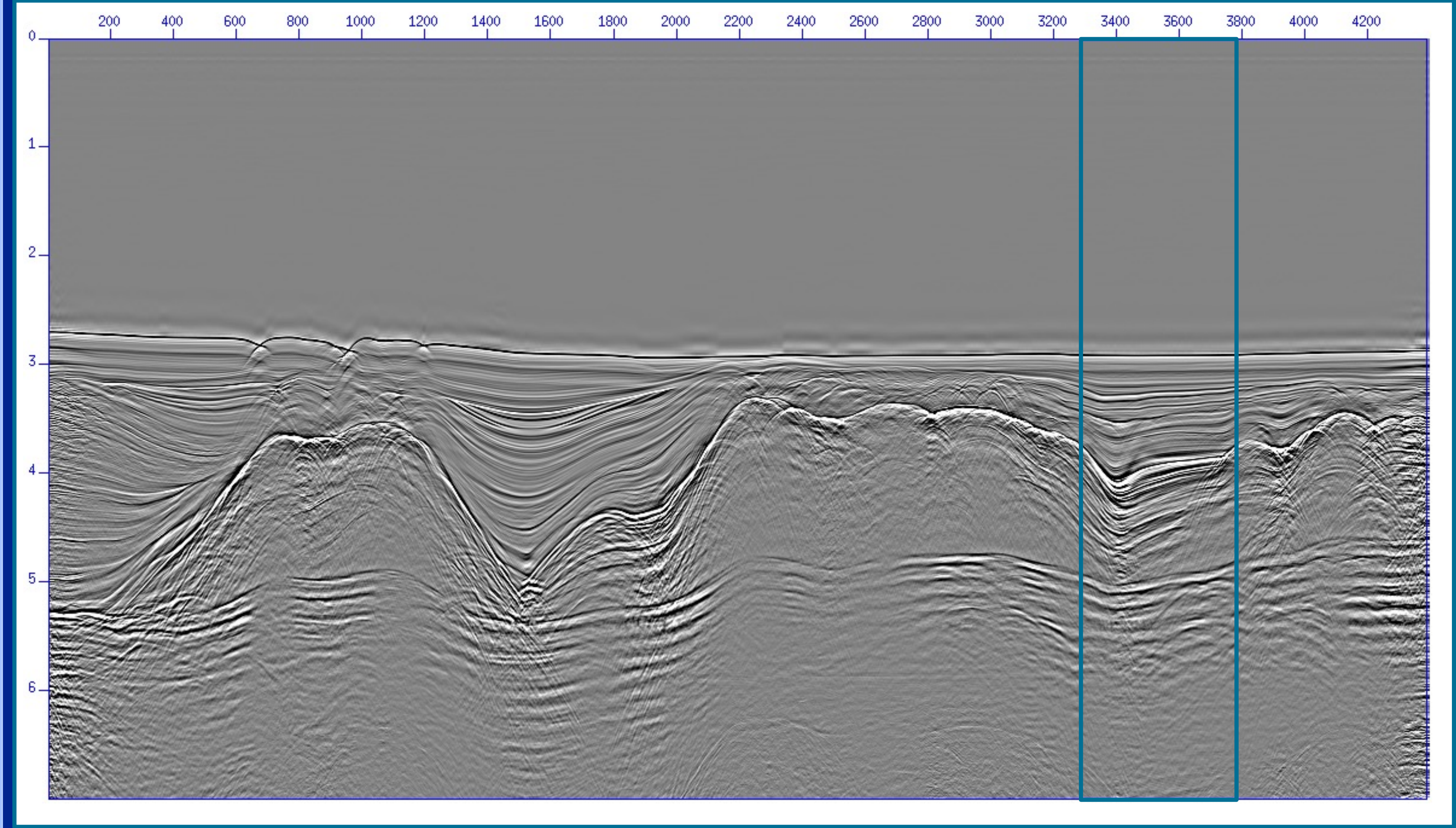
Multiple attenuation

- Internal multiple attenuation results (stacked sections)



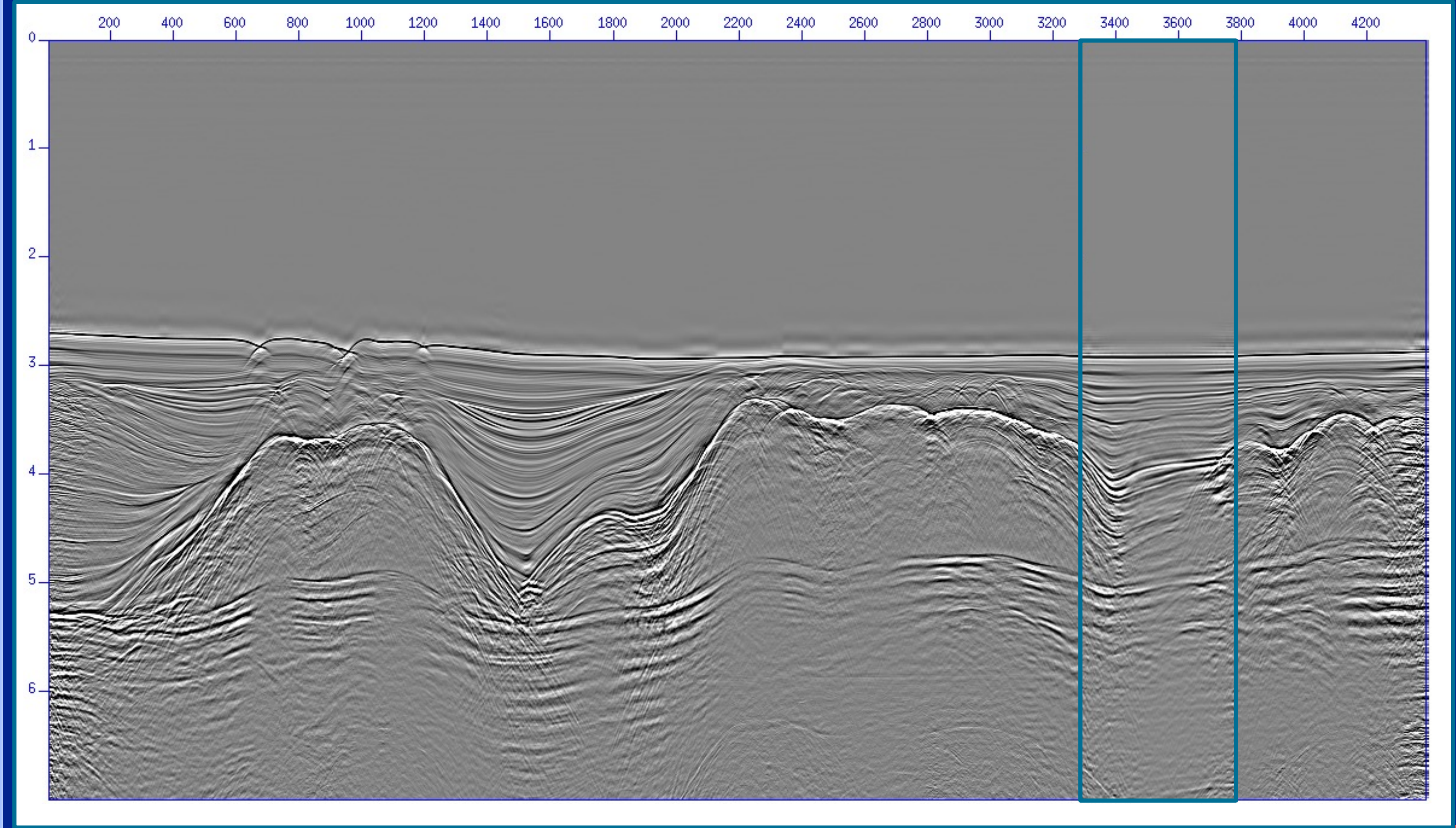
Multiple attenuation

- Internal multiple attenuation results (stacked sections)



Multiple attenuation

- Internal multiple attenuation results (stacked sections)



Conclusions

- Multiple removal/attenuation is a major problem in seismic exploration
- Free surface multiple removal results
  - Multiple prediction is excellent
    - Improved when source wavelet information was provided
  - Anti-alias filter application is important
  - Multiples from 3D structures are attenuated but not removed
  - Adaptive subtraction required
- Internal multiple attenuation results
  - Multiple prediction is excellent
  - Very high computer cost (both CPU time and memory)
  - Adaptive subtraction required

Conclusions

- ISS methods were able to attenuate both free surface and internal multiples in a very complex situation
  - No a priori information about the dataset is necessary
  - No other tested method was able to attenuate the sequence of internal multiples below the salt layers
  - High computer cost (internal multiples)
  - Adaptive subtraction requirement

Future work

- Test other adaptive subtraction methods
- Use well log information to map primaries close to the well
- Comparison between current result and results obtained by other multiple attenuation methods
- Internal multiple eliminator

Acknowledgments

- Dr. Weglein, Dr. Terengui and Dr. Hsu
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- University of Houston

Multiple attenuation

- Internal multiple attenuation