Multiples can be useful (at times) to enhance imaging, by providing an approximate image of an unrecorded primary, but it is both inaccurate and injurious to call that process “migrating multiples”

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Primaries are seismic reflection events with one reflection in their history, whereas multiples are events that have experienced more than one reflection.

Migration was originally, and remains today, basically and unequivocally about taking a primary event on a recorded seismic trace in time, and to locate where in space that reflection event was generated by a reflector; that concept assumes the event in time has only one reflection in its history. Hence, since, by definition, only primaries have experienced one reflector in their history; migration relates to and only has meaning for primaries. Migration has no meaning for multiples. We will see in this paper that not only did the original definition of migration only have meaning for primaries, but, in addition, when using the most complete physically interpretable and quantitative imaging condition for wave equation migration that only primaries contribute to the image at any reflector, in depth, and both free surface and internal multiples do not. Migration only has meaning for primaries, and primaries are the only seismic events that contribute to depth imaging and inversion at a reflector. Reminding us of that fundamental fact is one key message of this paper.

All of the wave theory and developments for migration, seek to mathematically capture and formulate that basic idea, and when appropriated applied, as intended, are consistent with that original time to depth thought, and relate to only primaries. To explain that consistency and to point out where things became somewhat confused and off track, regarding multiples, it will be important to briefly review the various wave theory migration concepts.

Both historically and currently, primaries are considered the seismic reflection events carrying information that can be reliably extracted and utilized for the purposes of locating and delineating hydrocarbon targets. That has been and remains the state of things. While multiples obviously carry subsurface information, they are considered a form of coherent noise, in terms of being directly useful in the sense that primaries are, as events, for determining subsurface properties. In this paper, we examine that proposition in light of the recent work on ‘migrating multiples’.

Migration and migration-inversion of seismic events are the workhorses of exploration seismology, used to transform recorded data into useful and reliable sub-surface information. They require an accurate velocity model to locate structure in migration, and, in addition to velocity, all other subsurface properties that impact wave field amplitude, to achieve migration-inversion.

In this paper, we briefly review these methods for migrating data inside a volume where waves are: (1) one way propagating and (2) two way propagating. Methods that use wave theory to migrate data have two ingredients, a wave propagation component and an imaging condition. We briefly discuss each of those two components here. There were three landmark imaging conditions introduced by Jon Claerbout (1971), Dan Lowenthal (1985) and Bob Stolt (1978) and their colleagues in the 1970's. Those three imaging conditions are: (1) the exploding reflector model, for zero offset data, (2) the space and time coincidence of up and down-going waves, and (3) predicting a coincident source and receiver experiment at depth and asking for time equals zero. We will refer to these three imaging conditions as Claerbout imaging I, II, and III, respectively. The third imaging condition predicts an actual seismic experiment at depth, and that predicted experiment consists of all the events that experiment would record, if you had a source and receiver at that subsurface location. That experiment would have its own recorded events, the primaries and multiples for that predicted experiment. Stolt and his colleagues (Clayton and Stolt, 1981; Stolt and Weglein, 1985; Stolt and Benson, 1986; Weglein and Stolt, 1999; Stolt and Weglein, 2012) then provided the extension, for one way waves, of the Claerbout source and receiver experiment imaging condition (Imaging condition III) to allow for non-coincident source and receiver at time equals zero, to realize both structural and inversion objectives. Due to causality, the offset dependence, at time equals zero, is highly localized about zero offset. The character of that singular function, sharply peaked in offset, is smooth in the Fourier conjugate space of offset wave-number, where the extraction of angle dependent plane wave reflection information naturally occurs. The latter extension and generalization produced migration-inversion (Stolt and Weglein, 1985), or first determining where anything changed (migration) followed by what specifically changed (inversion) at the image location. Recently, several papers by Weglein and his colleagues (Weglein et al., 2011a,b; Liu and Weglein, 2014) provided the next step in the evolution of migration based on the Claerbout predicted source and receiver experiment imaging condition (Imaging condition III), extending the prediction of the source and receiver experiment in a volume within which there are two way propagating waves. The latter method of imaging based on Claerbout Imaging condition III for a medium with two way propagating waves, plays a central role in the analysis of this paper. The predicted experiment, in the volume is realized by calling upon Green's theorem and a Green's function that along with its normal derivative vanishes on the lower portion of the closed surface.

Summary of wave equation migration for one way and two way propagating waves

Green's theorem based migration and migration-inversion require velocity information for location and velocity, density, absorption...for amplitude analyses at depth. When we say the medium is ``known,'' the meaning of known depends on the goal: migration or migration-inversion. Backpropagation and imaging each evolved and then extended/generalized and merged into migration-inversion (Figure 1).

For one-way wave propagation the double downward continued data, D is

$$D(\text{at depth}) = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} D \, dS_g \, dS_s$$

where D in the integrand is equal to the data on measurement surface. $\partial G_0^{-D} / \partial z_s =$ anticausal Green's function with Dirichlet boundary condition on the measurement surface, s = shot, and g = receiver. For two-way wave double downward continuation:

$$\begin{aligned} D(\text{at depth}) = \int_{S_s} \Bigg[& \frac{\partial G_0^{DN}}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g \\ & + G_0^{DN} \frac{\partial}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g \Bigg] dS_s, \end{aligned} \tag{1}$$

where D in the integrand is equal to the data on measurement surface. G_0^{DN} is neither causal nor anticausal. It is the Green's function needed for WEM RTM, that is RTM based on Claerbout Imaging Condition III. G_0^{DN} is the Green's function for the model of the finite volume that vanishes along with its normal derivative on the lower surface and the walls. If we want to use the anticausal Green's function of the two-way propagation with Dirichlet boundary conditions at the measurement surface then we can do that, but we will need measurements at depth and on the vertical walls. To have the Green's function for two-way propagation that doesn't need data at depth and on the vertical sides/walls, that requires a non-physical Green's function that vanishes along with its derivative on the lower surface and walls. Green's functions called upon in Green's theorem applications for migration are auxiliary functions and are specific point source wavefield solutions that satisfy the medium properties in the finite volume, and whose other properties are chosen for the convenience of the application. The commitment within Green's theorem applications is for the physical wavefield, $P(x, y, z, x_s, y_s, z_s)$ to relate to the physical reality and to have physical properties and boundary conditions.

Fang Liu and A. B. Weglein (2014) and Weglein (2015) take the next step to our goal and objective. Having established a Claerbout imaging III methodology (please see equation (1)) for a medium (a finite volume) with two-way propagating waves, we are in a position to predict source and receiver experiments at depth and then a Claerbout III imaging result for data consisting of primaries and multiples. For the 1D layered medium, and a normal incident wave that we are examining, the data (consisting of primaries and internal multiples) and the predicted source and receiver experiment at depth results and the migration algorithm's results are analytic, transparent and the conclusions unambiguous. The role of recorded primaries and multiples in contributing first to the predicted source and receiver experiment at depth, and then to the (Claerbout Imaging III) coincident source and receiver experiment at time equals zero provides a definitive response to whether or not multiples contribute to seismic imaging. Understanding the physics behind the mathematics for the case of primaries and internal multiples, allows for an immediate set of similar conclusions to be drawn for the role of free surface multiples in migration. In the references cited above, we provide the explicit Green's theorem source and receiver at depth prediction and then Claerbout III imaging for a general layered medium where the velocity and density vary and where the data consists of primaries and internal multiples.

We summarize the conclusion of those references (Fang Liu and A. B. Weglein (2014), Weglein (2015))

(1) All recorded events, primaries, internal multiples and free surface multiples contribute to the predicted coincident source and receiver experiment at depth

(2) Only the recorded primaries contribute to the image, that is once the time equal zero imaging condition is called on, only recorded primaries contribute to the image at any depth.

(3) The location of each reflector is determined, along with the reflection coefficient for the experiment both from above and from below each reflector (Figure (2)). The latter is not achievable using Claerbout Imaging II.

If you remove the multiples in the recorded data, the coincident source and receiver experiment at depth would change, but once the imaging condition is applied, the image’s location at the correct depth and its amplitude, the reflection coefficient, will not be affected. If, in these examples, your data consisted of only multiples, you will have no image at any depth. These conclusions are all shown in great detail in the above cited references.

Hence, for the purposes of imaging and inversion (and employing the most capable and quantitative imaging condition), primaries are the events that contribute to imaging and inversion and, hence, are considered signal and multiples are not.

Claerbout II and Claerbout III imaging results

The Claerbout imaging II, the time and space coincidence of up and down waves is formulated as

$$I(\vec{x}) = \sum_{\vec{x}_s} \sum_{\omega} D^{*}\left(\vec{x}, \vec{x}_s; \omega\right) U\left(\vec{x}, \vec{x}_s; \omega\right)$$

Where D is the downgoing wave and is U is the upgoing wave, respectively.

The sum over receivers for a given shot record realizes the Claerbout II imaging concept. The sum over sources is ‘introduced’ in an ad hoc manner to mitigate the inconsistent amplitude and phase of images, that can be clearly seen from imaging results with exact data and imaging a single horizontal reflector (please see the example in Chao Ma and Yanglei Zou(2015)). A comparison with a Claerbout imaging III result for the same reflector and the same data, produces an accurate and consistent reflection coefficient at every point on the reflector, for a single shot record.

Please compare Claerbout II imaging (above) with the one way and two way wave versions of Claerbout Imaging III, below.

$$D(\text{at depth}) = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} D \, dS_g \, dS_s$$

$$D(\text{at depth}) = \int_{S_s} \left[\frac{\partial G_0^{DN}}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g + G_0^{DN} \frac{\partial}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} D + \frac{\partial D}{\partial z_g} G_0^{DN} \right\} dS_g \right] dS_s,$$

For Claerbout III, the sum over receivers predicts the receiver experiment at depth for a source on the measurement surface, and then the sum over sources precisely then predicts the experiment with the source at depth, as well. The integration over receivers and over sources brings the source and receiver experiment to depth. There is nothing ad hoc or designed to fix something amiss (as though the data had random noise, to be mitigated by stacking). The noise is algorithmic, within Claerbout imaging II and is present with exact, analytic noise free data in the earlier integral over receivers as in Claerbout imaging II. This is why we say that Claerbout III is on the firmest physics foundation, with an interpretable, quantitative and consistent meaning to the image. And once again, it's why we adopt Claerbout III for the analysis of the role of primaries and multiples in imaging (in Fang Liu and Weglein (2014) and Weglein (2015)).

Claerbout imaging I, II and III give equivalent imaging results for a normal incident plane wave on a horizontal reflector. But as soon as you consider prestack data for even a single horizontal reflector, the significant differences in image interpretability and consistency become clear. Furthermore only Claerbout III can be readily and naturally extended for amplitude analysis at specular, curved surfaces and point diffractors/pinchouts and imaging both through, and from beneath, a discontinuous velocity model.

For our immediate purpose of examining how multiples can be used to provide an approximate image of an unrecorded primary, we look at Claerbout II with a few examples since the "migrating of multiples" activity is inspired and motivated by that algorithm with different up and down going wave chosen for different uses/objectives/purposes.

Imaging primaries with Claerbout imaging condition II

1D normal incident analytic example

In this section, we use a 1D normal incident analytic example to illustrate the idea of imaging a primary with Claerbout imaging condition II. Assume a down-going spike data that starts at $z = \varepsilon_s$ at $t = t_0 = 0$. The down-going wavefield from the source side that being forward propagated to depth z is $D = e^{i\omega[\frac{z-\varepsilon_s}{c_0}]}$, whereas the up-going wavefield from the receiver side that being back propagated to depth z is $U = R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{d-z}{c_0}]}$, where R_1 and d are the reflection coefficient and the depth of the reflector, respectively, see Figure 3. Applying the Claerbout imaging condition II we have

$$I_P = \int \left(e^{-i\omega\left[\frac{z-\varepsilon_s}{c_0}\right]}\right) \times \left(R_1 e^{i\omega\left[\frac{d-\varepsilon_s}{c_0}+\frac{d-z}{c_0}\right]}\right) d\omega = \int R_1\, e^{-i\omega\left[\frac{2d-2z}{c_0}\right]} d\omega$$
$$= \pi c_0 R_1 \delta(z-d)$$

We obtain the correct image location at depth d with an amplitude of $\pi c_0 R_1$.

Using a multiple to approximately image an unrecorded primary

1D normal incident analytic example

In this section, we apply Claerbout imaging condition II to a seismic data set that contains a first-order free-surface multiple. Similarly, assuming a down-going spike data starts at $z = \varepsilon_s$ at $t = t_0 = 0$ (see Figure 4). A first-order free-surface multiple is recorded at z_g. The down-going wavefield from a "virtual source" (represented by the dashed red line in Figure 4) that being forward propagated to depth z is $D = -R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{d+z}{c_0}]}$, whereas the up-going wavefield from the receiver side (represented by the yellow dashed line in Figure 4) that being back propagated to depth z is $U = -R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{2d}{c_0}+\frac{d-z}{c_0}]}$, where we have assumed the downward reflection coefficient at the free-surface to be -1 in deriving the up and down wavefield, see Figure 4. Applying the Claerbout imaging condition II, we have

$$I_M = \int \left(-R_1 e^{-i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{d+z}{c_0}]}\right) \times \left(-R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{2d}{c_0}+\frac{d-z}{c_0}]}\right) d\omega$$
$$= \int R_1^3\, e^{-i\omega\left[\frac{2d-2z}{c_0}\right]} d\omega = \pi c_0 R_1^3 \delta(z-d)$$

We obtain the correct image location at depth d, yet with a different amplitude of $c_0 R_1^3$, Hence, this produces an approximate image of the unrecorded primary.

The methods that seek to use multiples today as “signal” are really seeking to approximate images due to primaries that have not been recorded, due to limitations in acquisition, and to address the subsequent limited illumination that missing primaries can cause. They are not really using the multiple itself as an event to be followed into the subsurface for imaging purposes. The figure (5) illustrates the idea.

Assume a multiple is recorded, and a primary that is a sub-event is also recorded. The idea is to extract and predict the image due to an unrecorded subevent primary, from the recorded multiple and the recorded primary. All the various incarnations that are using multiples as “signal” are actually, and entirely after removing a recorded longer primary to have the approximate image of an unrecorded primary. It’s the missing image of unrecorded primaries that the method is seeking to produce and to utilize. The use of multiples procedure is itself is testament to the fact that a complete set of primaries is sufficient to image the subsurface, and that is an approximate image of an unrecorded primary that the method seeks to provide.

The recipe of taking the multiples back in time and the primaries forward in time and arranging for Imaging Condition II (not III) produces that output. However, that procedure is not migrating the multiples.

In a Recent Advances and the Road Ahead presentation, “Multiples: signal or noise?”, Weglein (2014) (please see https://vts.inxpo.com/scripts/Server.nxp?LASCmd=L:0&AI=1&ShowKey=21637&LoginType=0&InitialDisplay=1&ClientBrowser=0&DisplayItem=NULL&LangLocaleID=0&RandomValue=1415030021699) showed actual field data examples, from PGS, where there was clear added-value demonstrated for the image from actual primaries, plus the approximate images of unrecorded primaries, compared to the image from the original primaries.

There is another issue: in order to predict a free surface or internal multiple, the primary sub-events that constitute the multiple must be in the data, for the multiple prediction method to recognize an event as a multiple. If a primary is not recorded, the multiple that contains that unrecorded primary will not be predicted as a multiple. That issue and basic contradiction within the method is recognized by those who practice this method, and instead of predicting the multiple, they use all the events in the recorded data, primaries and multiples, and the multiples can be useful for predicting approximate images of missing primaries but the primaries in the data will cause artifacts. There are other artifacts that also come along with this method (from the inability to isolate primaries from multiples with unrecorded primaries) that have been noted in the literature.

There is a key assumption behind the use of source and receiver deghosted multiples to produce an approximate image of unrecorded primary, from the multiple that contains that primary as a subevent, and another recorded source deghosted but receiver ghost of the primary. It assumes that there is an unrecorded primary. If the recording of primary is adequate the procedure is not needed, and in fact, can cause only harm from its artifacts. The value of the procedure depends on the existence of an unrecorded primary. The methods that identify multiples require all primary subevents to be recorded. Hence, the required multiple the method depends on cannot be predicted. To address that dilemma, the entire data set is inserted where a multiple is required. Furthermore, on the other data input part of the algorithm a source deghosted primary is required with a receiver side ghost, assuming multiples have been removed, but that also cannot happen due to the unrecorded primary.

There are an enormous number and variety of false event generated by this procedure, among those false events is 'images of multiples', as figure 6 exemplifies. There are other false images, artifacts that do not correspond to a particular event (e.g., a multiple).

Since the procedure is ad hoc, a on top of an imaging condition for primaries which starts off as somewhat ad hoc (summing over sources),it cannot be easily or naturally improved because there is no framework without the artifacts that utilizes multiples for an enhanced image. One response to the artifacts is to collect the required primaries.

The reality of today’s methods for using multiples to predict missing “primaries” are aimed at structural improvement, at best, and are not claiming, seeking or delivering the amplitude and phase fidelity of the predicted primary. Those who go so far as to advocate using fewer sources and/or more widely separated cables because recorded multiples can produce “something like” a missing primary need to understand the deficits and costs including generating artifacts, less effectiveness with deeper primaries and the amplitude fidelity of the predicted primary. Whether the tradeoff makes sense ought to depend on, in part, the depth of the target, the type of play, and whether a structural interpretation or amplitude analysis is planned within a drilling program and decision.

By the way, this entire wave equation migration analysis (Claerbout Imaging Condition III) is ultimately based on the idea from Green (1828) that to predict a wave (an experiment) inside a volume you need to know the properties of the medium in the volume.

There is an alternative view: The inverse scattering series methods communicate that all processing objectives can be achieved directly and without subsurface information. The inverse scattering series treat multiples as a form of coherent noise, and provide distinct subseries to remove free surface and internal multiples before the inverse scattering subseries for imaging and inversion achieve their goals using only primaries Weglein et al. (2003) and Weglein et al. (2012). If the inverse scattering series needed multiples to perform migration and inversion, it would not have provided subseries that remove those multiply reflected events. With a velocity model (wave equation migration) or without a velocity model (inverse scattering series imaging) only primaries are signal , in the sense that they are the events need to locate and delineate targets. If you want to consider a multiple as a conditional ’signal’, that can at times enhance imaging, there is no harm in that. But to say that multiples are being migrated, and/or are the same footing as primaries, is simply not true and can relate more to confusions or marketing, than to a realistic view of the role that primaries and multiples play in seismic exploration. A complete set of recorded primaries, processed with a wave theory migration (versus asymptotic or ray migration) would not need or benefit from multiples. Multiples need to be removed before performing a velocity analysis using, e.g., tomography, CIG flatness or FWI. And a velocity model is required by all the methods that seek to use multiples to enhance imaging. Hence, multiples need to be removed in a step before this use of ‘multiples’ for imaging unrecorded primaries event gets started. Another question: what if the assumed unrecorded primary event in the method is actually recorded. Will the image of the recorded primary and the image of the approximate version of the recorded primary from the multiple damage the image of the actual primary, that has been assumed to not have been recorded?

Conclusion

Hence, primaries are signal and multiples can be useful, at times, for predicting the image of missing primaries. But it’s primaries that are signal, that we use for structure and inversion.

Primaries are signal for all methods that seek to locate and identify targets.

Given an accurate discontinuous velocity and density model, and data with primaries and multiples, then the Fang Liu and Weglein (2014), Weglein (2015) demonstrated that only primaries contributed to the images at every depth. If you predicted the source and receiver experiment at depth with a smooth velocity, it is possible to correctly locate (but not invert) each recorded primary event—but with a smooth velocity model every free surface and internal multiple will then produce a false image/artifact/event. If you removed the multiples first you can correctly locate structure from recorded primaries using a smooth velocity model.

Hence, we conclude that the inability, in practice, to provide an accurate discontinuous velocity model is why multiples need to be removed before imaging. That reality has been the case, is the case, and will remain true for the foreseeable future. Multiples need to be removed before velocity analysis and they need to be removed before imaging.

Many things are useful for creating primaries: money, the seismic boat, the air-guns, the observer, the cable, computers, etc., but we don't call all useful things signal.

Methods to provide a more complete set of primaries are to be supported and encouraged. Those methods include: (1) advances in and more complete acquisition, (2) interpolation and extrapolation methods, and (3) using multiples to predict missing primaries. However, a recorded primary is still the best and most accurate way to provide a primary, and the primary is the seismic signal.

A multiple can be useful, at times, for providing an unrecorded synthesized primary that is a subevent of the multiple. Given a data set consisting of: (1) the recorded primaries, (2) the synthesized primaries, (3) the free surface multiples, and (4) internal multiples, the practical necessity of using a smooth continuous velocity for migration demands that all multiples be removed before migration. In exploration seismology, migration and migration-inversion are methods we employ to locate and identify structure. Claerbout Imaging Condition III is the most definitive and quantitative migration concept and procedure. This paper reminds us that Claerbout Imaging Condition III clearly communicates that primaries are signal and multiples are noise. The original and intuitive migration idea that takes events in time traces to the location of structure in space, only has meaning for primaries. The most sophisticated and physically well-founded migration theory, based on Claerbout Imaging Condition III, agrees with that assessment and conclusion.

For the purposes of this paper the key point is that the migration result only depends on the primaries in your data, and that free surface and internal multiples play absolutely no role. None, whatsoever. If the multiples would have been removed the migration results with only primaries would be exactly the same, and if the data consisted of only multiples you would no image anywhere, a migration with a null result everywhere. The latter assumed an accurate discontinuous velocity model. If you used a smooth velocity model and your data consisted of primaries and multiples, then every multiple would produce a false image. Since in practice, we (almost) always use a smooth velocity model for migration, multiples need to be removed before migration, to avoid creating false reflectors in your migration result. With a smooth velocity model, migration will produce one false reflector for each free surface and internal multiple. That conclusion generalizes to multi-dimensions where only primaries are required for migration, assuming adequate acquisition, and a wave theory propagator rather than an asymptotic migration (e.g., Kirchhoff, Beam, and Paraxial Ray).

What if your acquisition of primaries is not adequate? In this paper , we show that multiples can be useful for imaging by providing an approximate image of an unrecorded primary, that is a sub-event of the multiple, but the multiple is itself not migrated.

Where did the misnomer "migrating multiples" come from? The somewhat vague nature and lack of a clear physical basis behind the Claerbout II Imaging condition leaves it vulnerable to being misrepresented, abused and misused. As with all of the three imaging principles they were strictly intended and meaningful for primaries, but somehow Claerbout II became the time and space coincidence of anything downgoing and anything else upgoing, became the "migration of the anything upgoing". If a chicken walked backwards and a frying pan went forward until they were time and space coincident, we could call that fried chicken a 'migrated chicken'. That fried chicken would have as much meaning regarding determining a structure map in the earth as a migrated multiple. That type of misnomer, misapplication, stretching, distortion and abuse of meaning is impossible with the definitiveness of the Claerbout III imaging condition, which is the reason we adopt it in our analysis of the role of primaries and multiples in imaging. There is a value in looking at the time and space coincidence of the up and down-going wavefields for other purposeful use, e.g., using multiples to find an approximate image of an unrecorded primary --- useful but it is not a migration of the multiple.

This article is presenting an alternative view of the recent attention and publications on the subject of "migrating multiples". The objective is to put this activity that can, at times, provide value on a reasonable measured and balanced perspective, of what this procedure is, and what it isn't. The method can at times help provide an improvement to imaging, but only when the collection of primaries is incomplete or inadequate, by extracting an approximate image of an unrecorded primary that is a sub-event of the multiple. The image obtained is an approximation to the image of the unrecorded primary, not the image of or "migration of the multiple".

That's another key point we wish to convey. Furthermore, the procedure of using multiples to find an approximate image of an unrecorded primary requires a velocity model, and all velocity analysis methods (e.g., tomography, CIG flatness and FWI) assume that multiples have been removed before the use of multiples step get started.

This is not a case of semantics but is important to understand for how in certain circumstances multiples can assist imaging, but avoiding over-selling and over-stating that value by mislabeling it as "migrating multiples". There are real, clear and evident signs of danger in that label, for interpreting multiples as the new primaries, and having been rehabilitated, multiples can now sit with primaries as good citizens that no longer need to be removed, no more than primaries need to be removed. There are damages in not recognizing the need to remove multiples before real migration and migration-inversion, and can distract from the serious and high priority unfinished business of the removal/elimination of multiples when they are proximal or interfering with primaries

Our purpose in this paper is to provide a clarity and perspective of this activity and a view of the serious value it represents, and the technical shortcomings, drawbacks, and problems it causes, as well. One serious problem and real danger is not in the procedure itself, but the serious misuse of the term migration as in referring to multiples being migrated. What's the problem with the label? We all know that primaries are migrated, and if multiples are now migrated as well, they must be on equal footing with primaries, and since they are now rehabilitated as good seismic citizens, we should no more seek to remove multiples than we seek to remove primaries. That is part of the danger of the misuse of the term migration in this process of trying to have a more complete and approximate set of primaries. This article will seek to disabuse anyone who thinks that multiples are being migrated and that we need no longer need to develop more effective methods to remove multiples.

The danger in this mislabeling and overselling in this case is two-fold , one is a discounting of the actual substantive value represented by the method , and avoiding disappointment and an inevitable back-lash, and the second is it can advertently or inadvertently distract from serious matters of substance(e.g, internal multiple elimination for offshore and onshore applications).

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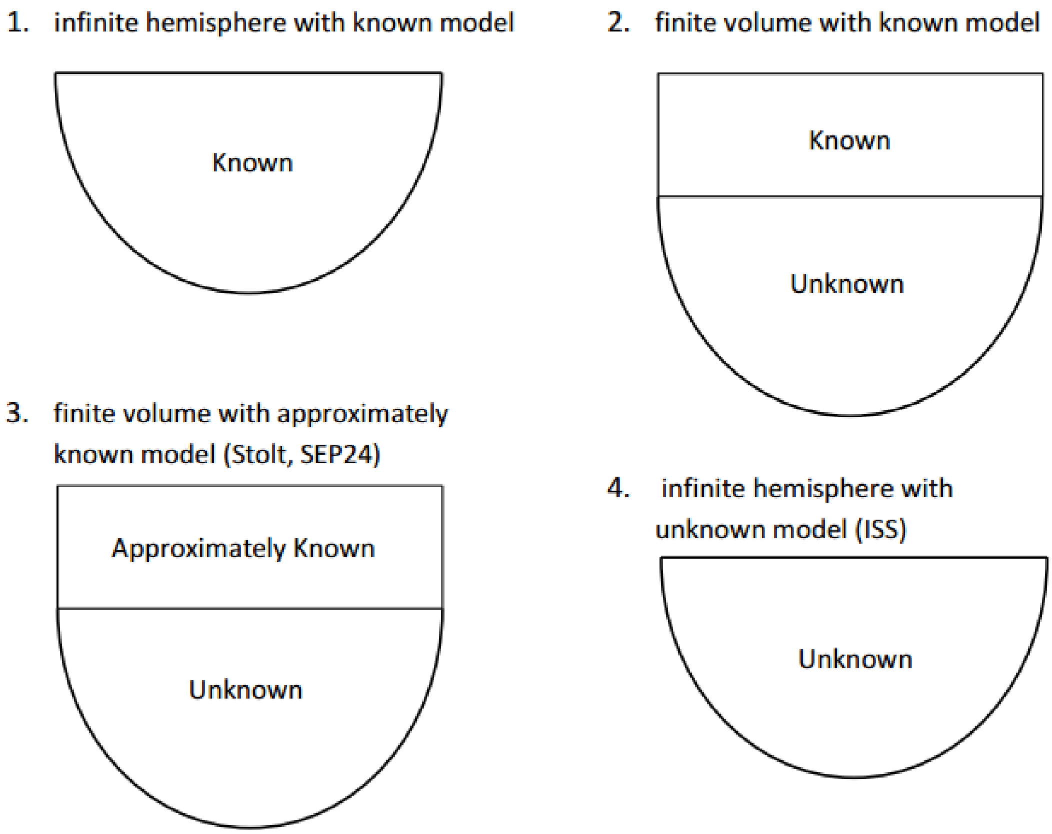

Figure 1: earth model and subsurface information: this figure describes the various earth models in the evolution of depth imaging. The first panel, the infinite hemisphere model was the first adopted by migration methods (1978). The second panel, the finite volume, where subsurface information is known only above any given reflector is the current industry standard. The fourth panel, the basis of the depth imaging with the inverse scattering series, where the velocity model is and remains unknown, everywhere, is the future model of seismic imaging. That fourth model is the model for the inverse scattering series free surface multiple elimination and internal multiple attenuation methods---today's industry standard.

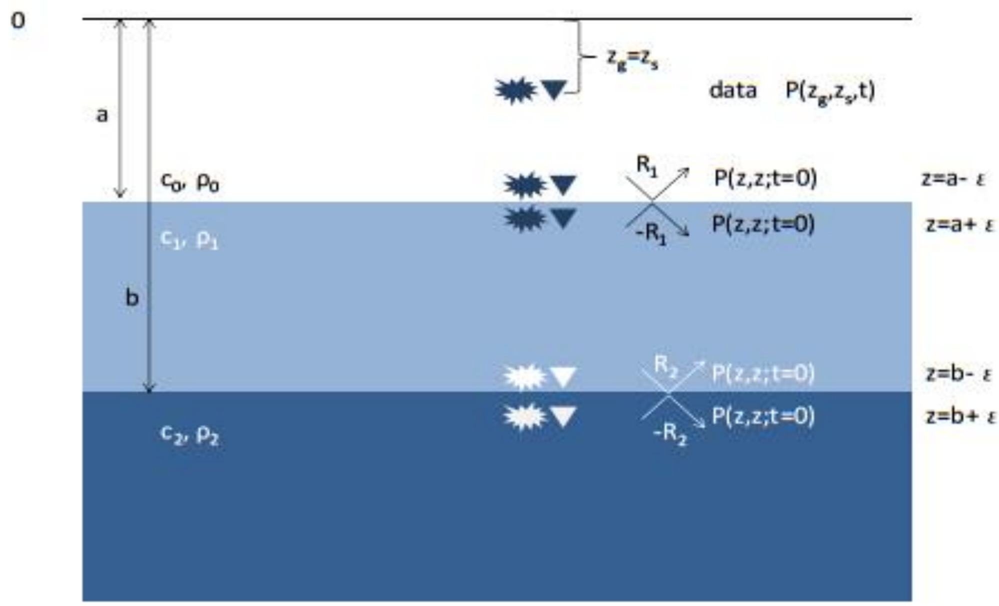

Figure 2: Green's theorem predicts the wavefield at an arbitrary depth z between the shallower depth a and deeper depth b. The experiment illustrated here corresponds to a plane wave normal incident on a layered medium with two reflectors. The measurement coordinates are zg and zs , the coincident source and receiver depths .a- ε, a+ε, b− ε, b+ε are the depth of the predicted source and receiver experiment at depths above and below the first reflector at z=a and the second reflector at z= b.

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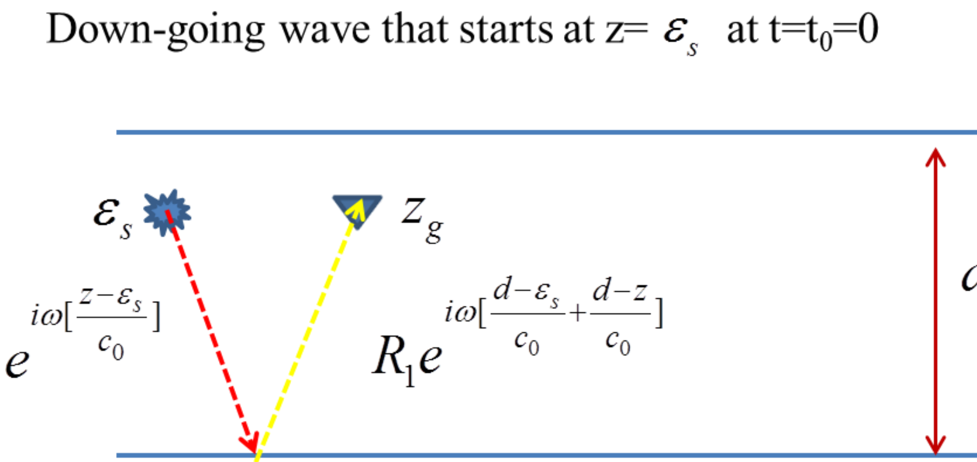

Figure 3: Use of a primary to find a image

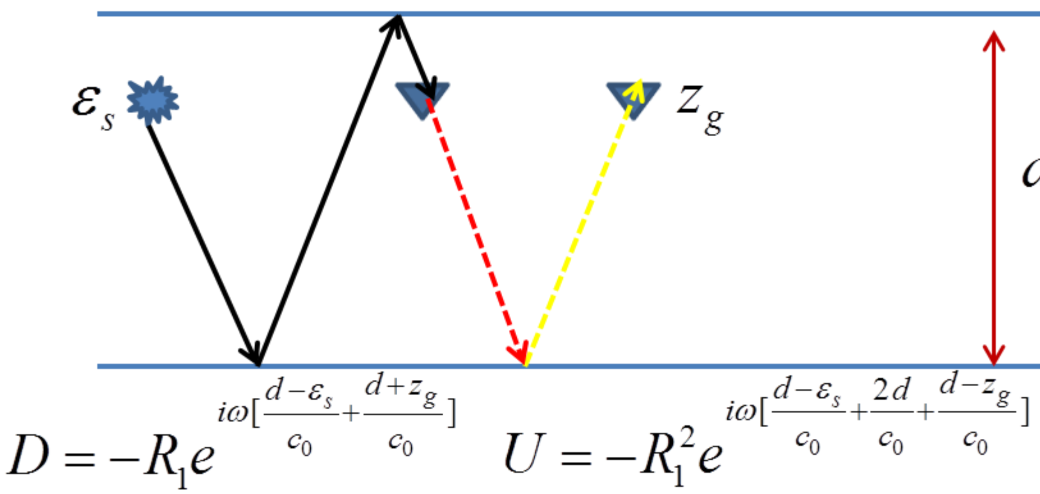

Figure 4: Use of a multiple to find an approximate image of an unrecorded primary.

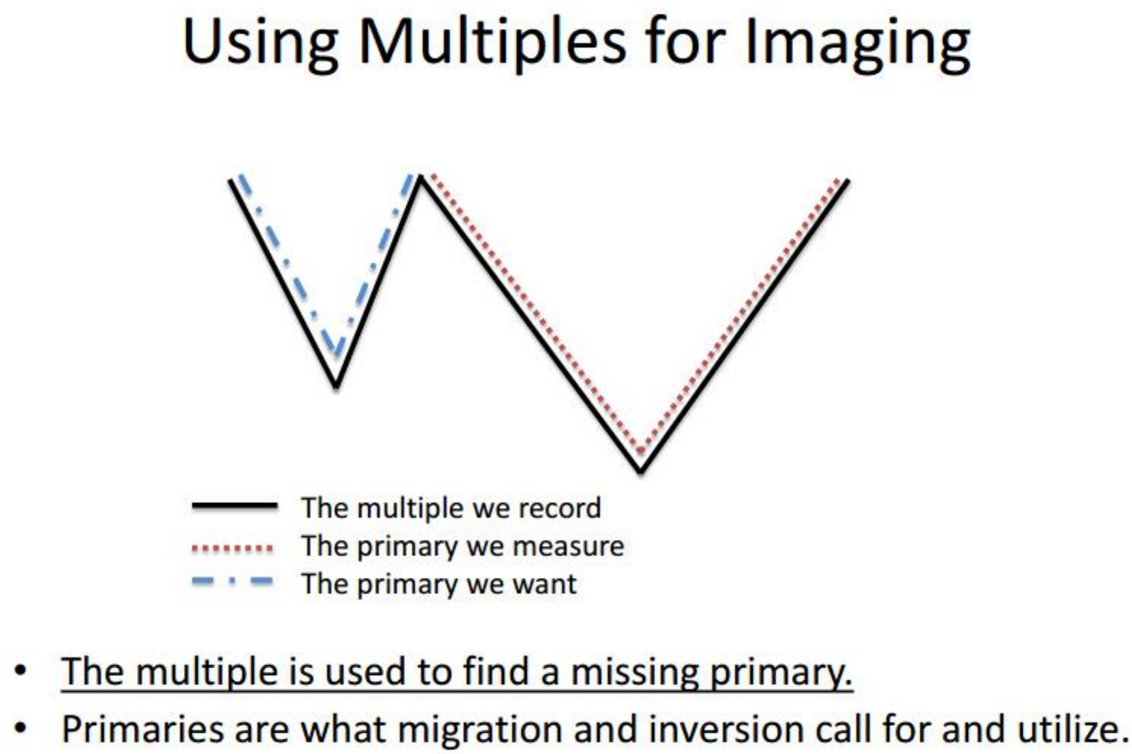

Figure 5: Using multiples for imaging.

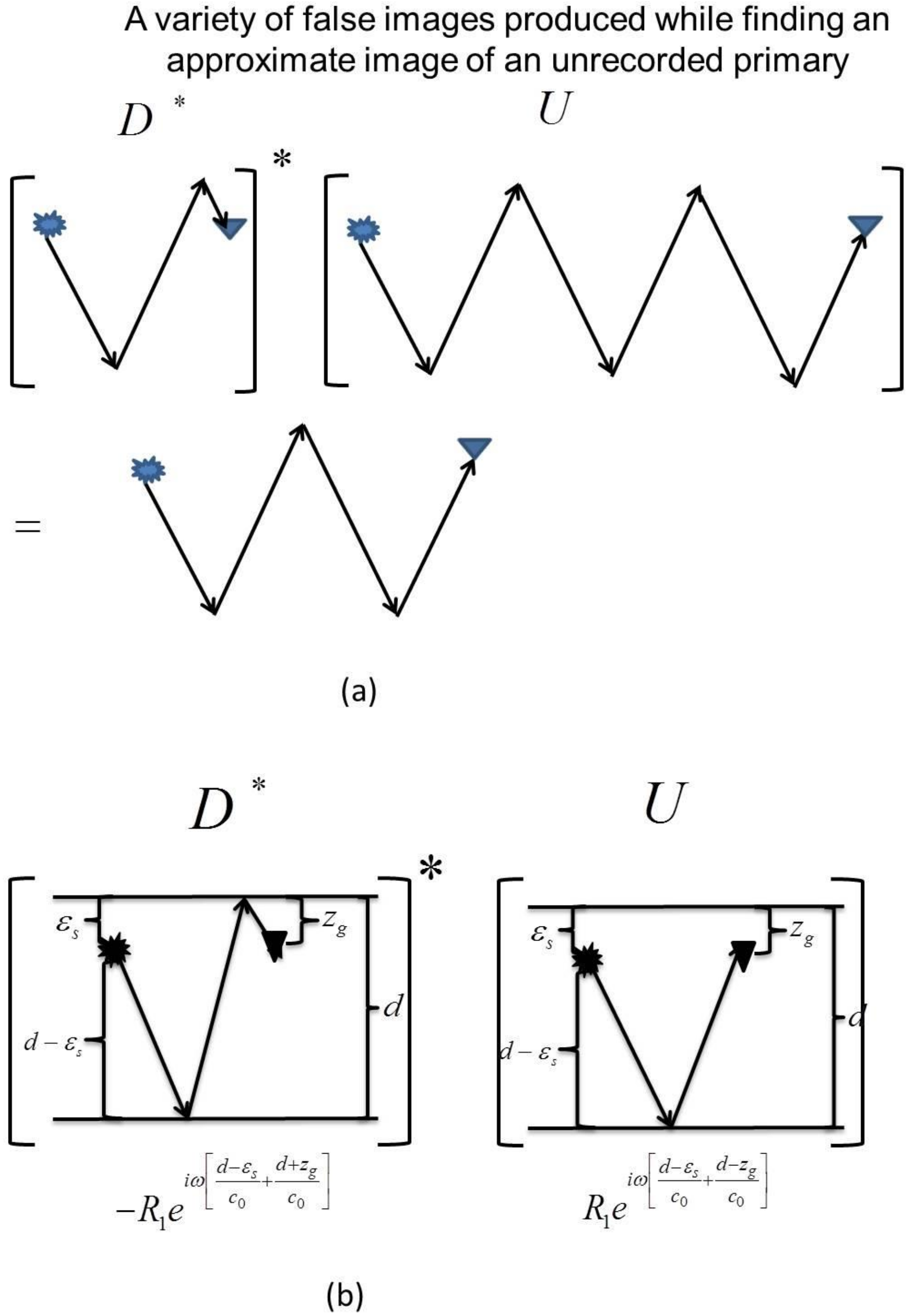

Figure 6: Examples of different types of false images generated by the use of multiples to predict the approximate image of an unrecorded primary. Figure (a) will produce an artifact due to an image of a multiples and (b) will produce an artifact at z=0 (the origin) that is beyond false image due to output mages of multiples.