Why exactly do multiples need to be removed in direct seismic processing methods? And what about indirect methods?

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June 30, 2021

Introduction

Multiple removal has been a long-term objective in seismic exploration. Recent methods that use multiples for different processing goals and objectives can be worthwhile. However, their use can also be a source of confusion as to whether the removal is no longer essential, a priority, or even necessary since some may now view multiples as "rehabilitated" and sit along side primaries as entirely useful events. One purpose of this paper is to disabuse us of that seriously flawed and erroneous thinking and to understand that the use of multiples and the removal of multiples have the same exact goal and objective: the imaging of primaries. We explain exactly why multiples need to be removed in all direct and indirect seismic processing methods — and therefore why multiple removal remains a key and central issue and challenge in seismic exploration.

Multiple Removal and Direct and Indirect Seismic Processing Methods

In what follows, we will explain why all direct and indirect seismic processing methods remove multiples at some step or stage within the method. Among references that define (and distinguish between) direct and indirect seismic processing methods are: Weglein [2013, 2017, 2018]. We can view this paper as the first of a two part set, with the first one here (basically asking "why") and the second paper asking "how?"

Direct Seismic Methods

For the purposes of this paper, it is useful to separate direct methods that require or do not require a velocity model.

Direct seismic methods with a velocity model — migration, homogeneous, continuous and discontinuous velocity models

Methods that employ the wave equation for migration have two ingredients: (1) a wave propagation concept and (2) an imaging condition. In Claerbout [1971], he described three imaging conditions for seismic migration. He combined these imaging conditions with one-way wave propagation concepts to determine structure at depth. The three imaging conditions are: (1) the exploding reflector model, (2) the space and time coincidence of an upgoing wave from the reflector and a downgoing wave from the source, and (3) the predicted coincident source and receiver experiment at depth, at time equals zero. We will refer to these original imaging conditions as Claerbout I, II and III. Claerbout I (the exploding reflector model) only relates to stacked or zero offset data. Claerbout II and Claerbout III are valid for prestack data. The third imaging condition, CIII, stood alone in terms of clarity and definitiveness and in its potential to be extended for complex structure and associated amplitude analysis. For example, Stolt and Weglein [1985] and Stolt and Weglein [2012] extended the original CIII for more physically complete and accommodating structural models, and in addition provide a detailed angle dependent amplitude analysis at the target, for both specular and non-specular reflection. We label the latter extension Stolt Claerbout III migration (or SCIII). Only CIII or SCIII could be extended to accommodate imaging beneath a discontinuous medium, the latter required to analyze (for the first time) and to unambiguously define the role of primaries and multiples in migration. The predicted coincident source and receiver experiment at depth consists of all the events that experiment would record, if you actually had a source and receiver at that subsurface location.

Weglein et al. [2016] provided that extension of SCIII to allow a discontinuous medium above a target and to image above and below each reflector, without any artifacts or issues such as “rabbit ears”. That new migration algorithm is represented in equation (3) below.

Stolt Claerbout III migration in homogeneous, smoothly varying or discontinuous media

For one-way wave propagation in a homogeneous or smoothly varying 2D medium the predicted source and receiver experiment at depth, $D(x_g, z_g, x_s, z_s, \omega)$

is

$$
\begin{aligned}
&D\left(x_g, z_g, x_s, z_s, \omega\right)(\text{at depth}) \\
&\qquad = \int_{S_s} \frac{\partial G_0^{-D}(x_s, z_s, x_s', z_s', \omega)}{\partial z_s'} \times \int_{S_g} \frac{\partial G_0^{-D}\left(x_g, z_g, x_g', z_g', \omega\right)}{\partial z_g'} \\
&\qquad \times D\left(x_g', z_g', x_s', z_s', \omega\right) dS_g \, dS_s, \qquad (1)
\end{aligned}
$$

where (x_g, z_g) and (x_s, z_s) are the coordinates of the predicted receiver and source at depth, and D in the integrand is the data, D(on the measurement surface), G_0^{-D} is the anticausal Green's function with Dirichlet boundary condition

on the measurement surface, s connotes shot, and g, receiver, respectively. For one way upgoing wave propagation, and this choice of Green's function, the lower surface doesn't contribute to the field inside the finite volume. For two-way propagation, in, e.g., a discontinuous medium above the image point (i.e., above the target reflector), we begin with the recorded data $D(x_g', z_g', x_s', z_s', \omega)$ on horizontal measurement surfaces, with z_g' = constant and z_s' = constant. The predicted experiment for the receiver at x_g, z_g, at depth, and the source at x_s', z_s' (on the measurement surface) is

$$
\begin{aligned}
&\int \{ \frac{\partial G_0^{DN}}{\partial z_g'} \left(x_g, z_g, x_g', z_g', \omega\right) D\left(x_g', z_g', x_s', z_s', \omega\right) \\
&- G_0^{DN}\left(x_g, z_g, x_g', z_g', \omega\right) \frac{\partial D}{\partial z_g'} \left(x_g', z_g', x_s', z_s', \omega\right) \underbrace{\{dx_g'\}}_{\{dS_g\}} \\
&= D\left(x_g, z_g, x_s', z_s', \omega\right) \qquad (2)
\end{aligned}
$$

A second application of Green's theorem inputs (2) and then predicts the experiment for both the receiver at x_g, z_g and the source at x_s, z_s, at depth, using equation (3) below

$$\int \{ D(x_g, z_g, x_s', z_s', \omega)\ \frac{\partial G_0^{DN}}{\partial z_s'}(x_s, z_s, x_s', z_s', \omega)$$

$$- G_0^{DN}(x_s, z_s, x_s', z_s', \omega)\ \frac{\partial D}{\partial z_s'}\left(x_g, z_g, x_s', z_s', \omega\right) \underbrace{\{dx_s'\}}_{\{dS_s\}}$$

$$= D\left(x_g, z_g, x_s, z_s, \omega\right) \qquad (3)$$

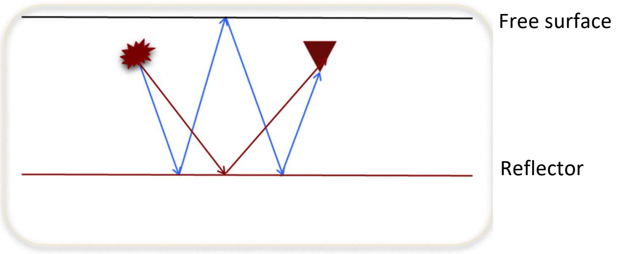
Equation (3) is the prediction required for Stolt Claerbout III migration for heterogeneous (and discontinuous) media. G_0^{DN} is the Green's function for wave propagation in the finite volume that vanishes along with its normal derivative on the lower surface of the finite volume. $dS_g = dx_g'$ and $dS_s = dx_s'$ in a 2D prediction. An integral of equation (3) over ω produces the predicted experiment at t=0 and SCIII migration.

Summary for Stolt CIII migration in a layered medium with data consisting of primaries and multiples

The analytic analysis of equation (3) for a layered medium is found in Weglein [2016], (following Liu and Weglein, 2014) and demonstrates, for the first time, how the actual individual recorded events (within the recorded data on the measurement surface) contribute to the predicted coincident source and receiver experiment at depth, and to each individual event in that predicted experiment. That analysis can output the source and receiver experiments predicted above and below each reflector.

At each depth, z, below the measurement surface, the predicted coincident source and receiver experiment cares about (depends on) <u>all</u> the actual recorded primary and multiple events on the measurement surface. At the predicted source and receiver experiment, any point at depth, that doesn't correspond to a location above or beneath a reflector, will produce a zero result when the $t=0^+$ imaging condition is applied. However, when using an accurate discontinuous velocity model and the imaging condition $t = 0^+$ is applied to the coincident source and receiver experiment at depth, z, above or beneath a reflector only the recorded primaries on the measurement surface contribute to the migration result.

The conclusion: multiples do not contribute to the image at any depth, when using an accurate discontinuous velocity model above the reflector to be imaged. That is, if we migrated data con- sisting of primaries and multiples with an accurate discontinuous velocity model, and used Stolt CIII migration for heterogeneous media, equation (3) at t=0, then multiples in the recorded data on the measurement surface will not contribute to the image above or below a reflector. For that (correct



Red event: primary
Blue event: free-surface multiple

Figure 1: *a primary and a free surface multiple (recorded data)*

discontinuous migration velocity) situation, multiples would not cause false images, and would not be harmful, or helpful. Please see figures 1-3 that illustrate these steps and conclusions for recorded data consisting of primaries and free surface multiples, and figures 4-7 for recorded data consisting of primaries and internal multiples, using equation (3) a Stolt Claerbout III migration for heterogeneous media. [Weglein et al., 2016]

However, if the migration (in a discontinuous medium) uses a smooth velocity for the data consisting of primaries and multiples, the "predicted" source and receiver experiment at depth will not be the actual source and receiver at depth. That difference and error results in every multiple causing a false image. Hence for a smooth velocity model, multiples must be removed. Since the industry leading edge migration velocity methods can at best find an improved smooth velocity model — and there are currently no candidates to produce an accurate discontinuous velocity model, recorded multiples must be removed now and for the foreseeable future. See e.g. the 2021 SEG/DGS Workshop on Velocity Model Building [Saad et al., 2021] and the final/wrap-up presentation by Weglein [Weglein, 2021]

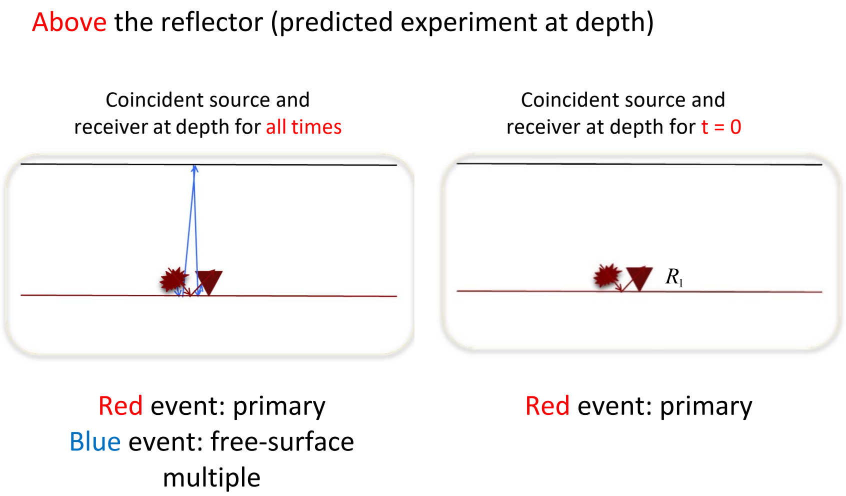


Figure 2: *the predicted experiment (and the t=0 image) at depth (above the reflector) from a recorded data consisting of a primary and a free-surface multiple*

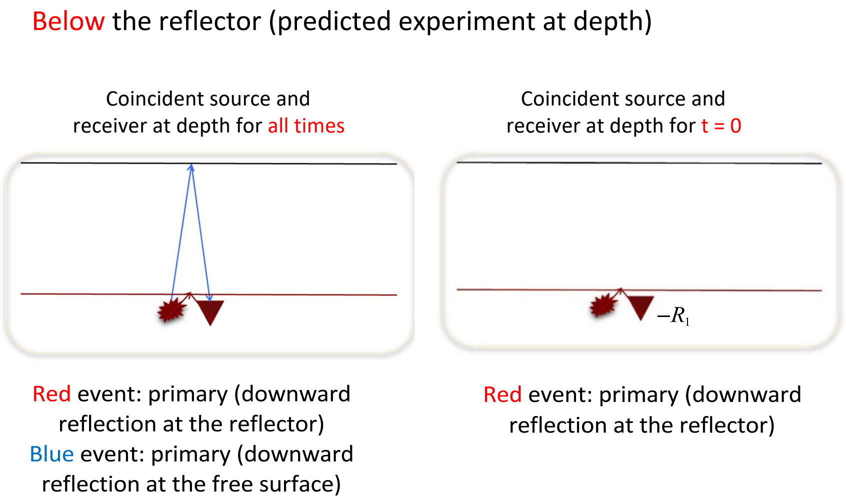
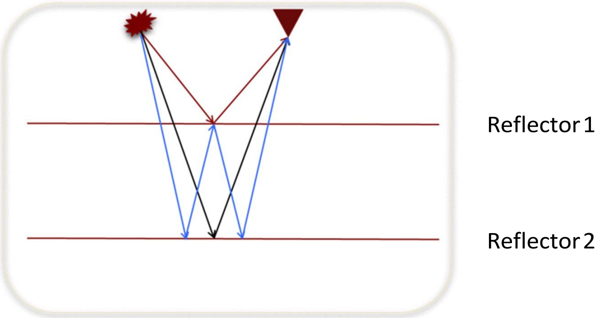


Figure 3: *the predicted experiment (and t=0 image) at depth beneath the reflector, from a recorded data consisting of a primary and free surface multiple*

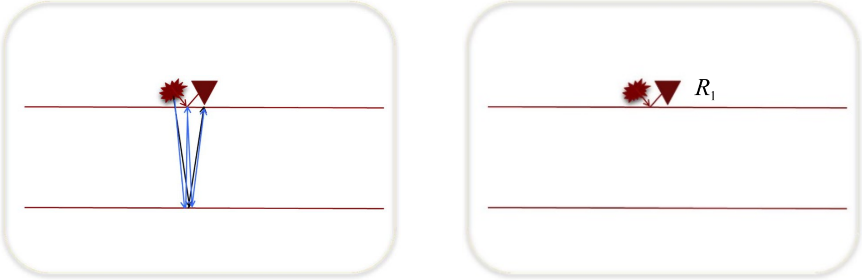


Red event: primary from the first reflector
Black event: primary from the second reflector
Blue event: internal multiple

Figure 4: *the recorded data consisting of two primaries and an internal multiple*

Coincident source and
receiver at depth for all times

Coincident source and
receiver at depth for t = 0



Red event: primary from the
first reflector
Black event: primary from the
second reflector
Blue event: internal multiple

Red event: primary from the
first reflector

Figure 5: *the predicted experiment (and image) above the first reflector, for a recorded data consisting of two primaries and an internal multiple*

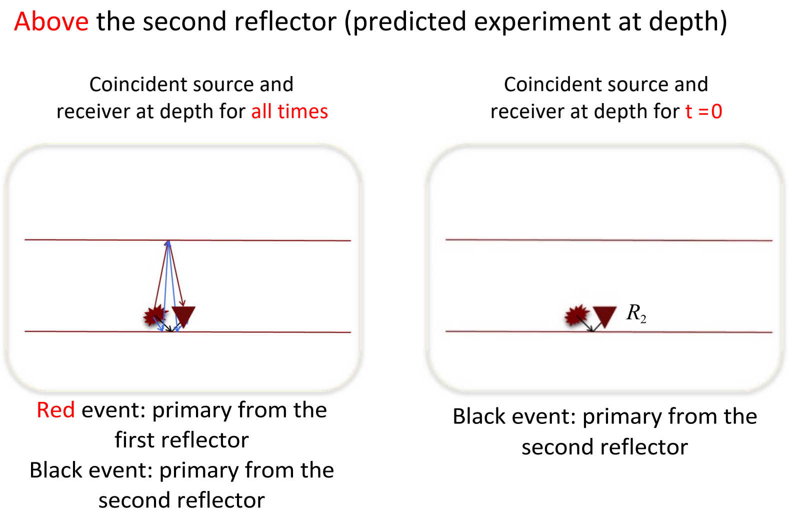


Figure 6: *The predicted experiment (and image at t=0) above the second reflector for a recorded data consisting of two primaries and an internal multiple*

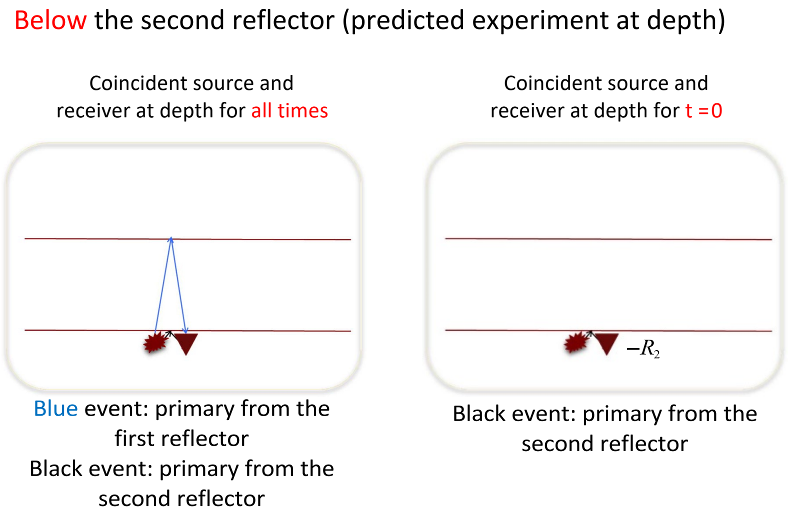


Figure 7: *the predicted experiment at depth (and image at t=0) beneath the second reflector for a recorded data consisting of two primaries and an internal multiple*

Direct methods with a velocity model: removing and using multiples

In the previous section we reviewed a new (and first) migration method (Weglein et al., 2016, Zou et al., 2017, Weglein, 2016) that can image above and below reflectors in a discontinuous medium, (without artifacts) and can accommodate primaries and multiples. The conclusion of that analysis is when using that new migration (SCIII that we label migration for heterogeneous media) with an accurate discontinuous media, multiples provide no harm or benefit, and there is no reason to remove them. However, when using a smooth velocity model, multiples will cause false images that can interfere with or masquerade as reflectors. The latter analysis is not possible with all current methods of migration, e.g., RTM, since RTM cannot image in a discontinuous medium (without artifacts) even with an accurate discontinuous velocity model. The methods that seek to remove these intrusive RTM artifacts (see e.g. Liu et al., 2011) have their own serious artifacts that damage the structural and amplitude fidelity of images. SCIII for heterogeneous media images in a discontinuous medium without artifacts.

Multiples can at times be useful, see e.g., Lu et al. [2011], Whitmore et al. [2011a,b], Ma and Zou [2015] but the migration of a multiple has no meaning [Weglein, 2019b]. Within the sphere of direct seismic methods with a velocity model, certain multiples can be useful to seek an approximate image of an unrecorded primary. Constrained by our ability to find (at most) a smooth velocity model for migration, the removal of recorded multiples is necessary to image recorded primaries, and the removal of unrecorded multiples is required to find an approximate image of an unrecorded primary. All multiples must be removed to image primaries, recorded and unrecorded primaries. Weglein [2018, 2017, 2019a]

All of the migration methods we have been discussing assume that the recorded data coverage is adequate to carry out their function. What about when the set of recorded primaries is inadequate?

The above methodology (of using multiples) assumes, e.g., that a recorded free surface multiple consists of two subevents, one that is recorded, and that the second subevent is a primary that is unrecorded. The idea is to extract and predict, from the recorded multiple and its recorded subevent, the approximate image of the unrecorded primary. If all the subevents of the multiple are recorded, the multiple has no use. This use of multiples is itself a testament to the fact that a complete set of recorded primaries is sufficient for imaging the subsurface.

We often hear that multiples can be useful to enhance illumination. To paraphrase Jon Claerbout “Waves, and the reflected seismic wavefield, are ubiquitous, and have no illumination issues. However, seismic processing methods that are asymptotic high frequency approximations, ray-like in nature, (e.g., Kirchhoff and RTM migration) (Weglein et al., 2016) can “squeeze” the wave into ray paths, that leave gaps and produce illumination issues and challenges.” In contrast, Stolt CIII migration [Weglein et al., 2016], equation (1) and equa- tion (3), are the only migration methods that make no high frequency ap- proximation in either the imaging condition or the propagation model.

The fact that our most capable migration velocity models (today and for the foreseeable future) are smooth and continuous, remains the key and central reason that all multiples must be removed for imaging and inversion when using any method that requires a velocity model.

Direct methods without a velocity model

The only direct method that can input primaries and multiples, and output all processing objectives without knowing, estimating or determining subsurface information (including velocity) is the isolated task subseries of the inverse scattering series. There are distinct subseries that directly remove free surface and internal multiples (e.g. Weglein et al., 2003, Zou et al., 2019). If multiples were needed to perform tasks such as depth imaging, Q compensation and parameter estimation the ISS would not have subseries whose entire purpose is to remove them. For direct seismic methods that do not require a velocity model multiples must be removed, with primaries (only) as input to the imaging and inversion subseries. See, e.g., Weglein et al. [2012], Zhang and Weglein [2009a,b], Liang et al. [2013], Zou and Weglein [2018].

Indirect seismic methods, e.g., CIG flatness and FWI

There are different types of indirect inverse methods. Among them are:
(1) seeking to satisfy a property that a inverse solution would possess;
(2) solving a forward problem in an inverse sense, and
(3) model matching.
The CIG flatness criteria is in the first category, while solving an elastic inverse in terms of PP data and FWI are in the second and third category, respectively. Why each of these is "indirect" is fully detailed in Weglein [2013, 2018, 2017, 2020].

Indirect methods like CIG flatness represent a necessary but not sufficient imaging condition that a correct migration velocity model has been found. The CIG flatness criteria assumes that the data consists of primaries and that multiples have been removed.

References for CIG are Anderson et al. [2012], Baumstein et al. [2009], Ben-Hadj-ali et al. [2008, 2009], Biondi and Sava [1999], Biondi and Symes [2004], Brandsberg-Dahl et al. [1999], Chavent and Jacewitz [1995], Fitchner [2011], Guasch et al. [2012], Kapoor et al. [2012], Rickett and Sava [2002], Sava et al. [2005], Sava and Fomel [2003], Sirgue et al. [2009, 2010, 2012], Symes and Carazzone [1991], Tarantola [1987], Zhang and Biondi [2013]. Many wrong velocity models can and will also satisfy a flat common-image-gather criterion, especially under complex imaging circumstances.

Another type of indirect method, FWI, is a model matching methodology that can input any data set, consisting of primaries, free surface multiples and internal multiples. Among FWI references are Brossier et al. [2009], Crase et al. [1990], Gauthier et al. [1986], Nolan and Symes [1997], Pratt [1999], Pratt and Shipp [1999], Sirgue et al. [2010], Symes [2008], Tarantola [1984, 1986], Valenciano et al. [2006], Vigh and Starr [2008], Zhou et al. [2012]. In practice, primaries are considered not enough, and primaries and all multiples too much to match. Internal multiples are first removed and then primaries and free surface multiples are matched. As was documented in a recent SEG/DGS Workshop on Velocity Model Building Saad et al. [2021] and the final/wrap-up presentation by Weglein [2021], FWI has been useful in providing an improved smooth velocity for migration. As we pointed out earlier in this paper, with a smooth migration velocity model, all multiples must be removed.

Therefore either initially or ultimately all multiples must be removed in all indirect seismic methods.

Conclusions

All current migration velocity analysis methods can (at best) produce a smooth continuous migration velocity model. For direct seismic methods, for example, migration with a smooth velocity model, all multiples will cause false images and artifacts — and need to be removed. To clearly analyze the role of primaries and multiples in imaging requires a new form of migration (that we label Stolt Claerbout III for heterogeneous media) that can image in a discontinuous medium without artifacts.

To use a recorded multiple, we assume it contains two subevents, one recorded and the other not recorded. Let's further assume that the unrecorded subevent is an unrecorded primary. When using a recorded multiple, and the recorded subevent of the multiple, to seek an estimate of the image of an unrecorded primary subevent of the multiple — to satisfy the latter assumption, unrecorded subevents of the recorded multiple, that are (not unrecorded primaries but rather) unrecorded multiples, must be removed. Furthermore the original recorded multiple must be removed to image recorded primaries. Hence, recorded and unrecorded multiples must be removed to image recorded and unrecorded primaries.

The inverse scattering series, is the only direct inversion method for a multi-dimensional earth that doesn't require any subsurface information (including velocity) to be known, estimated or determined. It contains distinct isolated task subseries that remove free surface and internal multiples. Only primaries are called for in task specific subseries for structure determination, parameter estimation and Q compensation without knowing, estimating or determining Q.

For indirect methods, based on a criteria that only relate to primaries, e.g., CIG flatness, multiples must first be removed. FWI is model matching of primaries and multiples and currently is able to output a smooth velocity for migration. Multiples must be removed when using a smooth velocity for migration. For the smooth migration velocity output of FWI to be useful, for imaging and inversion, multiples must first be removed.

Hence, all direct and indirect seismic processing methods require all multiples to be removed, either initially or eventually.

Acknowledgements

We appreciate the encouragement and support of M-OSRP sponsors. J. D. Mayhan is thanked for his assist in preparing this paper.

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