General theory for accommodating primaries and multiples in internal multiple algorithm: analysis and numerical tests

Hong Liang, Chao Ma and Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston*

SUMMARY

The inverse scattering series (ISS) predicts internal multiples directly and without subsurface information. This is achieved through a task-specific subseries within the overall ISS. The ISS leading-order attenuator of first-order internal multiple is the leading-order term in the subseries that contributes to the removal of first-order internal multiples. It has shown stand-alone capabilities for internal multiple prediction/attenuation for both marine and on-shore plays. The basic idea behind the leading-order attenuator is that all the events in the data are treated as subevents and combined nonlinearly (three data sets are involved), and among all the combinations first-order internal multiples can be predicted by the combination that has all subevents correspond to primaries. However, the entire data set, consisting of primaries and internal multiples, enters the algorithm. When internal multiples in the data themselves act as subevents, the leading-order attenuator produces not only first-order internal multiples, but also higher-order internal multiples and, at times, spurious events. The latter have been observed in the tests of Fu et al. (2010) and Luo et al. (2011). Weglein et al. (2011) have noted this and suggest that the resolution of the problem would reside in other terms of the ISS. Ma et al. (2012) describes the initial occurrence of the circumstance under which spurious event arises, and explains how to address that issue. This abstract extends the analysis in Ma et al. (2012) to more complex circumstances, and provide a description of the general arrival of spurious events. In this abstract we show how the ISS anticipates the issue due to spurious events and provides the response.

INTRODUCTION

The inverse scattering series can achieve all processing objectives directly and without subsurface information. Compared to the ISS free-surface multiple removal methods where the location and the properties of the free surface responsible for free-surface multiples are well-defined, the ISS internal multiple method does not require information concerning the properties of the Earth where internal multiples have experienced a shallowest downward reflection. It is data-driven and predicts internal multiples at all depths at once.

The ISS internal multiple attenuation algorithm was first proposed by Araújo et al. (1994) and Weglein et al. (1997). This algorithm is applicable for towed-streamer field data, land data, and ocean bottom data (Matson and Weglein, 1996; Matson, 1997) and can accommodate internal multiples with converted wave phases (Coates and Weglein, 1996). Ramírez and Weglein (2005) and Ramírez (2007) discuss early ideas to extend the attenuation algorithm towards an elimination method. The ISS internal multiple algorithm has shown encouraging results and differential added value when compared to other internal multiple methods (Fu et al., 2010; Hsu et al., 2011; Terenghi et al., 2011; Weglein et al., 2011; Luo et al., 2011; Kelamis et al., 2013).

Early analysis of the ISS leading-order attenuator focused on the performance of internal multiples prediction by using subevents that correspond to primaries. However, the input data contain both primaries and internal multiples and all events in the data will be treated as subevents. Under some circumstances treating internal multiples as subevents in the leading-order internal multiple algorithm can lead to spurious events. We show that spurious events can occur when more than two reflectors are involved in the data being processed, and explain how terms further in the ISS address and remove those spurious events. Following the suggestion of Weglein et al. (2011) Ma et al. (2012) derives the modified ISS internal multiple algorithm addressing the spurious event arising from the second of the three integrals of the ISS leading-order attenuator in a three-reflector medium. This paper evaluates that algorithm using numerical examples, and also extends the algorithm to a medium with a large number of reflectors.

THE LEADING-ORDER ISS INTERNAL MULTIPLE ATTENUATION ALGORITHM

The ISS internal multiple attenuation algorithm is a subseries of the inverse scattering series. The first term in the algorithm is the deghosted input data D from which the reference wavefield and free-surface multiples have been removed and source wavelet has been deconvolved. The second term in the algorithm is the leading-order attenuator of first-order internal multiples which attenuates first-order internal multiples (the order of an internal multiple is defined by the total number of downward reflections). The leading-order attenuator in a 2D earth is given by Araújo et al. (1994) and Weglein et al. (1997). For a 1D earth and a normal incidence wave the equation reduces to

$$b_3^{PPP}(k) = b_3(k) = \int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} b_1(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} b_1(z_3), \qquad (1)$$

where the deghosted data, $D(t)$, for an incident spike wave, satisfy $D(\omega) = b_1(2\omega/c_0)$, and where $b_1(z) = \int_{-\infty}^{\infty} e^{-ikz} b_1(k) dk$, $k = 2\omega/c_0$ is the vertical wavenumber, and $b_1(z)$ corresponds to an uncollapsed FK migration of an normal incident spike plane-wave data. For non-spike data, there is an obliquity factor in the relations between the data D and b_1 in the frequency domain (see Page R64 and R65 in Weglein et al. (2003)). Here, we introduce a new notation b_3^{PPP} where the superscript ("P" represents primary) indicates which events in the data input in each of the three integrals that

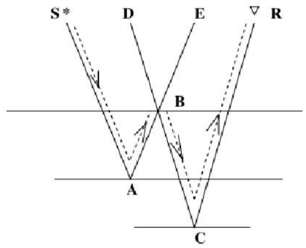


Figure 1: An internal multiple (dashed line) constructed by the "lower-higher-lower" pattern of three primary subevents (solid line). Figure adapted from Weglein et al. (2003).

we are focusing on towards the overall purpose of removing first-order internal multiples. The data with first-order internal multiples attenuated are

$$D(t) + D_3(t), \tag{2}$$

where $D_3(t)$ is the inverse Fourier transform of $D_3(\omega)$ and $D_3(\omega) = b_3(k)$ for an incident spike wave. Weglein and Matson (1998) showed that this algorithm can be interpreted using the subevent concept (see Figure 1).

THE GENERAL OUTPUT OF THE LEADING-ORDER ATTENUATOR WHEN AN INTERNAL MULTIPLE IS TREATED AS A SUBEVENT BY THE ALGORITHM

Early analysis focused exclusively on the performance of the algorithm for b_3 for the events in the data that correspond to primaries. However, seismic data contain not only primary events but also internal multiples. Zhang and Shaw (2010) have shown that higher-order internal multiples can be predicted by the leading-order attenuator using internal multiples as subevents in a two-interface example. However, the situation is considerably more complicated when the data from three or more reflectors are considered. In the latter case, spurious events can be predicted whose traveltimes do not correspond to an event in the data. In this section, we illustrate in a 1D earth the specific conditions under which the spurious events are produced by the leading-order attenuator using one internal multiple subevent.

An internal multiple subevent in the second integral in b_3

In Ma et al. (2012) it is shown that in a medium with three reflectors, and when an internal multiple acts as a subevent in the second of the three integrals (in equation 1) a spurious event can be produced. In this section, we interpret this diagrammatically using Figure 2 (pseudo-depth is determined by the water speed image, $b_1(z)$). An internal multiple has each of its downward reflections between two upward reflections. Then, in the diagrammatic representation of an internal multiple (Figure 2(a)) a higher red circle with a "-" sign should have lower blue circles with "+" signs on both side. However, in Figure 2(c) each of the two red circles has only one lower blue circle on one side, and one higher blue circle on the other side. Thus, this predicted event is neither an

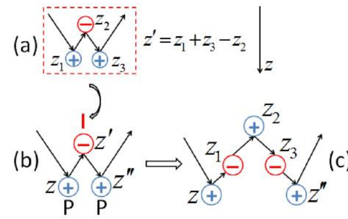


Figure 2: Diagrammatic illustration of the generation of a spurious event. (a) The diagram of a first-order internal multiple. The sign "+" ("-") means upward (downward) reflection or the pseudo-depth is added (subtracted). (b) Three subevents used by b_3: a primary ("P") with pseudo-depth z, an internal multiple ("I") with pseudo-depth z', and a primary with pseudo-depth z'', with $z' < z, z''$. (c) The produced spurious event with pseudo-depth $(z + z'' - (z_1 + z_3 - z_2))$.

internal multiple, nor a primary. The spurious event described here is generated by the leading-order attenuator using an internal multiple subevent in the second integral. The way it is generated suggests the way it can be removed. For the removal of this type of spurious events, substituting b_3 for the second b_1 in equation 1 leads to equation 3. The subevent combination of "primary–predicted internal multiple–primary" in equation 3 can be used to attenuate the spurious event. We examine one of the fifth order terms $(G_0V_1G_0V_3G_0V_1G_0)$ that satisfies the required Figure 2(c) geometry. The derivation and analytical examples are shown in Ma et al. (2012).

$$b_5^{PIP}(k) = \int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} b_3(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} b_1(z_3) \tag{3}$$

The output of the new ISS internal multiple algorithm for this three reflectors case is

$$D(t) + D_3(t) + D_5^{PIP}(t), \tag{4}$$

where $D_5^{PIP}(t)$ is the inverse Fourier transform of $D_5^{PIP}(\omega)$ and $D_5^{PIP}(\omega) = b_5^{PIP}(k)$ for spike data. The original algorithm (equation 2) attenuates the first-order internal multiples and preserve primaries but can also output spurious events. The modified algorithm in equation 4 provides the benefit of the original algorithm while addressing issues due to spurious events.

An internal multiple subevent in either of the outer integrals in b_3

The problem is yet more complicated when a first-order internal multiple subevent is in either of the outer integrals. As shown in the left panel of Figure 3, when an internal multiple with pseudo-depth z'' is in the rightmost integral $(z, z'' > z')$, we have $z'' = (z_1 + z_3 - z_2) > z'$ (this lower-higher-lower relationship in pseudo depth domain is required by b_3, and if it is not satisfied this kind of subevent combination will not occur in b_3). In such a case, there are several possible relations between z_1, z_2, z_3 and z', which are as follows:

- As shown by the first item in Figure 3, when $z_1 > z'$, the predicted event has the same pseudo-depth as a second order internal multiple. Its subevent construction is shown in Figure 4(a), and this occurs in a medium with number of reflectors $N \geq 2$.
- The second item in Figure 3 shows that when $z_1 = z'$, the predicted event has the same pseudo-depth as a first-order internal multiple. Figure 4(b) describes its subevent construction, which only happens in a medium with $N \geq 3$.
- The third item in Figure 3 shows that a spurious event is produced with $z_1 < z'$ and $z_3 < z'$ (the red circle at z' has only one lower blue circle on one side). Its subevent construction is illustrated by Figure 4(c). This type of spurious event can only be generated in a medium with $N \geq 4$.

Using the same logic and analysis as the previous section, we propose another method to address this type of spurious events by replacing the third b_1 in equation 1 with b_3, and the new term is shown in equation 5. Since this type of spurious event could be produced by the leading order attenuator using a first-order internal multiple subevent in either of the outer integrals (these two cases are equivalent), there is a leading coefficient 2 in the equation 5. This term is also identified from a portion of the fifth order term in the ISS (from the term $G_0V_1G_0V_1G_0V_3G_0$).

$$b_5^{PPI}(k) = 2\int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} b_1(z_2) \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} b_3(z_3) \qquad (5)$$

The new ISS internal multiple algorithm for this case with more than three reflectors is

$$D_1(t) + D_3(t) + D_5^{PIP}(t) + D_5^{PPI}(t). \qquad (6)$$

where $D_5^{PPI}(t)$ is the Fourier transform of $D_5^{PPI}(\omega)$ and $D_5^{PPI}(\omega) = b_5^{PIP}(k)$ for an incident spike wave. This modified general algorithm in equation 6 retains the strengths of the original algorithm while addressing issues due to spurious events.

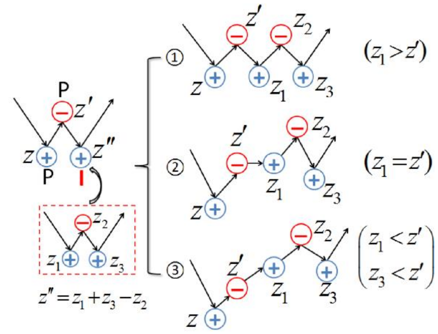


Figure 3: Diagrammatic illustration of predicted events when an internal multiple subevent is in either of the outer integrals.

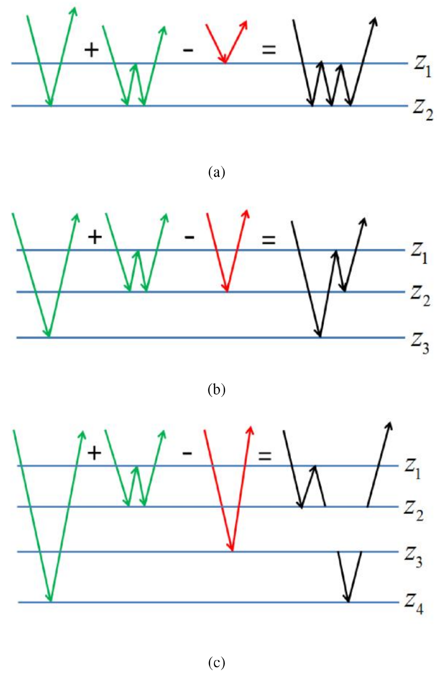
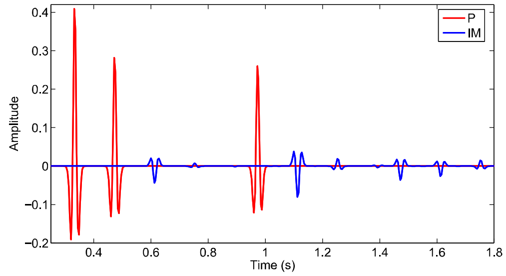


Figure 4: Events generated by the leading-order attenuator using an internal multiple subevent in either of the outer integrals: (a) a second order internal multiple, (b) a first order internal multiple, and (c) a spurious event, $2z_2 - z_1 > z_3$.

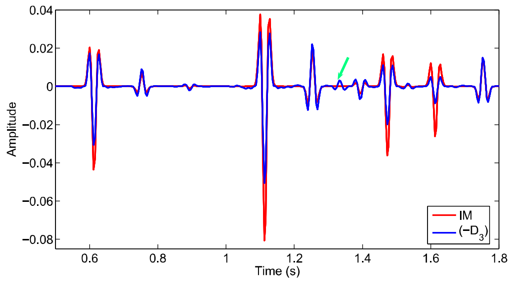
NUMERICAL EXAMPLES

In this section, we will compute and analyze the new terms for one dimensional, three reflector models. The spurious event would be produced when the internal multiple subevent is in the second of the three integrals. Thus, only the term in equation 3 will be tested in this section. In the previous section, the input data are assumed to be source wavelet. If the data are generated by using a source wavelet, then we have $D(\omega) = A(\omega)b_1(2\omega/c_0)$, and hence, $D_3(\omega) = A(\omega)b_3(\omega/c_0)$, and $D_5^{PIP}(\omega) = A(\omega)b_5^{PIP}(\omega/c_0)$.

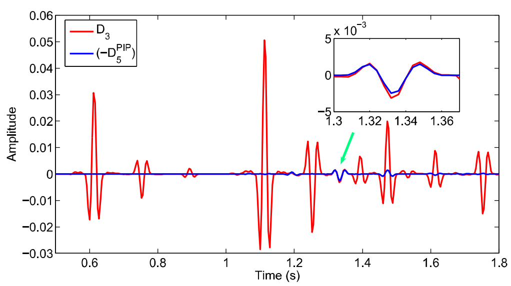
Figure 5(a) shows a 1D normal-incidence trace, which includes three primaries and all internal multiples. Figure 5(b) shows the comparison of the actual internal multiples in the data and the events produced by the leading-order attenuator. From the results we can see that by treating both primaries and internal multiples as subevents the leading-order attenuator can predict first-order and higher-order internal multiples, as well as the spurious event (pointed by the green arrow). Figure 5(c) shows the comparison of the spurious event generated by the leading-order attenuator and the one predicted by the higher-order term. By adding D_5^{PIP} to D_3 the spurious event is well attenuated and the internal multiple prediction is almost unchanged, as shown in Figure 6. From Figure 6 we can conclude that the modified algorithm in equation 4 provides the benefit of original algorithm (equation 2) while addressing the limitation due to spurious events.



(a)



(b)



(c)

Figure 5: (a) An input trace, including primaries (red) and all internal multiples (blue); (b) Actual internal multiples in the data (red), and events predicted by the ISS leading-order attenuator (blue) including predicted internal multiples and the spurious event (pointed by the green arrow); (c) comparison of the actual spurious event in D_3 and the predicted one in $(-D_5^{PIP})$ (pointed by the green arrow and and the close-up shown in the upper right box).

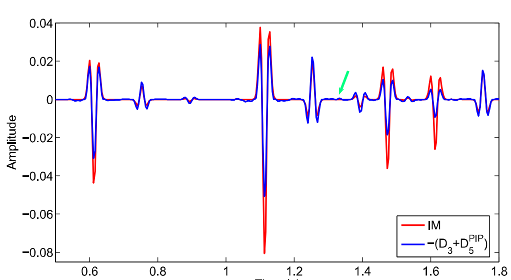
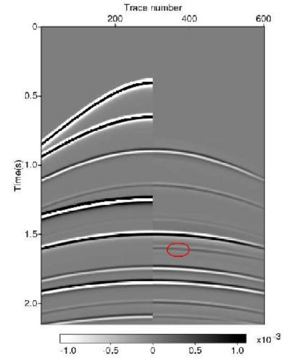
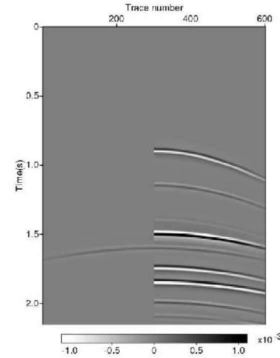


Figure 6: Actual internal multiples in the data (red) and the ones predicted by the modified algorithm (represented by $-(D_3 + D_5^{PIP})$). Green arrow points to the spurious events.



(a)



(b)

Figure 7: (a) The left are the input data (the first, second and fifth events are primaries), and the right are the events produced by b_3 (the spurious event is marked by the red circle); (b) The right are the events produced by b_3, and the left is the spurious event predicted by b_5^{PIP}.

The modified algorithm 4 can be also extended to 2D experiment in a 1D earth. Figure 7(a) shows a shot gather on the left, and on the right are the events predicted by the leading-order attenuator, in which the red circle mark the generated spurious event. In Figure 7(b), the right are still the events predicted by the leading-order attenuator, while the left shows the spurious event predicted by the higher-order term.

CONCLUSIONS

While the ISS leading-order attenuator has demonstrated its capability for internal multiple removal, it has strengths and limitations as implied by “leading order” and “attenuator”.The modified algorithm presented in this paper and Ma et al. (2012) addresses a shortcoming of the current leading-order ISS internal multiple attenuation algorithm that are observed in the examples of Fu et al. (2010) and Luo et al. (2011). Spurious events can be a particular problem if they are proximal to or interfere with primaries or multiples. If you suspect that this is the case, then the algorithm of this paper can remove the spurious event. The modified ISS internal multiple attenuation algorithm retains the benefit of the original algorithm while addressing one of its shortcomings. It now accommodates both primaries and internal multiples in the input data.

ACKNOWLEDGMENTS

We are grateful to all M-OSRP sponsors for their support of this research. Special thanks to Paolo Terenghi for assistance, Jim Mayhan for helpful comments and suggestions.

REFERENCES

Araújo, F. V., A. B. Weglein, P. M. Carvalho, and R. H. Stolt, 1994, Inverse scattering series for multiple attenuation: An example with surface and internal multiples: SEG Technical Program Expanded Abstracts, 1039–1041.

Coates, R. T., and A. B. Weglein, 1996, Internal multiple attenuation using inverse scattering: Results from prestack 1 & 2D acoustic and elastic synthetics: SEG Technical Program Expanded Abstracts, 1522–1525.

Fu, Q., Y. Luo, G. K. Panos, S. Huo, G. Sindi, S. Hsu, and A. B. Weglein, 2010, The inverse scattering series approach towards the elimination of land internal multiples: SEG Technical Program Expanded Abstracts, 3456–3461.

Hsu, S., P. Terenghi, and A. B. Weglein, 2011, The properties of the inverse scattering series internal multiple attenuation algorithm: Analysis and evaluation on synthetic data with lateral variations, choosing reference velocity and examining its sensitivity to near surface properties: Mission-Oriented Seismic Research Program (M-OSRP), Annual Report, 16–28.

Kelamis, P. G., Y. Luo, and A. Weglein, 2013, Strategies of land internal multiple elimination based on Inverse Scattering Series: Presented at the 6th International Petroleum Technology Conference.

Luo, Y., P. G. Kelamis, Q. Fu, S. Huo, G. Sindi, S. Hsu, and A. B. Weglein, 2011, Elimination of land internal multiples based on the inverse scattering series: The Leading Edge, 884–889.

Ma, C., H. Liang, and A. B. Weglein, 2012, Modifying the leading order iss attenuator of first-order internal multiples to accommodate primaries and internal multiples: fundamental concept and theory, development, and examples exemplified when three reflectors generate the data: M-OSRP 2011 Annual meeting.

Matson, K. H., 1997, An inverse-scattering series method for attenuating elastic multiples from multi-component land and ocean bottom seismic data: PhD thesis, University of British Columbia.

Matson, K. H., and A. B. Weglein, 1996, Removal of elastic interface multiples from land and ocean bottom data using inverse scattering: SEG Technical Program Expanded Abstract, 1526–1530.

Ramírez, A. C., 2007, I.-inverse scattering subseries for removal of internal multiples and depth imaging primaries; ii.-green's theorem as the foundation of interferometry and guiding new practical methods and applications: PhD thesis, University of Houston.

Ramírez, A. C., and A. B. Weglein, 2005, An inverse scattering internal multiple elimination method: beyond attenuation, a new algorithm and initial test: SEG Technical Program Expanded Abstracts, 2115–2118.

Terenghi, P., S. Hsu, A. B. Weglein, and X. Li, 2011, Exemplifying the specific properties of the inverse scattering series internal-multiple attenuation method that reside behind its capability for complex onshore and marine multiples: The Leading Edge, 876–882.

Weglein, A. B., F. V. Araújo, P. M. Carvalho, R. H. Stolt, K. H. Matson, R. T. Coates, D. Corrigan, D. J. Foster, S. A. Shaw, and H. Zhang, 2003, Inverse scattering series and seismic exploration: Inverse Problems, R27–R83.

Weglein, A. B., F. A. Gasparotto, P. M. Carvalho, and R. H. Stolt, 1997, An inverse-scattering series method for attenuating multiples in seismic reflection data: Geophysics, 1975–1989.

Weglein, A. B., S. Hsu, P. Terenghi, X. Li, and R. Stolt, 2011, Multiple attenuation: Recent advances and the road ahead 2011: The Leading Edge, 864–875.

Weglein, A. B., and K. H. Matson, 1998, Inverse scattering internal multiple attenuation: an analytic example and subevent interpretation: Mathematical Methods in Geophysical Imaging V, 108–117.

Zhang, H., and S. Shaw, 2010, 1-D analytical analysis of higher order internal multiples predicted via the inverse scattering series based algorithm: SEG Technical Program Expanded Abstracts, 3493–3498.