UNIVERSITY of HOUSTON

M-OSRP
Mission - Oriented Seismic Research Program

The first migration method that's equally effective for all acquired frequencies for imaging and inverting at the target

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Houston, Texas
December 14, 2015

Migration

was originally (and remains today) only meaningful for primaries

- The original purpose of migration was to map an event on a time section in (x,t) to a point on a structure map. That is, to a point on a reflector in x,z
  That concept only has meaning for primaries.
- We will show that that remains the meaning of migration today.

Wave theory methods used to migrate seismic data have two components

1. A wave propagation component
2. An imaging condition

Structure (migration)

- For one way propagating waves, Jon Claerbout (1971) introduced three imaging conditions
  (1) the exploding-reflector model,
  (2) time and space coincidence of up and downgoing waves, and
  (3) predicting a source and receiver experiment at a coincident-source-and-receiver subsurface point, and asking for time equals zero

Claerbout III migration

2D Claerbout III in the Fourier domain (Stolt migration)

1) Data on measurement surface

$$D(x_g, z_g = 0, x_s, z_s = 0, t)$$

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$$D(x_g, z_g = 0, x_s, z_s = 0, t)$$

2) Fourier transform

$$D(k_g, z_g = 0, k_s, z_s = 0, \omega) = \iiint dx_g dx_s dt e^{i(k_s x_s - k_g x_g + \omega t)} D(x_g, z_g = 0, x_s, z_s = 0, t)$$

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3) Data at depth

$$D(k_g, z, k_s, z, \omega) = D(k_g, z_g = 0, k_s, z_s = 0, \omega) e^{-i(q_g + q_s) z}$$

$$q_g = \sqrt{(\omega/c)^2 - k_g^2} \quad q_s = \sqrt{(\omega/c)^2 - k_s^2}$$

2D Claerbout III in the Fourier domain (Stolt migration)

4) Data at depth at t=0

$$D(k_g, z, k_s, z, t=0) = \int d\omega D(k_g, z, k_s, z, \omega)$$

2D Claerbout III in the Fourier domain (Stolt migration)

4) Data at depth at t=0

$$D(k_g, z, k_s, z, t=0) = \int d\omega D(k_g, z, k_s, z, \omega)$$

5) Change of variables

$$D(k_g, z, k_s, z, t=0) = D(k_m, z, k_h, z, t=0)$$

2D Claerbout III in the Fourier domain (Stolt migration)

4) Data at depth at t=0

$$D(k_g, z, k_s, z, t=0) = \int d\omega D(k_g, z, k_s, z, \omega)$$

5) Change of variables

$$D(k_g, z, k_s, z, t=0) = D(k_m, z, k_h, z, t=0)$$

6) Inverse Fourier transform

$$D(x_m, z, x_h, z, t=0)$$

Set x_g=x_s=x (x_m=x; x_h=0) original Stolt migration (Claerbout III)

2D Claerbout III in the Fourier domain (Stolt migration)

6) Inverse Fourier transform

$$D(x_m, z, x_h, z, t=0)$$

Set $x_g = x_s = x$ ($x_m = x$; $x_h = 0$) original Stolt migration (Claerbout III)

$$D(x_h) = \int e^{ik_h x_h} D(\mathrm{k}_h)\,\mathrm{d}\,\mathrm{k}_h$$

$$D(x_h = 0) = \int D(\mathrm{k}_h)\,\mathrm{d}\,\mathrm{k}_h$$

angle average plane wave reflection coefficient by summing over k_h

- Stolt extended (Claerbout III) migration retains k_h information at depth z, D(km,kh,kz) and that provides the plane wave reflection coefficient at a specular reflector

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- Stolt and collaborators further extended Claerbout III to provide a point reflectivity model that automatically images specular and non-specular reflections.

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- Stolt and collaborators further extended Claerbout III to provide a point reflectivity model that automatically images specular and non-specular reflections.

- <u>The latter is only possible for extensions/generalizations of Claerbout III(not for Claerbout II)</u>

Claerbout II imaging (A typical form)

$$I(\vec{x}) = \sum_{\vec{x}_s} \sum_{\omega} S'(\vec{x}_s, \vec{x}, \omega) R(\vec{x}_s, \vec{x}, \omega)$$

Where R is the reflection data (for a shot record), run backwards, and S is the source wavefield.

Claerbout II imaging

$$I(\vec{x}) = \sum_{\vec{x}_s} \sum_{\omega} S'(\vec{x}_s, \vec{x}, \omega) R(\vec{x}_s, \vec{x}, \omega)$$

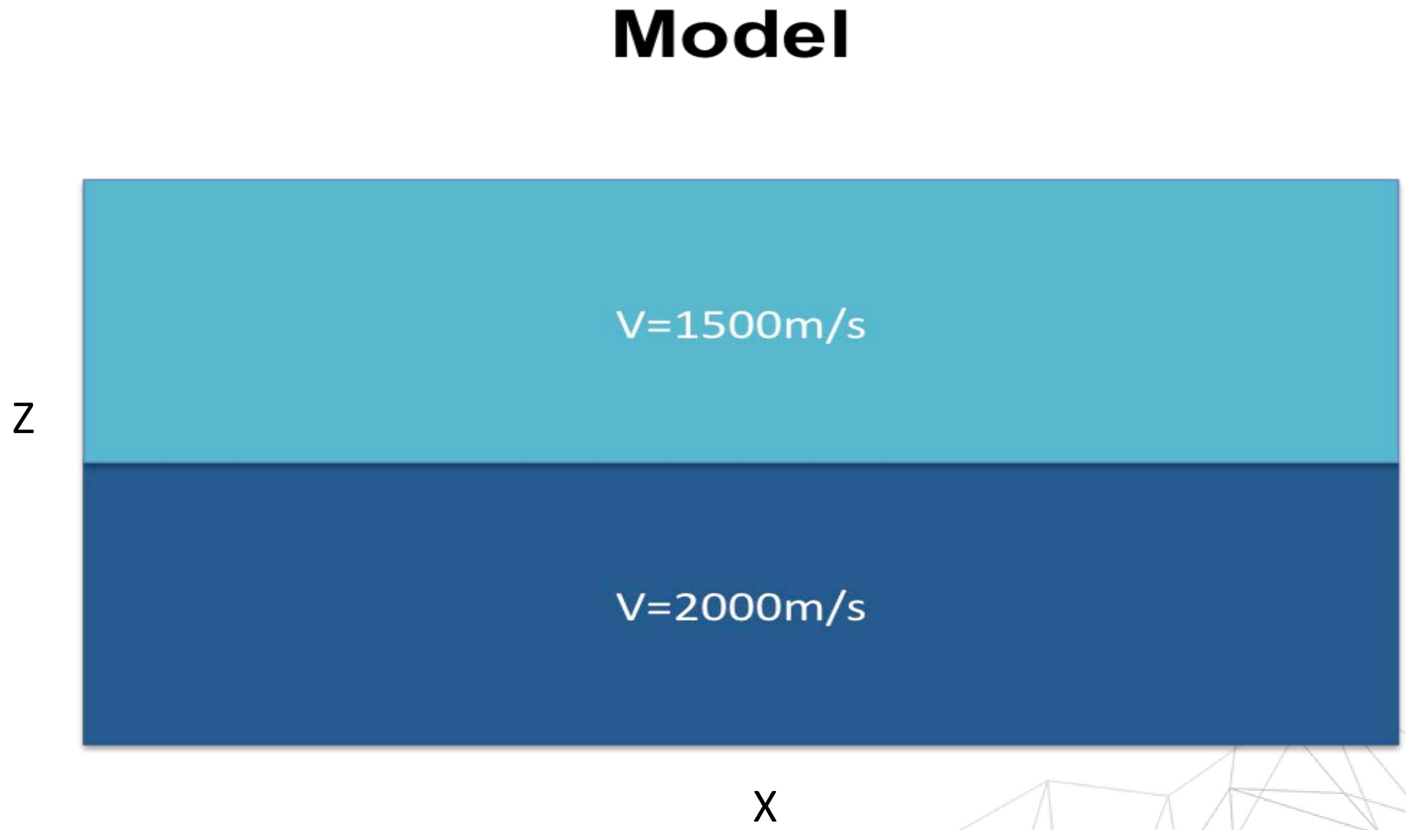
Claerbout III imaging

$$M^{stolt}(x,z) = \frac{1}{(2\pi)^3} \iiint d\omega dx_g dx_s dk_{sx} e^{-i(k_{sz}z + k_{sx}(x - x_s))} \int dk_{gx} e^{-i(k_{gz}z + k_{gx}(x - x_g))}$$

$$\int dt e^{i\omega t} D(x_g, x_s, t)$$

Let’s examine Claerbout II and III where only the imaging condition is the issue

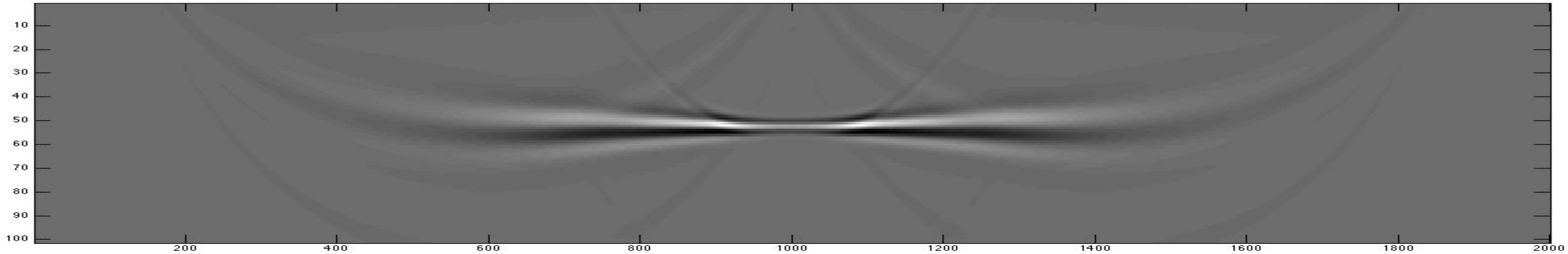
A numerical example of Claerbout II imaging (current leading edge RTM) for a single reflector with a homogeneous velocity model (one shot gather)



Yanglei Zou and Weglein, 2014

A numerical example of Claerbout II imaging (current leading edge RTM) for a single reflector with a homogeneous velocity model (one shot gather)

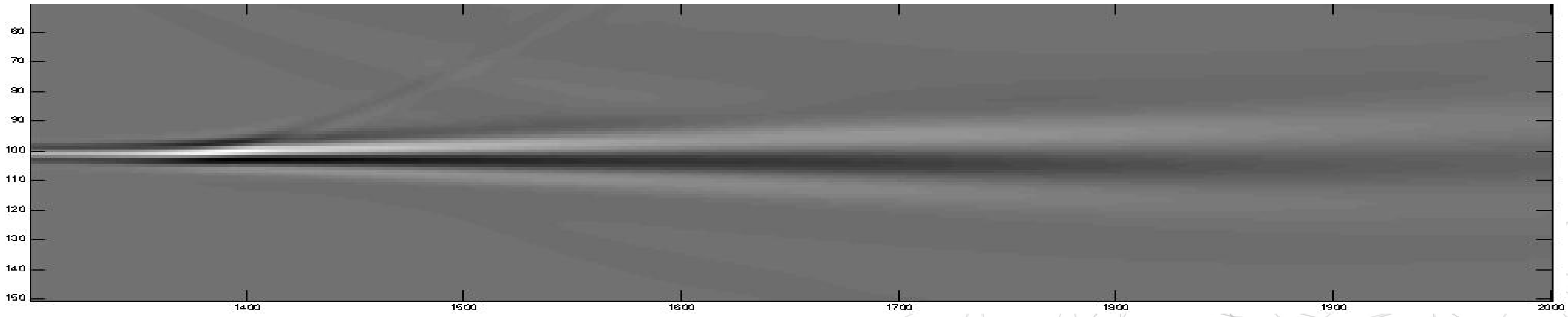
z(m)



x(m)

Zoom of the figure above (from x=1250m to x=1950m)

z(m)

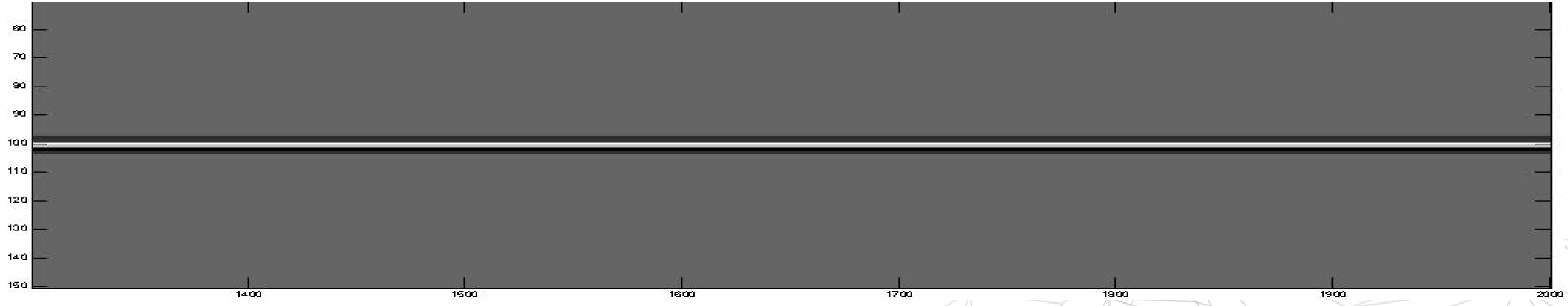


x(m)

Yanglei Zou and Weglein, 2014

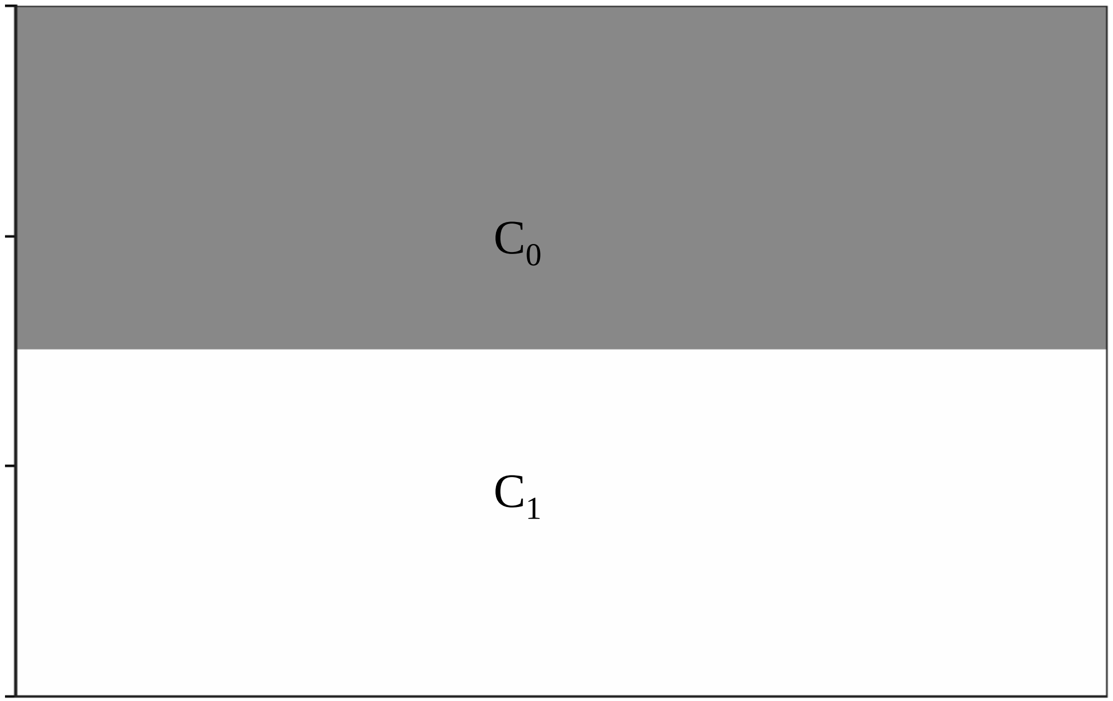
A numerical example of Claerbout III Stolt migration for a single reflector

Zoom of the figure above (from x=1250m to x=1950m)

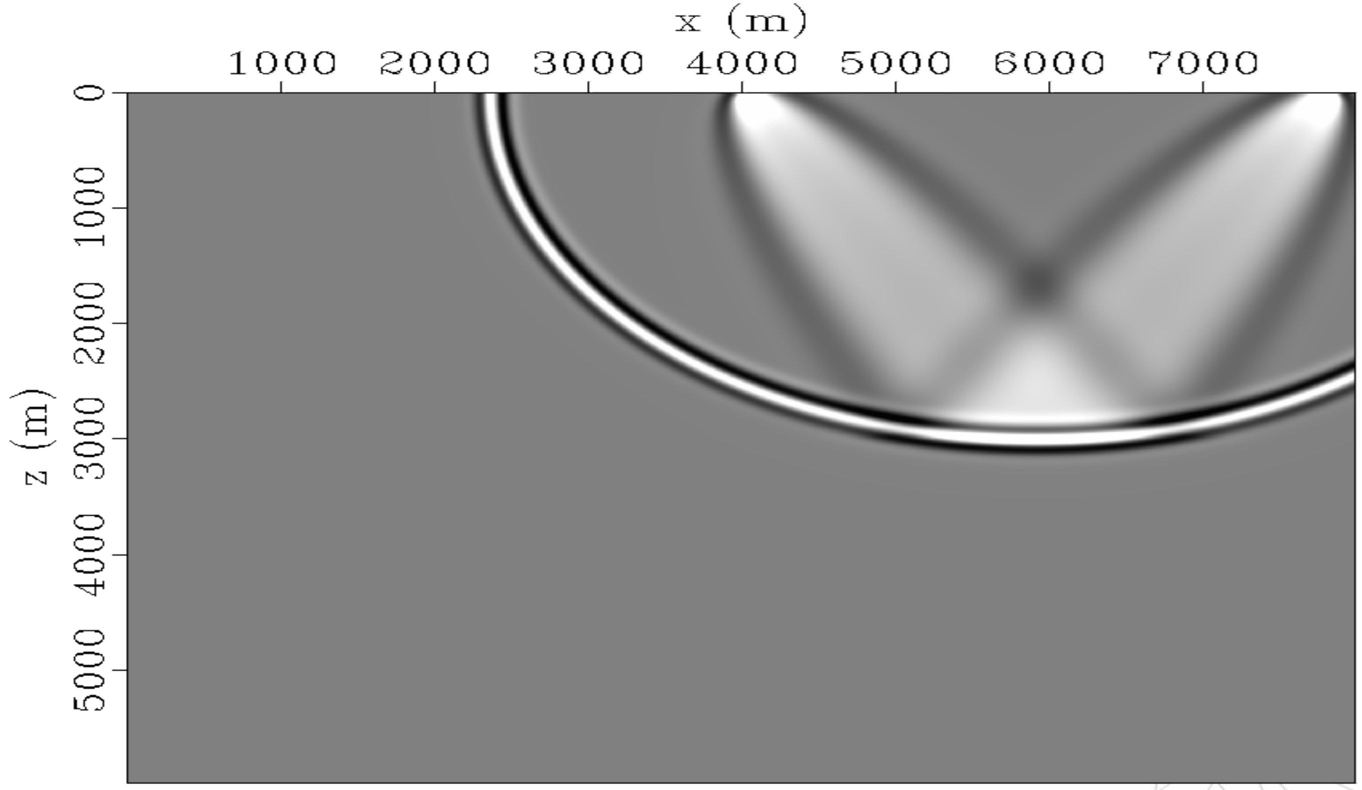


Yanglei Zou and Weglein, 2014

THE VELOCITY MODEL

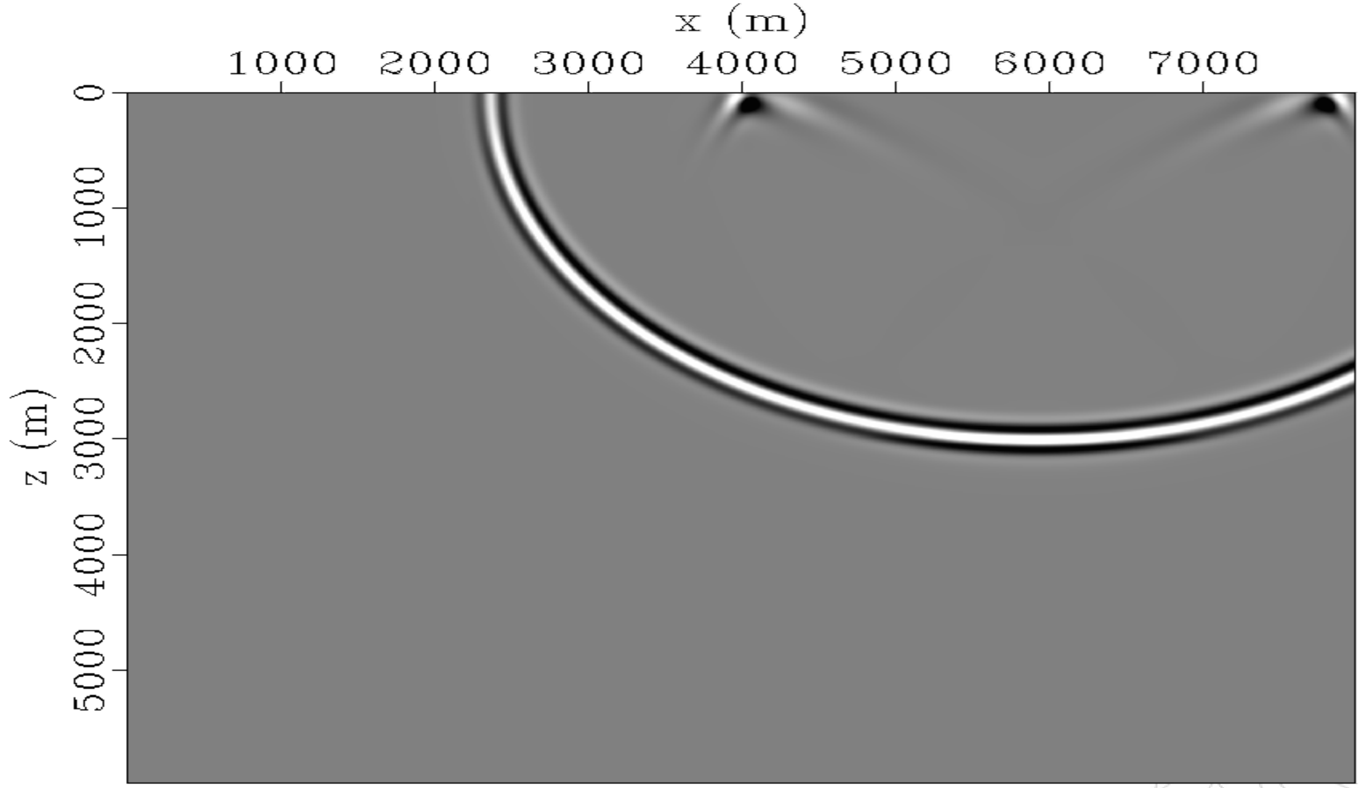


CLAERBOUT II RTM IMAGE FOR ONE TRACE



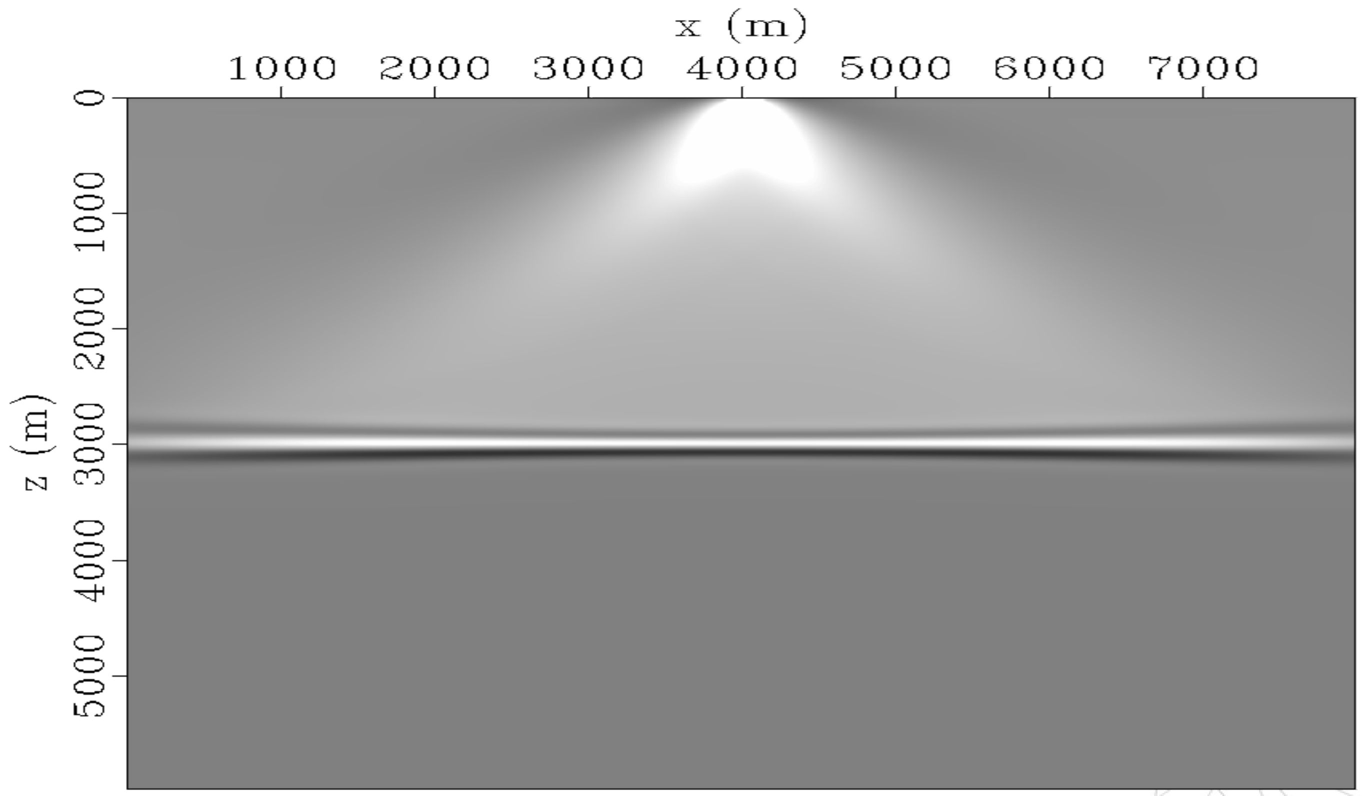
Qiang Fu

CLAERBOUT II RTM IMAGE AFTER ARTIFACTS REMOVAL



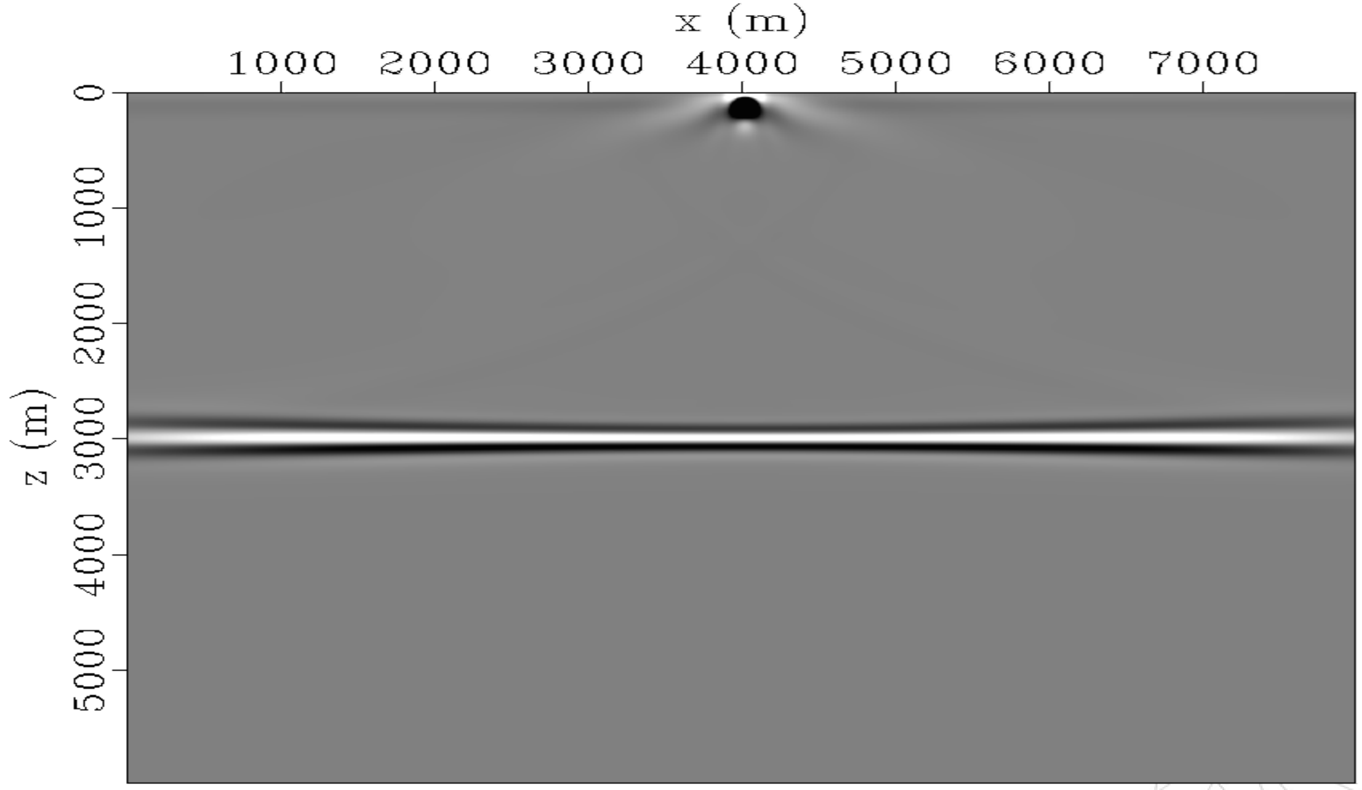
Qiang Fu

CLAERBOUT II RTM IMAGE FOR ONE SHOT GATHER



Qiang Fu

CLAERBOUT II RTM IMAGE AFTER ARTIFACTS REMOVAL



Qiang Fu

- Please note the inconsistent image along the reflector

THE NEW M-OSRP CLAERBOUT III (STOLT EXTENDED) MIGRATION FOR 2 WAY WAVE PROPAGATION

- The example with $\frac{c_0}{c_1}$ velocity
- The image both above and beneath the reflector

Qiang Fu

- **No "rabbit ears"**
- **Consistent image along the reflector**

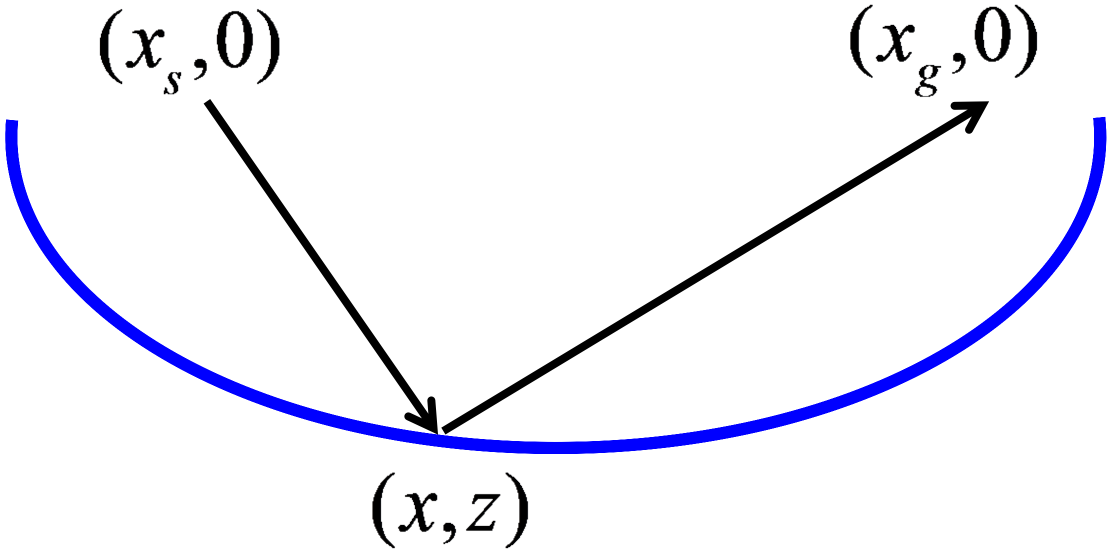
Light color – image from above
Dark color – image from below

- The Claerbout II imaging principle provides an image for one shot record. However, the resulting inconsistency in the image is mitigated by summing over shots. Claerbout II has a somewhat ad hoc origin and an ad hoc ‘fix’.

- In Claerbout III one shot record predicts the receiver at depth (not an image). The Green’s theorem weighted sum over shots then predicts the source at depth, leading to an image.
- There is nothing that’s inconsistent or being fixed with the sum over sources in Claerbout III.

How do you know if a migration method has made a high frequency approximation?

✓(1) If there is a travel time curve of candidate images within the method, it is a high frequency ‘ray theory’ approximation/ assumption.



$$t = r / c$$

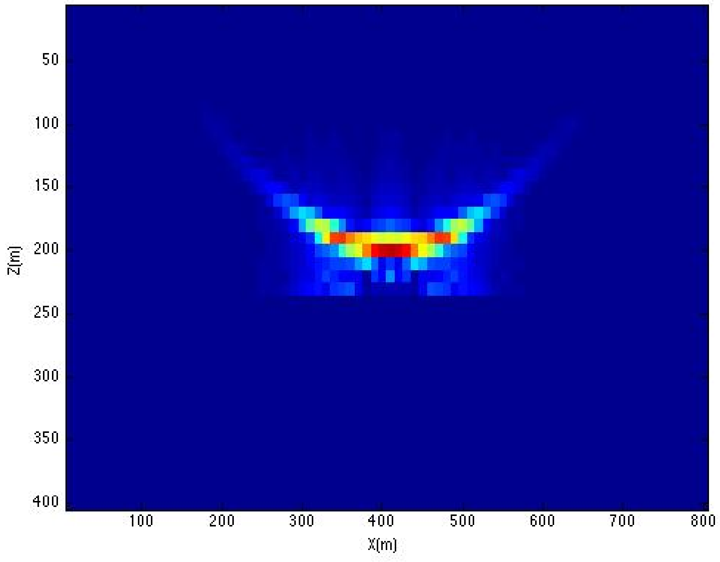
where,

$$r = r_g + r_s$$
$$= \sqrt{(x_g - x)^2 + z^2} + \sqrt{(x_s - x)^2 + z^2}$$

Imaging Conditions and High Frequency Assumptions

Claerbout III

Stolt migration: one source
one receiver

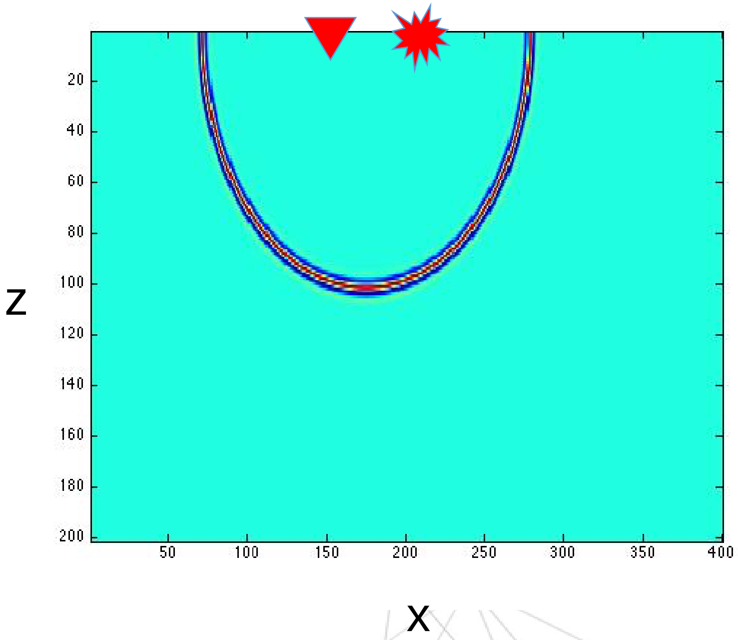


Yanglei Zou, 2015

No high frequency assumption

Claerbout II

RTM(2D)



High frequency assumption

How do you know if a migration method has made a high frequency approximation?

(2) A stationary phase of other high frequency approximation is employed within the method.

(3) a propagation model that assumes one way wave propagation (for anything other than a homogeneous subsurface) is a high frequency approximation

- high frequency approximation can enter migration methods through (1) the imaging condition (all Claerbout II) or (2) through the propagation component (or both), that is, for example, assuming a one way propagation for a slowly varying velocity in Claerbout III

2D Stolt migration

$$M^{Stolt}(x,z) = \frac{1}{(2\pi)^3} \int\int\int d\omega dx_g dx_s \int dk_{sx} e^{-i(k_{sz}z + k_{sx}(x-x_s))} \int dk_{gx} e^{i(k_{gz}z+k_{gx}(x-x_g))}$$
$$\int dt e^{i\omega t} D(x_g, x_s, t)$$

An asymptotic approximation can be made with the stationary phase approximation

$$\int dk_{sx} e^{-i(k_{sz}z+k_{sx}(x-x_s))} \simeq e^{-i\omega r_s/c} \sqrt{\frac{2\pi i \omega z^2}{c r_s^3}}$$

$$\int dk_{gx} e^{i(k_{gz}z+k_{gx}(x-x_g))} \simeq e^{-i\omega r_g/c} \sqrt{\frac{2\pi i \omega z^2}{c r_g^3}}$$

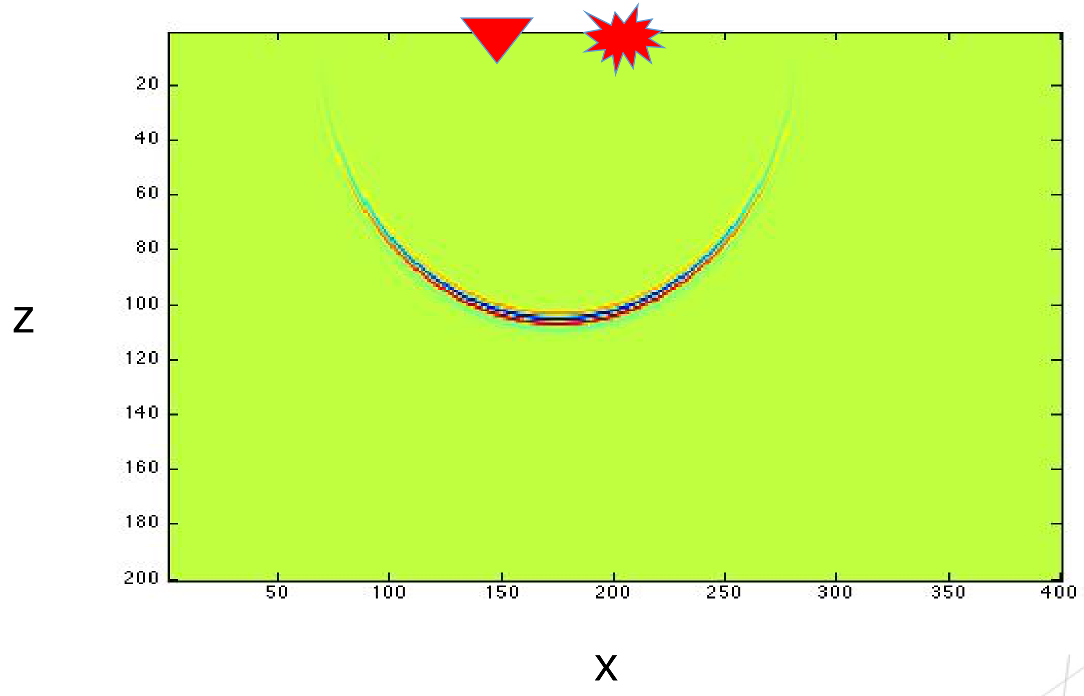
$$r_s = \sqrt{z^2 + (x-x_s)^2}$$
$$r_g = \sqrt{z^2 + (x-x_g)^2}$$
$$r = r_s + r_g$$

The Kirchhoff migration is an asymptotic approximation of Stolt migration

$$
\begin{aligned}
M^{Kirchhoff}(x,z) &= \frac{z^2}{(2\pi)^2 c}\int dx_g \int dx_s \int dt \frac{D(x_g,x_s,t)}{(r_s r_g)^{3/2}} \int d\omega i\omega e^{i\omega(t-r/c)} \\
&= \frac{z^2}{(2\pi)^2 c}\int dx_g \int dx_s \int d\omega i\omega e^{-i\omega r/c} \frac{D(x_g,x_s,\omega)}{(r_s r_g)^{3/2}} \\
&= \frac{z^2}{(2\pi)^2 c}\int dx_g \int dx_s \int d\omega i\omega \int dt e^{-i\omega t}\delta(t-r/c) \frac{D(x_g,x_s,\omega)}{(r_s r_g)^{3/2}} \\
&= \frac{z^2}{(2\pi)^2 c}\int dx_g \int dx_s \int dt \delta(t-r/c) \int d\omega i\omega e^{-i\omega t} \frac{D(x_g,x_s,\omega)}{(r_s r_g)^{3/2}} \\
&= -\frac{z^2}{2\pi c}\int dx_g \int dx_s \int dt \delta(t-r/c) \frac{\frac{d}{dt}D(x_g,x_s,t)}{(r_s r_g)^{3/2}} \\
&= -\frac{z^2}{2\pi c}\int dx_g \int dx_s \frac{\frac{d}{dt}D(x_g,x_s,t)|_{t=r/c}}{(r_s r_g)^{3/2}}
\end{aligned}
$$

Kirchhoff migration for a single source and receiver

Kirchhoff migration (2D)



Yanglei Zou et al, 2015

High Frequency approximation from a stationary phase approximation

Claerbout II and III have been extended and generalized

- For Claerbout II

  e.g., Yu Zhang, Sheng Xu and Norman Bleistein

  ----- introduce **a geometric optics reflection coefficient** model relating the reflection data and the incident source wavefield.

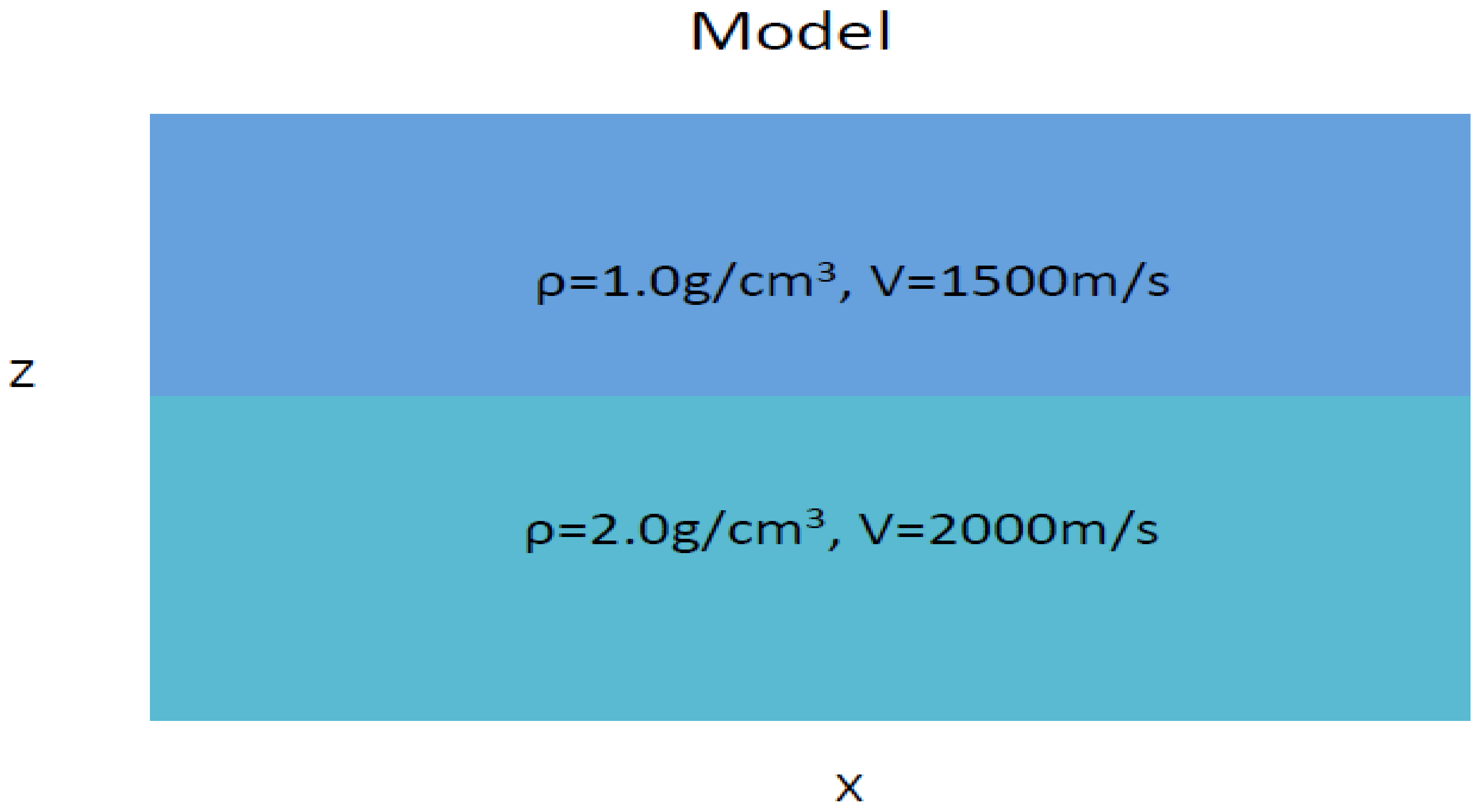
- For Claerbout III

  Stolt and collaborators

  ----- non-zero offset at t=0 provides amplitude information

  ----- outputs **plane wave reflection coefficient** or point scatterer reflectivity for specular and non-specular reflection

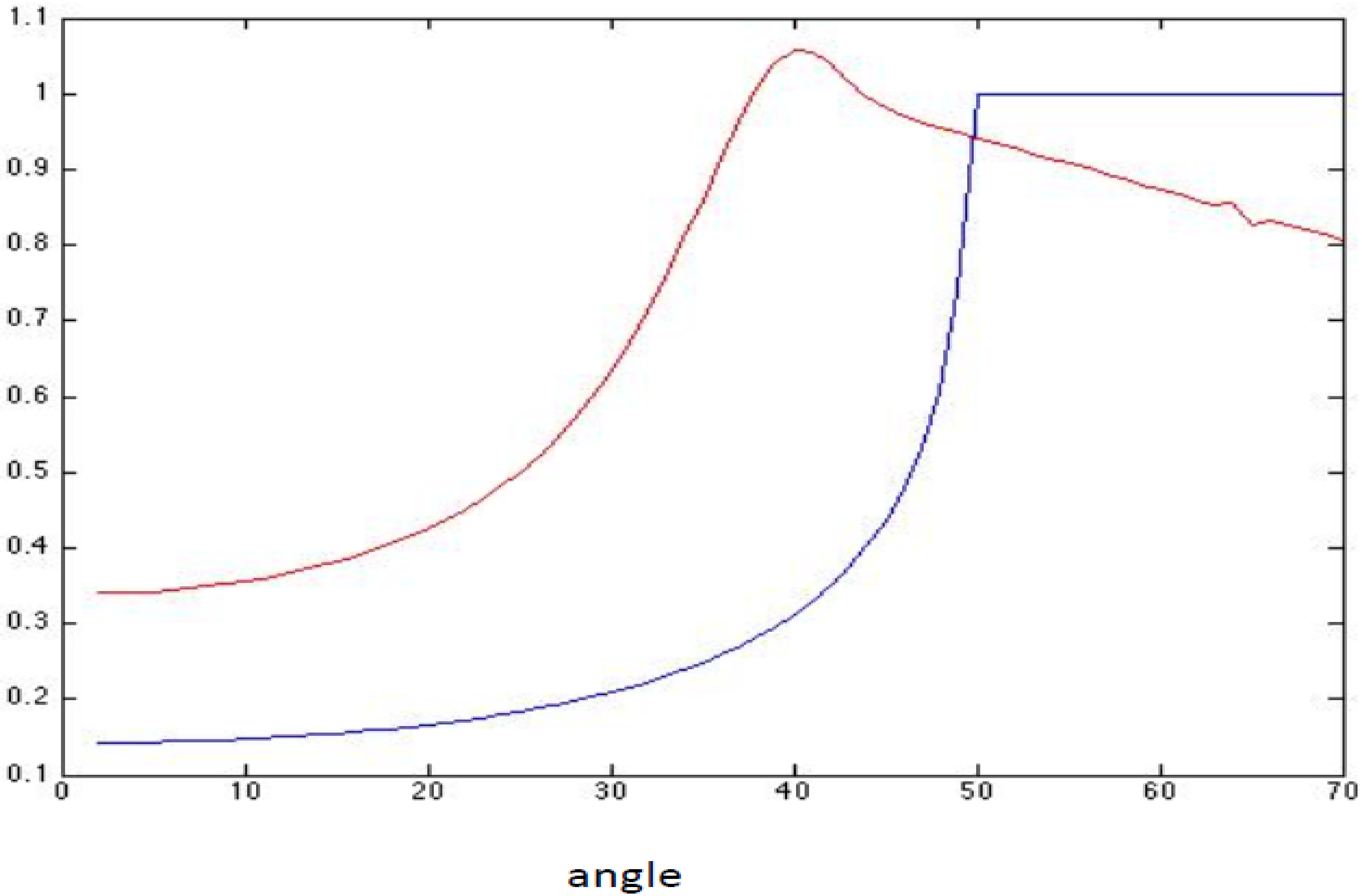
plane wave reflection coefficient and geometric optics approximate reflection coefficient



plane wave reflection coefficient and geometric optics approximate reflection coefficient

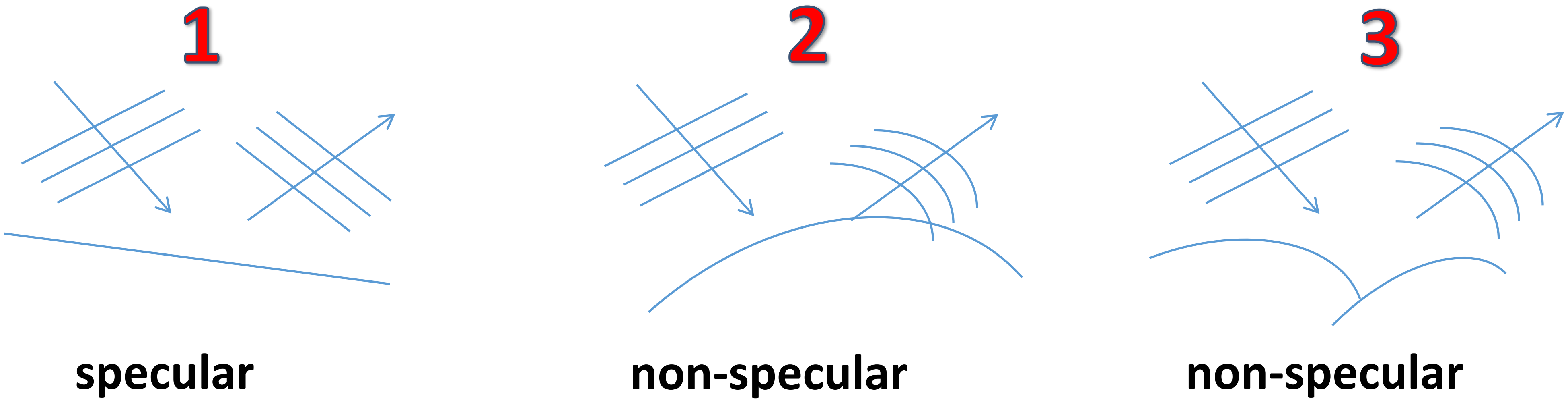
blue: plane wave reflection coefficient

red: geometric optics approximate reflection coefficient



Yanglei Zou and Weglein, 2015

Benefits of Claerbout III imaging (extended by Stolt and colleagues) for specular and non-specular imaging



1. Specular
outputs actual plane wave reflection coefficient data for specular reflection (unique to Claerbout III)

2. Non-Specular reflection
a point scatterer model for structure and inversion of non-specular reflections (unique to Claerbout III)

All current "RTM" methods (for two way waves) correspond to imaging condition II, and do **<u>not</u>** correspond to (nor do they provide the benefits of) a coincident source and receiver experiment at depth at time equals zero

- M-OSRP has recently pioneered and developed a new migration method that has Stolt's extension of Claerbout III imaging inside a medium with two way waves

- The new migration method provides **four benefits**

- Recent new imaging development from M-OSRP (Weglein, A.B., Stolt, R. H., Mayhan, J. D., "*Reverse-time migration and Green's theorem: Part I — The evolution of concepts, and setting the stage for the new {RTM} method*" Journal ofSeismic Exploration, 20, 73–90. February 2011; Weglein, A.B., Stolt, R. H., Mayhan, J. D., "*Reverse time migration and Green's theorem: Part II — A new and consistent theory that progresses and corrects current RTM concepts and methods*" Journal ofSeismic Exploration, 20, 135–159. May 2011; THE FIRST WAVE EQUATION MIGRATION RTM WITH DATA CONSISTING OF PRIMARIES AND INTERNAL MULTIPLES: THEORY AND 1D EXAMPLES FANG LIU and ARTHUR B. WEGLEIN JOURNAL OF SEISMIC EXPLORATION 23, 357-366 (2014) 357) provides the predicted source and receiver experiment at depth beneath an overburden that has two way wave propagation

  1. A tool to analyze the role of primaries and multiples in imaging

  2. A more effective and interpretable RTM by realizing Stolt extended Claerbout III providing a more complete and realistic model for imaging and inverting specular and non-specular reflectors

  3. Provides the first migration method that is equally effective at every frequency component of recorded data (avoids asymptotic high frequency approximations) in both the imaging condition and in how the imaging condition is implemented.

  4. Imaging and inverting from above (and from below) a reflector without backscatter.

Use the new imaging method with two way propagating waves to examine the role of primaries and multiples in imaging and inversion

- That provides a definitive response to the question of whether multiples are migrated

Predicting the experiment at depth

Green's theorem (1828) provides a way to predict an experiment inside a volume from measurements on a closed surface surrounding the volume

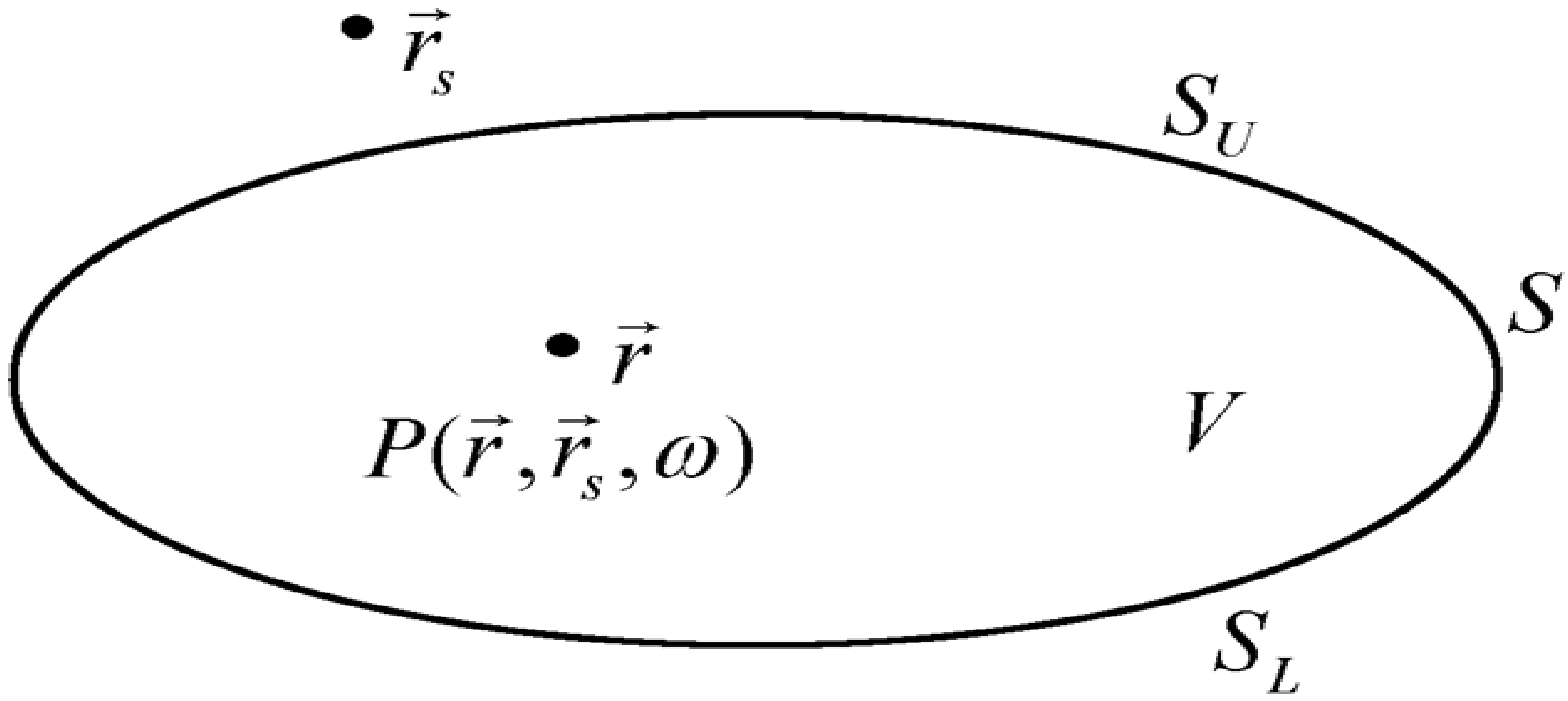
$$\underset{\vec{r}\text{ in the volume}}{P(\vec{r},\vec{r}_s,\omega)} = \oint_S (P\nabla G_0 - G_0\nabla P)\cdot\hat{n}\,ds \qquad \textbf{(1)}$$

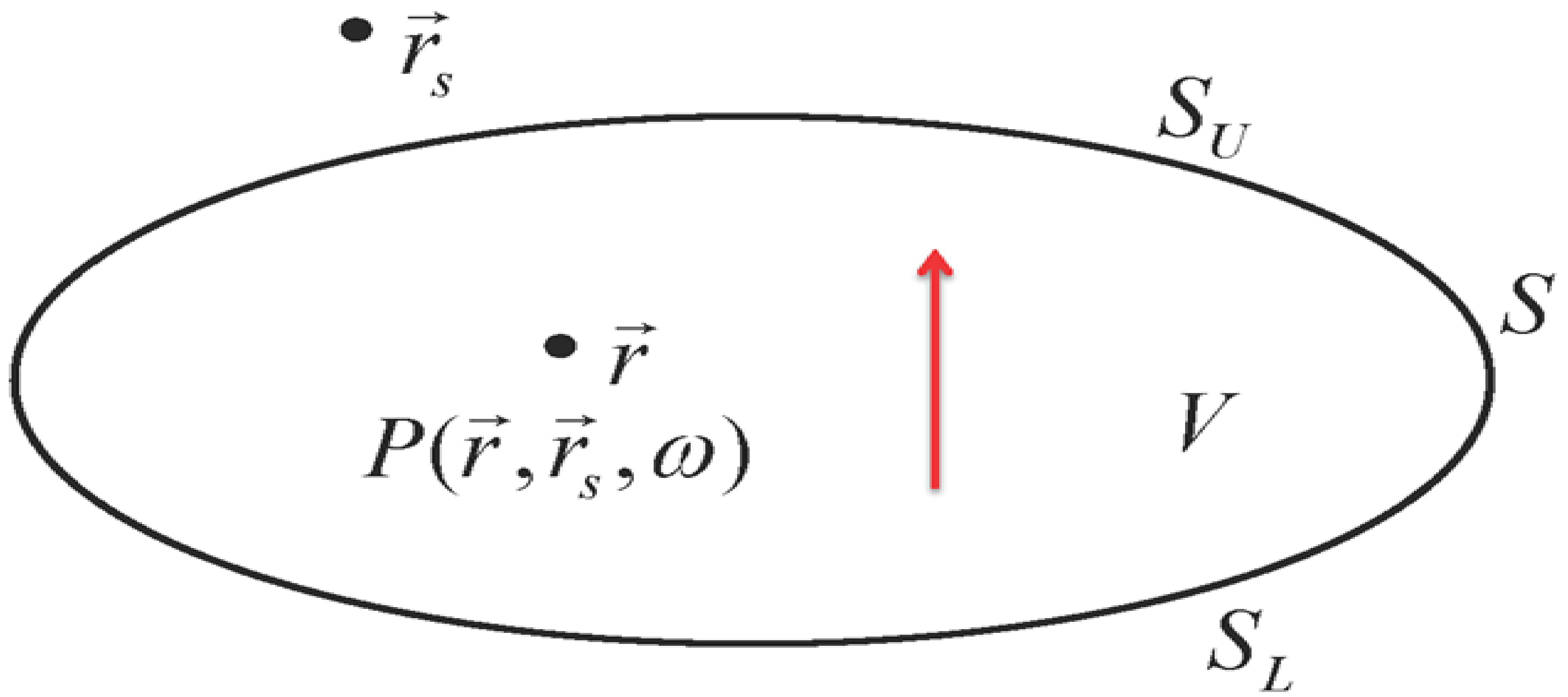
$$= \oiint \left\{ P(\vec{r}',\vec{r}_s,\omega)\nabla' G_0(\vec{r},\vec{r}',\omega) - G_0(\vec{r},\vec{r}',\omega)\nabla' P(\vec{r}',\vec{r}_s,\omega) \right\} \cdot \vec{n}\,dS$$

where $$\left(\nabla^2 + \frac{\omega^2}{c^2(\vec{r})}\right) P(\vec{r}, \vec{r}_s, \omega) = 0 \qquad \textbf{(2)}$$

$$\left(\nabla^2 + \frac{\omega^2}{c^2(\vec{r})}\right) G_0(\vec{r}, \vec{r}', \omega) = \delta(\vec{r} - \vec{r}') \qquad \textbf{(3)}$$

Where G_0 is **any** solution of (3) for $\vec{r}'$ on S and $\vec{r}$ in V


$\vec{r}_s$
S_U
S
$\vec{r}$
V
$P(\vec{r},\vec{r}_s,\omega)$
S_L



to predict P in the volume for one way propagating waves from only measurements on S_U use $G_0 = G_0^-$

$$P = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} P \, dS_g \, dS_s$$

(Green, 1-way waves)

Prestack Stolt migration

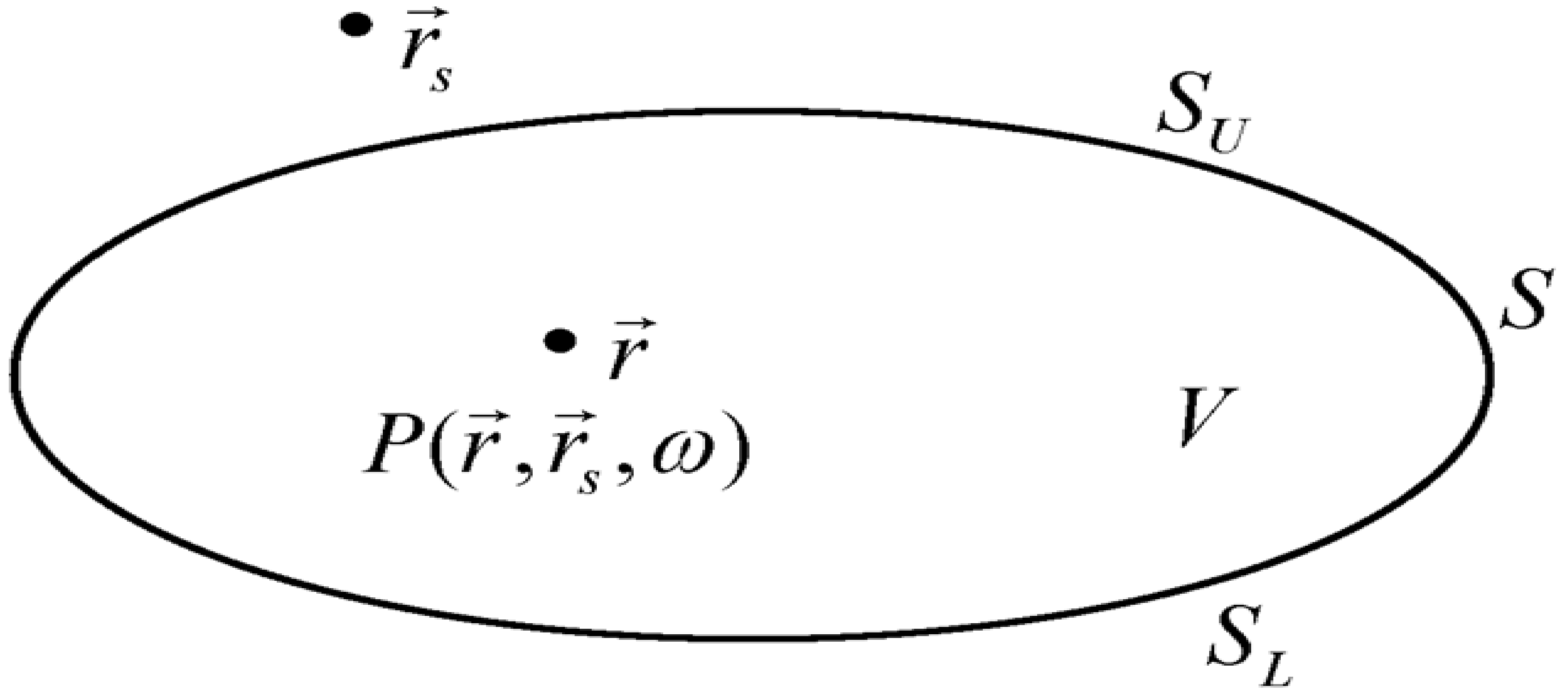
Dirichlet anti-causal Green's function

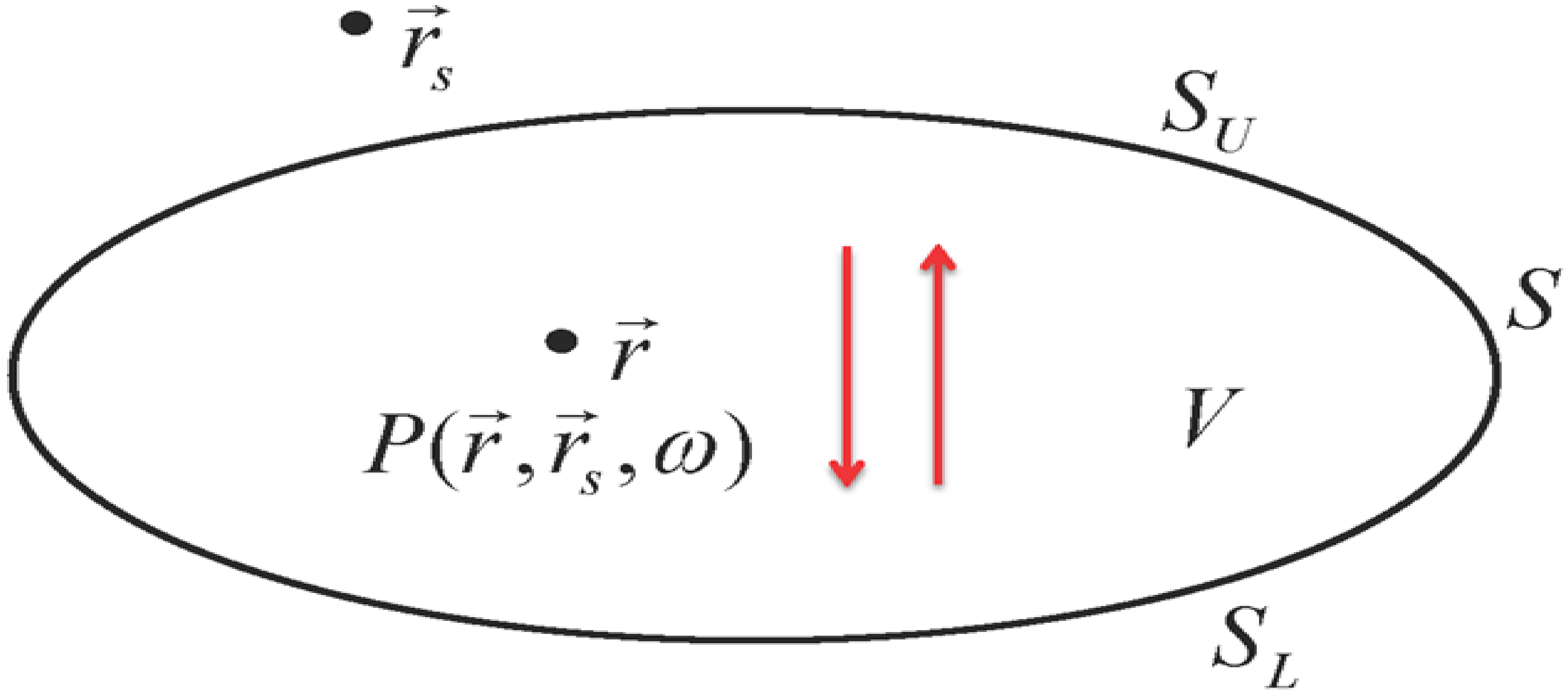
$$G_0^{-D} = -\frac{e^{-ik|z-z'|}}{2ik} - \left(-\frac{e^{-ik|z_I - z'|}}{2ik}\right),$$

where z_I is the image of z through $z' = a$.

$$P(z) = -\frac{dG_0^{-D}(z, z', \omega)}{dz'}\bigg|_{z'=z} P(a) = e^{-ik(z-a)} P(a),$$

which is in agreement with a simple Stolt FK phase shift migration.


$\vec{r}_s$
S_U
S
$\vec{r}$
$P(\vec{r}, \vec{r}_s, \omega)$
V
S_L



For two-way propagating waves, to predict the experiment from only measurements on S_U requires a Green's function (that along with its normal derivative) vanishes at the lower surface called G_0^{DN}

$$
\begin{aligned}
P = \int_{S_s} \Bigg[& \frac{\partial G_0^{DN}}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} P + \frac{\partial P}{\partial z_g} G_0^{DN} \right\} \, dS_g \\
& + \; G_0^{DN} \frac{\partial}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} P + \frac{\partial P}{\partial z_g} G_0^{DN} \right\} \, dS_g \Bigg] \, dS_s
\end{aligned}
$$

(Green, 2–way waves) (

for details see Weglein et al. (2011a,b) and F. Liu and Weglein (2014)

$$P = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} P \, dS_g \, dS_s$$

(Green, 1-way waves)

Prestack Stolt migration

$$P = \int_{S_s} \left[\frac{\partial G_0^{DN}}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} P + \frac{\partial P}{\partial z_g} G_0^{DN} \right\} dS_g \right.$$

$$\left. + \; G_0^{DN} \frac{\partial}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} P + \frac{\partial P}{\partial z_g} G_0^{DN} \right\} dS_g \right] dS_s$$

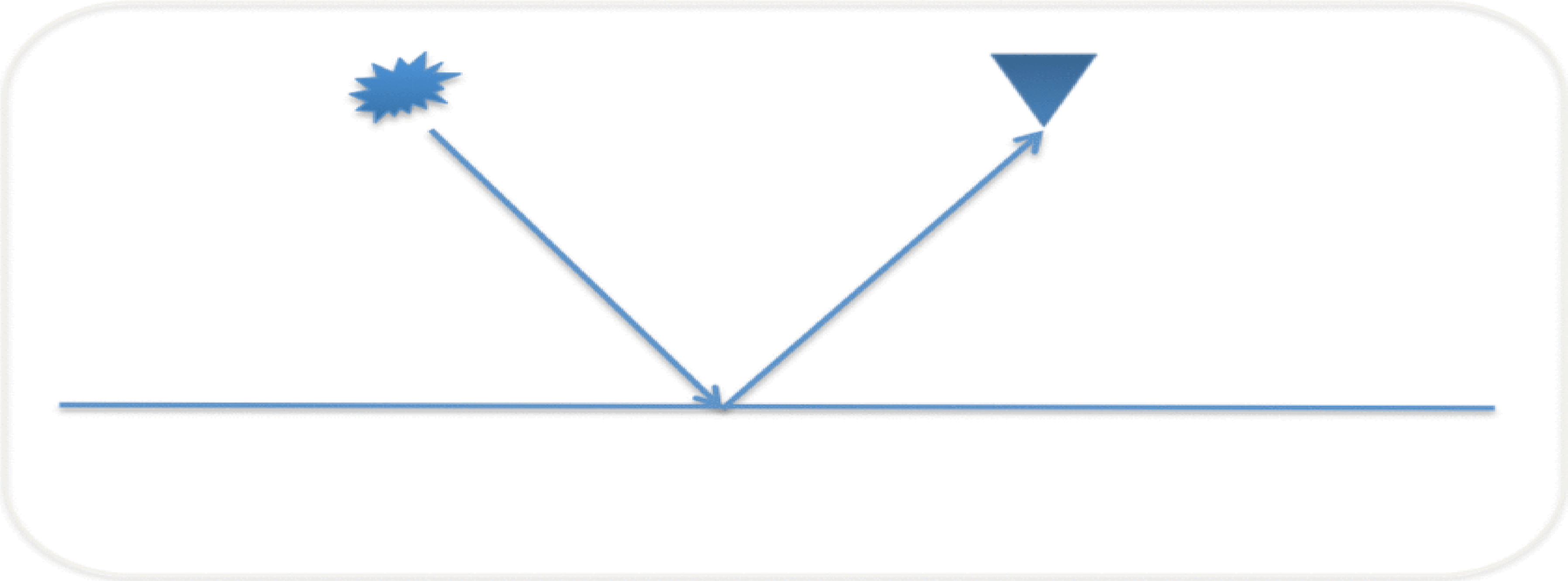
(Green, 2-way waves) (

for details see Weglein et al. (2011a,b) and F. Liu and Weglein (2014)

MIGRATING MULTIPLES?

- We have described how to predict an experiment at depth inside a volume where there are two way propagating waves**.** **For the first time, we follow and demonstrate how each individual surface recorded event** (primaries, free surface multiples and internal multiples) contributes to the predicted experiment at depth, and when the t=0 imaging condition, for imaging and inversion. **That will answer the question of whether multiples are signal or noise.**

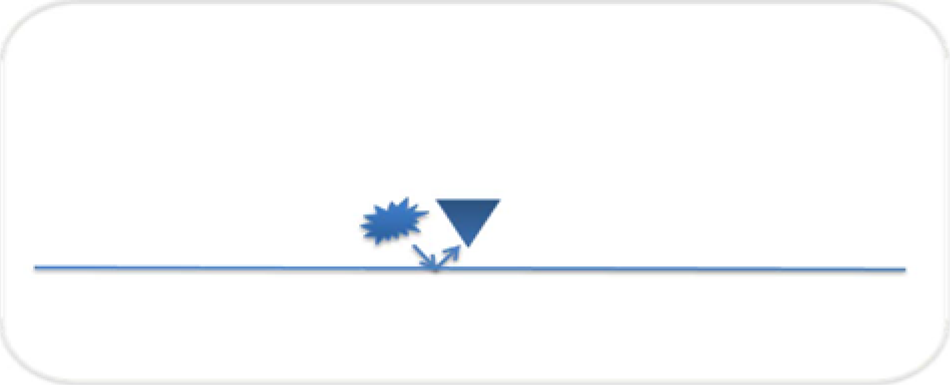
Case 1: a primary



Case 1: a primary from a single reflector

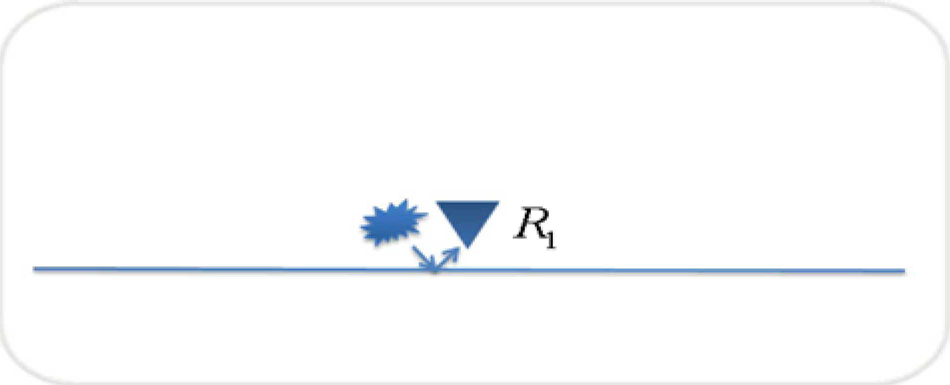
Above the reflector (predicted experiment at depth)

Coincident source and receiver at depth for all times



Blue event: primary

Coincident source and receiver at depth for $t = 0$



Blue event: primary

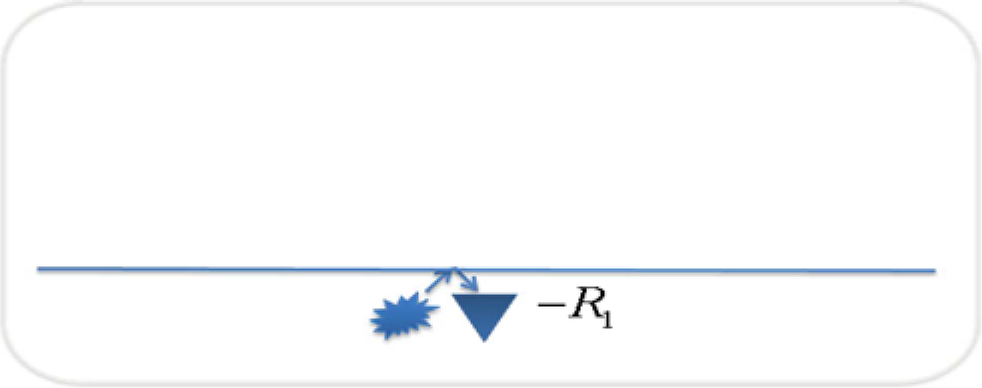
Case 1: a primary from a single reflector

Below the reflector (predicted experiment at depth)

Coincident source and receiver at depth for all times

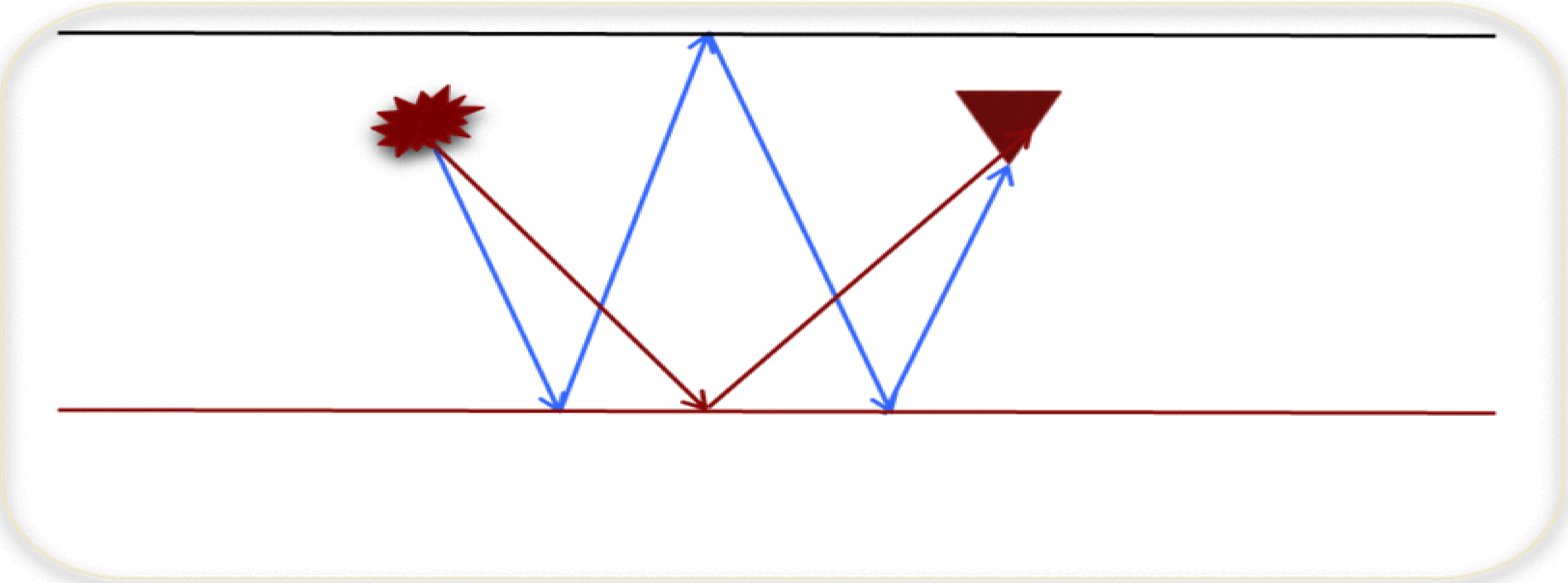
Blue event: primary

Coincident source and receiver at depth for t = 0



Blue event: primary

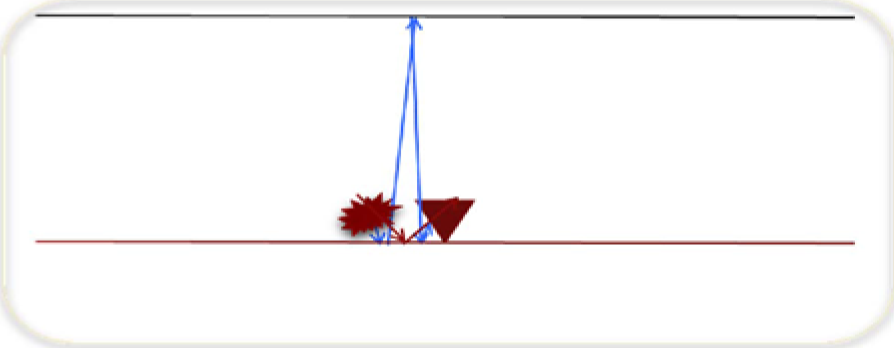
Case 2: a primary and free surface multiple



Case 2: a primary and a free-surface multiple

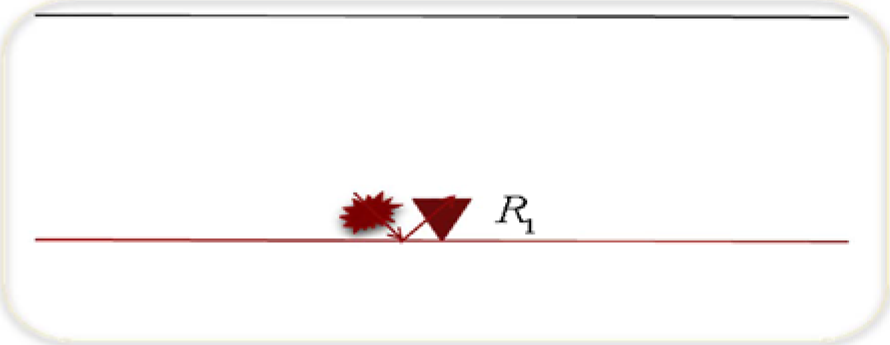
Above the reflector (predicted experiment at depth)

Coincident source and receiver at depth for all times



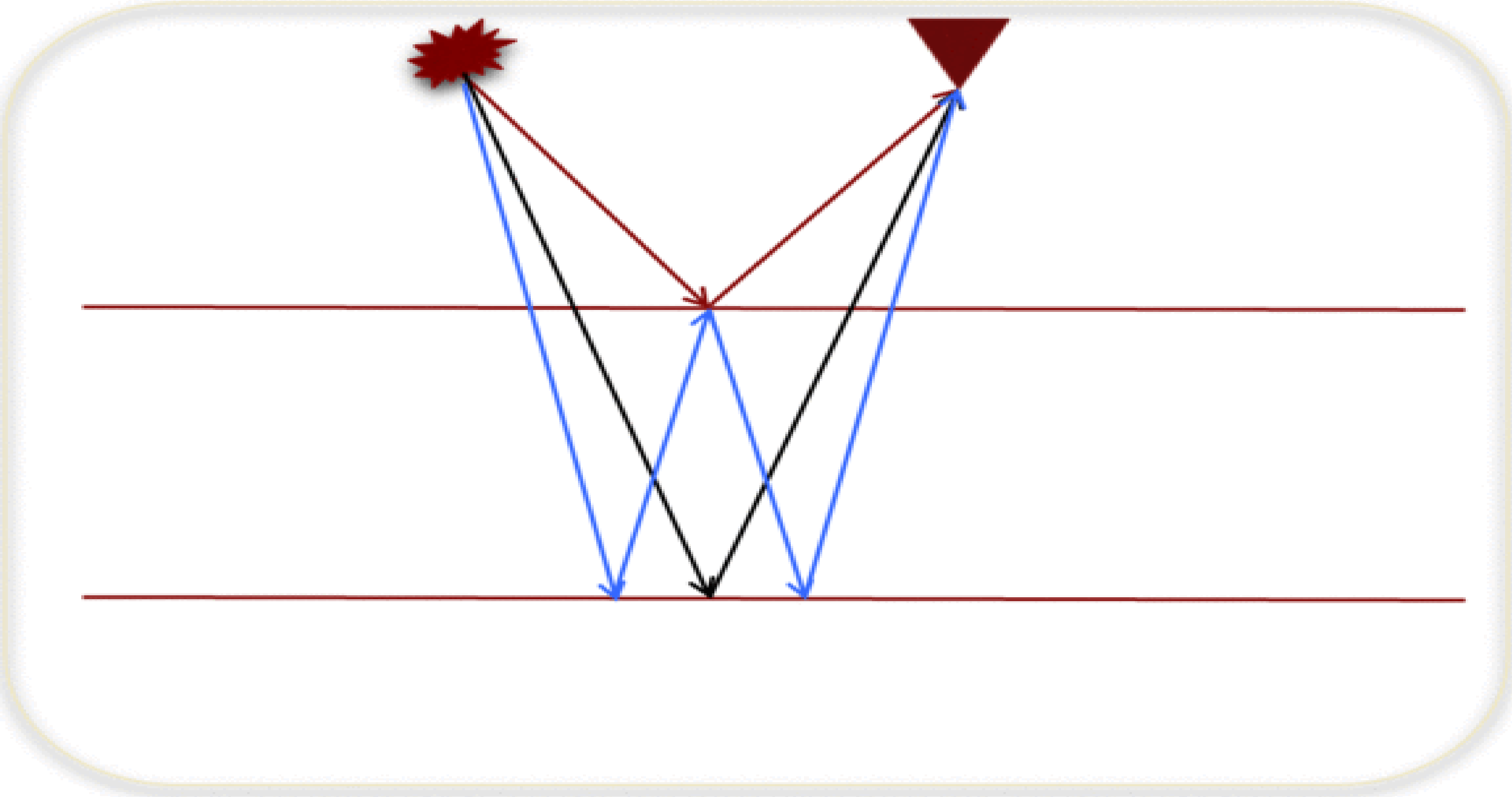
Red event: primary
Blue event: free-surface multiple

Coincident source and receiver at depth for t = 0



Red event: primary

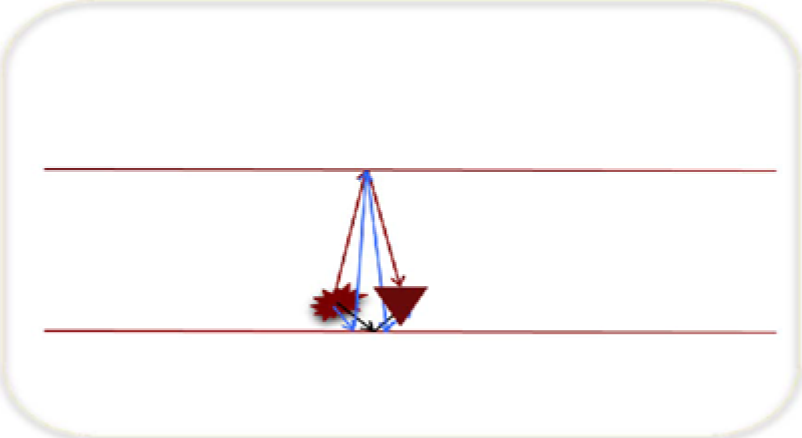
Case 3: two primaries and an internal multiples



Case 3: two primaries and an internal multiple

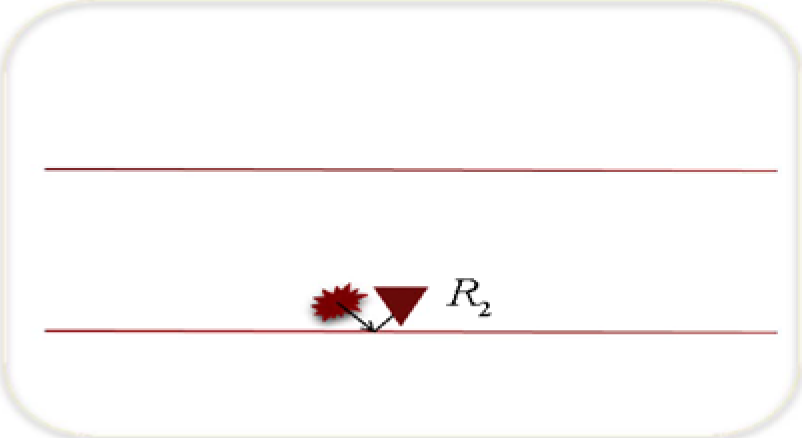
Above the second reflector (predicted experiment at depth)

Coincident source and receiver at depth for all times



Red event: primary from the first reflector
Black event: primary from the second reflector
Blue event: internal multiple

Coincident source and receiver at depth for t = 0

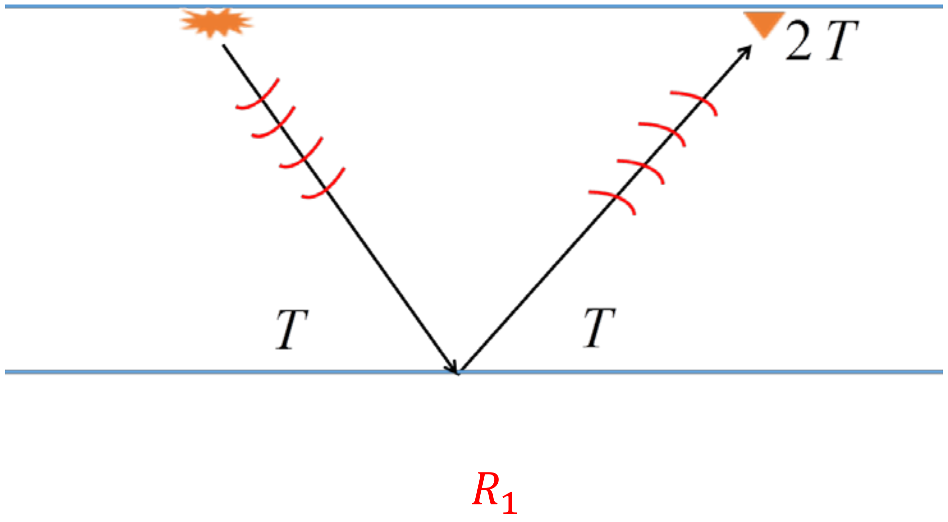


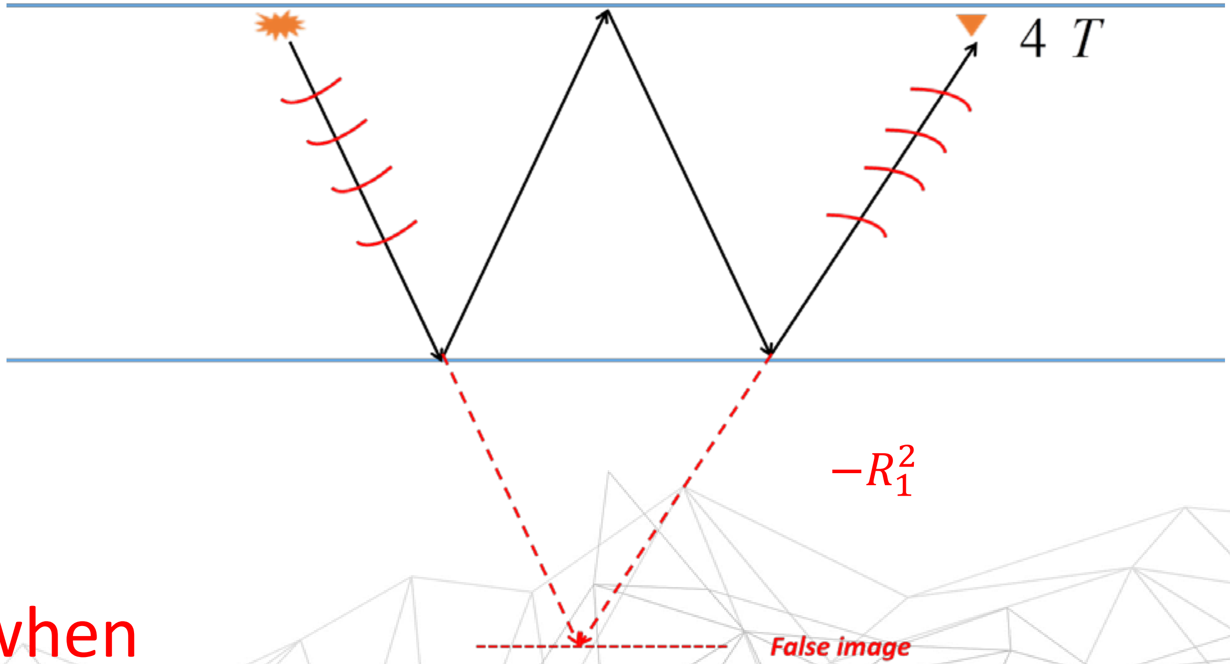
Black event: primary from the second reflector

Hence, only recorded primaries contribute to migration and inversion, and only primaries are signal. For a smooth velocity model it is possible to correctly locate primaries in depth, but all multiples (if not removed) will result in artifacts and spurious images.

What happens to multiples with methods that image recorded primaries

- Claerbout II





Multiples cause false images when migrating primaries

For smooth velocities, multiples produce false images and must be removed in any migration of primaries and multiples, the same is true for Claerbout III

- **What if the recorded primaries are insufficient/less than adequate?**
- **Given the velocity model of the unrecorded primary, a recorded multiple can be used to find an <u>approximate</u> image of an unrecorded primary**

Claerbout Imaging condition II

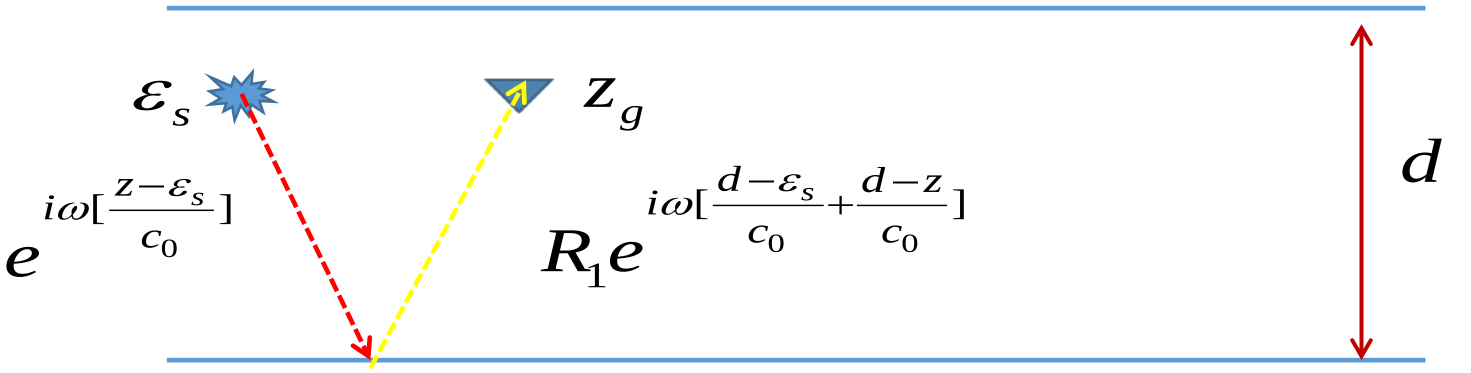
Space and time coincidence of upgoing and downgoing wavefield
(e.g., Claerbout, 1971; Whitmore, 1983)

$$I(x,z) = \int D^*(x,z,\omega)U(x,z,\omega)d\omega$$

$$I(x,z) = \int D(x,z,t)U(x,z,t)dt$$

Use of primaries for imaging with II

Down-going plane wave that starts at z = at t = t_0 = 0 ε_s



Downgoing	$D(z,\omega) = e^{i\omega[\frac{z-\varepsilon_s}{c_0}]}$
Upgoing	$U(z,\omega) = R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0}+\frac{d-z}{c_0}]}$

Use of primaries for imaging with II

- Applying the imaging condition $$I(z) = \int D^*(z,\omega) U(z,\omega) d\omega$$

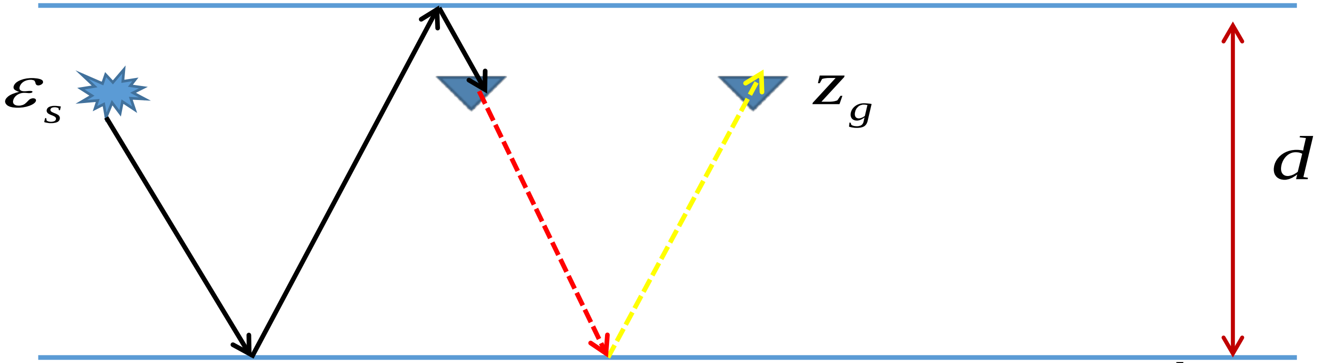
$$\int \left(e^{-i\omega[\frac{z-\varepsilon_s}{c_0}]} \right) \bullet \left(R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{d-z}{c_0}]} \right) d\omega$$

$$= \int \left(R_1 e^{i\omega[-\frac{z-\varepsilon_s}{c_0} + \frac{d-\varepsilon_s}{c_0} + \frac{d-z}{c_0}]} \right) d\omega$$

$$= \int R_1 e^{i\omega[\frac{2d-2z}{c_0}]} d\omega$$

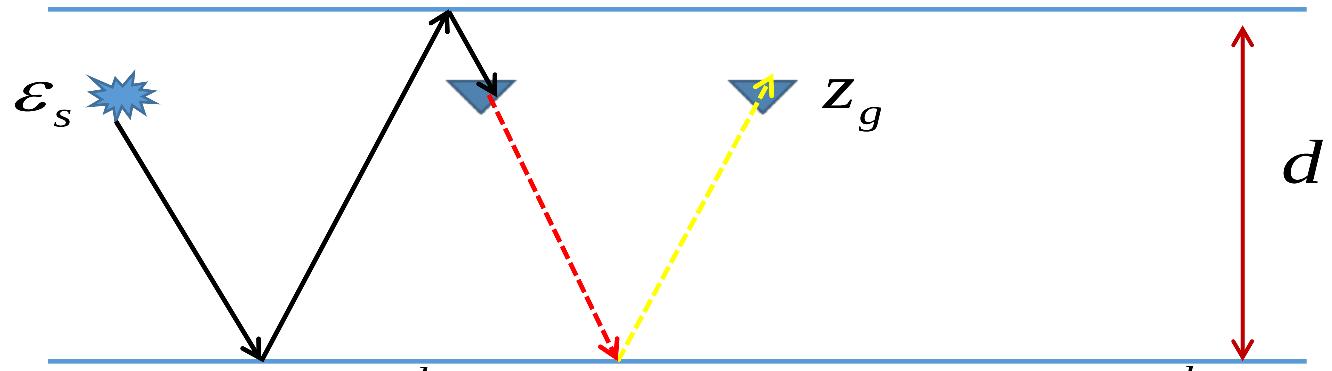
$$= \frac{c_0}{2} R_1 \delta(d-z)$$

Using multiples to enhance imaging



$$D = -R_1 e^{i\omega\left[\frac{d-\varepsilon_s}{c_0} + \frac{d+z_g}{c_0}\right]} \qquad U = -R_1^2 e^{i\omega\left[\frac{d-\varepsilon_s}{c_0} + \frac{2d}{c_0} + \frac{d-z_g}{c_0}\right]}$$

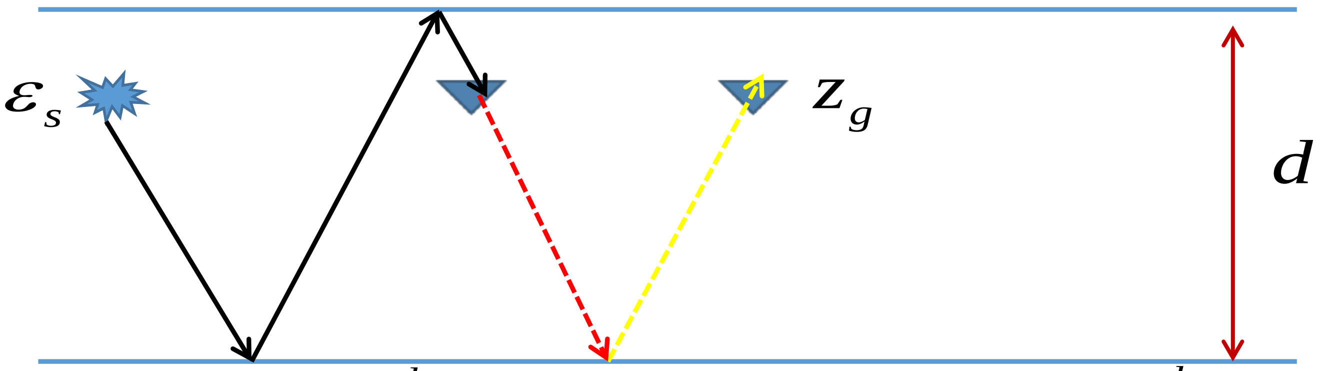
Using multiples to enhance imaging



$$D = -R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{d+z_g}{c_0}]} \qquad U = -R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{2d}{c_0} + \frac{d-z_g}{c_0}]}$$

Downgoing	$D(z,\omega) = -R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{d+z}{c_0}]}$
Upgoing	$U(z,\omega) = -R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{2d}{c_0} + \frac{d-z}{c_0}]}$

Using multiples to enhance imaging



$$D = -R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{d+z_g}{c_0}]} \qquad U = -R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{2d}{c_0} + \frac{d-z_g}{c_0}]}$$

Effective up & down wavefield separation

Downgoing	$D(z,\omega) = -R_1 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{d+z}{c_0}]}$
Upgoing	$U(z,\omega) = -R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{2d}{c_0} + \frac{d-z}{c_0}]}$

Using multiples to enhance imaging

- Applying the imaging condition $$I(z) = \int D^*(z,\omega) U(z,\omega) d\omega$$

$$\int \left(-R_1 e^{-i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{d+z}{c_0}]} \right) \bullet \left(-R_1^2 e^{i\omega[\frac{d-\varepsilon_s}{c_0} + \frac{2d}{c_0} + \frac{d-z}{c_0}]} \right) d\omega$$

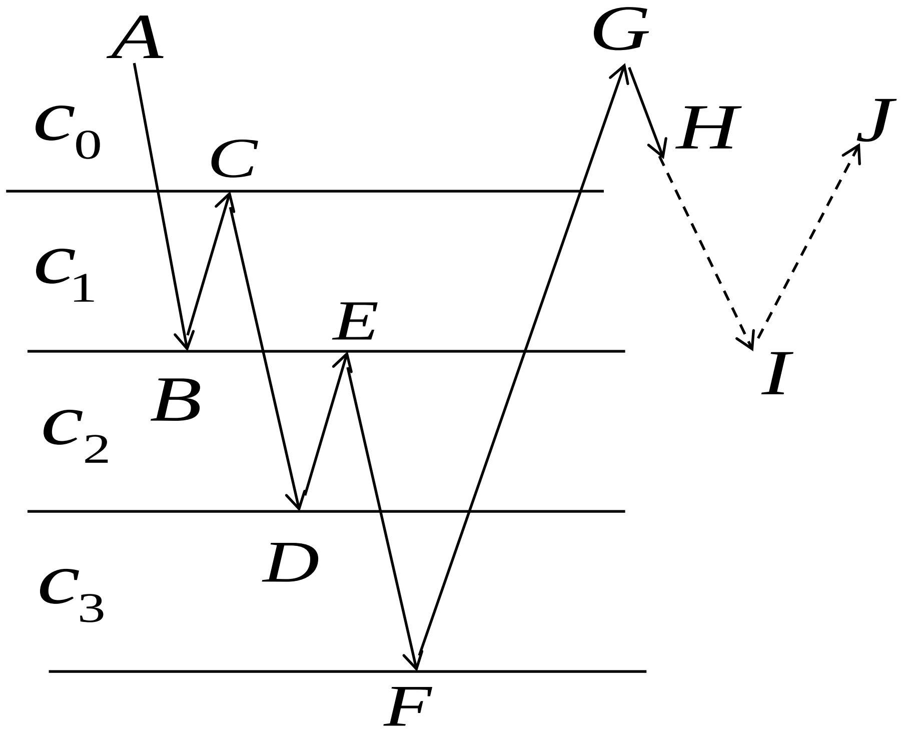
$$= \int \left(R_1^3 e^{i\omega[-\frac{d-\varepsilon_s}{c_0} - \frac{d+z}{c_0} + \frac{d-\varepsilon_s}{c_0} + \frac{2d}{c_0} + \frac{d-z}{c_0}]} \right) d\omega$$

$$= \int R_1^3 e^{i\omega[\frac{2d-2z}{c_0}]} d\omega$$

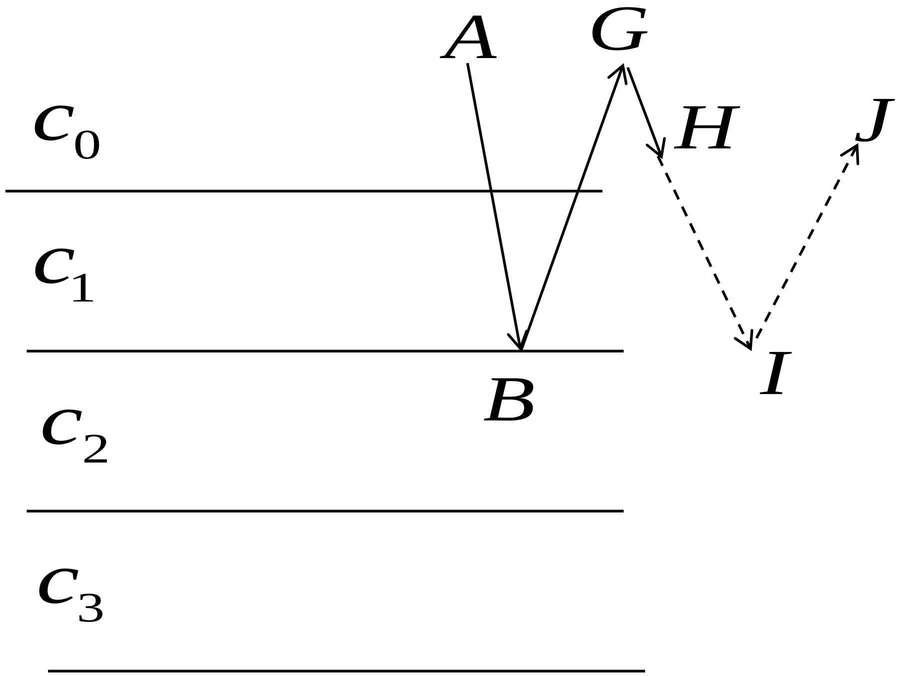
$$= \frac{c_0}{2} R_1^3 \delta(d-z)$$

- Amplitude of the image of unrecorded primary is R_1. The amplitude of the image found by using a multiple is R_1^3
- in addition, the approximate image is Claerbout II and carries the limitations we described above

- Amplitude of the image of unrecorded primary is R_1. The amplitude of the image found by using a multiple is R_1^3
- in addition, the approximate image is Claerbout II and carries the limitations we described above
- Only recorded primaries can achieve Claerbout III levels of structural fidelity and amplitude analysis



A
G
c_0
H
J
C
c_1
E
I
B
c_2
D
c_3
F



A
G
c_0
H
J
c_1
I
B
c_2
c_3

- These two figures have the same exact process in obtaining an approximate image of primary HIJ. To call that process migrating ABC…GIJ makes no sense. How can the migration of an event not care about the history of the event?

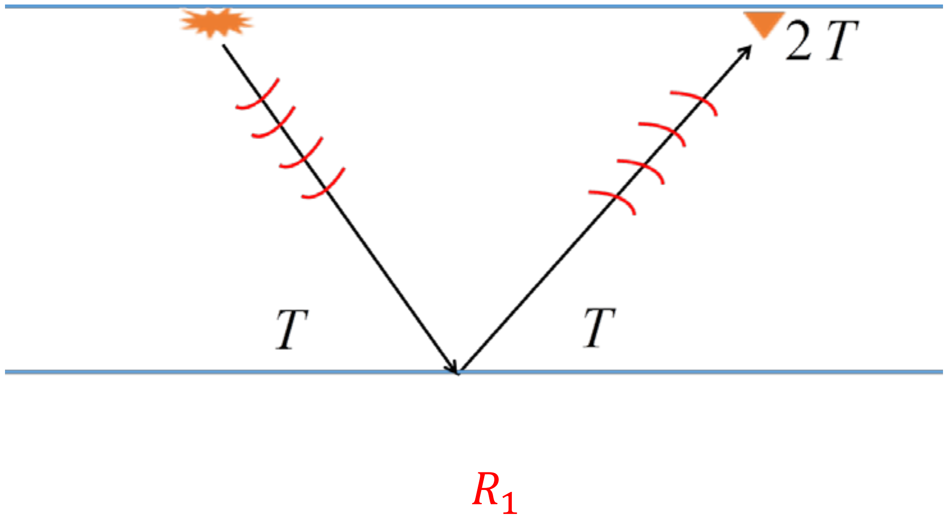
Imaging primaries

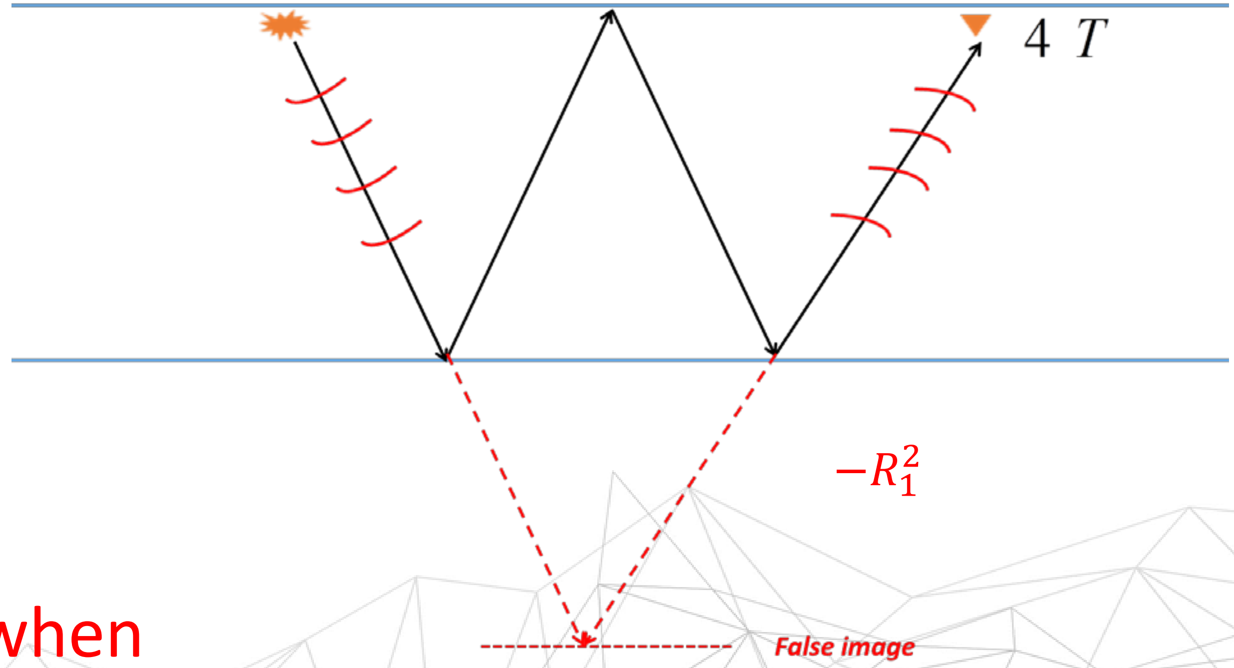
1. Imaging <u>recorded</u> primaries
2. Finding the approximate image of an <u>unrecorded</u> primary

1. Imaging recorded primaries

What happens to multiples with methods that image recorded primaries

- Claerbout II





Multiples cause false images when migrating primaries

For smooth velocities, multiples produce false images and must be removed in any migration of primary and multiples, the same is true for Claerbout III

- To image recorded primaries, using any imaging method, requires first effectively removing multiples without damaging primaries

- While tremendous progress has occurred in the last 20 years (from ISS methods for FS and internal multiples), significant high priority challenges remain

- Responding to those challenges is a strategic priority for the petroleum industry – if not, the information and value of recorded primaries will remain only partially accessible and delivered. Only recorded primaries can deliver Stolt extended Claerbout III imaging and inversion benefits.

2. Finding the approximate image of an unrecorded primary

Using a multiple to find an approximate image of an unrecorded primary

- Outputs a amplitude challenged Claerbout II image of the unrecorded primary (that's a very low bar for the claim of a new vision of seismic data). Since Claerbout II has an intrinsic high frequency approximation, the approximate image of unrecorded primaries will not only be amplitude and interpretation challenged but will not have equal effectiveness / fidelity at all frequencies. Only recorded primaries imaged with the Stolt extended Claerbout III imaging can have interpretable amplitude information and equal effectiveness at all frequencies.

- Hence, to seek images of recorded primaries plus an approximate image of unrecorded primaries, <u>requires,</u> as a critical step, a very effective multiple removal for the first, and predicting multiples for the second.

- The approximate image of unrecorded primaries cause cross-talk – since the method cannot predict a multiple unless all of its subevents are recorded – if all of its subevents are recorded the multiple is useless in providing an image of an unrecorded primary, since there are none – that circular reasoning leads to using the entire data set (that contain multiples) and the non-multiple events in the data are one of the causes of cross-talk.

- This 'use of multiples' is an ad-hoc method, that doesn't derive from a comprehensive framework, without issues where problems and shortcomings can be addressed in a systematic fashion, by returning to the theory without the issues. There is no theory that ‘uses’ multiples, without causing false events due to cross talk.

The effective removal of multiples remains a priority for those serious about imaging and inversion objectives ---- the improvement in structure from using multiples to obtain an approximate image of an unrecorded shallow primary can be useful ---- but doesn't represent a new and superseding view of primaries and multiples and their roles in migrating and inverting data.

Summary

- Claerbout II imaging is a high frequency approximation.

- Claerbout III imaging is not a high frequency approximation.

Summary

- The only medium that has one way wave propagation is a homogeneous medium.

- Assuming one way wave propagation for a non-homogeneous medium is a high frequency approximation.

- We have described and illustrated a new migration method that generalizes the (non-asymptotic) Stolt extended Claerbout III imaging principle and applies it for inhomogeneous media without high frequency/one way wave assumptions.

Summary

- That combination will have equal effectiveness at all frequencies for the image and inversion at the target/reservoir.
- Plan is to evaluate the added-value to structure and amplitude analysis and to deliver that capability in 2D and 3D to our sponsors.
- To migrate recorded primaries, it requires the removal of multiples.
- While there is progress in multiple removal, serious high priority challenges remain.

An important comment and noteworthy observation

In the inverse scattering series internal multiple attenuation algorithm method and algorithm ---- for example in 2D

$$D(x_g,\varepsilon_g,x_s,\varepsilon_s,t)$$

$$\downarrow$$

$$D(k_g,\varepsilon_g,k_s,\varepsilon_s,\omega)$$

Surface data

$$\downarrow$$

$$D(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z)$$

$\downarrow$ Change to incident plane wave data

$$b_1(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z) = -2iq_s D(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z)$$

$$\left(q_s = \sqrt{\left(\frac{\omega}{c_0}\right)^2 - k_s{}^2} \right)$$

$$b_1(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z) = -2iq_s D(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z)$$

$$\downarrow$$

Source and receiver at depth z $\longrightarrow$ $b_1(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z)e^{ik_z z}$

$$\left(k_z = \sqrt{\left(\frac{\omega}{c_0}\right)^2 - k_g{}^2} + \sqrt{\left(\frac{\omega}{c_0}\right)^2 - k_s{}^2} \right)$$

Time equal zero: $\int d\omega$

With change of variable $\omega \longrightarrow k_z$

$$\int dk_z \left(\frac{\partial \omega}{\partial k_z}\right) e^{ik_z z} b_1\left(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z\right)$$

Source and receiver experiment at depth, that retains the $x_h \neq 0$ information at $t = 0$

The Stolt extended and generalized Claerbout imaging III,

$$b_1(k_g, \varepsilon_g, k_s, \varepsilon_s, z)$$

And the ISS internal multiple attenuation algorithm:

$$b_3(k_g, \varepsilon_s, k_s, \varepsilon_g, q_g + q_s) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_1 e^{iq_1(\varepsilon_g - \varepsilon_s)} dk_2 e^{iq_2(\varepsilon_g - \varepsilon_s)}$$

$$\times \int_{-\infty}^{\infty} dz_1 e^{i(q_g + q_1)z_1} b_1(k_g, k_1, z_1)$$

$$\times \int_{-\infty}^{z_1 - \varepsilon} dz_2 e^{-i(q_1 + q_2)z_1} b_1(k_1, k_2, z_2)$$

$$\times \int_{z_2 + \varepsilon}^{\infty} dz_3 e^{i(q_2 + q_s)z_3} b_1(k_2, k_s, z_3)$$

Return to data

$$b_3(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z) = -2iq_s D_3(k_g,\varepsilon_g,k_s,\varepsilon_s,k_z)$$

$$\downarrow$$

$$D_3(x_g,\varepsilon_g,x_s,\varepsilon_s,t)$$

$$\downarrow$$

$$D(x_g,\varepsilon_g,x_s,\varepsilon_s,t) + D_3(x_g,\varepsilon_g,x_s,\varepsilon_s,t)$$

Data with internal multiple attenuated

An important comment and noteworthy observation

The well-documented stand-alone capability of the ISS internal multiple attenuator is due to **two factors:**

1)It’s the only method that is direct and doesn’t need and require any subsurface information (and without any interpretive input or intervention) and is model type independent

2)The water speed migration input to the algorithm (the Stolt-extended and generalized Claerbout imaging III) is the basic reason the algorithm can accurately and effectively accommodate subevents that are caused by, for example, specular and non-specular reflections, diffractions, pinchouts, refractions and head waves.

The new Stolt extended Claerbout III migration, being developed within M-OSRP, will provide benefit as:

1) a new and more capable method for migration and migration-inversion for both structural plays and amplitude analysis for conventional and unconventional reservoir identification and

2) as one of two distinct strategies to provide the next (and necessary) level of internal multiple removal, taking us from internal multiple attenuation to internal multiple elimination.

KEY DELIVERABLES FOR THE YEAR OF 2015-2016

1) On-shore preprocessing (wave separation, ground roll removal) where the near surface is complex and unknown

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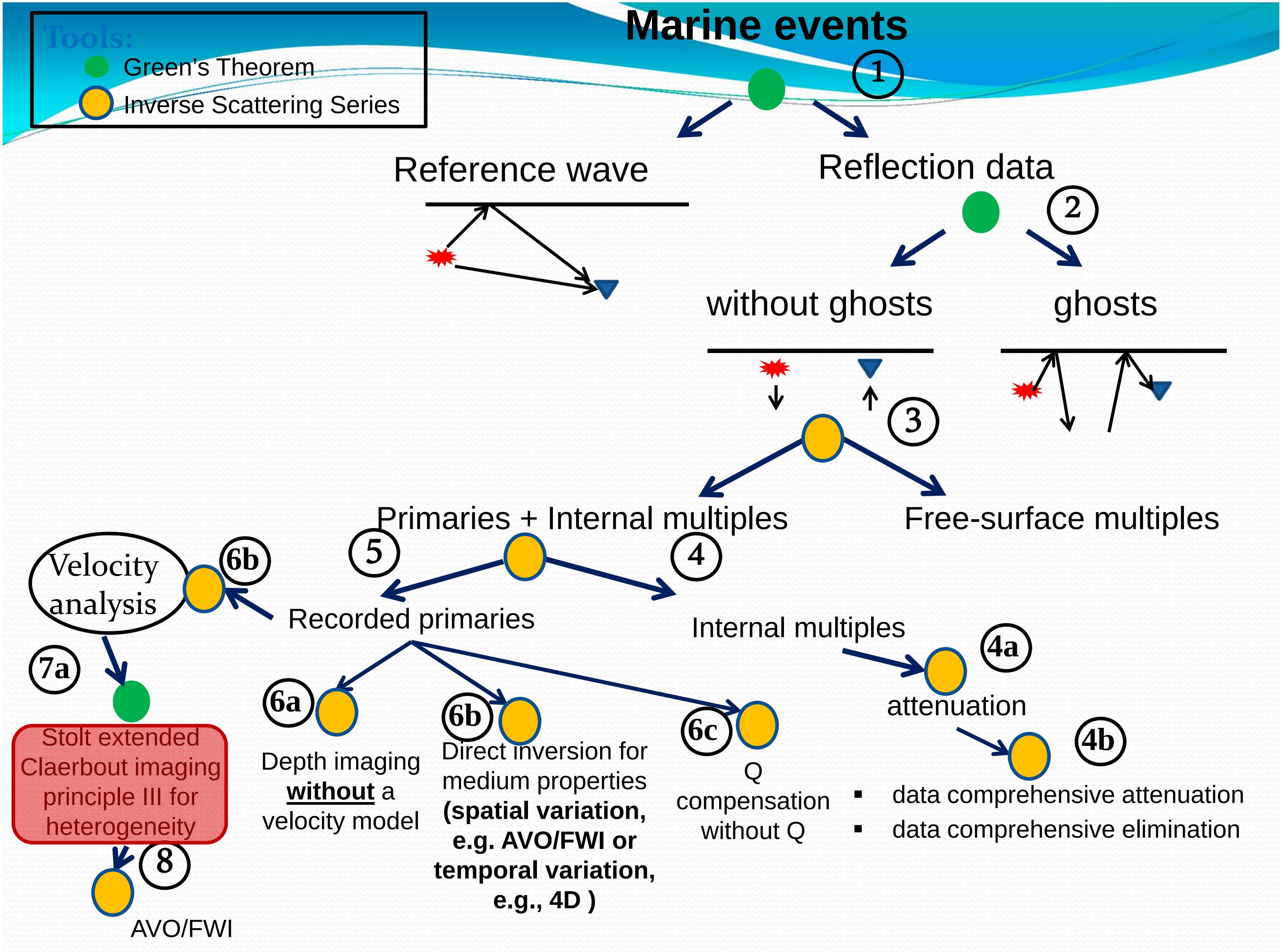
3) 2D and 3D ISS internal multiple elimination --- with two distinct strategies and deliverables

4) A new velocity analysis method based on ISS parameter estimation

5) Green's theorem wave separation for de-ghosting and P_0 and Ps, for marine and on-shore plays where the acquisition surface is not planar or horizontal

The comparison of structural resolution differences with conventional and broadband data between CII and CIII --Part I

Qiang Fu, Yanglei Zou, Chao Ma, and Arthur B. Weglein



Marine events
Tools:
Green's Theorem
Inverse Scattering Series
1
Reference wave
Reflection data
2
without ghosts
ghosts
3
Primaries + Internal multiples
Free-surface multiples
5
4
Velocity analysis
6b
Recorded primaries
Internal multiples
4a
attenuation
4b
7a
6a
6b
6c
Stolt extended Claerbout imaging principle III for heterogeneity
Depth imaging **without** a velocity model
Direct inversion for medium properties **(spatial variation, e.g. AVO/FWI or temporal variation, e.g., 4D)**
Q compensation without Q
data comprehensive attenuation
data comprehensive elimination
8
AVO/FWI

Implement Claerbout imaging principle III by Green's theorem

How to implement Claerbout imaging principle III by Green's theorem and special G_0^{DN}

Weglein A. B. et. al.(2011a),
Weglein A. B. et. al.(2011b)

First implementation for 1D medium for both above and beneath a reflector

Liu, Fang, and A. B. Weglein(2013)

Implementation for 1.5D medium for both above and beneath a reflector

Fu, Qiang, et. al. (2015)

First comparison of structural resolutions with conventional and broadband data for CII and CIII

Fu, Qiang, et. al. (2016)

Implementation for 2D medium (with lateral changing) for both above and beneath a reflector

Liu, Fang, et. al. (2015)
Progressing

Four comparisons on CII and CIII

1. Consistency of structural image
2. Interpretability of the amplitude
3. Migration with a discontinuous velocity model
4. Frequency fidelity

the image resolution at the target

The motivation of the comparison

- From previous presentations we know in principle CII and CIII should be different in terms of treatment of different frequencies
- But how significant are the differences?
- We would like to compare the structural resolution differences with conventional and broadband data between Claerbout II (CII) and Claerbout III (CIII) imaging principles and quantify the results for the first time

The design of the comparison

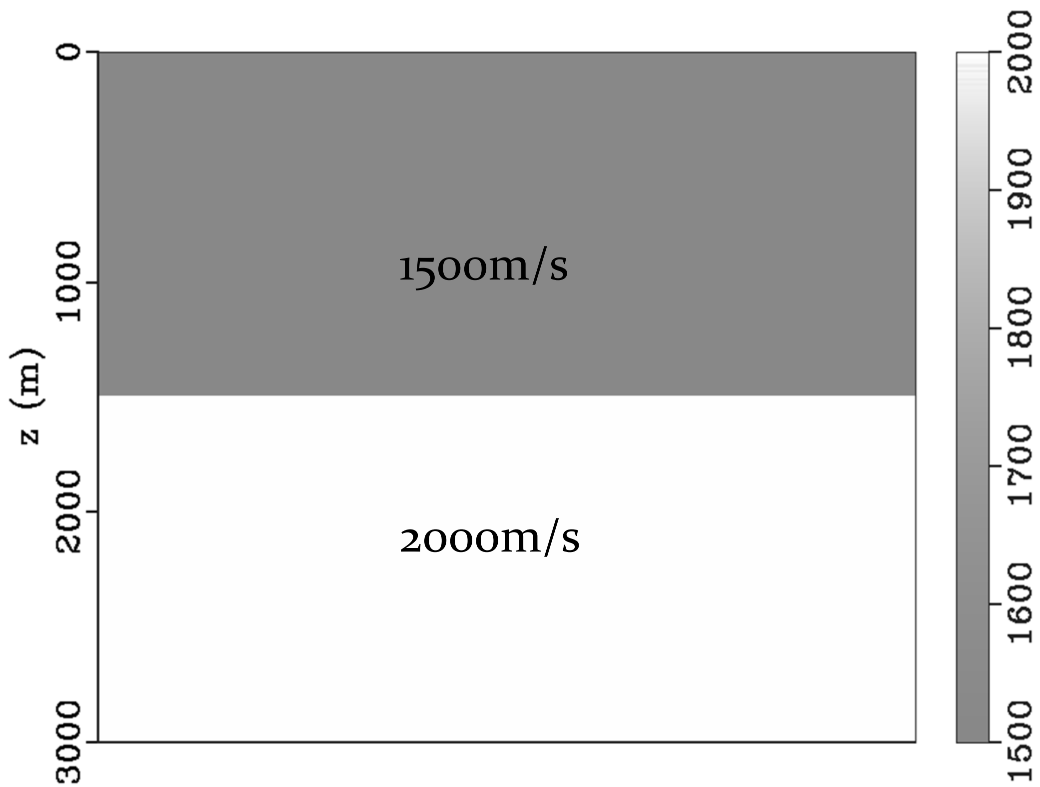
- Two factors may contribute to the imaging effectiveness
  - Imaging principle itself
  - Implementation of the imaging

  We would like to isolate the impact of imaging principle itself, so we use a velocity model which is as simple as possible; and implementations for CII and CIII as perfect as we can
- We focus on only the structural resolution differences

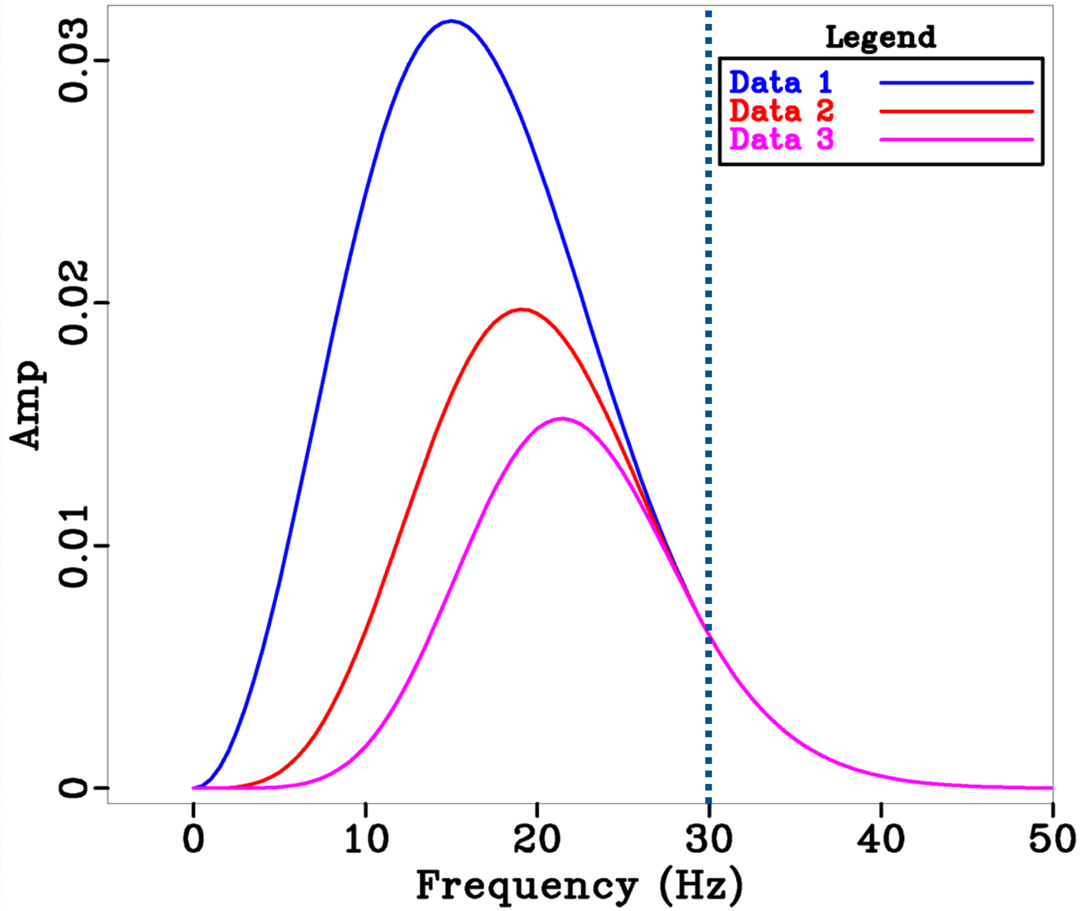
The scheme of the comparison

	CIII	CII
Conventional data (Less low frequencies)	The relative amplitude of the first side lobe	
Broadband data (More low frequencies)		

The velocity model of the data



The spectra of the synthetic data

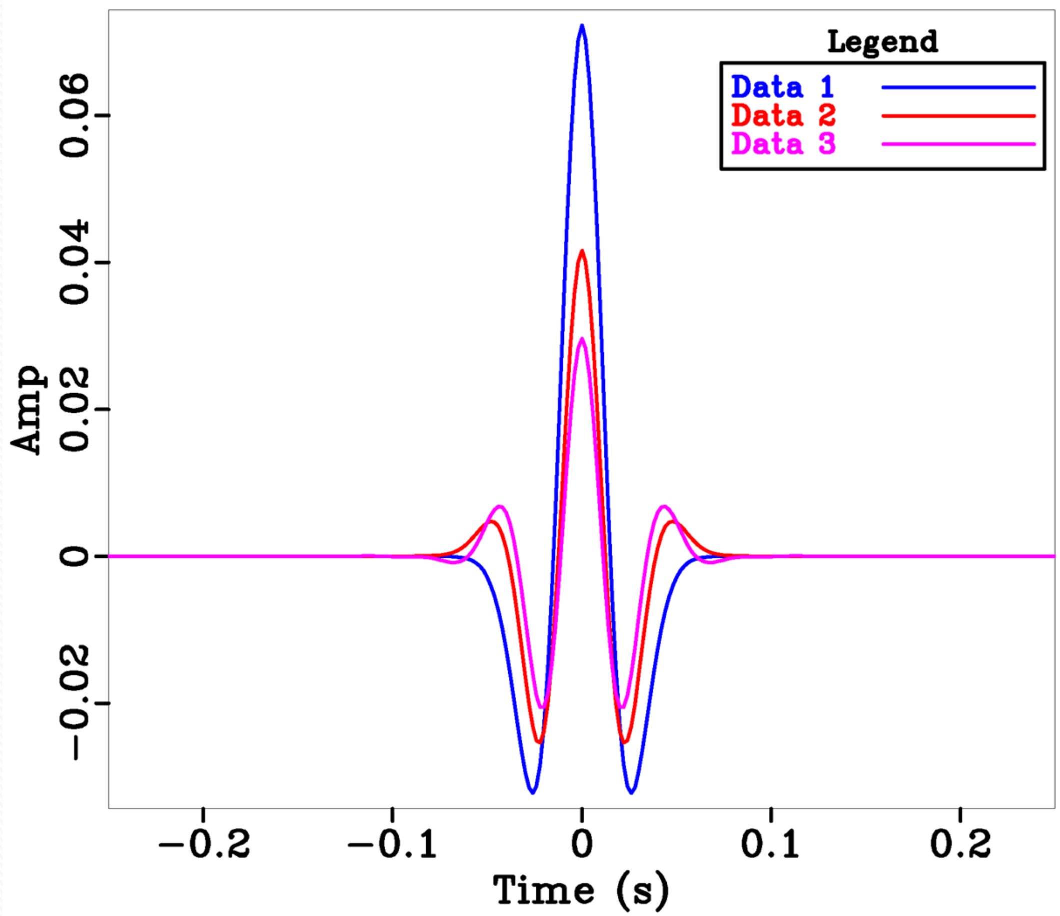


Data 1: Broadband data (More low frequencies)

Data 2: Conventional data (Less low frequencies)

Data 3: Even less low frequencies than Conventional data

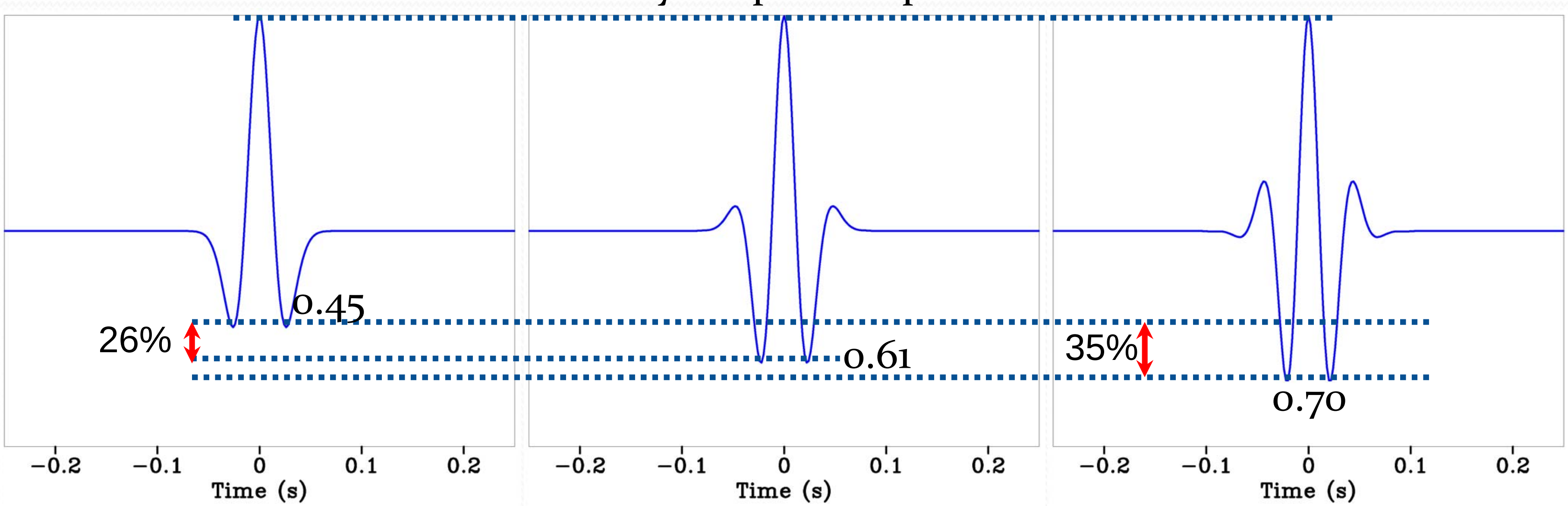
The difference in data (before imaging) – time domain



Data 1: Broadband data (More low frequencies)
Data 2: Conventional data (Less low frequencies)
Data 3: Even less low frequencies than Conventional data

Quantitative comparison of data

normalized by the peak amplitude

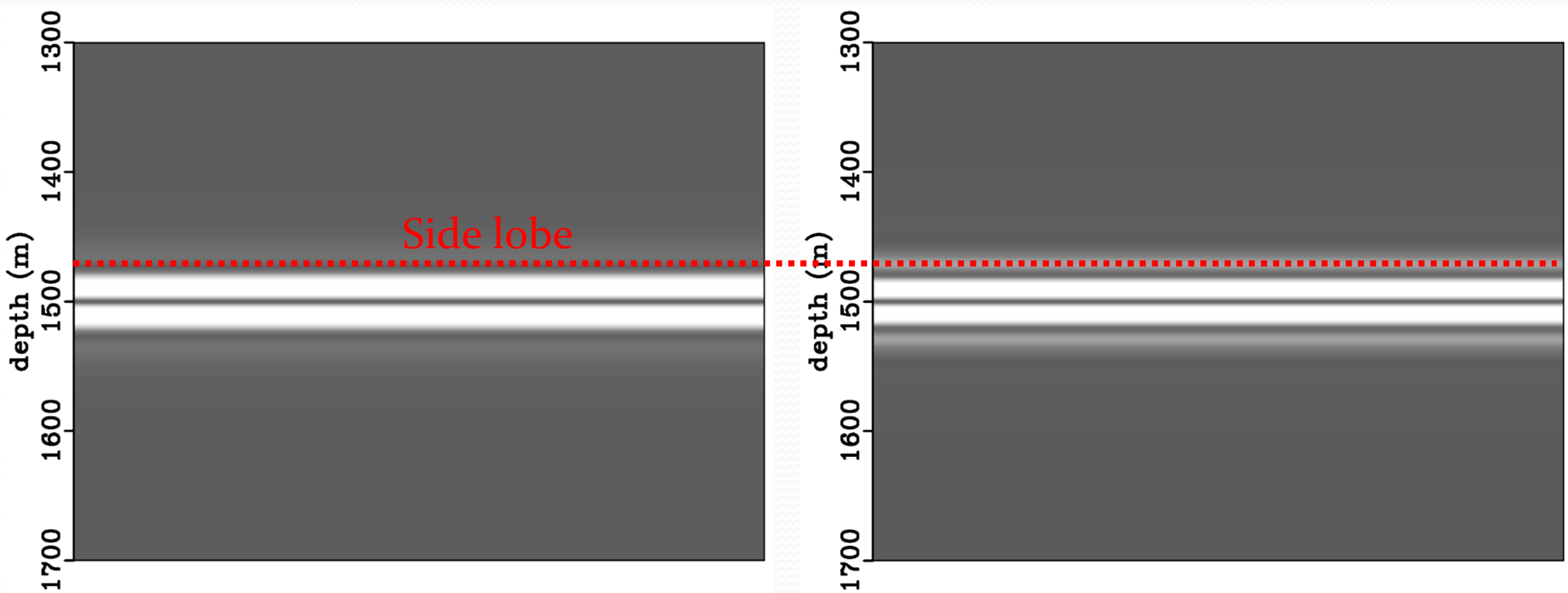


Data 1 Data 2 Data 3

Data 1: Broadband data (More low frequencies)
Data 2: Conventional data (Less low frequencies)
Data 3: Even less low frequencies than Conventional data

CIII imaging results

More low frequencies **Less low frequencies**

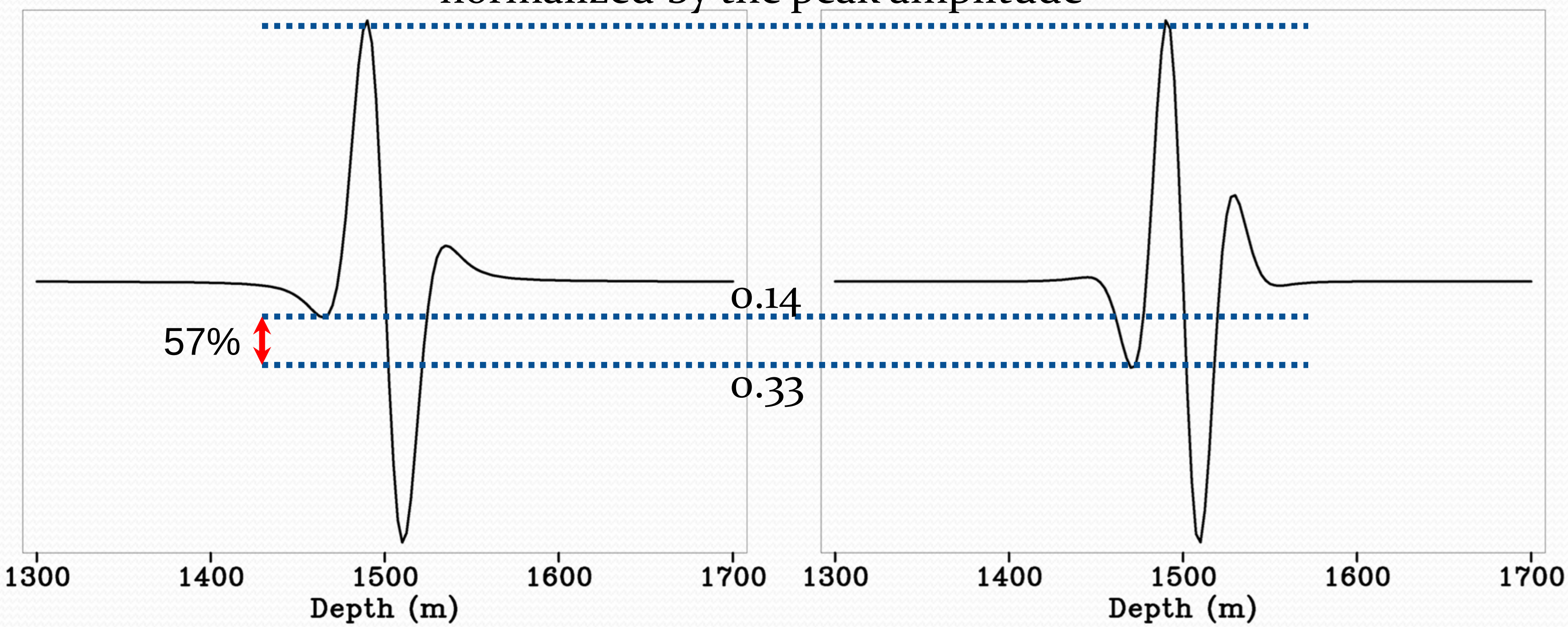


Note: The plots here are the absolute values of the images

Quantitative comparison of images

More low frequencies | **Less low frequencies**

normalized by the peak amplitude



Summary of part I

- For CIII, by including low frequencies in the input data, the side lobes due to the missing low frequencies reduced 57% (from 0.33 to 0.14)
- In the next part, My colleague Yanglei Zou will show you the comparison results for CII
- And finally he will compare the sensitivity to low-frequency component of structural resolution between CII and CIII imaging principles

Thank you

M-OSRP
Misson - Oriented Seismic Research Program
Solve The Right Problem

The comparison of structural resolution differences with conventional and broadband data between CII and CIII --Part II

Qiang Fu, Yanglei Zou*, Chao Ma and Arthur B. Weglein

June 9, 2016

Four comparisons

1) consistency of structural image

1) interpretability of the amplitude

1) migration with a discontinuous velocity model

1) frequency fidelity

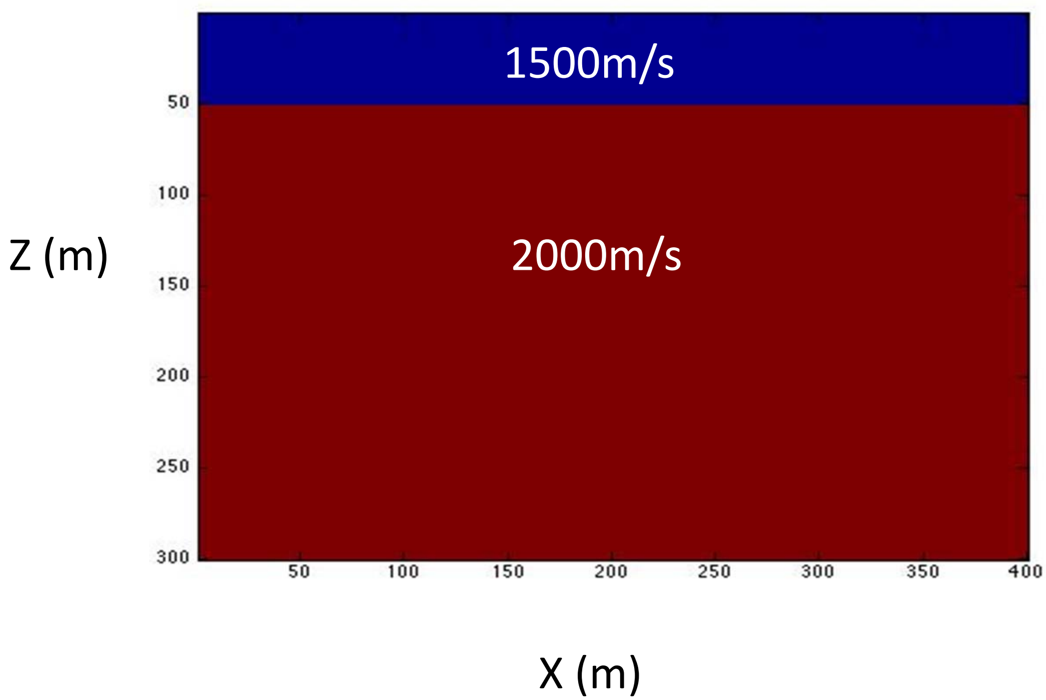
the image resolution at the target

Claerbout imaging condition II RTM formula

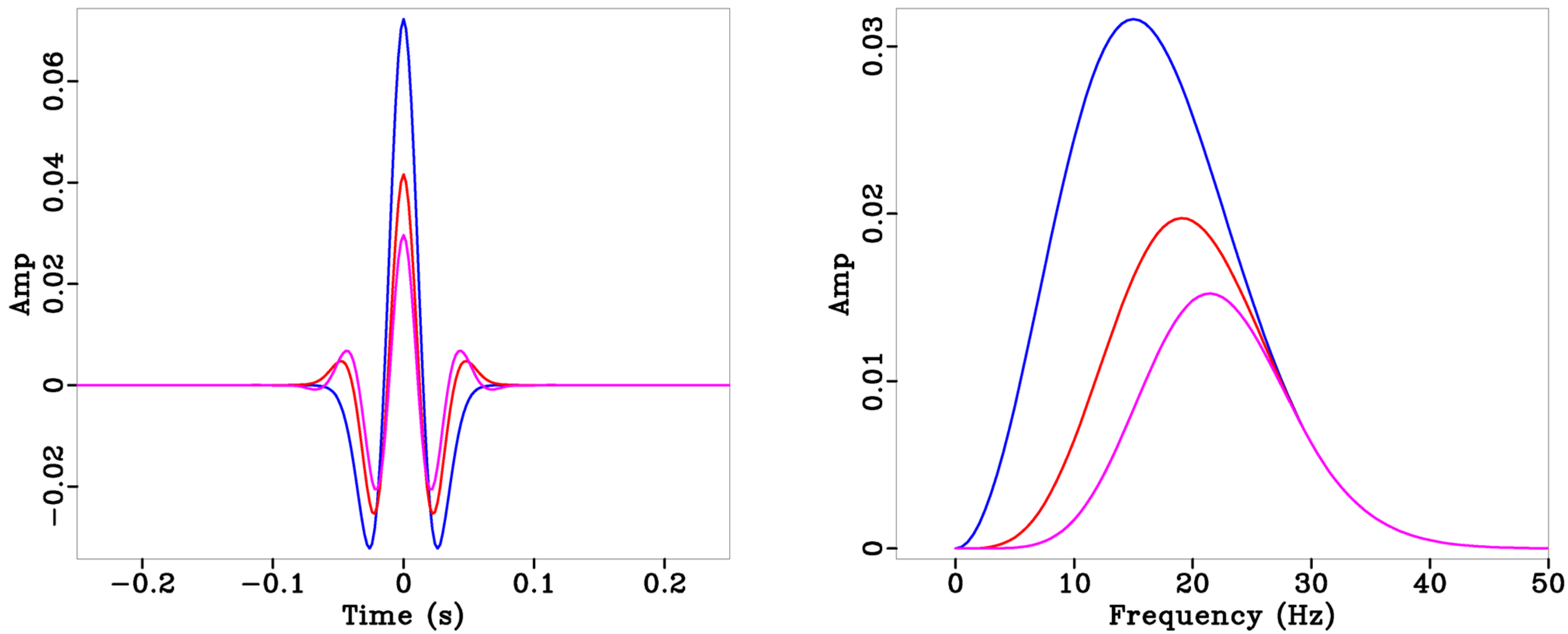
$$I(\vec{r}) = \sum_{x_s} \sum_{\omega} S^*(\vec{r}, x_s, \omega) R(\vec{r}, x_s, \omega)$$

(Baysal et al., 1983; Whitmore, 1983; McMechan, 1983)

The velocity model of the data



Wavelets of the input data



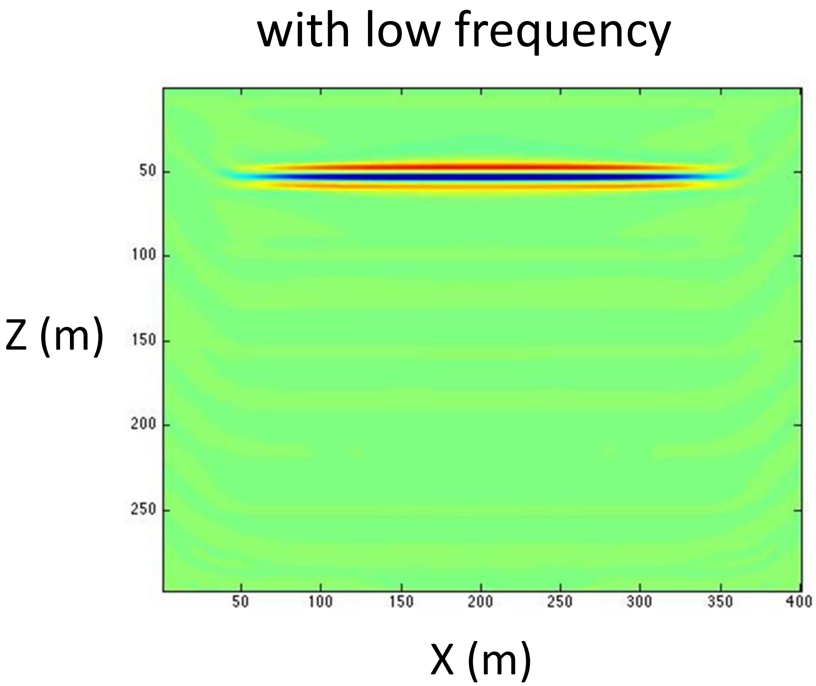
Blue line represents broadband data.

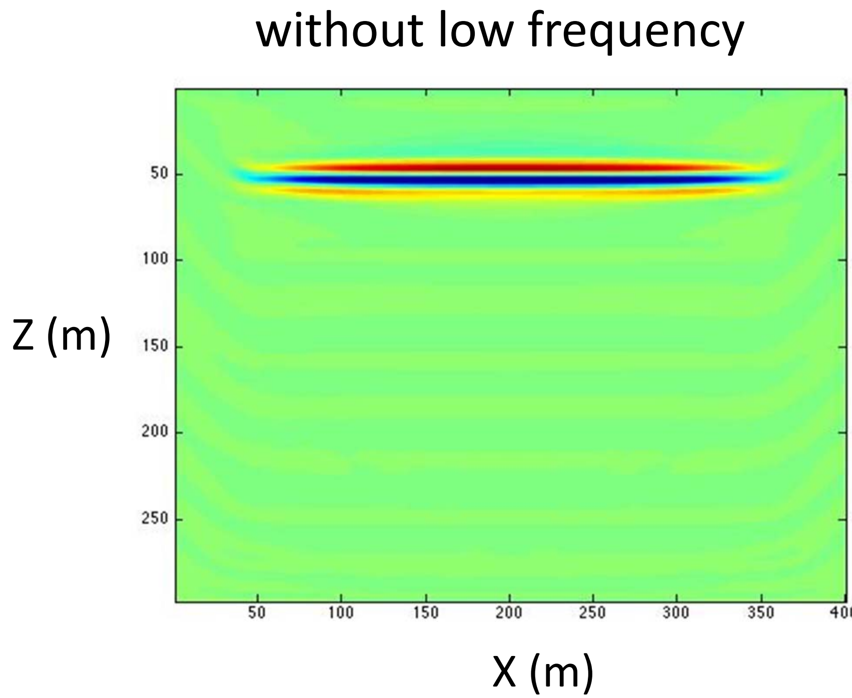
Red line represents conventional data.

Pink line represents even less low frequencies than conventional data.

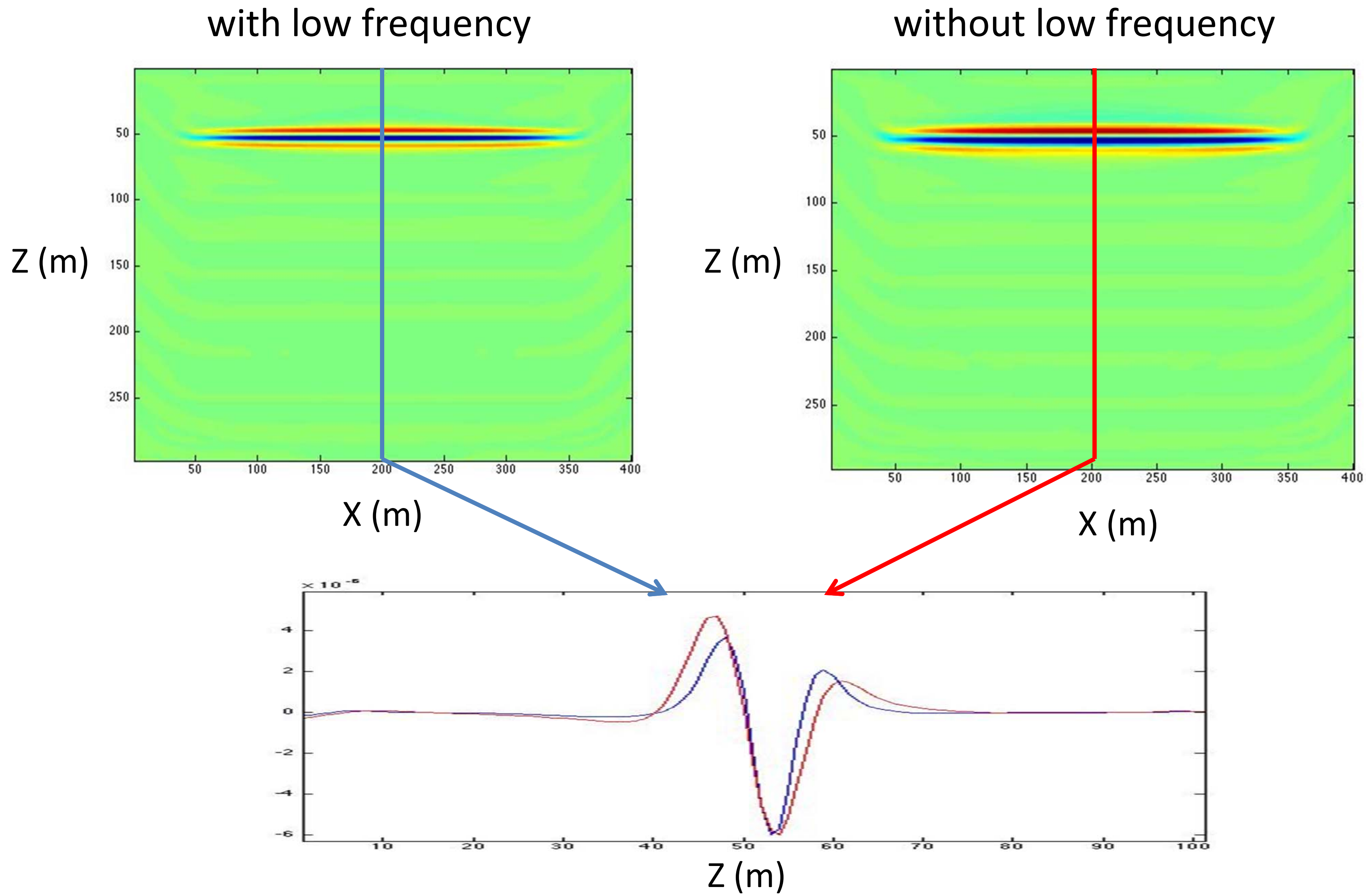
The reduction of the first side lobe amplitude with broadband data compared to conventional data is 26%.

Claerbout II RTM image

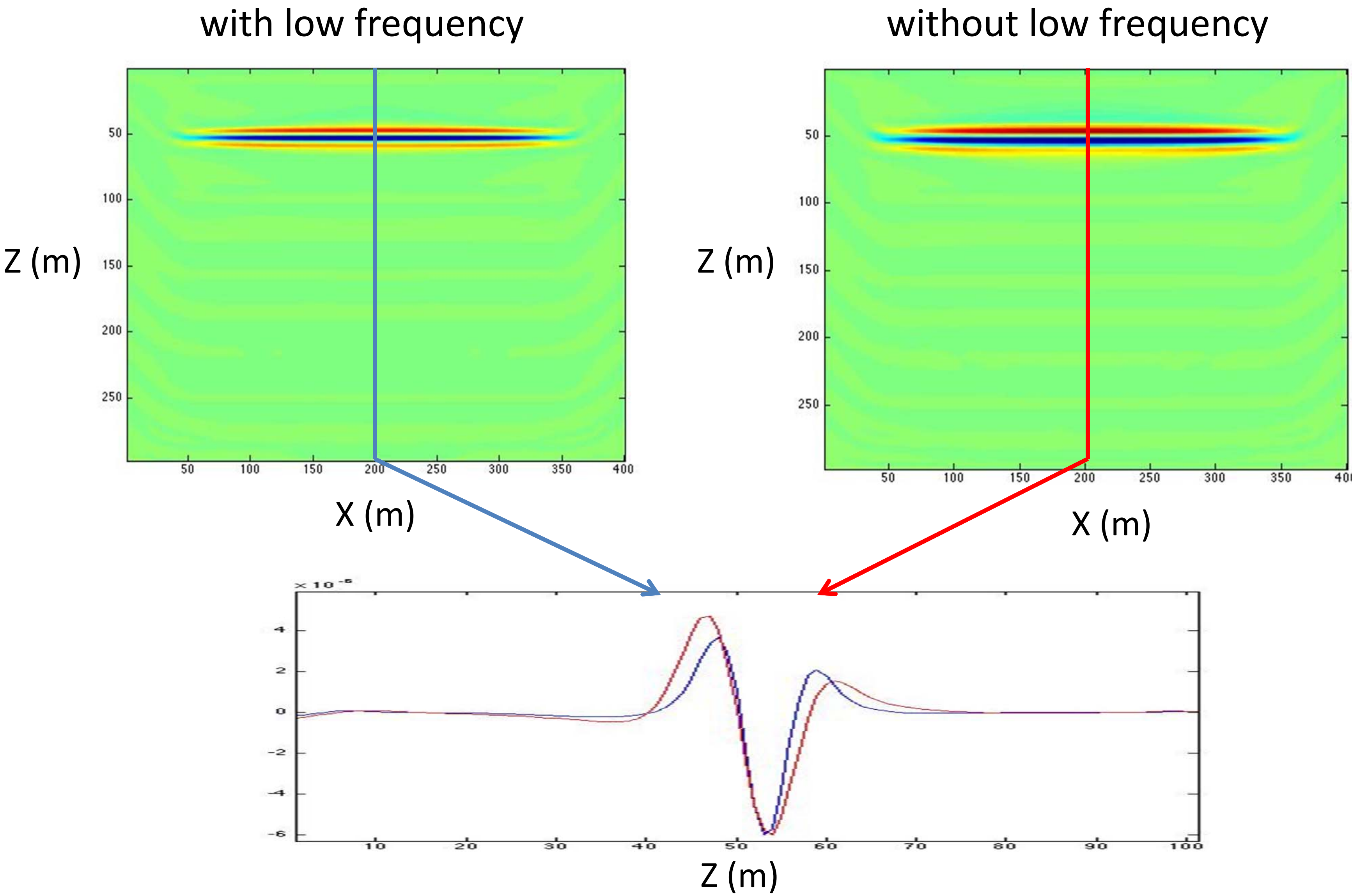




Claerbout II RTM image



Claerbout II RTM image



The reduction of the first side lobe amplitude with broadband data is only 20%.

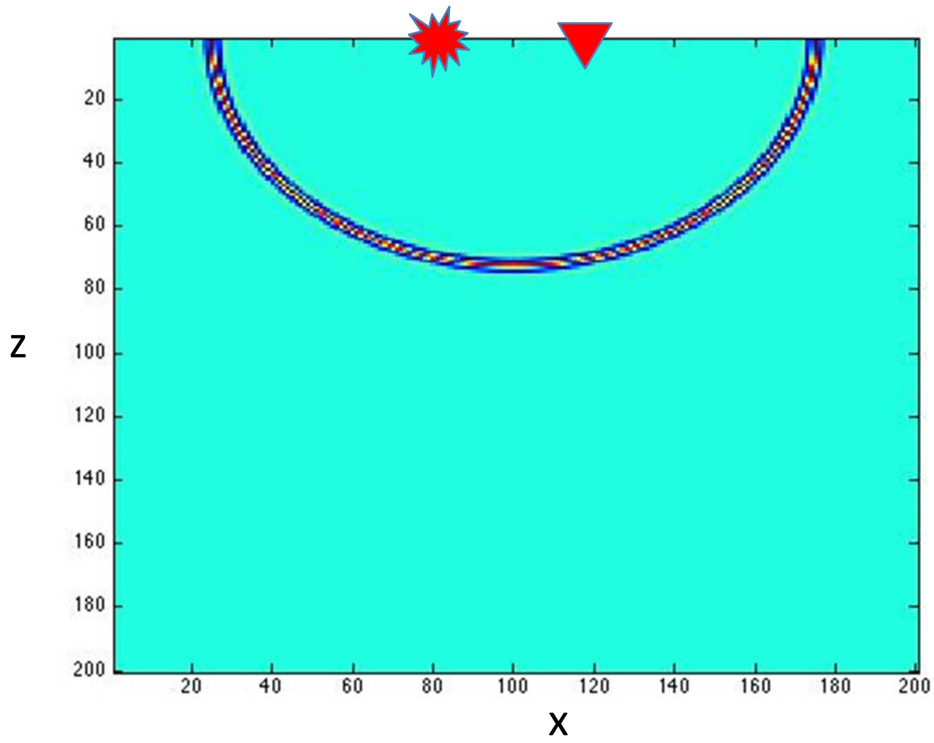
Conclusion

- All current industry leading edge migration methods (including RTM and all variants and extensions of RTM) are high frequency approximations, and hence do not treat low frequencies with equal fidelity as high frequencies at the target and image.

- M-OSRP has developed the first migration method that is equally effective at all recorded frequencies at the target and reservoir. This test shows our first results that quantify that difference and differential added-value.

- There are side lobes in the structural image due to the missing low frequencies. With the new imaging method and broadband data the side lobes reduced more than 50% whereas the conventional leading edge RTM only 20%. The new imaging method is able to benefit from broad band data for structural resolution improvement to a much greater extent than the current best industry standard.

Claerbout II (without using finite difference methods) : one source and one receiver image



$$\frac{1}{c^2}\frac{\partial^2 u(x,z,t)}{\partial t^2}=\frac{\partial^2 u(x,z,t)}{\partial x^2}+\frac{\partial^2 u(x,z,t)}{\partial z^2}$$

$$\frac{\partial^2 u(x,z,t)}{\partial t^2}\approx\frac{1}{\Delta t^2}[u(x,z,t-\Delta t)-2u(x,z,t)+u(x,z,t+\Delta t)]$$

$$\frac{\partial^2 u(x,z,t)}{\partial x^2}\approx\frac{1}{\Delta x^2}[u(x-\Delta x,z,t)-2u(x,z,t)+u(x+\Delta x,z,t)]$$

$$\frac{\partial^2 u(x,z,t)}{\partial z^2}\approx\frac{1}{\Delta z^2}[u(x,z-\Delta z,t)-2u(x,z,t)+u(x,z+\Delta z,t)]$$

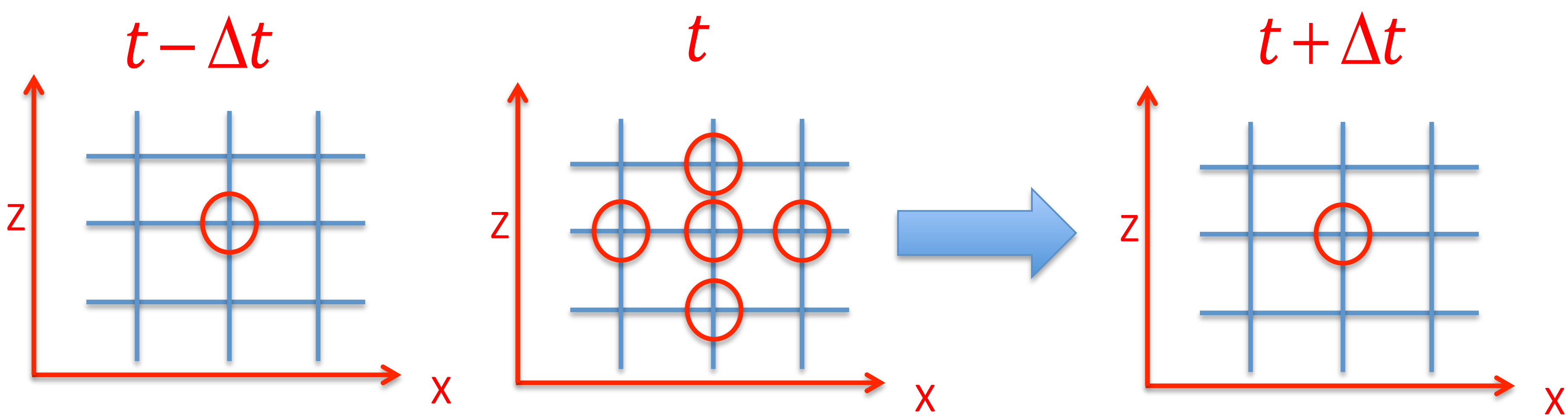
Forward propagate in time:

$$u(x,z,t+\Delta t)=$$

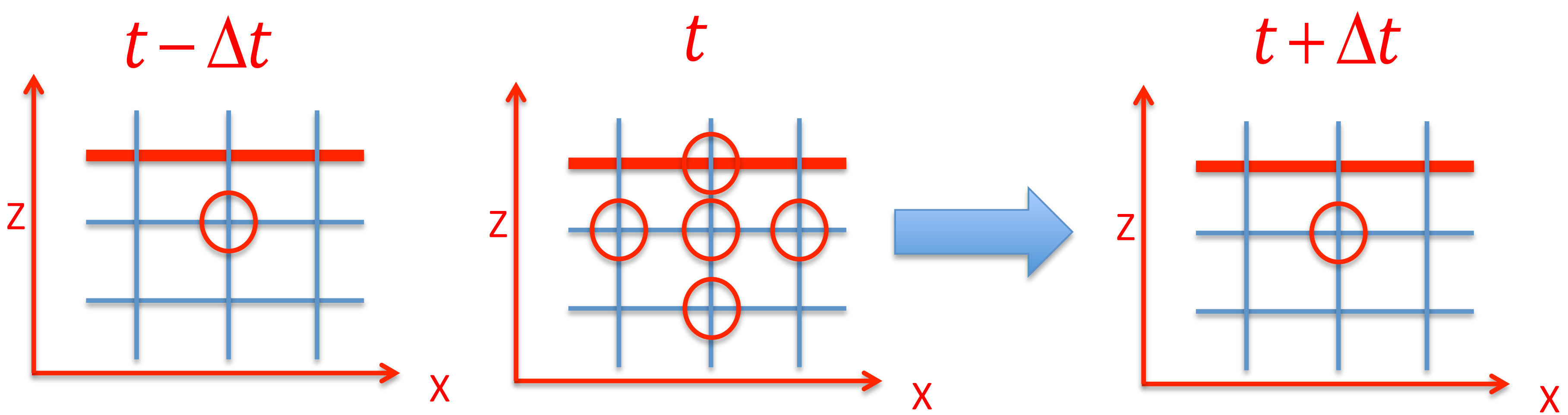
$$=c^2\Delta t^2\{\frac{1}{\Delta x^2}[u(x-\Delta x,z,t)-2u(x,z,t)+u(x+\Delta x,z,t)]$$

$$+\frac{1}{\Delta z^2}[u(x,z-\Delta z,t)-2u(x,z,t)+u(x,z+\Delta z,t)]\}$$

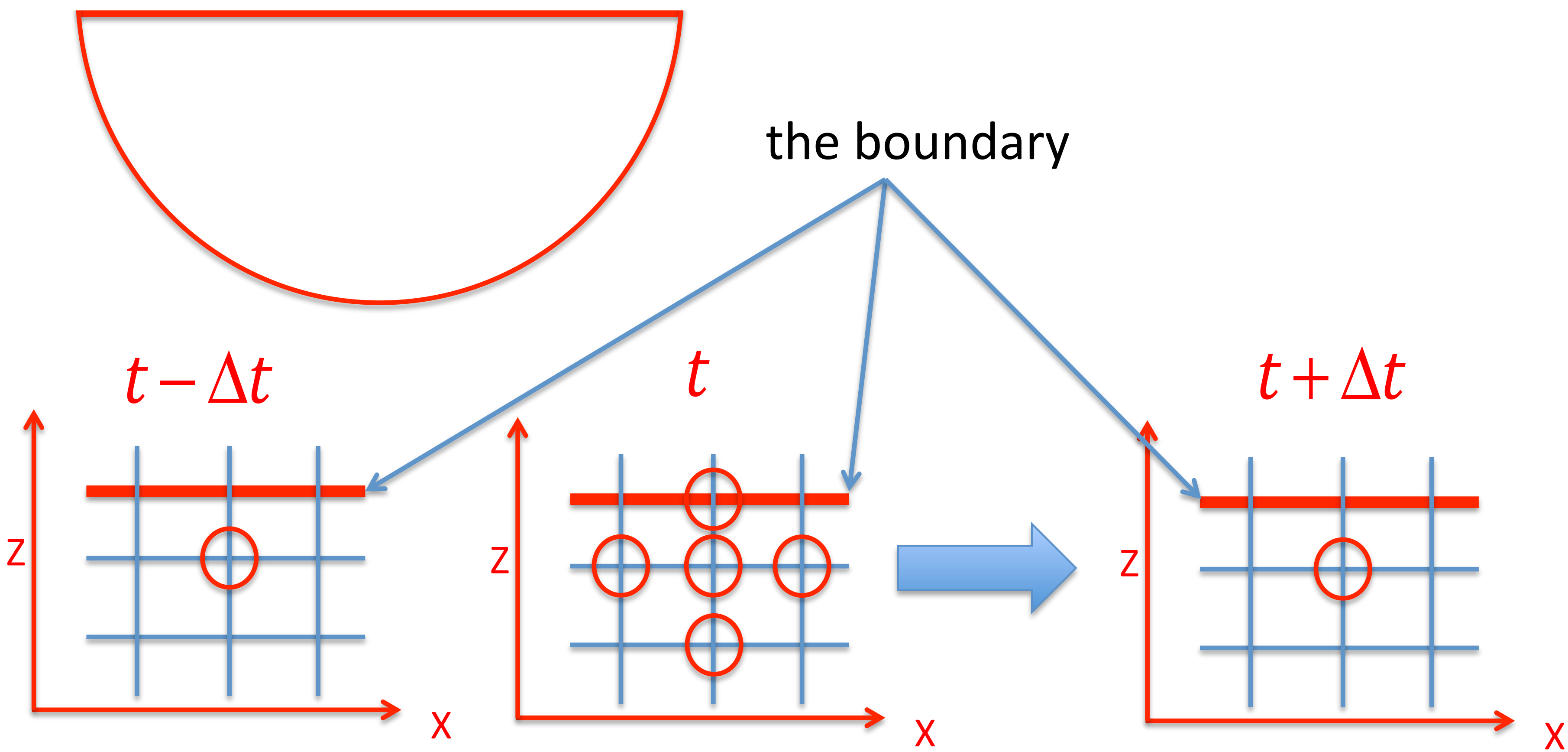
$$+2u(x,z,t)-u(x,z,t-\Delta t)$$



Forward propagate in time:

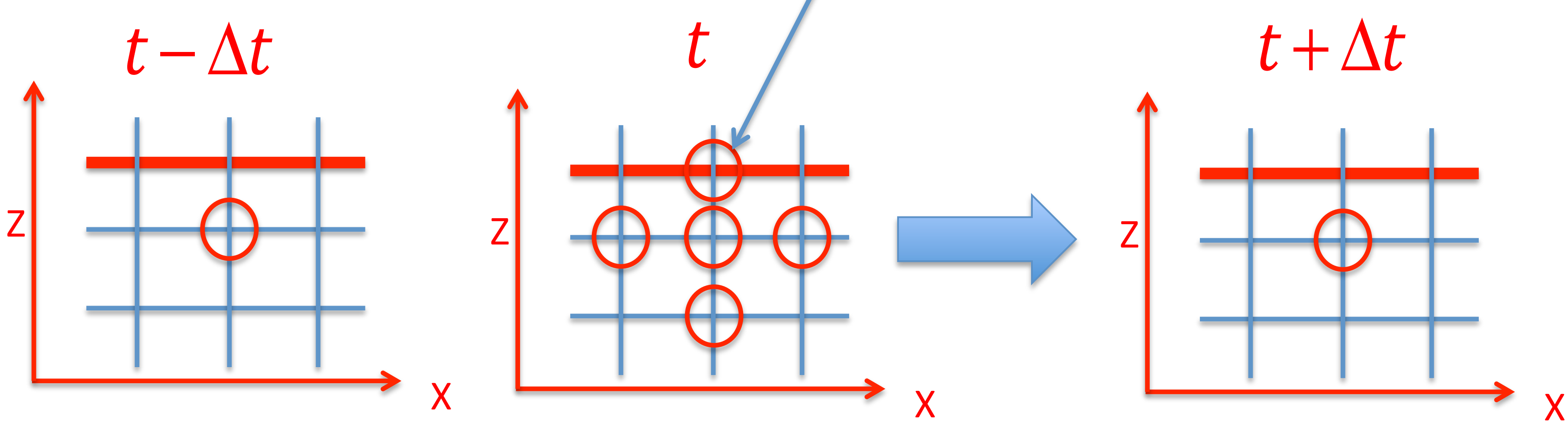


Forward propagate in time:



Forward propagate in time:

We have to give the value of the wavefield on the boundary.

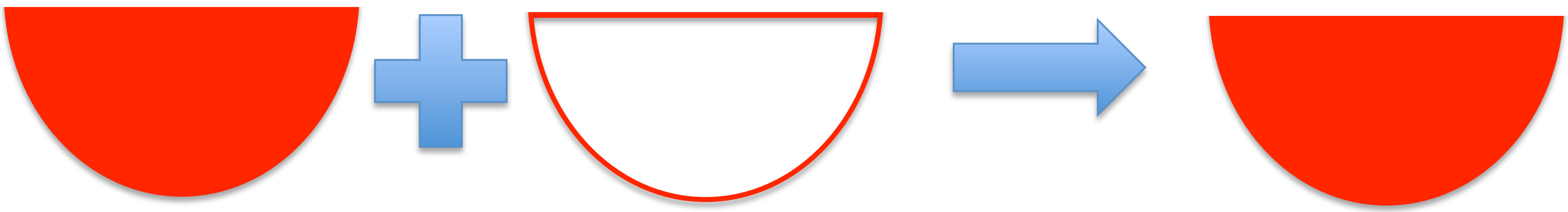


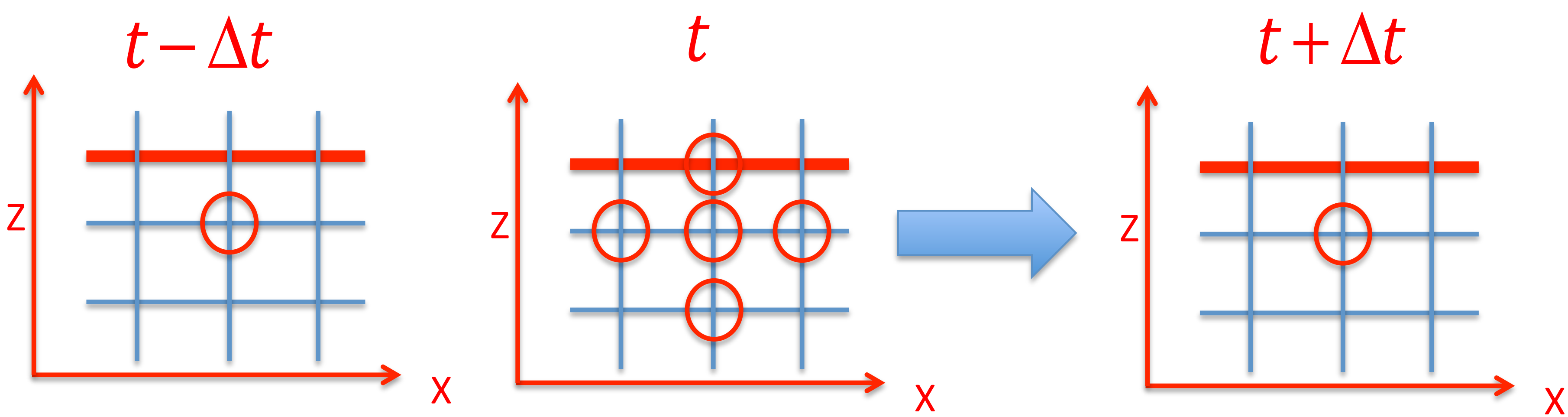
Forward propagate in time:

initial wavefield in the region t_0,t_0-Δt

wavefield on the boundary from t_0 to t_1 (t_1>t_0)

wavefield in the region from t_0 to t_1(t_1>t_0)





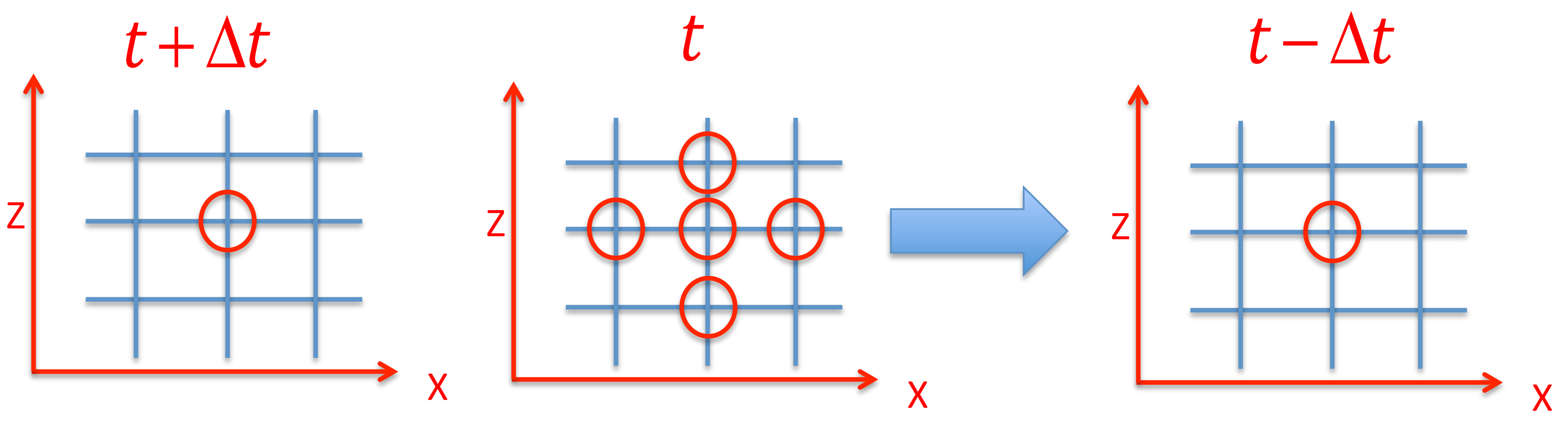
Back propagate in time:

$$u(x,z,t-\Delta t)=$$

$$=c^2\Delta t^2\{\frac{1}{\Delta x^2}[u(x-\Delta x,z,t)-2u(x,z,t)+u(x+\Delta x,z,t)]$$

$$+\frac{1}{\Delta z^2}[u(x,z-\Delta z,t)-2u(x,z,t)+u(x,z+\Delta z,t)]\}$$

$$+2u(x,z,t)-u(x,z,t+\Delta t)$$

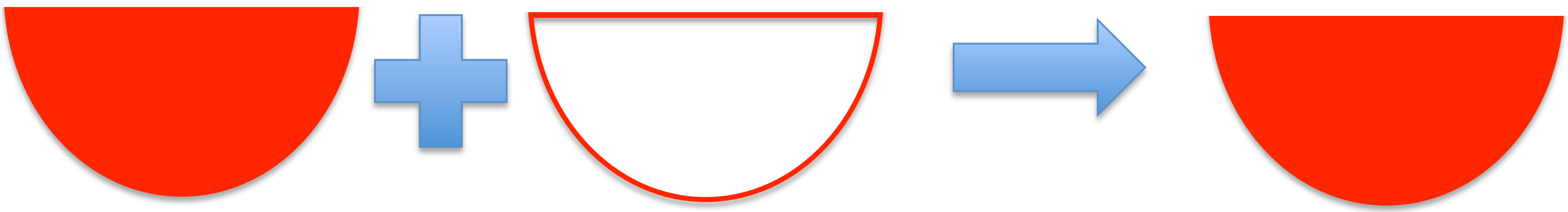


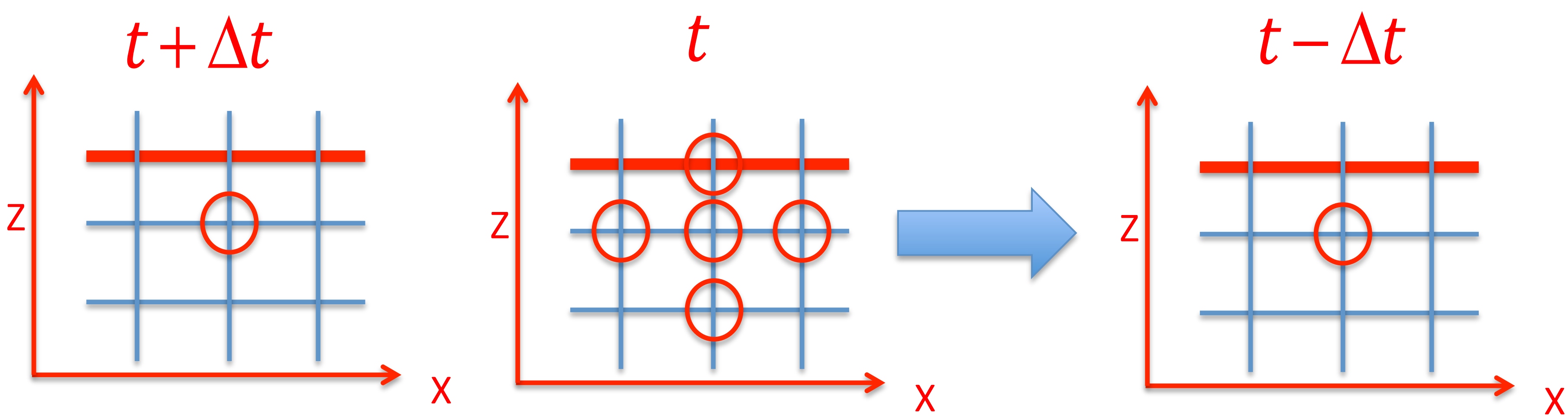
Back propagate in time:

initial wavefield
in the region
$t_1, t_1+\Delta t$

wavefield
on the boundary
from t_1 to t_0 ($t_1>t_0$)

wavefield
in the region
from t_1 to t_0($t_1>t_0$)





Back propagate in time: