A new Inverse Scattering Series (ISS) internal multiple attenuation algorithm responds to a limitation in the current algorithm: derivation for a three-reflector model and a test with analytic data

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SUMMARY

The Inverse Scattering Series (ISS) is a comprehensive framework for achieving seismic data processing goals without requiring subsurface information. Distinct isolated task-specific subseries can accomplish free surface multiple removal, internal multiple attenuation, depth imaging and inversion of primaries. The ISS can predict and eliminate internal multiples without a priori information. Although the leading order ISS internal multiple attenuation algorithm for the first order internal multiples has shown unmatched capability on complex synthetic and onshore data compared with other methods (e.g., Fu et al. (2010); Luo et al. (2011)), there are open issues to be addressed (e.g., Weglein et al. (2011)). For example, spurious events can be predicted in the first order attenuator (leading order prediction of the first order internal multiples) when there are both primaries and internal multiples in the input data. This paper and the companion paper (H.Liang et al., 2012) propose a new algorithm to directly respond to this issue. The new algorithm maintains the strength of the current algorithm and, in addition, can accommodate data consisting of both primaries and internal multiples.

INTRODUCTION

In seismic exploration, multiples are events which have experienced multiply reflections, and they are further classified by the location of downward reflection. Multiples which have at least one of downward reflections at the free surface (air-water or air-land) are free surface multiples. Multiples that have experienced all the downward reflections below the free surface are internal multiples. The order of an internal multiple depends on the number of downward reflections it has experienced. For example, the first order internal multiples have only one downward reflection below the free surface (dashed line in Figure 1). The primaries-only assumption in seismic data analysis requires multiple removal. The methods for multiple removal were classified as separation and wavefield prediction in Weglein (1999). The separation methods seek a characteristic to distinguish primaries from multiples, while the early wavefield prediction methods first modeled and then subtracted multiples. Both of these approaches have earned well deserved places in the seismic toolbox. However as seismic exploration moves to more complex areas these methods have limitations due to their assumptions and the requirements for subsurface information. The ISS free surface multiple removal algorithm (Carvalho (1992); Weglein et al. (1997)) and internal multiple attenuation algorithm (Araújo (1994); Weglein et al. (1997)) start by avoiding the assumptions of the earlier methods, e.g., they are completely multi-*D* and have no requirements for subsurface information. There are both separation and wavefield prediction ingredients in the ISS multiple removal methods and they can be viewed as a next step in the development of separation and wavefield prediction methods (Weglein et al., 2011). For example, the ISS free surface multiple separation distinguishes the free surface multiples from other events by the downward reflection at the free surface. In contrast, the ISS internal multiple separation is realized without any a priori information by understanding the difference in the construction of primaries and internal multiples in the forward series. As an example, the ISS leading order prediction for the removal of the first order internal multiple provides a “lower-higher-lower”relationship in the pseudo-depth domain and uses only primaries as subevents to predict the first order internal multiples from all reflectors, at all depths at once, and without any subsurface information.

However, when there are internal multiples in the input data, the ISS leading order prediction of internal multiples can produce spurious events. The leading order means it can effectively attenuate, not completely eliminate, internal multiples by itself. While we recognize the shortcomings of the current leading order ISS internal multiple attenuation algorithm, we also recognize that addressing them resides in the ISS (Weglein et al., 2011). Each term in the subseries achieves what the order of that term enables it to achieve. There are certain issues that a term of a given order can address, and other issues that require aid from higher order terms. The more difficult the task, the more complicated and more inclusive the subseries. For example, it requires an infinite series (in a closed form) to completely eliminate all first order internal multiples generated at the shallowest layer when the properties at and above that reflector are unknown (Ramírez and Weglein, 2005). Similarly, the internal multiple attenuation task is more difficult when the input data contains internal multiples as well as primaries than when the input data contains only primaries, so the ISS internal multiple attenuation algorithm needs to capture terms in order to address the spurious events. In this paper we provide an understanding of the issue of the leading order prediction of the first order internal multiples when the input data consists of both primaries and internal multiples. We also provide a new ISS internal multiple attenuation algorithm to address a particular type of spurious event that is predicted when the middle subevent in the first order attenuator is an internal multiple.

AN OVERVIEW OF THE ISS INTERNAL MULTIPLE ATTENUATION ALGORITHM

The leading term contribution to constructing a class of multiples in the forward series suggests the leading term contribution for their removal in the inverse series (Weglein et al., 2003). A subseries that focuses on internal multiple removal can be isolated from the inverse series. The ISS internal multiple attenuation algorithm starts with the input data, $D(k_g,k_s,\omega)$ which is the Fourier transformed prestack data that is deghosted, wavelet deconvolved and has free surface multiples removed. The leading order prediction of the first order internal multiples makes the leading term contribution to the removal of the first order internal multiples. In a $2D$ earth, it is,

$$b_3(k_g,k_s,\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 e^{-iq_1(z_g-z_s)} e^{iq_2(z_g-z_s)} \times \int_{-\infty}^{\infty} dz_1 b_1(k_g,k_1,z_1) e^{i(q_g+q_1)z_1} \times \int_{-\infty}^{z_1-\varepsilon} dz_2 b_1(k_1,k_2,z_2) e^{-i(q_1+q_2)z_2} \times \int_{z_2+\varepsilon}^{\infty} dz_3 b_1(k_2,k_s,z_3) e^{i(q_2+q_s)z_3}, \quad (1)$$

where ω is temporal frequency; k_s and k_g are the horizontal wavenumbers for the source and receiver coordinates, respectively; q_g and q_s are the vertical source and receiver wavenumbers defined by $q_i = sgn(\omega)\sqrt{\frac{\omega^2}{c_0^2} - k_i^2}$ for $i = (g,s)$; z_s and z_g are source and receiver depths; and z_i $(i = 1,2,3)$ represents pseudo-depth using reference velocity migration. The quantity $b_1(k_g,k_s,z)$ corresponds to an uncollapsed migration (Weglein et al., 1997) of an effective plane-wave incident data, and $b_1(k_g,k_s,q_g+q_s) = -2iq_s D(k_g,k_s,\omega)$.

With the input data and the leading order prediction of the first order internal multiples, we can obtain the data with the first order internal multiples attenuated,

$$D(k_g,k_s,\omega) + D_3(k_g,k_s,\omega) \quad (2)$$

Where $D_3(k_g,k_s,\omega) = (-2iq_s)^{-1} b_3(k_g,k_s,q_g+q_s)$.

For a $1D$ earth and a normal incident plane wave, equation 1 reduces to,

$$b_3(k) = \int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_1(z_1) \int_{-\infty}^{z_1-\varepsilon} dz_2 e^{-ikz_2} b_1(z_2) \times \int_{z_2+\varepsilon}^{\infty} dz_3 e^{ikz_3} b_1(z_3). \quad (3)$$

The leading order ISS internal multiple attenuation algorithm for the first order internal multiples for a $1D$ earth and an implusive incident plane wave is,

$$b_1 + b_3. \quad (4)$$

Note that the $(-2iq_s)$ factor is not needed here. However, in general the output of the ISS leading order removal of first the order internal multiples needs the $(-2iq_s)$ factor to take b to D as in equation 2.

The portion of the third order term of the ISS that predicts the first order internal multiple is isolated by requiring the "lower-higher-lower"relationship in pseudo-depth domain as shown in Figure 1. The assumption behind the first order internal multiple prediction in Figure 1 is all of the subevents have to be primaries for the prediction to be an internal multiple. There are circumtances, shown in the next section, where the "lower-higher-lower"template would produce spurious events when one of subevents is an internal multiple. However, these spurious events are fully anticipated and can be attenuated by other terms in the inverse series.

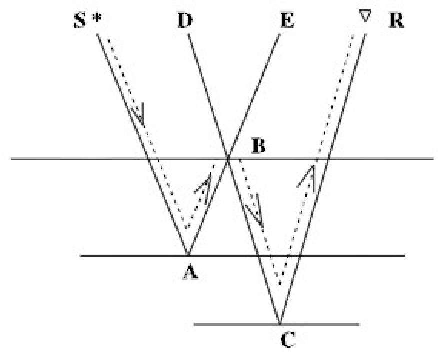

Figure 1: Combination of subevents for the first order internal multiple (dashed line), $(SABE)_{time} + (DBCR)_{time} - (DBE)_{time} = (SABCR)_{time}$, figure adapted from Weglein et al. (2003)

A NEW ISS INTERNAL MULTIPLE ATTENUATION ALGORITHM TO ATTENUATE THE SPURIOUS EVENT ARISING IN A THREE-REFLECTOR MODEL

Now we consider a three-reflector analytic example. We examine the prediction of the first order internal multiple attenuator using the input data which consists of three primaries and one specific internal multiple associated with the first two reflectors. For an impulsive incident wave $\delta(t-\frac{z}{c})$ the data is,

$$D(t) = R_1\delta(t-t_1) + R_2'\delta(t-t_2) + R_3'\delta(t-t_3) + R_4'\delta(t-(2t_2-t_1)) \quad (5)$$

where $R_2' = T_{01}R_2T_{10}$; $R_3' = T_{01}T_{12}R_3T_{21}T_{10}$; $R_4' = T_{01}R_2(-R_1)R_2T_{10}$, and t_i, R_i are two way times and reflection coefficients from the ith reflector respectively, and T_{ij} is the transmission coefficient between the ith and jth reflector.

Given this data, we find from equation 3,

$$\begin{aligned} b_3(t) = \\ & R_1(R_2')^2\delta(t-(2t_2-t_1)) + 2R_1R_2'R_3'\delta(t-(t_2+t_3-t_1)) \\ & +R_1(R_3')^2\delta(t-(2t_3-t_1)) + R_2(R_3')^2\delta(t-(2t_3-t_2)) \\ & +2R_1R_2'R_4'\delta(t-(3t_3-2t_1)) + R_2'(R_4')^2\delta(t-(3t_3-2t_2)) \\ & +2R_1R_3'R_4'\delta(t-(t_3+2t_2-2t_1)) + R_1(R_4')^2\delta(t-(4t_2-3t_1)) \\ & +2R_2'R_3'R_4'\delta(t-(t_3+t_2-t_1)) + (R_3')^2R_4'\delta(t-(2t_3-(2t_2-t_1))) \end{aligned} \quad (6)$$

We have assumed $t_3 > 2t_2 - t_1$ in deriving equation 6. In addition to the four first order internal multiples (first two

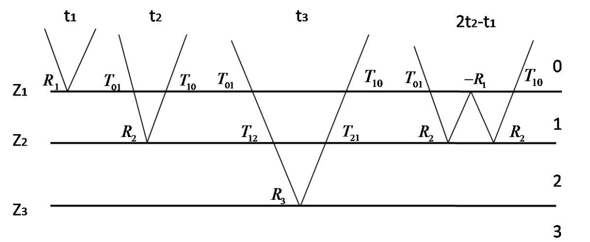

Figure 2: Three primaries and one internal multiple in a three-reflector model

rows in equation 6), the first order attenuator,b_3, predicts some additional events due to the specific intenral multiple in the input. Analysis of the traveltimes of these additional events shows each of them corresponds to one specific internal multiple of higher order with the exception of the last event $(R_3')^2 R_4' \delta(t-(2t_3-(2t_2-t_1)))$ which is a spurious event prediction.

An understanding of the properties of the first order attenuator when both primaries and internal multiples are input and act as subevents

When there are internal multiples in the data, there will be many other possible subevent combinations in the first order internal multiple atteuator, b_3. Since when

$$b_1 = P + I$$

it follows from equation 3 that

$$\begin{aligned} b_3^{IM} &= b_1 * b_1 * b_1 \\ &= (P+I)(P+I)(P+I) \\ &= PPP + PPI + PIP + IPP + PII + IPI + IIP + III \end{aligned}$$

where $*$ stands for nonlinear interaction between the data, P stands for primaries, and I stands for internal multiples. Besides the primary only subevent combination, *PPP*, there are subevent combinations involved with the internal multiple that produce the spurious event. A more detailed analysis shows that the spurious event $(R_3')^2 R_4' \delta(t-(2t_3-(2t_2-t_1)))$ in equation 6 comes from *PIP* as shown in Figure 3 .

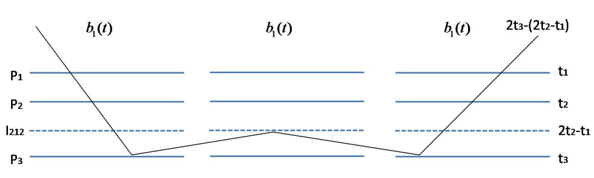

Figure 3: A analogous *W-like* configuration to produce the spurious event using the internal multiple as a subevent.

We use a diagram to illustrate the generation of the spurious event by *PIP* subevent combination. The diagram for *PIP* is shown in the left panel in Figure 4 which satisfies the "lower-higher-lower"relationship as required by the algorithm. Following the logic of predicting internal multiples by the "lower-higher-lower"pattern of three primary subevents, the *PIP* diagram will split into a "lower-higher-much higher-lower-much lower"configuration in the right panel of Figure 4. The resultant configuration does not agree with the double *W-like* configuration which constructs second order internal multiple using five primary subevents. The

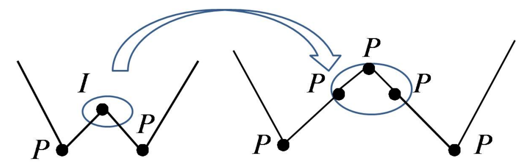

Figure 4: Separation for PIP into *W-like*

pseudo-depth of the two outermost P should be deeper than the effective pseudo-depth of the middle I to allow the *PIP* spurious events to happen, see Figure 3. In other words, the *PIP* spurious events can exist in a medium which has three or more reflectors. That explains the fact that there are no spurious events produced in a two-reflector example in H.Zhang and S.Shaw (2010) even though an internal multiple is included in their data.

When the internal multiple in *PPI* or *IPP* is separated into three "lower-higher-lower"primary subevents, it leads to a double *W-like* configuration which will predict the second order internal multiple as shown in Figure 5. This also explains the additional higher order internal multiple predictions in b_3 in our analytic example. It can be shown that there are circumstances where *PPI* produces spurious events in a medium which has more than three reflectors (H.Liang et al., 2012).

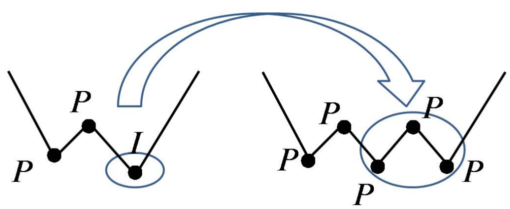

Figure 5: One possible separation for PPI into double *W-like*

Terms like *IIP* or *III* may also produce spurious events, when compared to the effects by terms like *PIP*, these terms can often be ignored in practice. However, the removal of the latter spurious events also resides in the higher order ISS terms, and beyond those considered and included in this paper.

A new term to attenuate the *PIP* spurious event

To remove the spurious events produced by the first order attenuator when using an internal multiple as the middle subevent, a new and higher order ISS term which has that capability is included in the current algorithm.

Guided by Figure 4 a portion of the fifth order term from the ISS $(G_0 V_1 G_0 V_3 G_0 V_1 G_0)$ can be employed to predict the *PIP* spurious events in $1D$,

$$b_5^{PIP}=\int_{-\infty}^{\infty}dz_1e^{ikz_1}b_1(z_1)\int_{-\infty}^{z_1-\varepsilon}dz_2e^{-ikz_2}b_3^{IM}(z_2) \times\int_{z_2+\varepsilon}^{\infty}dz_3e^{ikz_3}b_1(z_3) \tag{7}$$

$b_1(z)$ is an uncollapsed migration and $b_3(z)$ is the first order attenuator. Compared with equation 3, this equation also requires the "lower-higher-lower"relationship, but the middle b_1 becomes b_3 to obtain a prediction of the spurious event using the predicted internal multiple.

Then, adding equation 7 and equation 3 leads to our new algorithm for a $1D$ earth,

$$\begin{aligned}&b_1+b_3^{PPP}+b_5^{PIP}\\ =&b_1+\int_{-\infty}^{\infty}dz_1e^{ikz_1}b_1(z_1)\int_{-\infty}^{z_1-\varepsilon}dz_2e^{-ikz_2}(b_1(z_2)+b_3^{IM}(z_2))\\ &\times\int_{z_2+\varepsilon}^{\infty}dz_3e^{ikz_3}b_1(z_3)\end{aligned} \tag{8}$$

Where $b_3^{PPP}=b_3$. The superscript indicates the subevents combination that the algorithm can accommodate. Note that the $(-2iq_s)$ factor is needed in gennernal.

Compared with the original algorithm (equation 4), the new algorithm includes a portion of higher order term (b_5^{PIP}) only to attenuate the *PIP* spurious events predicted by b_3^{PPP} when internal multiples are in the data.

We use the same analytic example to test the new algorithm. Substituting $D(t)$ in equation 5 and b_3 in equation 6 into equation 7 produces,

$$\begin{aligned}b_5^{PIP}=&R_1(R_2')^2(R_3')^2\delta(t-(2t_3-(2t_2-t_1)))\\ &+(2R_1R_2'R_4'(R_3')^2+R_2'(R_4')^2(R_3')^2)\delta(t-(2t_3-(3t_2-2t_1)))\\ &+R_1(R_4')^2(R_3')^2\delta(t-(2t_3-(4t_2-3t_1)))\end{aligned} \tag{9}$$

The first term is the prediction of the spurious event. Substitution of $R_2'=T_{01}R_2T_{10}$ leads to,

$$(T_{01}T_{10})^2R_1(R_2)^2(R_3')^2\delta(t-(2t_3-(2t_2-t_1)))$$

The last term $(R_3')^2R_4'\delta(t-(2t_3-(2t_2-t_1)))$ in equation 6 is the spurious event. Substitution of $R_4'=T_{01}R_2(-R_1)R_2T_{10}$ leads to,

$$(-T_{01}T_{10})R_1(R_2)^2(R_3')^2\delta(t-(2t_3-(2t_2-t_1)))$$

When added to b_3, the first term in equation 9 will effectively attenuate the spurious event. The $T_{01}T_{10}$ error comes from the fact that b_5^{PIP} uses the predicted internal multiple as the middle subevent to predict the spurious event, while b_3 creates the spurious event using the actual internal multiple as the middle subevent (middle b_1) as shown in Figure 3 and Figure 6.

It is the geometric similarity (single *W-like*) between b_5^{PIP} and b_3 ,see Figure 4, that enables b_5^{PIP} to contribute to removing the spurious events produced in b_3. We note that each term in the inverse series does what the order of that term is capable of performing. Different portions of a given order term in the

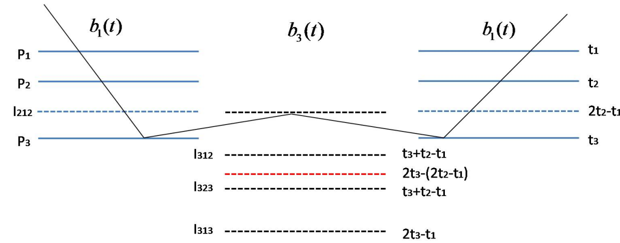

Figure 6: Illustration of the spurious event prediction in b_5^{PIP}. Notice the middle b_3 produces predicted internal multiples which have the opposite sign of the actual internal multiples. Only the first order predicted internal multiples (black dashed line) and spurious event (red dashed line) are shown.

ISS can contribute to different tasks. For example, in our case, although both the leading order prediction of the second order internal multiples b_5 (right panel in Figure 5) and b_5^{PIP} (right panel in Figure 4) come from the fifth order term in the inverse series, they have different tasks determined by their different geometries. b_5^{PIP} has a single *W-like* geometry that is capable of attenuating the spurious events while b_5 has a double *W-like* geometry which is capable of predicting second order internal multiples using primaries. Both are contained in the fifth order term in the ISS.

Therefore, by incorporating a higher order ISS term into the attenuator, equation 8 can effectively attenuate the *PIP* spurious events predicted by b_3.

DISCUSSION AND CONCLUSION

In this paper, we provide both: (1) an algorithm to address certain most significant spurious events observed in Fu et al. (2010) and Luo et al. (2011), and (2) a template for locating ISS terms addressing these more general spurious events that can arise from using a leading order internal multiple attenuation algorithm with a complex medium and a complex data. The ISS can remove all internal multiples without subsurface information and also remove spurious events that arise from using a complex data in a leading order algorithm. We exemplify that capability in this and the companion paper by H.Liang et al. (2012).

To conclude, the new algorithm in this paper retains the strength of the original algorithm while addressing a limitation in the current algorithm and provides an initial extention to accommodate data consisting of both primaries and internal multiples.

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