Inverse scattering internal multiple elimination

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Abstract

The inverse scattering processing methods use nonlinear combinations of measured data and propagation in a reference medium. The method for multiple attenuation, based on the inverse scattering series, first separates free-surface multiples from primaries and internal multiples, and, subsequently, primaries from internal multiples. The separation is performed through task specific series in terms of measured data and propagation in reference medium. These series result in distinct algorithms for free-surface and internal multiple removal and neither requires a model of the subsurface reflectors that generate the multiples. The free-surface algorithm predicts free-surface multiples from data composed of primaries, internal, and free-surface multiples; the internal-multiple algorithm predicts internal multiples from data that consists of primaries and internal multiples.

Internal multiples are distinguished from primaries in the measured wavefield because primaries only experience one upward reflection in the subsurface while internal multiples experience at least one downward reflection (two upward reflections) in the subsurface. The first term in the subseries for internal multiple elimination is an attenuator. It predicts the correct travel time and an amplitude always less than the true internal multiples' amplitude. The leading and higher order terms in the elimination series corrects the amplitude predicted by the attenuator moving the algorithm towards an eliminator. The leading order terms in this series are identified as terms in a subseries with nonlinear self-interactions at the generating reflector. The location where the downward reflection of the first order internal multiple takes place is called the genarating reflector. Adding the leading order terms, we obtain a leading order closed form that eliminates all internal multiples generated at the first reflector and improves the attenuation of the remaining multiples. A second subseries improved the elimination of internal multiples generated at deeper reflectors. The main part of this second subseries is summed to find a higher order closed form that eliminates the internal multiples generated at the second reflector and further improves the reduction of all internal multiples.

1 Introduction

There has been a renewed interest in the elimination of internal multiples from measured seismic data in the literature and in the energy industry. This interest concerns on possible ways to extend and advance beyond current capability. To that end it is natural to pursue an examination of the ideas behind the wave theoretic inverse scattering internal multiple algorithm. This data driven method is derived from a formalism based on the inverse scattering series, where a piece of the third equation, that allowed for a *lower-higher-lower* diagram resembling the ray-form of an internal multiple, was found to start a series that removes all internal multiples without any subsurface information (Weglein et al., 1981; Araújo, 1994; Weglein et al., 2003).

The first research efforts to attenuate internal multiples from marine seismic data without using any knowledge of the subsurface were done by Araújo et al. (1994) and Weglein et al. (1997). Attenuation refers to the amplitude reduction of certain event, or sets of events, in the seismic data. The algorithm derived by Araújo et al. (1994) and Weglein et al. (1997) is the first term in an infinite series that deals with internal multiple elimination. It is known as the attenuator. An analysis of the effectiveness of the attenuator shows that its travel time prediction is exact and the difference between elimination and attenuation resides in extra transmission coefficients in the prediction, when compared to the true multiple in the data(Weglein et al., 2003; Ramírez and Weglein, 2005b). The extra transmission coefficients correspond to coefficients down to, and including, the internal multiple's shallowest downward reflection. For example, an internal multiple having its shallowest downward reflection at the water bottom, in a marine experiment, has an error in its prediction related to the transmission coefficients at the water bottom. This error, or attenuation factor, is totally and completely independent of how many layers and how deep into the earth the multiple travels below the water bottom.

The first research efforts to address the complete removal of internal multiples from marine seismic data, without destroying primary reflections and without any subsurface information, were done by Ramírez and Weglein (2005a) (for the interested reader, a timetable with history highlights for the internal multiple is provided in Figure 1). Elimination refers to a complete removal of the amplitude of that event, or set of events, from the data. The overall purpose to develop a theory to eliminate internal multiples is to place internal multiples and free-surface multiples on the same footing (Carvalho, 1992; Weglein et al., 1997, 2003). The objective is to reach the same level of elimination effectiveness that the free-surface algorithm has, in which each single term completely removes all free-surface multiples of a certain order.

In seismic exploration, there are circumstances when: 1) internal multiple identification, or attenuation, is sufficient; and 2) when a residual, left from internal multiple attenuation, is a challenge and impediment for further processing. Depending on which of these circumstances are been faced (*e.g.* depending on the details of the geology and the level of ambition and demands of the processing and exploration objectives) an attenuator or an eliminator of internal multiples would be required. Among the circumstances when internal multiple elimination would provide value above that provided by an attenuator (for towed streamer marine data) are: 1) converted wave internal multiples; 2) proximal, or interfering, primaries and internal multiples at the target; 3) when there is a need to reduce the burden on adaptive subtraction to account for missing deterministic predictive capability. It is anticipated that internal multiple elimination will place greater demands on preprocessing steps such as data collection and wavelet estimation.

The methods presented here never move from not needing to need subsurface information when we progress from attenuate to eliminating internal multiples.

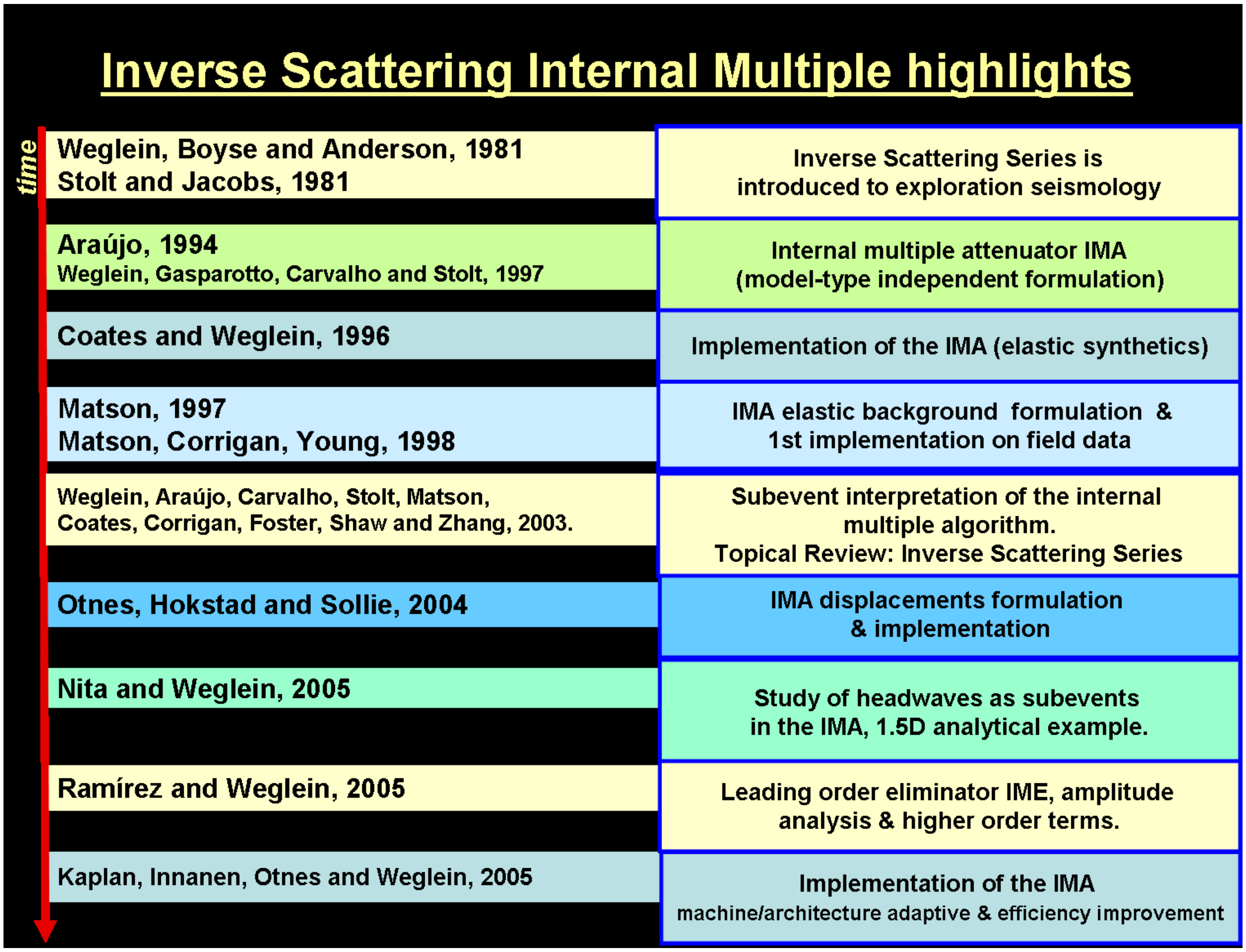

Figure 1: History highlights

Prerequisites for internal multiple elimination

The inverse scattering series task associated with internal multiple removal promises to accomplish its objective directly in terms of the measured data and reference propagation (Weglein et al., 2003). It never assumes that the reference medium is the actual. The reference medium is never moved or altered towards the actual. Inverse scattering internal multiple elimination is a wave equation demultiple approach that does not make assumptions about the earth below the receivers, nevertheless, it is subject to some constrains or prerequisites. It assumes that the data have been deghosted, there are no free-surface multiples and the source wavelet is known. For the most common practical implementations, the source wavelet is estimated during the multiple subtraction process by assuming that the source signature is the one that minimizes the energy in the demultipled wavefield. The energy minimizing adaptive subtraction, is often useful. However, it tends to fail precisely under the most complex circumstances where the underlying demultiple methods have their greatest strengths. For example, when interfering events and multiples of different orders are proximal to primaries, adaptive subtraction will eliminate the primary along with the multiple. Hence, the internal multiple elimination have a high bar on the source signature estimation as well as deghosting and free-surface multiple elimination. If we are able to satisfy this high bar of prerequisite, then the internal multiple elimination method would have the opportunity to reach its potential. The work that Guo et al. (2005) and Zhang and Weglein (2006) have pioneered provide new robust methods for wavelet estimation and deghosting, respectively. These new methods, and further developments, will help to satisfy the prerequisites of the internal multiple algorithm.

Internal multiple elimination

The third term in the inverse scattering series: $(G_0V_1G_0V_1G_0V_1G_0)$ contains the leading order contribution for the removal series of 1^{st} order internal multiples (Weglein et al., 1997). This leading order term is the internal multiple attenuator Araújo et al. (1994).

To simplify the analysis of the attenuator, a medium that only varies in depth will be assumed. The $1D$ earth and normal incidence wave version (Weglein et al., 2003) of the first order internal multiple attenuator is

$$b_1(k) = D(\omega), \tag{1}$$

$$b_3^{IM_1}(k) = \int_{-\infty}^{\infty} dz_1 e^{ikz_1} b_1(z_1) \int_{-\infty}^{z_1-\epsilon} dz_2 e^{-ikz_2} b_1(z_2) \int_{z_2+\epsilon}^{\infty} dz_3 e^{ikz_3} b_1(z_3), \tag{2}$$

where $k = 2\frac{\omega}{c_0}$ is the vertical wave number, $D(\omega)$ is the temporal Fourier transform of the measured scattered field (data), ϵ is a small positive parameter chosen to insure that the relations $z_1 > z_2$ and $z_3 > z_2$ are satisfied, the pseudodepths z_1 and z_2 are defined with the reference velocity c_0 to be $z_i = \frac{c_0 t_i}{2}$, and the superscript IM_1 refers to the $1st$ order internal multiple elimination series.

The attenuator's prediction is performed by a nonlinear combination of three sets of data. This nonlinear combination predicts the exact travel time and an amplitude estimate of the true internal multiple in the data. The estimate is always less than the actual amplitude. The difference between the estimate and the true amplitude is the attenuation factor given by the formula (Ramírez and Weglein, 2005b)

$$(AF_{P.IM})_j = \begin{cases} T_{01}T_{10} & \text{for } j=1 \\ \\ \Pi_{i=1}^{j-1}\left(T_{i\ i-1}^2 T_{i-1\ i}^2\right) T_{j\ j-1} T_{j-1\ j} & \text{for } 1<j<J \end{cases} \tag{3}$$

where j represents the generating reflector, $T_{j-1\ j}$ and $T_{j\ j-1}$ are the transmission coefficients going down and up through the interface j, respectively, and J is the total number of interfaces in the model. The interfaces are numbered with integers, starting with the shallowest location. In a single layer medium, the first order internal multiple has an amplitude of $-T_{01}R_2R_1R_2T_{10}$ and $b_3^{IM_1}$ predicts a first order internal multiple with an amplitude of $T_{01}T_{10}R_2R_1R_2T_{10}T_{01}$. In agreement with equation (3), the predicted internal multiple is attenuated by a factor of $T_{01}T_{10}$ when compared to the true internal multiple. The attenuation factor, equation(3), is affected by

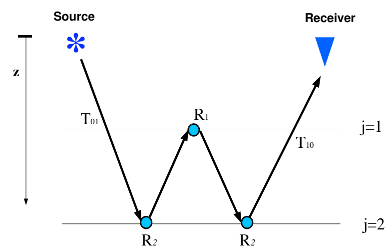

Figure 2: *First order internal multiple with downward reflection at $j = 1$.*

the history of the event down to and including only the depth of the generating reflector. It is a factor independent of the place where the two upward reflections occurred.

The first order internal multiple elimination series starts with $b_3^{IM_1}$. The multiples are predicted as traces in the same form as the effective data. The attenuation process is a simple addition $b_1 + b_3^{IM_1}$, since $b_3^{IM_1}$ contains internal multiples with opposite sign to the ones in the effective data. Each internal multiple predicted by $b_3^{IM_1}$, and generated at a certain j reflector, is attenuated by a factor of $(AF_{P.IM})_j$. The purpose of the higher order terms in the elimination series is to remove the effect of the attenuation factor. The higher order terms improve the effectiveness of the attenuator towards the objective of completely subtract the amplitude of multiples within the data. In order to achieve an elimination method, the inverse scattering subseries for internal multiple elimination must be able to predict the true amplitude for these events.

In the attenuator's prediction, the factor that multiplies the internal multiples generated at the first reflector, $(IM)_{j=1}$, is $T_{01}T_{10}$. Analytic analysis of this algorithm (Weglein et al., 2003; Ramírez and Weglein, 2005b) shows that this attenuation factor corresponds to the first term in the Taylor expansion of $(T_{01}T_{10})/(T_{01}T_{10}) = 1$,

$$\begin{aligned} T_{01}T_{10}\left(\frac{1}{T_{01}T_{10}}\right) &= T_{01}T_{10}\frac{1}{(1-R_1^2)} \\ &= T_{01}T_{10}\left(1 + R_1^2 + R_1^4 + R_1^6 + R_1^8 \cdots\right) \quad . \end{aligned} \tag{4}$$

Following the same analysis, it is found that the factor $(T_{01}T_{10})^2 T_{12}T_{21}$ multiplying the prediction of internal multiples generated at the second reflector, $(IM)_{j=2}$, corresponds to the first term in the more complicated geometric series for:

$$\begin{aligned} \frac{(T_{01}T_{10})^2 T_{12}T_{21}}{(T_{01}T_{10})^2 T_{12}T_{21}} &= (T_{01}T_{10})^2 T_{12}T_{21}\frac{1}{(1-R_1^2)^2(1-R_2^2)}, \\ &= (T_{01}T_{10})^2 T_{12}T_{21}\left(1 + 2R_1^2 + R_2^2 + 3R_1^4 + 2R_2^2R_1^2 + \cdots\right). \end{aligned} \tag{5}$$

Each one of the terms in the Taylor expansions, equations (4) and (5), are calculated by higher order terms in the inverse scattering internal multiple elimination series. Identifying and adding these higher order terms builds a sum of amplitude corrections that improves the prediction and subtraction of internal multiples from the data. The amplitude corrections are given by algorithms, found in the internal multiple elimination series $b_3^{IM_1} + b_5^{IM_1} + b_7^{IM_1} + \cdots$ (Ramírez and Weglein, 2005a), that only require measured values of the scattered field and the reference Green's function.

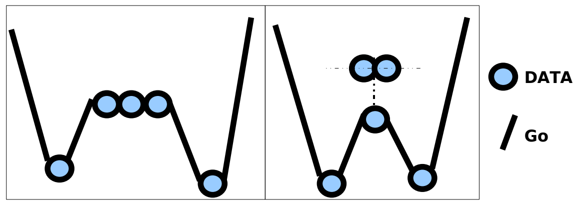

Figure 3: Diagrams for the fifth term in the internal multiple elimination series

The second term in the elimination series, ${b_5}^{IM_1}$, is fifth order in the data. It resides within the fifth term in the inverse series. It is the first step to move the attenuator towards an algorithm that eliminates 1^{st} order internal multiples. It is given by

$$\begin{aligned} b_5^{IM_1}(k) = & \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \\ & \times \int_{-\infty}^{z-\epsilon} dz' e^{-ikz'} \left[b_1(z')^3 + 2\, b_1(z') \int_{-\infty}^{z'-\epsilon} dz'''\, b_1(z''')^2 \right] \\ & \times \int_{z'+\epsilon}^{\infty} dz'' e^{ikz''} b_1(z''). \end{aligned} \tag{6}$$

and it can be separated in two parts (represented with the diagrams in Figure(3)),

$$\begin{aligned} b_{51}^{IM_1}(k) = & \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \\ \times & \int_{-\infty}^{z-\epsilon} dz' e^{-ikz'} b_1(z')^3 \int_{z'+\epsilon}^{\infty} dz'' e^{ikz''} b_1(z''), \end{aligned} \tag{7}$$

$$\begin{aligned} b_{52}^{IM_1}(k) = & \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \\ \times & \int_{-\infty}^{z-\epsilon} dz' e^{-ikz'} 2\, b_1(z') \int_{-\infty}^{z'-\epsilon} dz'''\, b_1(z''')^2 \int_{z'+\epsilon}^{\infty} dz'' e^{ikz''} b_1(z''). \end{aligned} \tag{8}$$

The diagram located on the left of Figure (3) corresponds to equation (7) and it belongs to a series that eliminates all 1^{st} order internal multiples that were downward reflected at the shallowest reflector. This term combines nonlinearly five sets of data. When added to the attenuator b_3 it provides extra amplitude information and the correct time of the internal multiples. The three hits in the diagram indicate triple self interaction at the generating reflector. Hence, the extra amplitude information given by the self-interacting data corresponds to powers of the reflection coefficient of each generating reflector, which is in agreement with the analysis in equations (4) and (5).

The analysis of the properties of this term, using its diagram representation and numerical examples, showed that it is the main contribution of $b_5^{IM_1}$ to the elimination of internal multiples (Ramírez and Weglein, 2005a). Its mathematical representation resembles the one of the attenuator, which is the leading order term of the series by itself. We can find the leading order terms by examining each term in the internal multiple elimination series and selecting the ones that only have data self-interactions at the generating reflector. The leading order terms are represented with the diagrams shown in Figure (4). The sum of these diagrams leads to the leading order closed form term

$$b_{LO}^{IM_1} = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \times \int_{-\infty}^{z-\epsilon} dz' e^{-ikz'} \left(\frac{1}{1-b_1(z')^2}\right) b_1(z') \int_{z'+\epsilon}^{\infty} dz'' e^{ikz''} b_1(z''). \tag{9}$$

This equation is the infinite sum of the main contributions in the inverse scattering internal multiple elimination series. It includes the first order term, the attenuator, and the main contribution from each subsequent term in the elimination series. The leading order eliminator, $b_{LO}^{IM_1}$, predicts all 1^{st} order internal multiples generated at the shallowest reflector without requiring a-priori information, nor a velocity model. The elimination step is performed in terms of the effective data, b_1, and the reference velocity contained in $k = \frac{2\omega}{c_0}$. Furthermore, the leading order eliminator helps to better attenuate all the internal multiples generated at deeper reflectors.

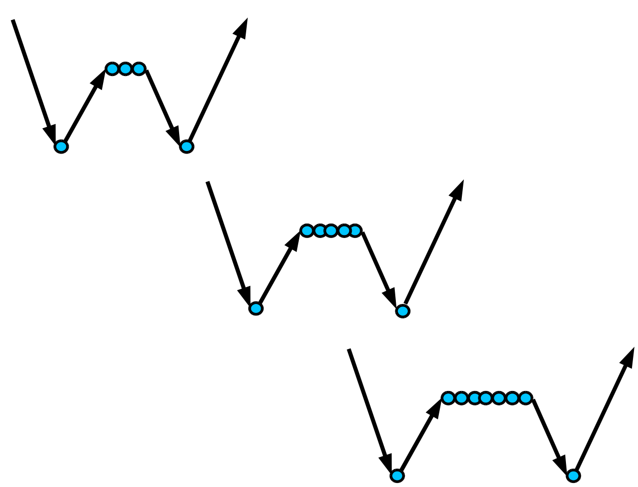

Figure 4: Leading order diagrams.

The diagram located on the right of Figure (3) represents equation (8). This equation contains $I = 2b_1(z')\int_{-\infty}^{z'-\epsilon} dz'''\, b_1(z''')^2$ in the middle integral. The term I, represented by the middle part of the diagram, has two self-interacting data within the overburden of the generating reflector. This double self interaction provides the series with second order corrections for any interface above the generating reflector and, it only acts on internal multiples downward reflected at interfaces below the shallowest reflector. The internal multiples generated at the shallowest reflector are completely eliminated with the leading order closed form term in equation (9). The double self-interacting diagram further attenuates all 1^{st} order internal multiples generated at deeper reflectors[1]. The main part of these second subseries can be summed in a higher order closed form term,

$$b_{HO}^{IM_1} = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z)$$

$$\int_{-\infty}^{z-\epsilon} dz' e^{-ikz'} \frac{2G_1(z') \int_{-\infty}^{z'-\epsilon} dz''' \; J(z''')}{1 - \int_{-\infty}^{z'} dz''' \; J(z''')} \int_{z'+\epsilon}^{\infty} dz'' e^{ikz''} b_1(z''). \tag{10}$$

$$J(z''') = \frac{b_1(z''')^2}{1 - b_1(z''')^2} \tag{11}$$

$$G_1(z') = \frac{b_1(z')}{1 - b_1(z')^2} \tag{12}$$

The higher order eliminator, assumes that the action of the leading order eliminator has taken effect prior to its calculation. The leading order closed form added to the effective data eliminates all multiples generated at the first reflector. The only task left, in terms of internal multiples, is to finish correcting the amplitude of the deeper internal multiples and eliminate them. This is the task performed by the higher order eliminator, $b_{HO}^{IM_1}$.

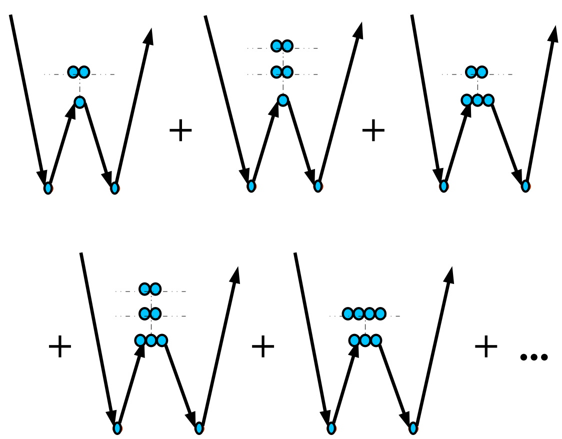

Figure 5: Higher order diagrams.

Some of the diagrams included in equation (10) are shown in Figure 5. Equation (10) is the infinite sum of the main terms in the higher order subseries of the internal multiple elimination series. The higher order eliminator includes diagrams that have extra data self-interactions above the generating reflector. The reason it is not including all the higher order terms is because, these terms in the inverse series for internal multiple elimination have different integer weights, which

[1]Where deeper refers to all reflector located below the shallowest one.

means that a specific higher order diagram is required to act more than once in the removal process. From the form of equation (10), the closed form only contains a weighting factor of 2 (please refer to the middle integral) in agreement to the weighting factor needed by equation 8. The first term included in the higher order closed form corresponds to equation 8, which is represented by the first diagram in Figure 5.

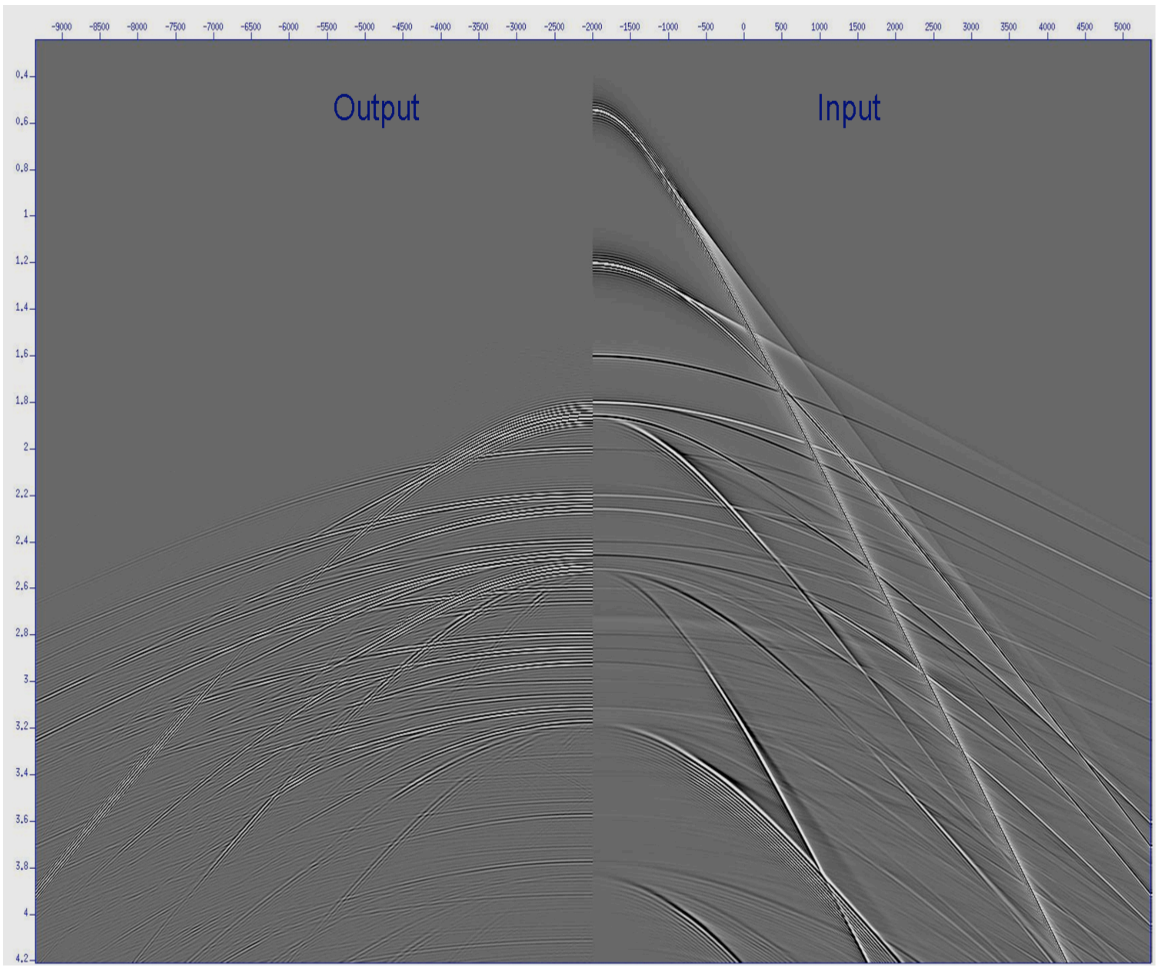

Figure 6: The left hand side shows the predicted internal multiples. The right hand side shows data containing primaries and internal multiples.

An elimination algorithm for internal multiples based on inverse scattering series has the potential of removing difficult internal multiples, leaving all primaries unaffected. Although the internal multiple amplitudes are reduced by the attenuator, $b_3^{IM_1}$, and substantially reduced (and a subset is eliminated) by the leading order closed form, $b_{LO}^{IM_1}$, there is in some cases an observable residual that can be further attenuated with the action of the higher order closed form, $b_{HO}^{IM_1}$. The higher order closed form term of the internal multiple elimination series complements the elimination of the amplitude of the remaining internal multiples by adding nonlinear contributions in terms of data and a reference Green's function. The combination of the leading order closed form with the higher order closed form term gives an improved algorithm for the removal of internal multiples. A $1.5D$ numerical example of the internal multiple prediction with $b_{LO}^{IM_1}$ in a half space of water and a horizontally layered elastic medium representing the earth, is shown in Figure 6. The finite differences synthetic data, on the right of this figure, contains primaries and internal multiples due to an elastic halfspace. The traces on the right show the predicted internal multiples. Notice that all multiples were predicted at the exact time. The data were deconvolved with an statistical estimate of the wavelet. The wavelet used to model the data was not used in the internal multiple prediction. Hence, the predicted multiples have a different wavelet. The fact that the internal multiple elimination algorithm, with an acoustic background, predicts internal multiples propagated in an elastic Earth is a remarkable result of the model-type independent nature of the algorithm.

2 2D extension of the algorithm

In the theory presented in the previous section, no assumptions about the earth below the receivers are made, this characteristic makes it ideal for addressing one of the current challenges in exploration seismology: removing multiples, locating and identifying targets in highly complex medium. When the medium is complicated, an accurate velocity model that would allow modeling and subtraction of internal multiples is unobtainable. Hence, the extension to a multidimensional earth is a necessary step.

The attenuation algorithm for a $2D$ earth, presented in Araújo (1994); Weglein et al. (1997) and Weglein et al. (2003), is

$$b_1(k_g, k_s, q_g + q_s) = -2iq_s D(k_g, k_s, \omega), \tag{13}$$

$$\begin{aligned} b_3^{IM_1}(k_g, k_s, q_g + q_s) = {} & \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s - x_g)} \int_{-\infty}^{\infty} dk_2 e^{iq_2(\epsilon_g - \epsilon_s)} \\ & \times \int_{-\infty}^{\infty} dz_1 e^{i(q_g + q_1)z_1} b_1(k_g, -k_1, z_1) \\ & \times \int_{-\infty}^{z_1 - \epsilon_2} dz_2 e^{i(-q_1 - q_2)z_2} b_1(k_1, -k_2, z_2) \\ & \times \int_{z_2 + \epsilon_1}^{\infty} dz_3 e^{i(q_2 + q_s)z_3} b_1(k_2, -k_s, z_3), \end{aligned} \tag{14}$$

where ω represents the temporal frequency, c_0 is the acoustic velocity of water; k_g and k_s are the horizontal wave numbers corresponding to receiver and source coordinates: x_g and x_s, respectively; the 2-D wave vectors: $\mathbf{k}_g = (k_g, -q_g)$ and $\mathbf{k}_s = (k_s, q_s)$ are constrained by $|\mathbf{k}_g| = |\mathbf{k}_s| = \frac{\omega}{c_0}$; the vertical wave numbers are $q_g = sign(\omega)\sqrt{(\frac{\omega}{c_0})^2 - {k_g}^2}$ and $q_s = sgn(\omega)\sqrt{(\frac{\omega}{c_0})^2 - {k_s}^2}$, and ϵ_i is a small positive parameter chosen to insure that the relations $z_1 > z_2$ and $z_3 > z_2$ are satisfied.

In equations (13) and (14), the effective data $b_1(k_g, k_s, q_g + q_s)$ is defined as a source obliquity factor times the $2D$ measured values of the scattered field, D. The variable z is the Fourier conjugate to the sum of the vertical wave numbers, $k_z = -(q_g + q_s)$. The attenuation of multiples is performed by adding the attenuator, $b_3^{IM_1}$, to the effective data, b_1.

As we showed in $1D$, the second term in the 1^{st} order internal multiple elimination series can be separated in two equations. The 2D form, of the first equation is

$$\begin{aligned}
b_{51}^{IM_1}(k_g, k_s, q_g+q_s) = & \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)} \int_{-\infty}^{\infty} dk_2 e^{iq_2(\epsilon_g-\epsilon_s)} \\
& \times \int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\
& \times \int_{-\infty}^{z_1-\epsilon_2} dz_2 e^{i(-q_1-q_2)z_2} \left[b_1(k_1,-k_2,z_2)\right]^3 \\
& \times \int_{z_2+\epsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3),
\end{aligned} \tag{15}$$

which have the same diagrammatic representation as shown in Figure (3). Studying the higher order terms in the inverse scattering internal multiple elimination series in a multidimensional model type independent form, we find that the form of the terms with self-interacting data at the generating reflector conserves the properties and characteristics found in the simple $1D$ case. Analogous to the $1D$ case, the first term in the leading order elimination series is the attenuator, equation (18), and the second term is given by equation (15). The next terms in the leading order series have the form:

$$\begin{aligned}
b_{51}^{IM_1}(k_g, k_s, q_g+q_s) = & \sum_{N=0}^{\infty}\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)} \int_{-\infty}^{\infty} dk_2 e^{iq_2(\epsilon_g-\epsilon_s)} \\
& \times \int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\
& \times \int_{-\infty}^{z_1-\epsilon_2} dz_2 e^{i(-q_1-q_2)z_2} \left[b_1(k_1,-k_2,z_2)\right]^{2N+1} \\
& \times \int_{z_2+\epsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3),
\end{aligned} \tag{16}$$

We can add the leading order terms in the multidimensional case to a closed form, which is given by,

$$b_1(k_g, k_s, q_g+q_s) = -2iq_s D(k_g,k_s,\omega), \tag{17}$$

$$\begin{aligned}
b_{LO}^{IM}(k_g, k_s, q_g+q_s) = & \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)} \int_{-\infty}^{\infty} dk_2 e^{iq_2(\epsilon_g-\epsilon_s)} \\
& \times \int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\
& \times \int_{-\infty}^{z_1-\epsilon_2} dz_2 e^{i(-q_1-q_2)z_2} \frac{b_1(k_1,-k_2,z_2)}{1-b_1(k_1,-k_2,z_2)^2} \\
& \times \int_{z_2+\epsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3),
\end{aligned} \tag{18}$$

This is a $2D$ model type independent leading order elimination algorithm for internal multiples. The leading order eliminator is a data-driven algorithm written in terms of effective data b_1 (see equation (17)). The leading order closed form, $b_{LO}^{IM_1}$, gives the main contribution towards eliminating internal multiples. It completely removes all 1^{st} order internal multiples generated at the first reflector and improves the attenuation of the remaining multiples. Leading order as an eliminator means it eliminates a class of internal multiples and further attenuates the rest. In a $2D$ medium, the multiples that have no cumulative transmission error (the ones with downward reflection at the shallowest reflector) are eliminated by the algorithm in equation (18), $b_1 + b_{LO}^{IM_1}$. The higher order closed form is being examined for a $2D$ extension. It is not always possible to generalize a $1D$ closed form to $2D$; an algorithm in $2D$ have more variables and different dependencies than the same algorithm in $1D$. However, we are studying the $2D$ expressions for the higher order terms in the elimination series. For a multidimensional world, the leading order eliminator provides the removal of all first order internal multiples generated at the first reflector and effectively attenuates the rest of the multiples.

There is an important subset of first order internal multiples that is now eliminated, and other internal multiples are reduced beyond attenuation. The former subset in practice can often be the most significant from a practical field viewpoint. The leading order elimination algorithm automatically eliminates those multiples that have their first reflection at the shallowest reflector, the water bottom, in marine exploration. The water bottom property is neither required nor determined for this eliminator algorithm, nor is information below the water bottom input to provide that ancillary benefit. The degree of the latter secondary benefit will vary but is always present. The fact that the new algorithm is not at all more expensive than the attenuator is worth noting. The sensitivity of the new algorithm for input wavelet is expected to be higher. In particular, an accurate estimation of the source wavelet will be needed to perform the division in the innermost integral. It will also allow convergence of the leading order closed form.

3 Internal multiples are predicted in terms of effective data

When the prerequisites of the internal multiple algorithm are satisfied, the predicted internal multiples can be attenuated/removed from the effective data b_1 by a simple addition $b_1 + b_3^{IM_1}$, for the attenuator, and $b_1 + b_{LO}^{IM}$, for the leading order eliminator. The output of this addition will be effective data with certain internal multiples removed and the rest attenuated. If instead of removing the internal multiples from effective data, one would like to remove them from measured data, then an extra obliquity factor is required as explained in the next lines.

The first inverse scattering equation,

$$D(k_g, k_s, \omega) = \frac{e^{-iq_g z_g}}{-2iq_g} V_1(k_g, k_s,_g +q_s) \frac{e^{-iq_s z_s}}{-2iq_s}, \tag{19}$$

is used to define the effective data as

$$b_1(k_g, k_s, q_g + q_s) = -2iq_s D(k_g, k_s, \omega), \tag{20}$$

or

$$b_1(k_g, k_s, q_g + q_s) = \frac{e^{-iq_g z_g}}{-2iq_g} V_1(k_g, k_s,_g +q_s) e^{-iq_s z_s}. \tag{21}$$

The internal multiple attenuation algorithm is found in the third inverse scattering equation, and is given by the algorithm in equation 14,

$$
\begin{aligned}
b_3^{IM_1}(k_g,k_s,q_g+q_s) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)}\int_{-\infty}^{\infty} dk_2 e^{iq_2(\epsilon_g-\epsilon_s)} \\
\times\int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\
\times\int_{-\infty}^{z_1-\epsilon_2} dz_2 e^{i(-q_1-q_2)z_2} b_1(k_1,-k_2,z_2) \\
\times\int_{z_2+\epsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3).
\end{aligned}
\tag{22}
$$

The left hand side of this equation, $b_3^{IM_1}$, is effective data consisting of internal multiples only. Therefore, its relation with measured data is

$$
b_3(k_g,k_s,q_g+q_s) = -2iq_s D^{IM}(k_g,k_s,\omega), \tag{23}
$$

where D^{IM} represents data consisting of internal multiples. In other words, the output of the attenuator needs to be divided by a source obliquity factor of $-2iq_s$ in order to have a prediction in terms of measured data. By induction, this analysis can be extended for the higher order terms in the internal multiple elimination series and leading order closed form.

We can predict a dataset consisting of internal multiples only by the attenuator or the leading order closed form by a factor of $-2iq_s$, and then subtract the multiples directly from the recorded data $D - D_3^{IM}$ or $D - D_{LO}^{IM}$, where

$$
\begin{aligned}
D_3^{IM}(k_g,k_s,q_g+q_s) = \frac{1}{(2\pi)^2}\frac{1}{-2iq_s}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)}\int_{-\infty}^{\infty} dk_2 e^{iq_2(\epsilon_g-\epsilon_s)} \\
\times\int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\
\times\int_{-\infty}^{z_1-\epsilon_2} dz_2 e^{i(-q_1-q_2)z_2} b_1(k_1,-k_2,z_2) \\
\times\int_{z_2+\epsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3).
\end{aligned}
\tag{24}
$$

and

$$
\begin{aligned}
D_{LO}^{IM}(k_g,k_s,q_g+q_s) = \frac{1}{(2\pi)^2}\frac{1}{-2iq_s}\int_{-\infty}^{\infty} dk_1 e^{iq_1(x_s-x_g)}\int_{-\infty}^{\infty} dk_2 e^{iq_2(\epsilon_g-\epsilon_s)} \\
\times\int_{-\infty}^{\infty} dz_1 e^{i(q_g+q_1)z_1} b_1(k_g,-k_1,z_1) \\
\times\int_{-\infty}^{z_1-\epsilon_2} dz_2 e^{i(-q_1-q_2)z_2} \frac{b_1(k_1,-k_2,z_2)}{1-b_1(k_1,-k_2,z_2)^2} \\
\times\int_{z_2+\epsilon_1}^{\infty} dz_3 e^{i(q_2+q_s)z_3} b_1(k_2,-k_s,z_3),
\end{aligned}
\tag{25}
$$

Conclusions

The first order term in the inverse scattering internal multiple series, known as the attenuator, provides an effective solution for many circumstances encountered in exploration seismology (Weglein et al., 2003). It predicts the correct arrival time and an estimate of the true amplitude of internal multiples in the data. There are circumstances when the attenuation, or identification, of internal multiples is not enough. An example of those possible circumstances (for towed streamer pressure measurements) is the possibility of either having a residual that is far from small (*e.g.* converted wave internal multiples) or where having a small residual interfering with a target primary, and the latter is itself small. In these cases, the attenuation is not enough and other algorithms need to be developed to extend the previous methods and advance beyond current capability. The internal multiple elimination series and closed forms aim to reduce residual internal multiples where the magnitude of the residual can be significant.

The higher order terms in the series add contributions to the attenuator to improve its effectiveness towards an elimination of internal multiples. The algorithm presented is based on inverse scattering, and it goes further in the removal of first order internal multiples. Two closed forms were obtained and used in examples. The first one, adds the leading order terms elimination subseries and it is an algorithm that completely eliminates all first order internal multiples generated at the first reflector. The second closed form adds the main contribution of the higher order terms. It shows a better estimate of the amplitudes, and provides an improvement towards the elimination of 1^{st} order internal multiples. In this theory, no assumptions about the earth below the receivers are made.

The extension to a multidimensional earth was achieved for the leading order closed form term. The leading order eliminator provides the removal of all first order internal multiples generated at the shallowest reflector and effectively attenuates the rest of the multiples. The extension to a multidimensional earth of the higher order terms as well as extensions of definitions is our current subject of study.

The output of the attenuator (Weglein et al., 1997), and the leading order eliminator, is a wavefield of internal multiples in terms of effective data. In order to have traces consisting of internal multiples in terms of data, the obliquity factor $-2iq_S$ needs to be deconvolved from the prediction.

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