Connections Among Synthetic Aperture Imaging Techniques

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Abstract

It is common knowledge that the disciplines of synthetic aperture radar and sonar have exchanged technological advances for decades. Furthermore, both have benefited from developments in the seismic imaging community. Such advances have occurred only sporadically as researchers have become aware of cross-discipline similarities that are not always obvious. This paper surveys several key synthetic aperture imaging fields and discusses fundamental commonalities among them. In particular, we consider synthetic aperture radar and sonar, X-ray computed tomography, seismic imaging, and radio astronomy.

1 Introduction

Interestingly, synthetic aperture radar (SAR) and sonar (SAS) belong to a rather distinguished family of Fourier imaging techniques that has garnered several Nobel Prizes:

- X-ray crystallography (Max von Laue, 1914, Physics)
- Holography (Dennis Gabor, 1971, Physics)
- Radio astronomy (Sir Martin Ryle and Antony Hewish, 1974, Physics)
- X-ray tomography (Allan Cormack and Godfrey Hounsfeld, 1979, Physiology or Medicine)
- Magnetic resonance imaging (Paul Lauterbur and Peter Mansfeld, 2003, Physiology or Medicine)

Synthetic aperture imaging practitioners have long recognized the existence of similarities between these disciplines and their own, and with the abundance of researchers, literature, and commercial products, it would seem as though ideas could flow freely among related technological specialties. The barrier to information exchange is, however, quite high in practice for three reasons. First, researchers tend to spend their entire career specializing in a single field. Second, the scientific literature is riddled with domain-specific jargon that is difficult for the uninitiated to negotiate. Thirdly, the governing physics differ for each application. What might be a fundamental limitation for one imaging mode could be a trivial concern for another. This paper begins to address the second and last of these concerns in hopes of creating an interest in deliberately working across disciplines to advance the collective state of the art.

2 Survey of Imaging Modalities

We approach our goal by first discussing the key technical challenges for several imaging techniques.

2.1 Synthetic Aperture Radar

It is difficult to succinctly characterize the variety of SAR instruments and applications in existence. Broadly speaking, SAR operates at ranges on the order of 1–1000 km, and typical frequencies used for imaging range between 4 and 35 GHz (C band to Ka band). Image sizes are on the order of 1–10 km, and fine resolution is generally understood to be around 0.3 m or better.

SAR collections fall into two categories, stripmap and spotlight [1, 2, 3]. Stripmap mode provides high area coverage rates, but it exhibits relatively coarse cross-range resolution because the integration angle is limited to the antenna beamwidth. This problem is overcome by spotlight mode imaging, in which the beam is continually slewed to point toward a fixed location on the ground during the collection. The integration angle may then be much larger than the beamwidth. The beamwidth therefore limits the scene size of a spotlight image, as opposed to its resolution.

Since its inception in the 1950s by Carl Wiley, SAR technology has had to overcome a number of technological obstacles. Notable examples are the transition from film recording systems to digital systems and the use of stretch receivers to overcome sampling rate limitations when employing large RF bandwidths. Today, there are no significant challenges to achieving the aforementioned fine resolution criteria of 0.3 m. Substantially improving upon this is a difficult task, however. For a given fractional bandwidth, increasing the transmitted RF bandwidth can be done by employing a higher center frequency. However, at high frequencies it becomes difficult to manufacture antennas and RF components capable of withstanding the power levels necessary for achieving good SNR.

2.2 Synthetic Aperture Sonar

Imaging underwater over any appreciable distance can only be achieved using sound waves. Light and radio waves attenuate far too quickly. Even high-frequency sound waves do not propagate very far. Fine resolu-

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tion real-beam imaging sonars have limited ranges for this reason, and synthetic aperture sonar is the preferred choice for creating very detailed images of the sea floor. SAS sensors typically operate somewhere in the band of 20–300 kHz. Typical range resolution is on the order of 3 cm, which is easy to achieve given current transducer technology and the fact that this resolution translates into only 25 kHz of transmitted bandwidth.

Two key innovations have made SAS a viable technology. The first is the use of the Vernier array, in which multiple receivers are used to increase the area coverage rate by allowing the sonar to extend its reception time [4, 5]. The increase in coverage rate is proportional to the number of receivers used. The second enabling technology is the use of redundant phase centers, in which a subset of the Vernier array is overlapped between successive pings [6]. The signals from the overlapping channels are cross-correlated, and the measured delay is used to infer the platform trajectory.

The most challenging problems for SAS come from operating in the extremes of ocean depth. Perhaps surprisingly, imaging in shallow water is very difficult. The environment is not free-space, as the sea surface acts as a constantly-moving mirror to produce multipath reflections that corrupt the data. Furthermore, shallow water may exhibit dramatic salinity and temperature gradients that cause refraction of the sound, whereas most reconstruction algorithms assume straight-line propagation. At the opposite end of the scale, it is difficult to manufacture vehicles and sensors that can travel to the deepest parts of the ocean. In addition to withstanding the extreme pressure, the unmanned vehicles that carry SAS must have enough on-board power to make the trip to and from the ocean floor while also having enough survey time to make the journey worthwhile. Acoustically speaking, however, operating in the deep ocean is relatively easy. The environment is free of surface reflections and the water is generally homogeneous, meaning that the actual propagation very closely matches the simple theoretical model used to derive reconstruction algorithms.

2.3 X-Ray Computed Tomography

X-ray CT is widely used for imaging the human body. It differs from SAR in that it measures transmitted, rather than reflected, energy. However, the underlying mathematics of image reconstruction are remarkably similar to spotlight mode SAR [7, 8]. Many of the challenges of creating a good CT image involve trade-offs associated with improving the data quality at the expense of exposing the patient to an increased dosage of radiation [9]. Current medical CT scanners offer a resolution limit on the order of less than 1 mm. Manufacturing smaller detectors would improve image resolution, but would also be more susceptible to noise, necessitating a higher source intensity and increased risk to the patient. Different facets of this balance between source flux and image quality shape many areas of active research and vendor intellectual property.

Apart from the hardware itself, another key component of successful CT imaging is the ability to calibrate the data. Within the industry, this process is known as *physical correction* and is quite complicated because of the sophistication and sensitivity of the equipment. In fact, the physical correction constitutes a significant fraction of the intellectual property associated with a commercial CT scanner.

Unwanted motion is another difficulty for CT reconstruction. Unlike SAR and SAS, the motion of the CT scanner is controlled and well-understood as it rotates around the patient. However, the motion of the body through breathing and heartbeat can easily corrupt the image. Tracking and correcting for this motion is an active area of research in the medical imaging community. Another serious problem for CT imaging is the presence of metal in the body, such as dental fillings and titanium replacement joints. These objects absorb the incident X-rays, effectively creating holes in the data that result in visual artifacts. Current research efforts are investigating methods of mitigating these artifacts and interpolating through the regions of missing data.

2.4 Seismic Imaging

Of all disciplines using synthetic aperture techniques, seismic imaging is probably the most mathematically sophisticated as well as the most difficult for outsiders to understand. The sophistication comes from the fact that the Earth's interior is a complex environment and is difficult to model, much less reconstruct from measurements. The steep learning curve results from the somewhat unusual circumstance that seismic imaging straddles pure science as well as heavy industry. The literature therefore combines deep concepts from physics and mathematics with the jargon of petroleum field engineers [10].

While other fields emphasize increasingly higher image fidelity with respect to the true scene properties (for example reflectivity or attenuation coefficient), the geophysical community is primarily interested in identifying the true structure of reflecting layers within the earth. It is less concerned with solving for the exact physical properties that cause the reflections. Furthermore, certain environments may favor certain reconstruction techniques. In contrast, other disciplines have a clearer path toward a single approach that will render the best possible image.

Some of the reconstruction methods used by the geophysics community overlap with other fields. In particular, what the SAR and SAS communities refer to as time-domain reconstruction, or backprojection, is known as Kirchhoff migration to geophysicists. Perhaps more significant is the ω-k algorithm developed by Stolt [11, 12]. It was a revolutionary step in that it achieves a mathematically exact reconstruction while employing the efficiency of the fast Fourier transform. It assumes a constant wave propagation speed, which is usually not a good fit to the subterranean environment. It is, however, an excellent model in the majority of radar and sonar applications. The ω-k algorithm therefore represents a transformational gift from the seismic community.

2.5 Radio Astronomy

Radio astronomy aperture synthesis is an impressive technology, if for no other reason than the sheer scales involved. While the previous collection methods involve forming a synthetic aperture using a moving aircraft or a rotating X-ray detector, radio astronomy forms apertures using the Earth's rotation [13]. Of the techniques discussed here, radio astronomy is the only one that is passive. Images are formed on the principle that signals from pairs of antennas, called baselines, can be correlated and the degree of coherence measured as the Earth rotates. The rotation of the baseline separation vectors determines the loci of the sampled data, resulting in a two-dimensional coherence, or visibility, map.

The Van Cittert–Zernike theorem from statistical optics states that the visibility is related to the image intensity through the Fourier transform. Radio astronomy image reconstruction therefore strongly resembles that of spotlight SAR via the polar formatting algorithm. In both cases, the measured data can be treated as samples within the frequency domain representation of the image sought. These samples are usually nonlinearly sampled, so reconstruction involves interpolating the available data onto a rectangular grid of samples that is amenable to inversion via the fast Fourier transform.

Radio astronomy spectra are also usually poorly sampled in some respect. This situation resulted in algorithms, such as CLEAN, that are used to improve the impulse response of the image. Some of these have migrated to the SAR and SAS communities. Geophysicists also use similar techniques for mitigating the effect of multiple reflections inside the earth.

3 Image Reconstruction

This section illustrates how reconstruction algorithms for the technologies listed above can all be traced back to a core set of mathematical principles, namely the adjoint and the pseudoinverse. Borrowing the notion of *exploding reflectors* from the seismic literature, we represent the scene as a continuum of sources all radiating simultaneously, combining the reflectivity and transmitted signal into a single function $f(\boldsymbol{x}, t)$. Reflection data can then be treated as though the signals originate at the reflectors, while traveling with half of the actual propagation speed. This general model allows us to ignore the reflection/transmission nature of the specific problem. We next introduce a linear operator T that maps the virtual source field $f(\boldsymbol{x}, t)$ into measurements $s(\boldsymbol{x}_\mathrm{m}, t)$ made at the set of observation points $\{\boldsymbol{x}_\mathrm{m}\}$. The mapping T is usually taken to be the convolution of $f(\boldsymbol{x}, t)$ with the free space Green's function. We will prefer the frequency domain version (usually associated with the Helmholtz equation) for the derivations to follow,

$$s(\boldsymbol{x}_\mathrm{m}, \omega) = Tf(\boldsymbol{x}', \omega) = \int_X \frac{e^{-i2kR}}{4\pi R} f(\boldsymbol{x}', \omega) d\boldsymbol{x}', \quad (1)$$

where $R = |\boldsymbol{x}' - \boldsymbol{x}_\mathrm{m}|$ and $k = \omega/c$.

We next introduce the adjoint operator $T^\dagger$ associated with T and defined by the relationship $\langle T^\dagger s, f\rangle = \langle s, Tf\rangle$, where the angle brackets denote the inner product. We can find the adjoint operator by substituting the forward mapping into this definition and rearranging the result:

$$\begin{aligned}\langle s, Tf\rangle &= \int_\Omega s^*(\boldsymbol{x}_\mathrm{m}, \omega) \int_X f(\boldsymbol{x}', \omega) \frac{e^{-i2kR}}{4\pi R} d\boldsymbol{x}' d\omega \\ &= \int_X f(\boldsymbol{x}', \omega) \int_\Omega s^*(\boldsymbol{x}_\mathrm{m}, \omega) \frac{e^{-i2kR}}{4\pi R} d\omega d\boldsymbol{x}' \\ &= \int_X f(\boldsymbol{x}', \omega) \int_\Omega \left\{ s(\boldsymbol{x}_\mathrm{m}, \omega) \frac{e^{i2kR}}{4\pi R} \right\}^* d\omega d\boldsymbol{x}' \\ &= \langle T^\dagger s, f\rangle, \qquad (2)\end{aligned}$$

where the adjoint operator, applied to $s(\boldsymbol{x}_\mathrm{m}, \omega)$, is seen to be

$$T^\dagger s(\boldsymbol{x}_\mathrm{m}, \omega) = \int_\Omega \frac{e^{i2kR}}{4\pi R} s(\boldsymbol{x}_\mathrm{m}, \omega) d\omega. \quad (3)$$

The operator $T^\dagger$ is identified as the backprojection operator since it maps points s in the measurement space back into the space of the scene. The corresponding reconstruction algorithm pseudocode is as follows:

```
Initialize the output image
for Each point $x$ in the reconstructed image do
  for Each measured signal in the set $\{s(x_m, \omega)\}$ do
    · Backproject all frequencies $\omega$ from $x_m$ to $x$ using the adjoint operator $T^\dagger s(x_m, \omega)$;
    · Accumulate the resulting scalar value into the output image at location $x$;
  end for
end for
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While the adjoint operator (3) backprojects the measured data as desired, it does not compensate for effects such as spherical spreading. In fact, it introduces another spreading term. A filtered backprojection solution based on the pseudoinverse performs the desired compensation and also serves as the least squares solution when the measurements are subject to noise. The least squares solution is given by $f = (T^\dagger T)^+ T^\dagger s$, where the superscript $+$ denotes the pseudoinverse. This solution backprojects and then filters, while most practical implementations reverse this order. The operators T and $T^\dagger$ can be expanded as sets of orthonormal basis function functions, allowing us to rearrange the solution to obtain $f = T^\dagger (TT^\dagger)^+ s$ [14]. This is the desired outcome, applying filtering before

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backprojection. For the forward model given by (1) and noiseless data s, the term $(TT^{\dagger})^{+}$ reduces to $(4\pi R)^2$, providing the necessary compensation for the spreading associated with T and $T^{\dagger}$.

The preceding result matches backprojection as commonly performed for SAS, SAR, and also basic seismic reconstruction. Furthermore, the well-known Stolt, or ω–k, algorithm can be derived from (1) [11]. An important branch of reconstructions can be developed when the imaged scene is interrogated by plane waves or parallel rays. This provides the desired connection to X-ray CT, radio astronomy, and spotlight mode SAR.

For spotlight SAR, the far-field assumption means that the sample in any given range bin is proportional to the ground reflectivity integrated over a plane, at that range, normal to the radar's line of sight [3]. The collection of sampled bins is called a range profile. The X-ray CT problem is one of transmission, not reflection, and scanners operate on the principle that detectors measure the amount of radiation passing through the patient along straight-line rays from the source. The set of measurements provided at a single time by the detector array is called a projection, or view. According to the projection slice theorem, the 1D Fourier transform of a spotlight range profile or a CT view provides the samples representing a single slice through the 2D Fourier transform of the desired image. The connection to radio astronomy is made at this point, because radio interferometry arrays directly measure the 2D Fourier transform of the intensity of the portion of sky being imaged. Reconstruction techniques for all three imaging modalities involve algorithms for taking the Fourier transform of irregular or non-equispaced sampled spectra. This often means interpolating the frequency-domain samples onto a regular sampling grid, suitable for applying the FFT.

4 Conclusion

This paper offers a high-level survey of several major technologies that employ aperture synthesis to reconstruct imagery. All can be traced to common mathematical underpinnings, yet the examples of significant technology sharing are relatively sporadic. It is hoped that studying the synthetic aperture family tree will aid researchers interested in accessing the literature outside their primary field and will expose opportunities for future cross-discipline innovation.

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