Mission - Oriented Seismic Research Program

SEG/DGS WORKSHOP

Closing Address Presentation: the Status of Velocity Model Building-and Consequences for Seismic Processing Methods and Objectives

Arthur B. Weglein

Thursday, March 11, 2021
SEG/DGS Workshop

SEG | DGS Workshop: Challenges & New Advances in Velocity Model Building 9-11 March 2021 | Virtual Workshop

Velocity model building is a long term interest and challenge in seismic exploration

Why are there challenges?

- All seismic methods make assumptions (and have prerequisites) – and when assumptions are satisfied they are effective and when they are violated the methods can (and will) fail.
- That failure can contribute to dry hole exploration wells or suboptimal development well placement.

That breakdown in effectiveness and capability defines a seismic challenge

Seismic E&P challenges

- Among assumptions:
  - Acquisition
  - Compute power
  - Innate algorithmic assumptions/requirements

Innate algorithmic assumptions

- Many processing methods require subsurface information
- In complex and ill-defined areas that requirement can be difficult or impossible to satisfy

Responding to seismic challenges

- Two ways to address:
  - Find a more effective way to satisfy the assumption or prerequisite—thereby removing the violation
  - Find a way to remove the violation of the assumption by removing the assumption—find a new algorithm that achieves that same exact processing objective without that assumption

Responding to seismic challenges

- There are times when one or the other of these two responses to a particular challenge can be indicated and appropriate choice

Velocity plays different roles in seismic exploration:

- As a direct interpretation tool and value
- As an essential prerequisite for other methods to be effective, e.g., imaging and inversion

In addition, ‘velocity’ comes in different forms and varieties:

- As migration methods evolved and became more effective there was a concomitant need for a more accurate and realistic velocity model
  - Stacking velocity (for post-stack migration)
  - Velocity for pre-stack time migration
  - Velocity for depth migration
    - RMS velocity
    - Interval velocity
    - P and S wave velocity
  - Velocity (as one among parameters) for inversion at depth

- Among methods that are employed :
  - Stacking
  - CIG flatness
  - Tomography
  - FWI
- Not a closed subject

There are different issues in seismic processing for on-shore and offshore plays—and that includes for velocity analysis

The state of velocity analysis is directly related to (and impacts) other seismic issues and challenges

Those seismic processing methods that depend on an adequate velocity model are in turn used (as a metric) to define the accuracy of the model.

That measure of the adequacy of a velocity model deserves our critical attention and analysis—we will discuss that in what follows.

That leads us to discuss <u>seismic migration</u>—a topic intimately related to velocity model building and the purpose of this Workshop.

Wave equation migration has two ingredients

1. A propagation model
2. An imaging principle

Three Imaging Principles
Claerbout 1971

1. Exploding reflector (for post-stack data)
2. Time and space coincidence of the down wave from the source and the upwave from a reflector
3. Predicting a coincident source and receiver experiment at depth and asking for t = 0

Imaging Principles Continued

- We label these imaging principles Claerbout I, II and III (CI, CII, CIII)
- Only CII and CIII for pre-stack data

Imaging Principles Continued

- CII→RTM—assumes (at most) planar specular reflection
  - provides a geometric optics (high frequency approximate reflection coefficient)
  - CII cannot accommodate a discontinuous velocity model
- CIII→Stolt Claerbout III to image and invert specular and non-specular (curved, diffractive, pinchout reflectors)
  - Recently extended by M-OSRP to accommodate discontinuous media
    - Weglein et al 2016 SEG Abstract
    - Ecopetrol paper
    - Y. Zou et al Wedge paper 2017

When using the most capable migration methods, Stolt CIII migration for heterogeneous media and a smooth velocity for migration

- Multiples will cause artifacts (false images) and hence must be removed.

When using an accurate discontinuous velocity (and density etc.) in Stolt CIII migration

- Multiples provide no benefit and cause no harm
- No need to remove
- See, e.g. Weglein et al 2016 SEG Abstract and the references and links below

- See link: **SEG/KOC Workshop: Seismic Multiples, the Challenges and the Way Forward" in Kuwait December 3-5 2019 .**
- " A new perspective on removing and using multiples | they have the same exact goal | imaging primaries, recorded and unrecorded primaries | Recent advances in multiple removal".
- https://youtu.be/sD89_418h1A

The first migration method that is equally effective for all acquired frequencies for imaging and inverting at the target and reservoir

Arthur B. Weglein, Yanglei Zou*, Qiang Fu*, Fang Liu*, Jing Wu*, Chao Ma*, Robert H. Stolt†, Xinglu Lin*, and James D. Mayhan**

Summary

There is an industry wide interest in acquiring lower frequency seismic data. There is also an interest in assuring that the broadband data provides added value in processing and interpretation, to better resolve structure and to provide improved amplitude analysis at the target and at the reservoir. There are industry reports that when comparing the new and more expensively acquired broadband lower frequency data with conventional recorded

this new migration method. In that paper, the new migration method is used to provide a definitive response to the role of primaries and multiples in seismic processing. This paper focuses on the frequency fidelity properties of all current and the new migration method.

For the imaging principle component, a good reference to start with is Jon Claerbout's 1971 landmark contribution. He listed three imaging principles: the exploding-reflector model which is for stacked or zero offset data. We call this Claerbout imaging principle I. The second imaging prin-

A wedge resolution comparison between RTM and the first migration method that is equally effective at all frequencies at the target: tests and analysis with both conventional and broadband data

Yanglei Zou, Qiang Fu, and Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston

SUMMARY

Acquiring lower-frequency seismic data is an industry-wide interest. There are industry reports that (1) when comparing the new and more expensively acquired broad-band lower-frequency data with conventional recorded data, taken over a same region, these two data sets have the expected difference in frequency spectrum and appearance, but (2) they often provide less than the hoped for difference in structural resolution improvement or added benefit for amplitude analysis at the target and reservoir. In Weglein et al. (2016) and Q. Fu et al. (2017), they demonstrate that all current migration and Claerbout imaging principle II (CII). Waves propagate down from the source, are incident on the reflector, and the reflector generates a reflected upgoing wave. According to CII, the reflector exists at the location in space where the wave that is downward propagating from the source and the upwave from the reflector are at the same time and space. All RTM methods are based on RTM (CII) imaging principle and we after refer to RTM in this paper as RTM (CII). The third is Claerbout imaging principle III (CIII), which starts with surface source and receiver data and predicts what a source and receiver would record inside the earth. CIII then arranges the predicted source and receiver to be coincident and asks for $t = 0$. If the pre-

- At this moment, (and for the foreseeable future) the high water mark of velocity analysis capability (including tomography and FWI) is to improve a smooth velocity for migration—while that can be adequate for a structure map—that requires that all multiples must be removed

- However, when going beyond a structure map for parameter estimation at depth with migration-inversion a multiparameter elastic (anelastic) migration, with known discontinuous properties above the target is needed for a direct non-linear inversion of mechanical properties at the target.

- References:
  - Stolt and Weglein, 1985 Geophysics
  - Stolt and Weglein Cambridge University Press 2012 SII
  - Haiyan Zhang Thesis
  - Haiyan Zhang and Weglein 2009
  - Hong Liang and Weglein 2013
  - Hong Liang Thesis

- As the industry trend moved to more complex offshore and on-shore plays challenges arose from the inability to provide an adequate velocity model for migration and other seismic objectives.

One Response: BETTER SATISFY THE ASSUMPTIONS

- Although there has been progress in that endeavor (that this Workshop reports and recognizes) there are serious and significant challenges that remain—we encourage further research investment along that path of providing ever more effective velocity model building—there is no final and ultimate solution—always a work in progress.

A Second Response: REMOVE THE ASSUMPTIONS

- At the same time—in the late 1980's-1990's a second response to this same industry trend to ever more complex overburdens and inaccessible targets motivated a campaign to directly achieve all processing objectives directly and without knowing, estimating, or determining subsurface properties.

Up to this point in this presentation, we assumed that information in the overburden (above a target) is achievable

Innate algorithmic assumptions

- The inverse scattering series states that all processing objectives can be achieved directly and without any subsurface information.
- Among processing objectives
  1. Free surface multiple removal
  2. Internal multiple removal
  3. Depth imaging
  4. Non-linear AVO
  5. Q compensation

  All without subsurface information

Multiple Removal

- The impact of the inverse scattering series on removing free surface and internal multiples is well documented in the seismic literature—as the only methods that can predict the accurate amplitude and phase of all free surface and internal multiples at all offsets, automatically accommodating specular and non-specular generators, without any subsurface information (known, estimated, or determined) and without adaptive subtraction

- A few references:
  - Chao Ma et al 2018 Geophysics (ISS FSME compared to SRME)
  - Yanglei Zou et al 2019 SEG Abstract (ISS internal multiple elimination)
  - Yi Luo et al TLE 2011 (on-shore application)
  - Weglein, Jing Wu and Fred Melo 2021 EAGE, Workshop on Multiples (a toolbox of multiple removal methods)

GEOPHYSICS, VOL. 84, NO. 5 (SEPTEMBER-OCTOBER 2019); P. S459–S478, 51 FIGS.
10.1190/GEO2018-0411.1

Comparison of the inverse scattering series free-surface multiple elimination (ISS FSME) algorithm with the industry-standard surface-related multiple elimination (SRME): Defining the circumstances in which each method is the appropriate toolbox choice

Chao Ma[1], Qiang Fu[1], and Arthur B. Weglein[1]

ABSTRACT

The industry-standard surface-related multiple elimination (SRME) method provides an approximate predictor of the amplitude and phase of free-surface multiples. This approximate predictor then calls upon an energy-minimization adaptive subtraction step to bridge the difference between the SRME prediction and the actual free-surface multiple. For free-surface multiples that are proximal to other events, the criteria behind energy-minimization adaptive subtraction can be invalid. When applied under these circumstances, a proximal primary can often be damaged. To reduce the dependence on the adaptive process, a more accurate free-surface multiple prediction is required. The inverse scattering series (ISS) free-surface multiple elimination (FSME) method predicts free-surface multiples with accurate time and accurate amplitude of free-surface multiples for a multidimensional earth, directly and without any subsurface information. To quantify these differences, a comparison with analytic data was carried out, confirming that when a free-surface multiple interferes with a primary, applying SRME with adaptive subtraction can and will damage the primary, whereas ISS free-surface elimination will precisely remove the free-surface multiple without damaging the interfering primary. On the other hand, if the free-surface multiple is isolated, then SRME with adaptive subtraction can be a cost-effective toolbox choice. SRME and ISS FSME each have an important and distinct role to play in the seismic toolbox, and each method is the indicated choice under different circumstances.

A new multidimensional method that eliminates internal multiples that interfere with primaries, without damaging the primary, without knowledge of subsurface properties, for off-shore and on-shore conventional and unconventional plays

Yanglei Zou, Chao Ma, and Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston

SUMMARY

Multiple removal is a longstanding problem in exploration seismology. Many methods have been developed including: stacking, FK filter, Radon transform, deconvolution and Feedback loop. They make statistical assumptions, assume move-out differences, or require knowledge of the subsurface and the generators of the multiples (e.g., Foster and Mosher, 1992; Verschuur et al., 1992; Berkhout and Verschuur, 1997; Jakubowicz, 1998; Robinson and Treitel, 2008; Wu and Wang, 2011; Meles et al., 2015; da Costa Filho et al., 2017; Lomas and Curtis, 2019). As the industry moved to deep water and more complex on-shore and off-shore plays, these methods bumped up against their assumptions. The Inverse Scattering Series (ISS) internal-multiple-attenuation algorithm (Araújo et al., 1994, Weglein et al., 1997 and Weglein et al., 2003) made none of the assumptions of previous methods (listed above) and stands alone, and is unique in its effectiveness when the subsurface and generators are complicated and unknown. It is the only multi-dimensional internal-multiple-removal method that can predict all internal multiples with exact arrival time and approximate amplitude without requiring any subsurface information. When internal multiples and primaries are isolated, the ISS internal-multiple-attenuation algorithm is usually combined with an energy-minimization adaptive subtraction to re- multiple-elimination algorithm is more effective and more compute-intensive than the current most capable ISS attenuation-plus-adaptive-subtraction method. We provide it as a new capability in the multiple-removal toolbox and a new option for circumstances when this type of capability is called for, indicated and necessary. That can frequently occur in offshore and onshore conventional and unconventional plays. We are exploring methods to reduce the computational cost of these ISS attenuation and elimination algorithms, without compromising effectiveness.

INTRODUCTION

The ISS (Inverse-Scattering-Series) allows all seismic processing objectives, e.g., free-surface-multiple removal and internal-multiple removal, depth imaging, non-linear amplitude analysis and Q compensation to be achieved directly in terms of data, without any need for, or determination of subsurface properties (e.g., Weglein et al., 2012; Zhang and Weglein, 2009a,b; Zou and Weglein, 2018). The ISS internal-multiple attenuation algorithm is the only method today that can predict the correct time and approximate amplitude for all first-order internal multiples generated from all reflectors, at once, without any subsurface information. If the multiple to be removed is iso-

Elimination of land internal multiples based on the inverse scattering series

Yi Luo, Panos G. Kelamis, Qiang Fu, ShouDong Huo, and Ghada Sindi, Saudi Aramco
Shih-Ying Hsu and Arthur B. Weglein, University of Houston

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Despite the explosion of new, innovative technologies in the area of multiple identification and subsequent attenuation, their applicability is mostly limited to marine environments especially in deep water. In land seismic data sets however, the application of such multiple-elimination methodologies is not always straightforward and in many cases poor results are obtained. The unique characteristics of land seismic data (i.e., noise, statics and coupling) are major obstacles in multiple estimation and subsequent elimination. The well-defined surface multiples present in marine data are rarely identifiable in land data. Particularly in desert terrains with a complex near surface and low-relief structures, surface multiples hardly exist. In most cases, we are dealing with so called "near-surface-related multiples." These are primarily internal multiples generated within the complex near surface.

In this paper, we employ theoretical concepts from the inverse scattering series (ISS) formulation and develop computer algorithms for land internal multiple elimination. The key characteristic of the ISS-based methods is that they do not require any information about the subsurface: i.e., they are fully data-driven. Internal multiples from all possible generators are computed and adaptively subtracted from the input data. These methodologies can be applied prestack and poststack the main internal multiple generators. Thus, some advanced knowledge of the main multiple generators is required. On land, as shown by Kelamis et al. (2006), the majority of internal multiples are generated by a series of complex, thin layers encountered in the near surface. Thus, the applicability of the CFP-based layer/boundary approach is not always straightforward because it requires the definition of many phantom layers. In contrast, the ISS theory does not require the introduction of phantom layers/boundaries. Instead, it computes all possible internal multiples produced by all potential multiple generators. Therefore, fully automated internal multiple-elimination algorithms can be developed in the prestack and poststack domains.

Basic principles of ISS technology

The ISS-based formulation for internal multiple attenuation (Araújo et al., 1994; Weglein et al., 1997) is a data-driven algorithm. It does not require any information about the reflectors that generate the internal multiples or the medium through which the multiples propagate and, in principle, it does not require moveout differences or interpretive intervention. The algorithm predicts internal multiples for all horizons at once.

Multiples: towards a toolbox perspective on assumptions, challenges and options (an Invited Presentation)

Arthur B. Weglein[1], Jing Wu[2], Frederico Xavier de Melo[2]
[1]M-OSRP/Physics Department/University of Houston
[2]WesternGeco/Schlumberger

Introduction

All seismic methods make assumptions; when the assumptions are satisfied, methods are effective, and when assumptions are violated, the methods can have difficulty and can fail. Seismic challenges arise when the assumptions behind the algorithm are violated. The three links provided at the end of this Abstract provide: (1) a more extensive and detailed version of this Abstract with a context, motivation and perspective (and the references cited within this abstract) and (2) the various types of assumptions behind seismic processing algorithms.

They are all important. For example, a seismic processing method is not an isolated entity, but rather a link in a sequence of processing steps. All the earlier steps in the chain, are important assumptions, prerequisites and requirements for the effectiveness of later steps.

A critically important assumption for a given link in the chain is the need for subsurface information. The industry trend to deep and complex offshore and onshore plays made the need for adequate subsurface information increasingly difficult or impossible to satisfy, and that inability remains the situation today. That reality drove the interest in developing methods that did not need to know, to estimate or to determine subsurface information.

JOURNAL OF SEISMIC EXPLORATION 27, 593-608 (2018)

ISS *Q* COMPENSATION WITHOUT KNOWING, ESTIMATING OR DETERMINING *Q* AND WITHOUT USING OR NEEDING LOW AND ZERO FREQUENCY DATA

YANGLEI ZOU and ARTHUR B. WEGLEIN

M-OSRP, Physics Department, University of Houston, Houston, TX 77204, U.S.A.

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ABSTRACT

Zou, Y. and Weglein, A.B., 2018. ISS *Q* compensation without knowing, estimating or determining *Q* and without using or needing low and zero frequency data. *Journal of Seismic Exploration*, 27: 593-608.

INVERSE SCATTERING SERIES DIRECT DEPTH IMAGING WITHOUT THE VELOCITY MODEL: FIRST FIELD DATA EXAMPLES

ARTHUR B. WEGLEIN[1], FANG LIU[1], XU LI[1], PAOLO TERENGHI[1], ED KRAGH[2], JAMES D. MAYHAN[1], ZHIQIANG WANG[1], JOACHIM MISPEL[3], LASSE AMUNDSEN[3], HONG LIANG[1], LIN TANG[1] and SHIH-YING HSU[1]

[1] *M-OSRP, University of Houston, 617 Science & Research Bldg. 1, Houston, TX 77004, U.S.A.*
[2] *SCR/Schlumberger, Schlumberger Cambridge Research Center High Cross, Madingley Road, Cambridge CB3 0EL, U.K.*
[3] *Statoil ASA, Statoil Forskningssenter, Arkitekt Ebbells veg 10, 7053 Ranheim, Norway.*

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ABSTRACT

Weglein, A.B., Liu, F., Li, X., Terenghi, P., Kragh, E., Mayhan, J.D., Wang, Z., Mispel, J., Amundsen, L., Liang, H., Tang, L. and Hsu, S.-Y., 2012. Inverse scattering series direct depth imaging without the velocity model: First field data examples. *Journal of Seismic Exploration*, 21: 1-28.

Now returning to our earlier slide: Responding to seismic challenges

- Two ways to address:
  - Find a more effective way to satisfy the assumption—thereby removing the violation
  - Find a way to remove the assumption or prerequisites

We encourage investment and support for both ways to address the open issues in velocity model building

- Develop fundamentally new and more effective methods to satisfy the assumptions, finding an adequate velocity model
- Further develop and deliver imaging methods that do not need to know, estimate, or determine a velocity model

In Conclusion

I. Velocity model building:

1. Improvements reported in providing a smooth velocity model
2. Purpose: to serve and improve structural imaging (migration)
3. For any velocity model—we recommend using Stolt Claerbout III imaging rather than RTM, to provide differential added value for imaging and inversion of specular and complex nonspecular and diffractive (pinchout) mapping, and for improved illumination and target and reservoir resolution

Conclusion (continued)

- We suggest using the most effective migration method, Stolt CIII for heterogeneous media (rather than RTM) in judging the efficacy of a velocity model—it makes no high frequency (ray-theory) assumption and can automatically accommodate every type of structure.
- Weglein et al 2016 SEG and Y. Zou et al 2017 SEG Wedge Model

Conclusion (continued)

- <u>What about Illumination?</u>
- To paraphrase Jon Claerbout "Waves are ubiquitous and do not have illumination issues." "Seismic processing methods that make asymptotic or high frequency approximation will introduce illumination issues," e.g., like Kirchhoff migration and RTM, that impose 'ray-like' constraints on imaging paths and can result in illumination issues. An interest in illumination points to using Stolt Claerbout III migration for heterogeneous media.

II. Velocity model building (continued)
   1. Providing an accurate discontinuous velocity (and density etc.) model is beyond all current capability.
   2. Direct determination of changes in earth mechanical properties at the imaged target is beyond current capability

III. Items I and II above directly communicate that removing multiples will remain a high priority and pressing need now and for the foreseeable future.

Conclusion (continued)

IV. What about using multiples?

1. Removing and using multiples are after the <u>same</u> exact objective, imaging primaries
2. Recorded and unrecorded primaries
3. A recorded multiple can be useful
   - When it consists of two subevents, one recorded and the second subevent an unrecorded primary

- After the recorded multiple is ‘used’ it must be removed to image recorded primaries
- To find an approximate image of an unrecorded primary, unrecorded multiples must be removed
- Summary:
  - To image recorded primaries, recorded multiples must be removed
  - To image unrecorded primaries, unrecorded multiples must be removed

- All the above conclusions on the need to remove recorded and unrecorded multiples derive from the current capability of (at most) determining a smooth velocity model for migration.

Thank you to Fons Ten Kroode and the entire SEG/DGS Workshop organizing committee for the honor and privilege of delivering this closing presentation.

Thanks to the sponsors of M-OSRP for their encouragement and support of this research.

A. B. W. thanks Samuel Oedi, M-OSRP/Physics Dept./UH for his assist in the preparation of these slides.

A selection among of additional references for the topics and discussion in this presentation.

- See link: **SEG/KOC Workshop: Seismic Multiples, the Challenges and the Way Forward" in Kuwait December 3-5 2019 .**
  - " A new perspective on removing and using multiples | they have the same exact goal | imaging primaries, recorded and unrecorded primaries | Recent advances in multiple removal".
  - https://youtu.be/sD89_418h1A
- http://www.mosrp.uh.edu/people/faculty/arthur-weglein
- **Google Scholar:** Arthur Weglein
- **Recent advances in the physics of imaging and potentially game- changing Q compensation without knowing, estimating or determining Q** (for improved resolution, amplitude analysis and illumination) assuring increased low and high frequency benefit for petroleum exploration
  - http://www.uh.edu/nsm/physics/news-events/stories/2018/0525-seismic-processing.php
- **WebTalk - Séries de Conversas Geofísicas com Dr. Arthur Weglein e Msc. Odette Aragão – YouTube**
  - https://www.youtube.com/watch?v=iir4cuk50Cw

- For the specific interest of this Workshop
  - **Game-changing migration**
  - **Petrobras invited us to present at a game changing seminar series- thought that might be of interest**
    - M-OSRP Invited Presentation at the Petrobras Workshop on Game Changing Seismic Technology
    - http://mosrp.uh.edu/news/invited-presentation-petrobras-workshop-aug-2016
  - **On direct inversion for FWI objectives**
    - Key–note address, Abu Dhabi, March 31st , 2015 presented at the SEG FWI, Workshop Filling the gaps in Abu-Dhabi
  - **Q compensation without Q**
    - Yanglei Zou and Weglein JSE 2018
    - **Recent advances in the physics of imaging and potentially game- changing Q compensation without knowing, estimating or determining Q** (for improved resolution, amplitude analysis and illumination) assuring increased low and high frequency benefit for petroleum exploration
    - http://www.uh.edu/nsm/physics/news-events/stories/2018/0525-seismic-processing.php

DIRECT NON-LINEAR ACOUSTIC AND ELASTIC INVERSION: TOWARDS FUNDAMENTALLY NEW COMPREHENSIVE AND REALISTIC TARGET IDENTIFICATION

A Dissertation

Presented to

the Faculty of the Department of Physics

University of Houston

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

By

Haiyan Zhang

December 2006

GEOPHYSICS, VOL. 74, NO. 6 (NOVEMBER-DECEMBER 2009); P. WCD29–WCD39, 6 FIGS.
10.1190/1.3256283

Direct nonlinear inversion of 1D acoustic media using inverse scattering subseries

Haiyan Zhang[1] and Arthur B. Weglein[2]

ABSTRACT

A task-specific, multiparameter (more than one mechanical property changes across a reflector), direct nonlinear inversion subseries of the inverse-scattering series is derived and tested for an acoustic medium in which velocity and density vary vertically. Task-specific means that terms in the distinct subseries corresponding to tasks for imaging only and inversion only are identified and separated. Direct means there are formulas that solve explicitly for and output the physical properties, without, e.g., search algorithms, model matching and optimization schemes, and proxies that typically characterize indirect methods. Numerical test results with analytic data indicate that one term beyond linear provides added value beyond standard linear techniques, and the improved estimates are valid over a larger range of angles. The direct acoustic inversion is nonlinear. It serves as an important step for new concepts and methods to guide the much more complicated and minimally realistic elastic inverse for exploration seismology target identification purposes.

sumptions of the former methods (like small-contrast assumptions) often are violated in practice and can cause erroneous predictions. The latter category usually involves a significant and often daunting computation effort (especially in multidimensional cases) and sometimes can report erroneous or ambiguous results.

To provide more accurate and reliable target identification, especially with large-contrast, large-angle target geometry, we develop a more comprehensive multiparameter, multidimensional, direct-nonlinear-inversion framework based on the inverse-scattering task-specific subseries (see, for example, Weglein et al., 2003). The inverse-scattering series has a tremendous generality and comprehensiveness, allowing many distinct traditional processing objectives to be achieved within a single framework, but without the traditional need to provide information about the properties that govern actual wave propagation in the earth.

It begins with scattering theory, which is the relationship between the perturbation or alteration in the properties of a medium and the concomitant perturbation or change in the wavefield. The relationship between those two changes is always nonlinear. Any change in a medium will result in a change in the wavefield that is nonlinearly related to that physical property change. Here we examine the relationship between the perturbation in a medium and the perturbation in a

ADDRESSING SEVERAL KEY OUTSTANDING ISSUES AND EXTENDING THE CAPABILITY OF THE INVERSE SCATTERING SUBSERIES FOR INTERNAL MULTIPLE ATTENUATION, DEPTH IMAGING, AND PARAMETER ESTIMATION

A Dissertation

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By

Hong Liang

December 2013

The first migration method that is equally effective for all acquired frequencies for imaging and inverting at the target and reservoir

Arthur B. Weglein, Yanglei Zou*, Qiang Fu*, Fang Liu*, Jing Wu*, Chao Ma*, Robert H. Stolt†, Xinglu Lin* and James D. Mayhan**

Summary

There is an industry wide interest in acquiring lower frequency seismic data. There is also an interest in assuring that the broadband data provides added value in processing and interpretation, to better resolve structure and to provide improved amplitude analysis at the target and at the reservoir. There are industry reports that when comparing the new and more expensively acquired broadband lower frequency data with conventional recorded data, taken over a same region, that these two datasets have the expected difference in frequency spectrum and appearance, but they provide little or no difference in structural improvement or added benefit for amplitude analysis at the target and reservoir. The methods that take recorded data and determine structure and perform amplitude analysis are migration and migration-inversion, respectively. There are two objectives of this paper: (1) to demonstrate that all current migration and migration inversion methods make high frequency asymptotic assumptions, that consequently do not provide for equal effectiveness at all recorded frequencies, at the target and reservoir. The consequence is that in the process of migration, they lose or discount the information in the newly acquired lowest frequency components in the broad band data, and (2) we address that problem, with the first migration method that will be equally effective at all frequencies at the target and reservoir, and will allow the broad band lower frequency data to provide improved structure and more effective amplitude analysis.

Seismic acquisition and seismic processing must be consistent and aligned to provide interpretive value from broad band data.

Introduction

Migration methods that use wave theory for seismic imaging have two components: (1) a wave propagation model, and (2) an imaging condition.

We will examine each of these two components in this paper. After a brief general introduction, the focus will be on the specific topic of this paper: the frequency fidelity of migration algorithms.

That analysis leads to a new and first migration that is equally effective at the target and/or the reservoir. A paper, Weglein (2016), provides a detailed development of this new migration method. In that paper, the new migration method is used to provide a definitive response to the role of primaries and multiples in seismic processing. This paper focuses on the frequency fidelity properties of all current and the new migration method.

For the imaging principle component, a good reference to start with is Jon Claerbout's 1971 landmark contribution. He listed three imaging principles: the exploding-reflector model which is for stacked or zero offset data. We call this Claerbout imaging principle I. The second imaging principle is the time space coincidence of up and downgoing waves. Waves propagate down from the source, are incident on the reflector and the reflector then generates a reflected up-going wave. According to Claerbout II (CII), the reflector exists at the location in space where the wave that is downward propagating from the source and the up wave from the reflector are at the same time and space.

Claerbout III (CIII) imaging starts with surface source and receiver data, and predicts what a source and receiver would record inside the earth. The CIII imaging principle then arranges the predicted source and receiver to be coincident and asks for $t = 0$. If the predicted coincident source and receiver experiment at depth is proximal to a reflector you get a non-zero result at time equals zero. CIII provides a direct and definitive yes or no at every subsurface point.

These three imaging conditions will give exactly the same result for a normal incident spike plane wave on a single horizontal reflector.

Claerbout II and III are of central industry interest today, since we currently process pre-stacked data. Imaging condition II and III will produce different results for a separated source and receiver located in a homogeneous half space above a single horizontal reflector. That difference forms a central and key message of this paper.

Before we undertake that comparison, let us take a look at a realization of the CIII imaging principle. Stolt's 1978 landmark contribution realized CIII imaging in the Fourier domain.

Stolt FK migration is

$$
\begin{aligned}
M^{stolt}(x,z) = {} & \frac{1}{(2\pi)^3}\iiint d\omega dx_g dx_s dk_{sx} \\
& \times \exp(-i(k_{sz}z + k_{sx}(x - x_s))) \\
& \times \int dk_{gx} \exp(-i(k_{gz}z + k_{gx}(x - x_s))) \\
& \times \int dt \exp(i\omega t) D(x_g, x_s, t). \qquad (1)
\end{aligned}
$$

*M-OSRP, 617 Science & Research Bldg. 1, University of Houston, Houston, Texas 77204.

†2622 Mockingbird Drive, Ponca City, Oklahoma 74604.

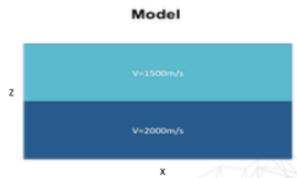

Fig. 1: *A numerical example of Claerbout II imaging (current leading edge RTM) for a single reflector with a homogeneous velocity model (one shot gather) (Yanglei Zou and Weglein, 2014).*

The weighted sum of recorded data, summed over receivers basically predicts the receiver experiment at depth, for a source on the surface. The sum over sources predicts the source in the subsurface, as well. Then the predicted source and receiver experiment is output for a coincident source and receiver, and at time equals zero; it defines a CIII image. Each step (integral) in this Stolt Fourier form of CIII has a specific physically interpretable purpose towards the CIII image.

Stolt made two extensions to Claerbout III. One was retaining the k_h information, angle dependent information at the target for structure and amplitude analysis, and in addition, introduced a point reflectivity. That point reflectivity automatically provides the specular reflection coefficient if there is one. It also provides a point reflectivity, an operator, which you can use for structure, which is non-planar, and to perform subsequent amplitude analysis. Those two extensions to get plane wave reflection coefficients and point reflectivity are only realizable in Claerbout III. Claerbout II cannot be extended and generalized in these two ways. Claerbout II is the basis and starting point for all current RTM methods. Hence, all RTM methods have certain intrinsic limitations, in terms of the ability to interpret images.

Claerbout II imaging

$$I(\vec{x}) = \sum_{\vec{x}_s}\sum_{\omega} S'(\vec{x}_s, \vec{x}, \omega) R(\vec{x}_s, \vec{x}, \omega). \qquad (2)$$

R is the reflection data (for a shot record), run backwards, and S is the source wavefield.

The CII imaging is somewhat ad-hoc and not nearly on the same firm physical foundation and as interpretable as CIII.

We compare the CII and CIII where it is not a propagation issue, and where the structure is simple, that is, we consider a homogeneous medium above a single horizontal reflector. We will apply Claerbout II and Claerbout III and examine the differences.

In Figure 1, we see the model. The migration velocity here is $1500 m/s$.

Figure 2 is the result from Claerbout II for one shot record. There is an inconsistency in the image. The bottom image in Figure 2 shows a blow up to see the lateral inconsistency in the CII image. If you want to associate the image with something like structure and/or reflectivity, you are not obtaining something that is consistent in the simplest possible example.

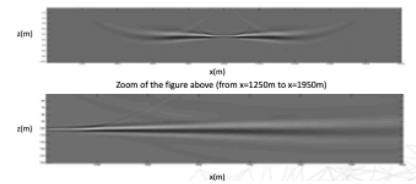

Fig. 2: *A numerical example of Claerbout II imaging (current leading edge RTM) for a single reflector with a homogeneous velocity model (one shot gather) (Yanglei Zou and Weglein, 2014).*

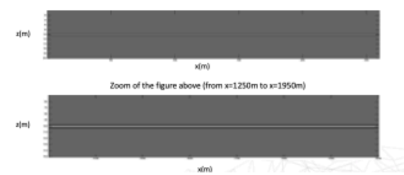

Fig. 3: *A numerical example of Claerbout III Stolt migration for a single reflector (Yanglei Zou and Weglein, 2014).*

Figure 3 shows the equivalent one shot record image of the Claerbout III Stolt migration.

Stolt's CIII produces a consistent and interpretable image. What people do in practice, with CII for one shot record is they stack over sources. They treat the CII algorithm as if it was intrinsically flawed and noisy. In Claerbout III, the sum over receivers, dx_g, is required to bring the receiver down, the sum over sources, dx_s, is required to bring the source down.

The sum over sources in Claerbout III is not fixing something that is inconsistent and intrinsically amiss, as the sum over sources seeks to mitigate in Claerbout II. There is no physics in CII to the sum over sources.

Now set the migration velocity be a discontinuous function c_0 over c_1 in Figure 4. In Figure 5, we perform Claerbout II for one trace, one source and one receiver and output the result. You find this ellipse and these (in)famous rabbit ears due to the c_0, c_1 migration velocity model. Faqi Liu, et al, have provided a method to remove rabbit ears in Claerbout II when imaging with a discontinuous velocity.

Even for the single horizontal reflector and with rabbit ear removal the CII image is not consistent. And in fact,

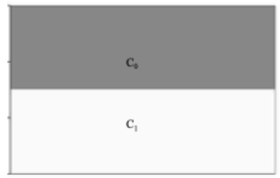

Fig. 4: *The velocity model.*

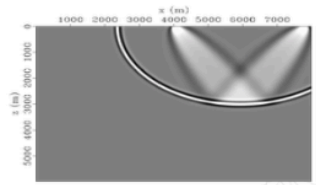

Fig. 5: *Claerbout II RTM image for one trace.*

the reduction or removal of the rabbit ears can have a negative impact on the image itself. Let's compare this to Claerbout III.

The new CIII migration in Figure 6 for two way propagating waves (from equation 4) produces the coincident source and receiver above and below the reflector with a light and dark amplitude for R_1 and $-R_1$, respectively. There are no rabbit ears in the new CIII (equation 4). With this new two way wave propagating CIII migration, you can, e.g., obtain the reflection coefficient from above and from below a top salt reflector.

How do high frequency approximations/ assumptions enter a migration algorithm?

How do you know if a migration method has made a high

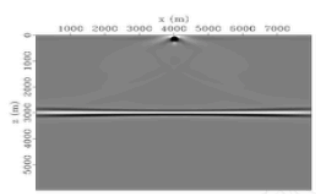

Fig. 6: *Claerbout II RTM image after artifacts removal. Please note the inconsistent image along the reflector.*

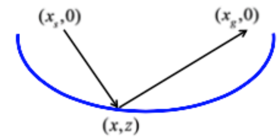

Fig. 7: *(1) If there is a travel time curve of candidate images within the method, it is a high frequency "ray theory" approximation/assumption.* $t = r/c$ *where,* $r = r_g + r_s = \sqrt{(x_g - x)^2 + z^2} + \sqrt{(x_s - x)^2 + z^2}$.

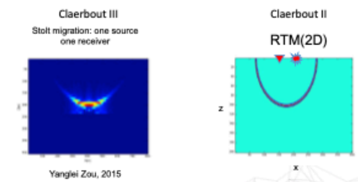

Fig. 8: *Imaging Conditions and High Frequency Assumptions. Left panel: No high frequency assumption. Right panel: High frequency assumption.*

frequency approximation?

If you have a picture shown in Figure 7 (a set of candidate images in the migration process) at any step or stage in the migration method, then the migration method has made an asymptotic high frequency assumption/approximation. As we saw for Claerbout II, for one source and one receiver, the image is an ellipse. If you have a travel time ellipse of candidate images, that's an absolute and definite indication that the migration method has made a high-frequency approximation. This picture (Figure 7) is a ray-theory picture.

In Figure 8, we compare the results of CII and III for one source and one receiver, CII provides an ellipse while CIII does not. CIII provides a local image. In CII, in this simplest case, where the data is perfect and the medium is homogeneous, the contribution from one source and one receiver, you obtain a set of candidates. CIII will never provide candidates. CIII will bring you to a point in the earth where you have a coincident source and receiver experiment. At time equals zero, if there is a non zero result, you are at a reflector, there is a structure there, not a possible or candidate structure. The result from Claerbout II is a set of candidates of possible structure. That's intrinsic to Claerbout II, hence intrinsic to all current RTM. So, if you are doing RTM today and any extension of it, understand that you have made a high frequency approximation in your migration methods. Similarly Kirchhoff migration is an asymptotic high frequency approximate of Stolt CIII (see Figure 9)

There are other ways that high frequency approximations can enter migration methods. If you made a stationary phase approximation, the migration method is a high fre-

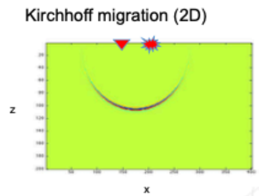

Fig. 9: *Kirchhoff migration for a single source and receiver (Yanglei Zou et al, 2015). High Frequency approximation from a stationary phase approximation.*

quency approximation.

There is another more subtle way that high frequency approximations can enter migration methods. Let's say, we are in Claerbout III (we are predicting the source and receiver experiment at depth) and lets assume a smooth velocity model. If in that smooth velocity model, you were assuming at every point, that the wave is moving in one direction, then you have made a high frequency approximation, even though you are using Claerbout III imaging. The only time that the wave is moving in one direction at a given point is in a homogeneous medium. As soon as you have any deviation from homogeneous, at every point in that medium, part of that wave is moving down and part of wave is moving up. If you are assuming it is only moving in one direction at one point (e.g., using WKBJ or diving waves), you have made a high frequency approximation.

All CII imaging, i.e., all RTM methods today are from the imaging principle itself, high frequency approximations/assumptions regardless of how they are implemented. Equation 3 represents a Green's theorem formulation of CIII for one way waves and is equivalent to Stolt migration equation 1. G_0^{-D} is an anticausal Green's function that vanishes on the measurement surface. For a heterogeneous medium assuming one way propagation, at a point (even if you assume its overall downgoing and then upgoing, e.g., between source and reflector, and then separately, first downgoing and then upgoing from reflector to receiver) is a high frequency approximation, even if you are adopting a CIII imaging principle.

$$P = \int_{S_s} \frac{\partial G_0^{-D}}{\partial z_s} \int_{S_g} \frac{\partial G_0^{-D}}{\partial z_g} P dS_g dS_s \qquad (3)$$

Prestack Stolt migration (Green, 1-way waves)

Equation 4 is the new migration method of this paper. It is a CIII imaging for a heterogeneous medium, that doesn't assume one-way propagation at either a point or separately, overall between source and reflector, and, reflector to receiver. G_0^{DN} is the Green's function for the heterogeneous medium that vanishes along with its normal derivative at the lower surface of the migration volume (Weglein et al., 2011b).

Fig. 10: *The new M-OSRP Claerbout III (Stolt extended) migration for 2 way wave propagation. The example with c_0/c_1 velocity. The image both above and beneath the reflector. No "rabbit ears". Consistent image along the reflector. Light color image from above. Dark color image from below. (Qiang Fu and Weglein, 2015)*

Equation 4 is the first migration method that is equally effective at all frequencies at the target and at the reservoir. Equation 4 was used in obtaining the result above and below the reflector in Figure 10.

$$P = \int_{S_s} \left[\frac{\partial G_0^{DN}}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} P + \frac{\partial P}{\partial z_g} G_0^{DN} \right\} dS_g \right.$$
$$\left. + G_0^{DN} \frac{\partial}{\partial z_s} \int_{S_g} \left\{ \frac{\partial G_0^{DN}}{\partial z_g} P + \frac{\partial P}{\partial z_g} G_0^{DN} \right\} dS_g \right] dS_s \qquad (4)$$

(Green, 2-way waves) for details see Weglein et al. (2011a,b) and F. Liu and Weglein (2014)

Conclusion

To obtain broad band benefit and added value requires effective deghosting and a migration method that treats all frequencies with equal effectiveness at the target and reservoir. This paper provides a first migration algorithm with those qualities and benefits. Seismic acquisition and processing must be consistent and aligned to provide interpretive value at the target and reservoir from broadband data.

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Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2016 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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A wedge resolution comparison between RTM and the first migration method that is equally effective at all frequencies at the target: tests and analysis with both conventional and broadband data

Yanglei Zou, Qiang Fu, and Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston

SUMMARY

Acquiring lower-frequency seismic data is an industry-wide interest. There are industry reports that (1) when comparing the new and more expensively acquired broad-band lower-frequency data with conventional recorded data, taken over a same region, these two data sets have the expected difference in frequency spectrum and appearance, but (2) they often provide less than the hoped for difference in structural resolution improvement or added benefit for amplitude analysis at the target and reservoir. In Weglein et al. (2016) and Q. Fu et al. (2017), they demonstrate that all current migration and migration-inversion methods make high-resolution asymptotic assumptions. Consequently, in the process of migration, they lose or discount the information in the newly acquired lowest-frequency components in the broadband data. The new Stolt extended Claerbout III migration for heterogeneous media (We-

Claerbout imaging principle II (CII). Waves propagate down from the source, are incident on the reflector, and the reflector generates a reflected upgoing wave. According to CII, the reflector exists at the location in space where the wave that is downward propagating from the source and the upwave from the reflector are at the same time and space. All RTM methods are based on RTM (CII) imaging principle and we after refer to RTM in this paper as RTM (CII). The third is Claerbout imaging principle III (CIII), which starts with surface source and receiver data and predicts what a source and receiver would record inside the earth. CIII then arranges the predicted source and receiver to be coincident and asks for $t = 0$. If the predicted coincident source and receiver experiment at depth is proximal to a reflector one gets a non-zero result at time equals zero. Stolt and his colleagues provided several major extensions of CIII and we refer to that category of imaging principles/methods as Stolt extended CIII.

A direct inverse method for subsurface properties: The conceptual and practical benefit and added value in comparison with all current indirect methods, for example, amplitude-variation-with-offset and full-waveform inversion

Arthur B. Weglein[1]

Abstract

Direct inverse methods solve the problem of interest; in addition, they communicate whether the problem of interest is the problem that we (the seismic industry) need to be interested in. When a direct solution does not result in an improved drill success rate, we know that the problem we have chosen to solve is not the right problem — because the solution is direct and cannot be the issue. On the other hand, with an indirect method, if the result is not an improved drill success rate, then the issue can be either the chosen problem, or the particular choice within the plethora of indirect solution methods, or both. The inverse scattering series (ISS) is the only direct inversion method for a multidimensional subsurface. Solving a forward problem in an inverse sense is not equivalent to a direct inverse solution. All current methods for parameter estimation, e.g., amplitude-variation-with-offset and full-waveform inversion, are solving a forward problem in an inverse sense and are indirect inversion methods. The direct ISS method for determining earth material properties defines the precise data required and the algorithms that directly output earth mechanical properties. For an elastic model of the subsurface, the required data are a matrix of multicomponent data, and a complete set of shot records, with only primaries. With indirect methods, any data can be matched: one trace, one or several shot records, one component, multicomponent, with primaries only or primaries and multiples. Added to that are the innumerable choices of cost functions, generalized inverses, and local and global search engines. Direct and indirect parameter inversion are compared. The direct ISS method has more rapid convergence and a broader region of convergence. The difference in effectiveness increases as subsurface circumstances become more realistic and complex, in particular with band-limited noisy data.

Introduction

Seismic processing is an inverse problem to determine the properties of a medium from measurements of a wavefield exterior to the medium. The ultimate inversion objective of seismic processing in seismic exploration is to use recorded reflection data to extract useful subsurface information that is relevant to the location and production of hydrocarbons. There is typically a coupled chain of intermediate steps and processing that takes place toward that objective, and I refer to each of those intermediate steps, stages, and tasks as objectives "associated with inversion" or inverse tasks toward the ultimate subsurface information extraction goal and objective. All seismic processing methods that are used to extract subsurface information make assumptions and have prerequisites.

A seismic method will be effective when those assumptions/conditions/requirements are satisfied. When those assumptions are not satisfied, the method can have difficulty and/or will fail. That failure can and will contribute to processing and interpretation difficulties with subsequent dry-hole exploration well drilling or drilling suboptimal appraisal and development wells.

Challenges in seismic processing and seismic exploration and production are derived from the violation of assumptions/requirements behind seismic processing methods. Advances in seismic processing effectiveness are measured in terms of whether the new capability results in/contributes to more successful plays and better informed decisions and an increased rate of successful drilling.

[1]University of Houston, M-OSRP/Physics Department, Houston, Texas, USA. E-mail: aweglein@uh.edu.

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The purpose of seismic research is to identify and address seismic challenges and to thereby add more effective options to the seismic processing toolbox. These new options can be called upon when indicated, appropriate, and necessary as circumstances dictate.

No toolbox option is the appropriate choice under all circumstances. For example, the most effective method, from a technical perspective, might be more than is necessary and needed, under a given circumstance, and a less effective and often less costly option could be the appropriate and indicated choice. Under other more complex and daunting circumstances, the more effective and (perhaps) more costly option will be the only possible choice that is able to achieve the objective of that processing task and interpretation goal. The objective is to expand the number of options in the seismic toolbox to allow a capable response to a larger number of circumstances. As I will point out below, "identify the problem" is the first, the essential, and sometimes the most difficult (and often the most ignored and/or underappreciated) aspect of seismic research.

Identifying and delineating the violation of assumptions behind seismic processing methods is an absolutely essential first step in a strategy and plan for developing a response to prioritizing and pressing seismic exploration challenges. This paper provides a new insight, and advance for the first and critical step of addressing seismic processing challenges: problem identification.

I explain in detail and exemplify why only a direct inversion method can help us to decide whether the problem we (the seismic industry) are interested in addressing is, in fact, the problem we need to address.

Seismic processing methods can be classified as based on either statistical models and principles or wave-theory concepts and approaches. Wave-theory concepts used in seismic processing can be further catalogued as modeling and inversion.

In the next section, I describe these two wave theory approaches to seismic processing, that is, modeling and inversion, and I will further distinguish between direct and indirect inversion methods. That clarification represents a central theme and objective of this paper.

Modeling and inversion

Modeling, as a seismic processing tool, starts with a prescribed wavefield source mechanism and a model type (e.g., acoustic, elastic, anisotropic, or anelastic), and then properties are defined within the model type for a given medium (e.g., velocities, density, and attenuation Q). The modeling procedure then provides the seismic wavefield that the energy source produces at all points inside and outside the medium.

Inversion also starts with an assumed known and prescribed energy source outside the medium. In addition, the wavefield outside the medium is assumed to be recorded and known. The objective of seismic inversion is to use the latter source description and wavefield measurement information to make inferences about the subsurface medium that are relevant to the location and production of hydrocarbons.

Direct and indirect inversion

Inversion methods can be classified as direct or indirect. A direct inversion method solves an inverse problem (as its name suggests) directly. On the other hand, an indirect inversion method seeks to solve an inverse problem circuitously through indirect approaches that often call up assumed aligned objectives or conditions. There are times when the indirect approach will seek to satisfy necessary (but typically not sufficient) conditions, and properties, and it is often mistakenly considered and treated as though it was equivalent to a direct method and solution. Indirect methods come in many varieties; some are obvious, and others are more subtle and harder to identify as being indirect. Among indicators, identifiers, and examples of "indirect" inverse solutions (Weglein, 2015a) are (1) model matching, (2) objective/cost functions, (3) local and global-search algorithms, (4) iterative linear inversion, (5) methods corresponding to necessary but not sufficient conditions, e.g., common-image gather flatness as an indirect migration velocity analysis method, and (6) solving a forward problem in an inverse sense, e.g., amplitude-variation-with-offset (AVO) and full-waveform inversion (FWI). Regarding the last indirect indicator, item (6), I will show that solving a forward problem in an inverse sense is not equivalent to a direct inverse solution for those same objectives.

As a simple illustration, a quadratic equation

$$ax^2 + bx + c = 0 \tag{1}$$

can be solved through a direct method as

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{2}$$

or it can be solved by an indirect method searching for x, such that, e.g., some functional of

$$(ax^2 + bx + c)^2 \tag{3}$$

is a minimum.

In the next section, this example will be further discussed and examined as a way to introduce and develop fundamental concepts in a simple and transparent context. The lesson gleaned from that simple example will later (in this paper) be extended and applied to the more complicated and relevant seismic inverse formulations and methods. In Weglein (2013), there is an introduction to the subject of direct and indirect inverse solutions, which provides a useful background reference for this paper and contains several indirect inversion references (Blum, 1972; Keys and Weglein, 1983; Gauthier et al., 1986; Tarantola, 1986, 1987; Crase et al., 1990; Symes and Carazzone, 1991; Chavent and Jacewitz, 1995; Matson, 1997; Nolan and Symes, 1997; Weglein and Matson, 1998; Biondi and Sava, 1999;

Brandsberg-Dahl et al., 1999; Pratt, 1999, Pratt and Shipp, 1999; Rickett and Sava, 2002; Weglein et al., 2002, 2011; Sava and Fomel, 2003; Biondi and Symes, 2004; Sava et al., 2005; Valenciano et al., 2006; Iledare and Kaiser, 2007; Ben-Hadj-ali et al., 2008; Symes, 2008; Vigh and Starr, 2008; Baumstein et al., 2009; Ben-Hadj-ali et al., 2009; Brossier et al., 2009; Hawthorn, 2009; Sirgue et al., 2009, 2010, 2012; Liang et al., 2010; Ferreira, 2011; Fichtner, 2011; Li et al., 2011, Luo et al., 2011; Anderson et al., 2012; Guasch et al., 2012, Kapoor et al., 2012; Weglein, 2012a, 2012b; Zhou et al., 2012; Zhang and Biondi, 2013).

The important quadratic equation example

The direct quadratic formula solution equation 2 explicitly and directly outputs the exact roots (for all values of a, b, and c) when the roots are real and distinct, a real double root, and imaginary and complex roots. The quadratic equation and quadratic solution provide a very simple and insightful example. How would a search algorithm know after a double root is found that it is the only root and not to keep looking and searching forever for a second, nonexistent, root? How would a search algorithm know to search for only real or for real and complex roots? How would a search algorithm accurately locate an irrational root such as $\sqrt{3} \cong 1.732\ldots$, as $x = (-b \pm \sqrt{b^2 - 4ac})/2a$ would directly and precisely and immediately produce? Indirect methods such as model matching and seeking and searching and determining roots as in equation 3 are ad hoc and do not derive from a firm framework and foundation and never provide the confidence that we (the seismic industry) are actually solving the problem of interest.

What is the point in discussing the quadratic formula? And what is the practical big deal about a direct solution?

How can this example and discussion of the quadratic equation possibly be relevant to exploration seismology? Please imagine for a moment that equation 1 $ax^2 + bx + c = 0$ was an equation whose inverse and solution for x given by equation 2 $x = (-b \pm \sqrt{b^2 - 4ac})/2a$ had seismic exploration drill location prediction consequence. And furthermore, suppose that this direct solution for x did not lead to successful and/or improved drilling decisions. Under the latter circumstance, we (the seismic industry) could not blame or question the method of solution of equation 1 because equation 2 is direct and unquestionably solving equation 1. If equation 2 was not producing useful and beneficial results, we know that our starting equation 1 is the issue, and we have identified the problem. The problem we thought we need to solve (equation 1) is not the problem we need to solve. In contrast with equation 3, an indirect method, any lack of drilling prediction improvement and added value or other negative exploration consequences could be due to either the equation you are seeking to invert and/or the boundless, unlimited selection, and the plethora of indirect methods using either partial or full recorded wavefields.

That lack of clarity and definitiveness within indirect methods obfuscates the underlying issue and makes identification of the problem (and what is behind a seismic challenge) considerably more difficult to identify and to define. Indirect methods with search engines, such as equation 3, lead to "workshops" for solving equation 1 and grasping at mega high-performance computing (HPC) straws (and capital expenditure investment for buildings full of HPC) that are required to search, seek, and locate "solutions." The more HPC we invest in, and is required, the more we are literally "buying-in," and as stake-holders, we become committed and therefore convinced of the unquestioned validity of the starting point and our indirect thinking and methodology.

Therefore, beyond the benefit of a direct method, such as equation 2 providing assurance that we are actually solving the problem of interest (equation 1), there is the unique problem location and identification benefit of a direct inverse when a seismic analysis, processing, and interpretation produces unsatisfactory E&P results.

To bring this (quadratic equation example) closer to the seismic experience, please imagine hypothetically that we are not satisfied (in terms of improved drill location and success rate) with a direct inverse of the elastic-isotropic equation for amplitude analysis. Because we were using a direct inversion solution, we know we need to go to a different starting point, perhaps with a more complete and realistic model of wave propagation because we can exclude the direct inverse solution method as the problem and issue. That is an example of determining that a problem of interest is not the same problem we need to be interested in.

How to distinguish between the "problem of interest" and the problem we need to be interested in

Direct inverse methods provide value for knowing that you have actually solved the problem of interest. Furthermore, with direct inverse solutions, there is the enormous additional value of determining whether our starting point, the problem of interest, is in fact the problem we need to be interested in.

Scattering theory and the forward and inverse scattering series: The basis of direct inversion theory and algorithms

Scattering theory is a form of perturbation theory. It provides a direct inversion method for all seismic processing objectives realized by a distinct isolated task subseries of the inverse scattering series (ISS) (Weglein et al., 2003). Each term in the ISS (and the distinct and specific collection of terms that achieve different specific inversion associated tasks) is computable (1) directly and (2) in terms of recorded reflection data and without any subsurface information known, estimated, or determined before, during, or after the task is performed and the specific processing objective is achieved.

For certain distinct tasks, and subseries, e.g., free-surface multiple elimination and internal multiple attenuation, the algorithms not only do not require subsurface information but in addition possess the absolutely remarkable property of being independent of the earth model type (Weglein et al., 2003). That is, the distinct ISS free-surface and internal multiple algorithms are unchanged, without a single line of code having the slightest change for acoustic, elastic, anisotropic, and anelastic earth models (Weglein et al., 2003; Wu and Weglein, 2014). For those who subscribe to indirect inversion methods as, e.g., the "be-all and end-all" of inversion with various model matching approaches, it would be a useful exercise for them to consider how they would formulate a model-type-independent model-matching scheme for free-surface and internal multiple removal. It is not conceivable, let alone realizable, to have a model-type-independent model matching scheme.

For the specific topic and focus of this paper, the inversion task of parameter estimation, there is an obvious need to specify the model type and what parameters are to be determined. Hence, it is for that parameter estimation/medium property objective, and that model-type-specific ISS subseries, that the difference between the problem of interest and the problem that we need to be interested in, is relevant, central, and significant. Only direct inversion methods for earth mechanical properties provide that assumed earth model-type evaluation, clarity, and distinction.

The basic operator identity that relates a change in a medium and the change in the wavefield

A direct inverse solution for parameter estimation can be derived from an operator identity that relates the change in a medium's properties and the commensurate change in the wavefield. That operator identity is general and can accommodate any seismic model type, for example, acoustic, elastic, anisotropic, heterogeneous, and anelastic earth models. That operator identity can be the starting point and basis of (1) perturbative scattering-theory modeling methods and (2) a firm and solid math-physics foundation and framework for direct inverse methods.

Theory

Let us consider an energy source that generates a wave in a medium with prescribed properties. With the same energy source, let us consider a change in the medium and the resulting change in the wavefield inside and outside the medium. Scattering theory is a form of perturbation theory that relates a change (or perturbation) in a medium to a corresponding change (or perturbation) in the original wavefield. When the medium changes, the resulting wavefield changes. The direct inverse solution (Weglein et al., 2003; Zhang, 2006) for determining earth mechanical properties is derived from the operator identity that relates the change in a medium's properties and the commensurate change in the wavefield within and exterior to the medium. Let L_0, L, G_0, and G be the differential operators and Green's functions for the reference and actual media, respectively, that satisfy

$$L_0 G_0 = \delta \quad \text{and} \quad LG = \delta, \tag{4}$$

where δ is a Dirac delta function. I define the perturbation operator V and the scattered wavefield ψ_s as follows:

$$V \equiv L_0 - L \quad \text{and} \quad \psi_s \equiv G - G_0. \tag{5}$$

The operator identity

The relationship (called the Lippmann-Schwinger or scattering theory equation)

$$G = G_0 + G_0 V G \tag{6}$$

is an operator identity that follows from

$$L^{-1} = L_0^{-1} + L_0^{-1}(L_0 - L)L^{-1}, \tag{7}$$

and the definitions of L_0, L, and V.

Direct forward series and direct inverse series

The operator identity equation 6 (for a fixed-source function) is the exact relationship between changes in a medium and changes in the wavefield; it is a relationship between those quantities and not a solution. However, the operator identity equation 6 can be solved for G as

$$G = (1 - G_0 V)^{-1} G_0, \tag{8}$$

and expanded as

$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots . \tag{9}$$

The forward modeling of the wavefield G from equation 9 for a medium described by L is given in terms of the two parts of L, that is, L_0 and V. The differential operator L_0 enters through G_0, and V enters as V itself. Equation 9 communicates that modeling using scattering theory requires a complete and detailed knowledge of the earth model type and medium properties within the model type. Equation 9 communicates that any change in medium properties, V, will lead to a change in the wavefield, $G - G_0$ that is always nonlinearly related to the medium property change, V. Equation 9 is called the Born or Neumann series in the scattering theory literature (see, e.g., Taylor, 1972). Equation 9 has the form of a generalized geometric series

$$G - G_0 = S = ar + ar^2 + \cdots = \frac{ar}{1-r} \quad \text{for } |r| < 1, \tag{10}$$

where I identify $a = G_0$ and $r = VG_0$ in equation 9, and

$$S = S_1 + S_2 + S_3 + \cdots, \tag{11}$$

where the portion of S that is linear, quadratic, ... in r is

$$S_1 = ar,$$
$$S_2 = ar^2, \quad (12)$$
$$\vdots$$

and the sum is

$$S = \frac{ar}{1-r}, \quad \text{for } |r| < 1. \quad (13)$$

Solving equation 13 for r, in terms of S/a produces the inverse geometric series

$$r = \frac{S/a}{1+S/a} = S/a - (S/a)^2 + (S/a)^3 + \cdots$$
$$= r_1 + r_2 + r_3 + \cdots, \quad \text{when } |S/a| < 1, \quad (14)$$

where r_n is the portion of r that is nth order in S/a. When S is a geometric power series in r, then r is a geometric power series in S. The former is the forward series, and the latter is the inverse series. That is exactly what the inverse series represents: the inverse geometric series of the forward series equation 9. This is the simplest prototype of an inverse series for r, i.e., the inverse of the forward geometric series for S.

For the seismic inverse problem, I associate S with the measured data (see, e.g., Weglein et al., 2003)

$$S = (G - G_0)_{ms} = \text{Data}, \quad (15)$$

and the forward and inverse series follow from treating the forward solution as S in terms of V, and the inverse solution as V in terms of S (where S corresponds to the measured values of $G - G_0$). The inverse series is the analog of equation 14, where $r_1, r_2, \ldots$ are replaced with $V_1, V_2, \ldots$:

$$V = V_1 + V_2 + V_3 + \cdots, \quad (16)$$

where V_n is the portion of V that is nth order in the measured data D. Equation 9 is the forward-scattering series, and equation 16 is the ISS. The identity (equation 6) provides a generalized geometric forward series, a very special case of a Taylor series. A Taylor series of a function $S(r)$

$$S(r) = S(0) + S'(0)r + \frac{S''(0)r^2}{2} + \cdots$$
$$\text{and } s(r) = S(r) - S(0) = S'(0)r + \frac{S''(0)r^2}{2} + \cdots, \quad (17)$$

whereas the geometric series is

$$S(r) - \underbrace{S(0)}_{a} = ar + ar^2 + \cdots. \quad (18)$$

The Taylor series equation 17 reduces to the special case of a geometric series equation 18 if

$$S(0) = S'(0) = \frac{S''(0)}{2} = \cdots = a. \quad (19)$$

The geometric series equation 18 has an inverse series, whereas the Taylor series equation 17 does not. In general, a Taylor series does not have an inverse series. That is the reason that inversionists committed to a Taylor series starting point adopt the indirect linear updating approach, where a linear approximate Taylor series is inverted. They attempt through updating to make the linear form an ever more accurate approximate — and its premise and justification is entirely indirect and hence ad hoc — in the sense that some sort of iterative linear updating of a reference medium and model matching seek to satisfy a property that a solution might "reasonably" satisfy.

The relationship 9 provides a geometric forward series that honors equation 6 in contrast to a truncated Taylor series that does not.

All conventional current mainstream parameter estimation inversion, including iterative linear inversion, AVO, and FWI, are based on a forward Taylor series description of given data (where the chosen data can often be fundamentally and intrinsically inadequate from a direct inversion perspective), that do not honor and remain consistent with the identity equation 9.

Solving a forward problem in an inverse sense is not the same as solving an inverse problem directly

I will show that, in general, solving a forward problem in an inverse sense is not the same as solving an inverse problem directly. The exception is when the exact direct inverse is linear, as for example, in the theory of wave-equation migration (see, e.g., Claerbout, 1971; Stolt, 1978; Stolt and Weglein, 2012; Weglein et al., 2016). For wave-equation migration, given a velocity model, the migration and structure map output is a linear function of the input recorded reflection data.

To explain the latter statement, if I assume $S = ar$ (i.e., that there is an exact linear forward relationship between S and r), then $r = S/a$ is solving the inverse problem directly. In that case, solving the forward problem in an inverse sense is the same as solving the inverse problem directly; i.e., it provides a direct inverse solution.

However, if the forward exact relationship is nonlinear, for example,

$$S_n = ar + ar^2 + \cdots + ar^n,$$
$$S_n - ar - ar^2 - \cdots - ar^n = 0, \quad (20)$$

and solving the forward problem 20 in an inverse sense for r will have n roots, $r_1, r_2, \ldots, r_n$. As $n \to \infty$, the number of roots $\to \infty$. However, from the direct nonlinear forward problem $S = ar/(1-r)$, I found that the direct inverse solution $r = S/(a+S)$ has one real root.

This discussion above provides an extremely simple, transparent, and compelling illustration of how solving a forward problem in an inverse sense is not the same as solving the inverse problem directly when there is a nonlinear forward and nonlinear inverse problem. The difference between solving a forward problem in an inverse sense (e.g., using equation 9 to solve for V) and solving an inverse problem directly (e.g., equations 21–23) is much more serious, substantive, and practically significant the further I move away from a scalar single component acoustic framework. For example, it is hard to overstate the differences when examining the direct and indirect inversion of the elastic heterogeneous wave equation for earth mechanical properties and the consequences for structural and amplitude analysis and interpretation. This is a central flaw in many inverse approaches, including AVO and FWI (see Weglein, 2013).

The expansion of V in equation 16, in terms of G_0 and $D = (G - G_0)_{ms}$, the ISS (Weglein et al., 2003) can be obtained as

$$G_0 V_1 G_0 = D, \tag{21}$$

$$G_0 V_2 G_0 = -G_0 V_1 G_0 V_1 G_0, \tag{22}$$

$$\begin{aligned} G_0 V_3 G_0 = & - G_0 V_1 G_0 V_1 G_0 V_1 G_0 \\ & - G_0 V_1 G_0 V_2 G_0 - G_0 V_2 G_0 V_1 G_0, \\ & \vdots \end{aligned} \tag{23}$$

To illustrate how to solve equations 21–23, for V_1, V_2, and V_3, consider the marine case with L_0 corresponding to a homogeneous reference medium of water. Here, G_0 is the Green's function for propagation in water; D is the data measured, for example, with towed streamer acquisition; G is the total field that the hydrophone receiver records on the measurement surface; and G_0 is the field that the reference wave (due to L_0) would record at the receiver. The differential operator V then represents the difference between earth properties L and water properties L_0. The solution for V is found using

$$V = V_1 + V_2 + V_3 + \cdots, \tag{24}$$

where V_n is the portion of V that is nth order in the data D. Substituting equation 24 into the forward series equation 9, then evaluating equation 9 on the measurement surface and setting terms that are equal order in the data equal, I find equations 21–23. Solving equation 21 for V_1 involves the data D and G_0 (water-speed propagator) and solving for V_1 is analytic, and corresponds to a prestack water-speed Stolt *f-k* migration of the data D.

Hence, solving for V_1 involves an analytic water-speed *f-k* migration of data D. Solving for V_2 from equation 22 involves the same water-speed analytic Stolt *f-k* migration of $-G_0 V_1 G_0 V_1 G_0$, a quantity that depends on V_1 and G_0, where V_1 depends on data and water speed and G_0 is the water-speed Green's function. Each term in the series produces V_n as an analytic Stolt *f-k* migration of a new "effective data," where the effective data, the right side of equations 21–23, are multiplicative combinations of factors that only depend on the data D and G_0. Hence, every term in the ISS is directly computed in terms of data and water speed. That is the direct nonlinear inverse solution.

There are closed-form inverse solutions for a 1D earth and a normal incident plane wave (see, e.g., Ware and Aki, 1969), but the ISS is the only direct inverse method for a multidimensional subsurface.

The ISS provides a direct method for obtaining the subsurface properties contained within the differential operator L, by inverting the series order-by-order to solve for the perturbation operator V, using only the measured data D and a reference Green's function G_0, for any assumed earth model type. Equations 21–23 provide V in terms of $V_1, V_2, \ldots$, and each of the V_i is computable directly in terms of D and G_0. There is one equation (equation 21) that exactly produces V_1, and V_1 is the exact portion of V that is linear in the measured data D. The inverse operation to determine $V_1, V_2, V_3, \ldots$ is analytic, and it never is updated with band-limited data D. The band-limited nature of D never enters an updating process as occurs in iterative linear inversion, nonlinear AVO, and FWI.

The ISS and isolated task subseries

I can imagine that a set of tasks needs to be achieved to determine the subsurface properties V from recorded seismic data D. These tasks are achieved within equations 21–23. The inverse tasks (and processing objectives) that are within a direct inverse solution are (1) free-surface multiple removal, (2) internal multiple removal, (3) depth imaging, (4) Q compensation without Q, and (5) nonlinear direct parameter estimation. Each of these five tasks has its own task-specific subseries from the ISS for $V_1, V_2, \ldots$, and each of those tasks is achievable directly and without subsurface information (see, e.g., Weglein et al., 2003, 2012; Innanen and Lira, 2010). In Appendix A, I review the details of equations 21–23 for a 2D heterogeneous isotropic elastic medium.

Direct inverse and indirect inverse

Because iterative linear inversion is the concept and thinking behind many inverse approaches, I determined to make explicit the difference between that approach and a direct inverse method. The direct 2D elastic isotropic inverse solution described in Appendix A is not iterative linear inversion. Iterative linear inversion starts with equation 21. In that approach, I solve for V_1 and then change the reference medium iteratively. The new differential operator L_0' and the new reference medium G_0' satisfy

$$L_0' = L_0 - V_1 \quad \text{and} \quad L_0' G_0' = \delta. \tag{25}$$

In the indirect iterative linear approach, all steps basically relate to the linear relationship equation 21 with a new reference background medium, with differential operator L_0' and a new reference Green's function G_0', where in terms of the new updated reference L_0' equation 21 becomes

$$G_0'V_1'G_0' = D' = (G - G_0')_{ms}, \tag{26}$$

where V_1' is the portion of V linear in data $(G - G_0')_{ms}$. We can continue to update L_0' and G_0', and we hope that the indirect procedure is solving for the perturbation operator V. In contrast, the direct inverse solution (equations 16 and A-6) calls for a single unchanged reference medium for computing $V_1, V_2, \ldots$. For a homogeneous reference medium, $V_1, V_2, \ldots$ are each obtained by a single unchanged analytic inverse. We remind ourselves that the inverse to find V_1 from data is the same exact unchanged analytic inverse operation to find $V_2, V_3, \ldots$ from equations 21, 22, ..., which is completely distinct and different from equations 25 and 26 and higher iterates.

For ISS direct inversion, there are no numerical inverses, no generalized inverses, no inverses of matrices that are computed from and contain noisy band-limited data. The latter issue is terribly troublesome and difficult and is a serious practical problem, which does not exist or occur with direct ISS methods. The inverse of operators that contain and depend on band-limited noisy data is a central and intrinsic characteristic and practical pitfall of indirect methods, model matching, updating, and iterative linear inverse approaches (e.g., AVO and FWI).

Are there any circumstances in which the indirect iterative linear inversion and the direct ISS parameter estimation would be equivalent?

Are there any circumstances in which the ISS direct parameter inversion subseries would be equivalent to and correspond to the indirect iterative linear approach? Let us consider the simplest acoustic single-reflector model and a normal incident plane-wave reflection data experiment with ideal full band-width perfect data. Let the upper half-space have velocity c_0 and the lower half-space have velocity c_1 and then analyze these two methods (direct ISS parameter estimation and indirect iterative linear inversion) to use the reflected data event to determine the velocity of the lower half-space, c_1. Yang and Weglein (2015) examine and analyze this problem and compare the results of the direct ISS method and the indirect iterative linear inversion. They show that the direct ISS inversion to estimate c_1 converged to c_1 under all circumstances and all values of c_0 and c_1. In contrast, the indirect linear iterative inversion had a limited range of values of c_0 and c_1 where it converged to c_1, and in that range, it converged much slower than the direct ISS parameter estimation for c_1. The iterative linear inverse simply shut down and failed when the reflection coefficient R was greater than 1/4 (see Appendix B and Yang, 2014).

The direct ISS parameter estimation method converged to c_1 for any value of the reflection coefficient R. Hence, under the simplest possible circumstance, and providing the iterative linear method with an analytic Fréchet derivative, as a courtesy from and a gift delivered to the linear iterative from the ISS direct inversion method, the ranges of usefulness, validity, and relative effectiveness were never equivalent or comparable. With band-limited data and more complex earth models (e.g., elastic multiparameter), this gap in the range of validity, usefulness, and effectiveness will necessarily widen (see Zhang, 2006; Weglein, 2013). The indirect iterative linear inversion and the direct ISS parameter-estimation method are never equivalent, and there are absolutely no simple or complicated circumstances in which they are equally effective. The distinct ISS free-surface-multiple elimination subseries and internal-multiple attenuation subseries are not only not dependent on subsurface properties, but they are precisely the same unchanged algorithms for any earth model type.

There was an earlier time when free-surface multiples were modeled and subtracted. Multiple-removal methods have moved on. Parameter-estimation methods continue to be firmly connected to model matching and subtraction. That stark and immense difference between iterative linear updating model matching and the direct inversion inverse scattering methods is an essential point to consider and comprehend for those interested in understanding these methodologies and their seismic processing and interpretation consequences and value. It is not conceivable to even formulate an iterative linear model matching method that is not dependent on a specified model type — let alone to compare it with ISS model-type-independent algorithms.

Direct ISS parameter inversion: A time-lapse application

The direct inverse ISS elastic parameter estimation method (equation A-6) was successfully applied (Zhang et al., 2006) in a time-lapse sense to discriminate between pressure and fluid saturation changes. Traditional time-lapse estimation methods were unable to predict and match that direct inversion ISS discrimination.

Further substantive differences between iterative linear model matching inversion and direct inversion from the Lippmann-Schwinger equation and the ISS

The difference between iterative linear and the direct inverse of equation A-6 is much more substantive and serious than merely a different way to solve $G_0V_1G_0 = D$ (equation 21), for V_1. If equation 21 is someone's entire basic theory, you can mistakenly think that

$$\hat{D}^{\mathrm{PP}} = \hat{G}_0^{\mathrm{P}}\hat{V}_1^{\mathrm{PP}}\hat{G}_0^{\mathrm{P}} \tag{27}$$

is sufficient to update (generalizing equations 25 and 26)

$$\hat{D}'^{\mathrm{PP}} = \hat{G}_0'^{\mathrm{P}}\hat{V}_1'^{\mathrm{PP}}\hat{G}_0'^{\mathrm{P}}. \tag{28}$$

Please note that ^ indicates that variables are transformed to PS space. This step loses contact with and violates the basic operator identity $G = G_0 + G_0VG$ for the elastic wave equation. The fundamental identity $G = G_0 + G_0VG$ for the elastic wave equation is a nonlinear multiplicative matrix relationship. For the forward and inverse series, the input and output variables are matrices. The inverse solution for a change in an earth mechanical property has a nonlinear coupled dependence on all the data components

$$\begin{pmatrix} D^{PP} & D^{PS} \\ D^{SP} & D^{SS} \end{pmatrix}, \tag{29}$$

in 2D and the P, SH, SV 3×3 generalization in 3D (Stolt and Weglein, 2012, chapter 7).

A unique expansion of VG_0 in orders of measurement values of $(G - G_0)$ is

$$VG_0 = (VG_0)_1 + (VG_0)_2 + \cdots . \tag{30}$$

The scattering-theory equation allows that forward series form the opportunity to find a direct inverse solution. Substituting equation 30 into equation 9 and setting the terms of equal order in the data to be equal, I have $D = G_0V_1G_0$, where the higher order terms are $V_2, V_3, \ldots$, as given in Weglein et al. (2003, p. R33, equations 7–14).

For the elastic equation, V is a matrix and the relationship between the data and V_1 is

$$\begin{pmatrix} D^{PP} & D^{PS} \\ D^{SP} & D^{SS} \end{pmatrix} = \begin{pmatrix} G_0^P & 0 \\ 0 & G_0^S \end{pmatrix} \begin{pmatrix} V_1^{PP} & V_1^{PS} \\ V_1^{SP} & V_1^{SS} \end{pmatrix} \begin{pmatrix} G_0^P & 0 \\ 0 & G_0^S \end{pmatrix}, \tag{31}$$

$$V_1 = \begin{pmatrix} V_1^{PP} & V_1^{PS} \\ V_1^{SP} & V_1^{SS} \end{pmatrix}, \tag{32}$$

$$V = \begin{pmatrix} V^{PP} & V^{PS} \\ V^{SP} & V^{SS} \end{pmatrix}, \tag{33}$$

$$V = V_1 + V_2 + \cdots, \tag{34}$$

where V_1, V_2 are linear, quadratic contributions to V in terms of the data

$$D = \begin{pmatrix} D^{PP} & D^{PS} \\ D^{SP} & D^{SS} \end{pmatrix}. \tag{35}$$

The changes in elastic properties and density are contained in

$$V = \begin{pmatrix} V^{PP} & V^{PS} \\ V^{SP} & V^{SS} \end{pmatrix}, \tag{36}$$

and that leads to direct and explicit solutions for the changes in mechanical properties in orders of the data

$$D = \begin{pmatrix} D^{PP} & D^{PS} \\ D^{SP} & D^{SS} \end{pmatrix}, \tag{37}$$

$$\frac{\Delta\gamma}{\gamma} = \left(\frac{\Delta\gamma}{\gamma}\right)_1 + \left(\frac{\Delta\gamma}{\gamma}\right)_2 + \cdots, \tag{38}$$

$$\frac{\Delta\mu}{\mu} = \left(\frac{\Delta\mu}{\mu}\right)_1 + \left(\frac{\Delta\mu}{\mu}\right)_2 + \cdots, \tag{39}$$

$$\frac{\Delta\rho}{\rho} = \left(\frac{\Delta\rho}{\rho}\right)_1 + \left(\frac{\Delta\rho}{\rho}\right)_2 + \cdots, \tag{40}$$

where γ, μ, and ρ are the bulk modulus, shear modulus, and density, respectively.

The ability of the forward series to have a direct inverse series derives from (1) the identity among G, G_0, and V provided by the scattering equation and then (2) the recognition that the forward solution can be viewed as a geometric series for the data D, in terms of VG_0. The latter derives the direct inverse series for VG_0 in terms of the data.

Viewing the forward problem and series as the Taylor series

$$D(m) = D(m_0) + D'(m_0)\Delta m + \frac{D''(m_0)}{2}\Delta m^2 + \cdots, \tag{41}$$

in which the derivatives are Fréchet derivatives, in terms of Δm, does not offer a direct inverse series, and hence there is no choice but to solve the forward series in an inverse sense. It is that fact that results in all current AVO and FWI methods being modeling methods that are solved in an inverse sense. Among references that solve a forward problem in an inverse sense in P-wave AVO are Clayton and Stolt (1981), Shuey (1985), Stolt and Weglein (1985), Boyse and Keller (1986), Stolt (1989), Beylkin and Burridge (1990), Castagna and Smith (1994), Goodway et al. (1997), Burridge et al. (1998), Smith and Gidlow (2000), Foster et al. (2010), and Goodway (2010). The intervention of the explicit relationship among G, G_0, and V (the scattering equation) in a Taylor series-like form produces a geometric series and a direct inverse solution.

The linear equations are

$$\begin{pmatrix}\hat{D}^{PP} & \hat{D}^{PS}\\ \hat{D}^{SP} & \hat{D}^{SS}\end{pmatrix}=\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}_1^{PP} & \hat{V}_1^{PS}\\ \hat{V}_1^{SP} & \hat{V}_1^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}, \tag{42}$$

$$\hat{D}^{PP}=\hat{G}_0^P\hat{V}_1^{PP}\hat{G}_0^P, \tag{43}$$

$$\hat{D}^{PS}=\hat{G}_0^P\hat{V}_1^{PS}\hat{G}_0^S, \tag{44}$$

$$\hat{D}^{SP}=\hat{G}_0^S\hat{V}_1^{SP}\hat{G}_0^P, \tag{45}$$

$$\hat{D}^{SS}=\hat{G}_0^S\hat{V}_1^{SS}\hat{G}_0^S, \tag{46}$$

$$\tilde{D}^{PP}(k_g,0;-k_g,0;\omega)=-\frac{1}{4}\left(1-\frac{k_g^2}{\nu_g^2}\right)\tilde{a}_\rho^{(1)}(-2\nu_g) -\frac{1}{4}\left(1+\frac{k_g^2}{\nu_g^2}\right)\tilde{a}_\gamma^{(1)}(-2\nu_g)+\frac{2k_g^2\beta_0^2}{(\nu_g^2+k_g^2)\alpha_0^2}\tilde{a}_\mu^{(1)}(-2\nu_g), \tag{47}$$

$$\tilde{D}^{PS}(\nu_g,\eta_g)=-\frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)\tilde{a}_\rho^{(1)}(-\nu_g-\eta_g) -\frac{\beta_0^2}{2\omega^2}k_g(\nu_g+\eta_g)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)\tilde{a}_\mu^{(1)}(-\nu_g-\eta_g), \tag{48}$$

$$\tilde{D}^{SP}(\nu_g,\eta_g)=\frac{1}{4}\left(\frac{k_g}{\nu_g}+\frac{k_g}{\eta_g}\right)\tilde{a}_\rho^{(1)}(-\nu_g-\eta_g) +\frac{\beta_0^2}{2\omega^2}k_g(\nu_g+\eta_g)\left(1-\frac{k_g^2}{\nu_g\eta_g}\right)\tilde{a}_\mu^{(1)}(-\nu_g-\eta_g),\ \text{and} \tag{49}$$

$$\tilde{D}^{SS}(k_g,\eta_g)=\frac{1}{4}\left(1-\frac{k_g^2}{\eta_g^2}\right)\tilde{a}_\rho^{(1)}(-2\eta_g) -\left[\frac{\eta_g^2+k_g^2}{4\eta_g^2}-\frac{2k_g^2}{\eta_g^2+k_g^2}\right]\tilde{a}_\mu^{(1)}(-2\eta_g), \tag{50}$$

where $a_\gamma^{(1)}$, $a_\mu^{(1)}$, and $a_\rho^{(1)}$ are the linear estimates of the changes in bulk modulus, shear modulus, and density, respectively. Here, k_g is the Fourier conjugate to the receiver position x_g and ν_g and η_g are the vertical wavenumbers for the P- and S-reference waves, respectively, where

$$\nu_g^2+k_g^2=\frac{\omega^2}{\alpha_0^2}, \tag{51}$$

$$\eta_g^2+k_g^2=\frac{\omega^2}{\beta_0^2}, \tag{52}$$

and α_0 and β_0 are the P- and S-velocities in the reference medium, respectively. The direct quadratic nonlinear equations are

$$\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}_2^{PP} & \hat{V}_2^{PS}\\ \hat{V}_2^{SP} & \hat{V}_2^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix} =-\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}_1^{PP} & \hat{V}_1^{PS}\\ \hat{V}_1^{SP} & \hat{V}_1^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}_1^{PP} & \hat{V}_1^{PS}\\ \hat{V}_1^{SP} & \hat{V}_1^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}, \tag{53}$$

$$\hat{G}_0^P\hat{V}_2^{PP}\hat{G}_0^P=-\hat{G}_0^P\hat{V}_1^{PP}\hat{G}_0^P\hat{V}_1^{PP}\hat{G}_0^P-\hat{G}_0^P\hat{V}_1^{PS}\hat{G}_0^S\hat{V}_1^{SP}\hat{G}_0^P, \tag{54}$$

$$\hat{G}_0^P\hat{V}_2^{PS}\hat{G}_0^S=-\hat{G}_0^P\hat{V}_1^{PP}\hat{G}_0^P\hat{V}_1^{PS}\hat{G}_0^S-\hat{G}_0^P\hat{V}_1^{PS}\hat{G}_0^S\hat{V}_1^{SS}\hat{G}_0^S, \tag{55}$$

$$\hat{G}_0^S\hat{V}_2^{SP}\hat{G}_0^P=-\hat{G}_0^S\hat{V}_1^{SP}\hat{G}_0^P\hat{V}_1^{PP}\hat{G}_0^P-\hat{G}_0^S\hat{V}_1^{SS}\hat{G}_0^S\hat{V}_1^{SP}\hat{G}_0^P, \tag{56}$$

$$\hat{G}_0^S\hat{V}_2^{SS}\hat{G}_0^S=-\hat{G}_0^S\hat{V}_1^{SP}\hat{G}_0^P\hat{V}_1^{PS}\hat{G}_0^S-\hat{G}_0^S\hat{V}_1^{SS}\hat{G}_0^S\hat{V}_1^{SS}\hat{G}_0^S. \tag{57}$$

Because $\hat{V}_1^{PP}$ relates to $\hat{D}^{PP}$, $\hat{V}_1^{PS}$ relates to $\hat{D}^{PS}$, and so on, the four components of the data will be coupled in the nonlinear elastic inversion. I cannot perform the direct nonlinear inversion without knowing all components of the data. Thus, the direct nonlinear solution determines the data needed for a direct inverse. That, in turn, defines what a linear estimate means. That is, a linear estimate of a parameter is an estimate of a parameter that is linear in data that can directly invert for that parameter. Because D^{PP}, D^{PS}, D^{SP}, and D^{SS} are needed to determine a_γ, a_μ, and a_ρ directly, a linear estimate for any one of these quantities requires simultaneously solving equations 47–50 (for further details, see, e.g., Weglein et al., 2009).

Those direct nonlinear formulas are like the direct solution for the quadratic equation mentioned above and solve directly and nonlinearly for changes in the velocities, α, β, and the density ρ in a 1D elastic earth. Stolt and Weglein (2012) present the linear equations for a 3D earth that generalize equations 47–50. Those formulas prescribe precisely what data you need as input, and they dictate how to compute those sought-after mechanical properties, given the necessary data. There is no search or cost function, and the unambiguous and unequivocal data needed are full-multicomponent data — PP, PS, SP, and SS — for all traces in each of the P- and S-shot records. The direct algorithm determines first the data needed and then the appropriate algorithms for using those data to directly compute the sought-after changes in the earth's mechanical properties. Hence, any method that calls itself inversion (let alone full-wave inversion) for determining changes in elastic properties, and in particular, the P-wave velocity α, and that inputs only P-data, is more off base, misguided, and lost than the methods that sought two or more functions of depth from a single trace. You can model-match P-data ad nauseum, which takes a lot of computational effort and people with advanced degrees in math and physics computing Fréchet derivatives, and it requires sophisticated L_P-norm cost functions and local or global search engines, so it must be reasonable, scientific, and worthwhile. Why can I not use just PP-data to invert for changes in V_P, V_S, and density because Zoeppritz says that I can model PP from those quantities and because I have, using PP-data with angle variation, enough dimension? As stated above, data dimension is good, but it is not good enough for a direct inversion of those elastic properties.

Adopting equations 27 and 28 as in AVO and FWI, there is a violation of the fundamental relationship between changes in a medium and changes in a wavefield, $G = G_0 + G_0VG$, which is as serious as considering problems involving a right triangle and violating the Pythagorean theorem. That is, iteratively updating PP data with an elastic model violates the basic relationship between changes in a medium V and changes in the wavefield $G - G_0$ for the simplest elastic earth model.

This direct inverse method for parameter estimation provides a platform for amplitude analysis and a solid framework and direct methodology for the goals and objectives of indirect methods such as AVO and FWI. A direct method for the purposes of amplitude analysis provides a method that derives from, respects, and honors the fundamental identity and relationship $G = G_0 + G_0VG$. Iteratively inverting multicomponent data has the correct data, but it does not correspond to a direct inverse algorithm. To honor $G = G_0 + G_0VG$, you need the data and the algorithm that the direct inverse prescribes. Not recognizing the message that an operator identity and the elastic wave equation unequivocally communicate is a fundamental and significant contribution to the gap in effectiveness in current AVO and FWI methods and application (equation A-6). This analysis generalizes to 3D with P, SH, and SV data.

The role of direct and indirect methods

There is a role for direct and indirect methods in practical real-world applications. In our view, indirect methods are to be called upon for recognizing that the world is more complicated than the physics that we assume in our models and methods. For the part of the world that you are capturing in your model and physics, nothing compares to direct methods for clarity and effectiveness. An optimal indirect method would seek to satisfy a cost function that derives from a property of the direct method. In that way, the indirect and direct methods would be aligned, consistent, and cooperative for accommodating the part of the world described by your physical model (with a direct inverse method) and the part that is outside (with an indirect method).

The indirect method of model matching primaries and multiples (so-called FWI)

All model matching inverse approaches are indirect methods. Iterative linear inversion model matching is an indirect search methodology, which is ad hoc and without a firm and solid foundation and theoretical and conceptual framework. Nevertheless, we can imagine and understand that model matching primaries and multiples, rather than only primaries, could improve upon matching only primaries. However, model matching primaries and multiples remains ad hoc and indirect and is always on much shakier footing than direct inversion for the same inversion goals and objectives. Direct ISS inversion for parameter estimation only requires and inputs primaries.

For all multidimensional seismic applications, the only direct inverse solution is provided by the operator identity equation 6 and is in the form of a series of equations 21–23, the ISS (Weglein et al., 2003). It can achieve all processing objectives within a single framework and a single set of equations 21–23 without requiring any subsurface information. There are distinct isolated-task inverse scattering subseries derived from the ISS, which can perform free-surface multiple removal (Carvalho et al., 1992; Weglein et al., 1997), internal multiple removal (Araújo et al., 1994; Weglein et al., 2003), depth imaging (e.g., Shaw, 2005; Liu, 2006; Weglein et al., 2012), parameter estimation (Zhang, 2006; Li, 2011; Liang, 2013; Yang and Weglein, 2015), and Q compensation without needing, estimating, or determining Q (Innanen and Weglein, 2007; Lira, 2009; Innanen and Lira, 2010), and each achieves its objective directly and without subsurface information. The direct inverse solution (e.g., Weglein et al., 2003, 2009) provides a framework and a firm math-physics foundation that unambiguously defines the data requirements and the distinct algorithms to perform each and every associated task within the inverse problem, directly and without subsurface information.

Having an ad hoc, indirect method as the starting point places a cloud over issue identification when less-than-satisfactory results arise with field data. In addition, we saw that direct inversion parameter estimation has a significantly lower dependence on the low-frequency data components in comparison with indirect methods such as nonlinear AVO and FWI.

Only a direct solution can provide algorithmic clarity, confidence, and effectiveness. The current industry-standard AVO and FWI, using variants of model-matching and iterative linear inverse, are indirect methods, and iteratively linearly updating P data or multicomponent data (with or without multiples) does not correspond to, and will not produce, a direct solution.

All direct inverse methods for structural determination and amplitude analysis require only primaries

In Weglein (2016), the role of primaries and multiples in imaging is examined and analyzed. The most capable and interpretable migration method derives from predicting a source and receiver experiment at depth. For data consisting of primaries and multiples, a discontinuous velocity model is needed to achieve that predicted experiment at depth. With that discontinuous velocity model, free-surface and internal multiples play no role in the migration and the exact same image results with or without multiples (see Weglein, 2016). For a smooth velocity model, multiples will result in false and misleading images and must be removed before the migration and migration-inversion of primaries.

In Weglein et al. (2003), ISS direct depth imaging (without a velocity model or subsurface information) removes free-surface and internal multiples prior to the distinct subseries that input primaries and perform depth imaging and amplitude analysis, respectively, each directly and without subsurface information and only using and requiring primaries.

Hence, all direct inversion methods, those with and those without subsurface/velocity information, require only primaries for complete structural determination and amplitude analysis. Methods that seek to use multiples to address issues from less than a complete acquisition of primaries are seeking an appropriate image of an unrecorded primary.

Indirect methods are ad hoc without a clear or firm math-physics foundation and framework, and they start without knowing whether "the indirect solution" is in fact a solution. A more complete or fuller data set being matched between model data and field data, each with primaries and multiples, could at times improve upon matching only primaries, but the entire approach is indirect and ad hoc with or without multiples, and it lacks the benefits of a direct method. With indirect methods, there is no framework and theory to rely on, and no confidence that a solution is forthcoming under any circumstances.

If I seek the parameters of an elastic heterogeneous isotropic subsurface, then the differential operator in the operator identity is the differential operator that occurs in the elastic, heterogeneous, isotropic wave equation. From 40 years of AVO and amplitude analysis application in the petroleum industry, the elastic isotropic model is the baseline minimally realistic and acceptable earth model type for amplitude analysis, for example, for AVO and FWI. Then, taking the operator identity (called the Lippmann-Schwinger, or scattering theory, equation) for the elastic-wave equation, I can obtain a direct inverse solution for the changes in the elastic properties and density. The direct inverse solution specifies the data required and the algorithm to achieve a direct parameter estimation solution. In this paper, I explain how this methodology differs from all current AVO and FWI methods, which are, in fact, forms of model matching. Multicomponent data consisting of only primaries are needed for a direct inverse solution for subsurface properties. This paper focuses on one specific inverse task, parameter estimation, within the overall and broader set of inversion objectives and tasks. Furthermore, the impact of band-limited data and noise are discussed and compared for the direct ISS parameter estimation and indirect (AVO and FWI) inversion methods.

In this paper, I focused on analyzing and examining the direct inverse solution that the ISS inversion subseries provides for parameter estimation. The distinct issues of (1) data requirements, (2) model type, and (3) inversion algorithm for the direct inverse are all important (Weglein, 2015b). For an elastic heterogeneous medium, I show that the direct inverse requires multicomponent/PS (P- and S-component) data and prescribes how that data are used for a direct parameter estimation solution (Zhang and Weglein, 2006).

Conclusion

In this paper, I describe, illustrate, and analyze the considerable conceptual, substantive, and practical benefit and added value that a direct parameter inversion from the ISS provides in comparison with all current indirect inverse methods (e.g., AVO and FWI) for amplitude analysis goals and objectives. A direct method provides (1) a solution that we (the seismic industry) can have confidence that it is in fact solving the defined problem of interest and (2) in addition, when the method does not improve the drilling decisions, then we know that the issue is that the problem of interest is not the problem that we need to be interested in. On the other hand, indirect methods such as AVO and FWI have a plethora of approaches and paths, and when less-than-satisfactory results occur, we do not know whether the issue is the chosen problem of interest or the choice among innumerable indirect solutions, or both.

All scientific methods make assumptions — and seismic processing and interpretation methods are no exception. When the assumptions behind seismic methods are satisfied, the methods are useful and effective and can support successful drill decisions. When the assumptions are not satisfied, the methods can have difficulty or can fail. The latter breakdown can contribute to unsuccessful ill-informed drill decisions, dry-hole drilling, or suboptimal appraisal and development wells.

The objective of seismic research is to provide new and effective toolbox capability for processing and interpretation that will improve the drill success rate and reduce dry-hole and suboptimal drilling decisions. Toward that end, the starting point in seismic research is to identify the outstanding prioritized problems and challenges that need to be addressed and solved.

The ability to clearly and unambiguously define the origin and root cause behind seismic issues, problems, breakdown, and challenges is an essential and critically important step in designing and executing a strategy to provide new and more capable methods to the seismic processing and interpretation toolbox.

Direct inversion methods can provide that problem definition and clarity. They are also unique in providing the confidence that the problem of interest is actually being addressed. For ISS parameter estimation, although the recorded data are of course band limited, the band-limited data are never used to compute the updated inverse operator for the next iterated linear step because the inverse operator is fixed and analytic for every term in the ISS. That is one of several important and substantive differences pointed out in this paper between the direct inverse ISS parameter estimation method and all indirect inversion methods, e.g., AVO and FWI. I provide an explicit analytic example and comparison between direct ISS parameter estimation and the indirect linear updating model matching concepts behind AVO and FWI.

All seismic processing methods depend on the amplitude and phase of seismic data. Different processing methods that seek to achieve a certain specific processing goal can have different relative sensitivities to noise and bandwidth. Amplitude analysis for determining earth mechanical property changes is one of the most sensitive. Methods that achieve seismic goals as a sequence of separate intermediate steps have a natural advantage over methods that seek to combine goals. Achieving an intermediate, easier goal that is less demanding can significantly enhance the ability to achieve the subsequent more demanding seismic processing objectives. The indirect methods that seek to locate structure and identify changes in earth mechanical properties at once have a terrible dependence on missing low-frequency data. However, if I first locate a structure by wave-equation migration (a process that is insensitive to missing low frequency data), then in principle, I can determine the earth mechanical property changes with a single frequency within the bandwidth. The ISS direct amplitude analysis method described, exemplified, tested, and compared in this paper assumes that a set of less-daunting seismic processing tasks, using an ISS task specific subseries, has been achieved (e.g., multiple removal, depth imaging) before this task is undertaken. To have a fair comparison, the indirect model matching method is tested with a data with a well-located single reflector, and hence there are no imaging issues or multiples in the problem. That allows a pristine, clear, and definitive comparison of the amplitude analysis — parameter estimation function of the prototype direct ISS method and the corresponding indirect model-matching iterative updating approach. There are important issues of resolution and illumination, which will impact the results of this paper, with advances in migration theory and algorithms that avoid all high-frequency approximations in the imaging principles and wave-propagation models that can improve resolution and illumination.

Direct and indirect methods can play an important role and function in seismic processing, in which the former accommodates and addresses the assumed physics and the latter provides a channel for real-world phenomena beyond the assumed physics. Both are called for within a comprehensive and effective seismic processing and interpretation strategy.

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Appendix A

The operator identity and direct inverse solution for a 2D heterogeneous isotropic elastic medium

I describe the forward and direct inverse method for a 2D elastic heterogeneous earth (see Zhang, 2006).

The 2D elastic wave equation for a heterogeneous isotropic medium (Zhang, 2006) is

$$L\mathbf{u} = \begin{pmatrix} f_x \\ f_z \end{pmatrix} \quad \text{and} \quad \hat{L}\begin{pmatrix} \phi^{\mathrm{P}} \\ \phi^{\mathrm{S}} \end{pmatrix} = \begin{pmatrix} F^{\mathrm{P}} \\ F^{\mathrm{S}} \end{pmatrix}, \tag{A-1}$$

where $\mathbf{u}$, f_x, and f_z are the displacement and forces in displacement coordinates and ϕ_{P}, ϕ_{S}, and F^{P}, F^{S} are the P– and S-waves and the force components in P- and S-coordinates, respectively. The operators L and L_0 in the actual and reference elastic media are

$$L = \left[\rho\omega^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \partial_x\gamma\partial_x + \partial_z\mu\partial_z & \partial_x(\gamma-2\mu)\partial_z + \partial_z\mu\partial_x \\ \partial_z(\gamma-2\mu)\partial_x + \partial_x\mu\partial_z & \partial_z\gamma\partial_z + \partial_x\mu\partial_x \end{pmatrix}\right], \tag{A-2}$$

$$L_0 = \left[\rho\omega^2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \gamma_0\partial_x^2 + \mu_0\partial_z^2 & (\gamma_0-\mu_0)\partial_x\partial_z \\ (\gamma_0-\mu_0)\partial_x\partial_z & \mu_0\partial_x^2 + \gamma_0\partial_z^2 \end{pmatrix}\right], \tag{A-3}$$

and the perturbation V is

$$V \equiv L_0 - L = \begin{bmatrix} a_\rho\omega^2 + \alpha_0^2\partial_x a_\gamma\partial_x + \beta_0^2\partial_z a_\mu\partial_z & \partial_x(\alpha_0^2 a_\gamma - 2\beta_0^2 a_\mu)\partial_z + \beta_0^2\partial_z a_\mu\partial_x \\ \partial_z(\alpha_0^2 a_\gamma - 2\beta_0^2 a_\mu)\partial_x + \beta_0^2\partial_x a_\mu\partial_z & a_\rho\omega^2 + \alpha_0^2\partial_z a_\gamma\partial_z + \beta_0^2\partial_x a_\mu\partial_x \end{bmatrix}, \tag{A-4}$$

where the quantities $a_\rho \equiv \rho/\rho_0 - 1$, $a_\gamma \equiv \gamma/\gamma_0 - 1$, and $a_\mu \equiv \mu/\mu_0 - 1$ are defined in terms of the bulk modulus, shear modulus, and density (γ_0, μ_0, ρ_0, γ, μ, ρ) in the reference and actual media, respectively.

The forward problem is found from the identity equation 9 and the elastic wave equation A-1 in PS-coordinates as

$$\hat{G}-\hat{G}_0=\hat{G}_0\hat{V}\hat{G}_0+\hat{G}_0\hat{V}\hat{G}_0\hat{V}\hat{G}_0+\cdots,$$

$$\begin{pmatrix}\hat{D}^{PP} & \hat{D}^{PS}\\ \hat{D}^{SP} & \hat{D}^{SS}\end{pmatrix}=\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}^{PP} & \hat{V}^{PS}\\ \hat{V}^{SP} & \hat{V}^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}$$
$$+\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}^{PP} & \hat{V}^{PS}\\ \hat{V}^{SP} & \hat{V}^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}^{PP} & \hat{V}^{PS}\\ \hat{V}^{SP} & \hat{V}^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}+\cdots, \quad \text{(A-5)}$$

and the inverse solution, equations 21–23, for the elastic equation A-1 is

$$\begin{pmatrix}\hat{D}^{PP} & \hat{D}^{PS}\\ \hat{D}^{SP} & \hat{D}^{SS}\end{pmatrix}=\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}_1^{PP} & \hat{V}_1^{PS}\\ \hat{V}_1^{SP} & \hat{V}_1^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix},$$
$$\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}_2^{PP} & \hat{V}_2^{PS}\\ \hat{V}_2^{SP} & \hat{V}_2^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}$$
$$=-\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}_1^{PP} & \hat{V}_1^{PS}\\ \hat{V}_1^{SP} & \hat{V}_1^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix}\begin{pmatrix}\hat{V}_1^{PP} & \hat{V}_1^{PS}\\ \hat{V}_1^{SP} & \hat{V}_1^{SS}\end{pmatrix}\begin{pmatrix}\hat{G}_0^P & 0\\ 0 & \hat{G}_0^S\end{pmatrix},$$
$$\vdots \quad \text{(A-6)}$$

where $\hat{V}^{PP}=\hat{V}_1^{PP}+\hat{V}_2^{PP}+\hat{V}_3^{PP}+\cdots$ and any one of the four matrix elements of V requires the four components of the data

$$\begin{pmatrix}\hat{D}^{PP} & \hat{D}^{PS}\\ \hat{D}^{SP} & \hat{D}^{SS}\end{pmatrix}. \quad \text{(A-7)}$$

The 3D heterogeneous isotropic elastic generalization of the above 2D forward and direct inverse elastic isotropic method begins with the linear 3D form found in Stolt and Weglein (2012, p. 159).

In summary, from equation A-5, $\hat{D}^{PP}$ can be determined in terms of the four elements of V. The four components $\hat{V}^{PP}$, $\hat{V}^{PS}$, $\hat{V}^{SP}$, and $\hat{V}^{SS}$ require the four components of D. That is what the general relationship $G=G_0+G_0VG$ requires; i.e., a direct nonlinear inverse solution is a solution order-by-order in the four matrix elements of D (in 2D). The generalization of the forward series equation A-5 and the inverse series equation A-6 for a direct inversion of an elastic isotropic heterogeneous medium in 3D involves the 3×3 data, D, and V matrices in terms of P, SH, and SV data and start with the linear $G_0V_1G_0=D$ (Stolt and Weglein, 2012, p. 179).

Appendix B

Numerical examples for a 1D normal incident wave on an acoustic medium

Numerical examples for a 1D normal incident wave on an acoustic medium are shown in this section. First, I examine and compare the convergence of the ISS direct inversion and iterative inversion. Second, the rate of convergence of the ISS inversion subseries is examined and studied using an analytic example, where the ISS method converges and the iterative linear method does not and where both methods converge.

The operator identity for a 1D acoustic medium

For a normal incidence plane wave on a 1D acoustic medium (where only the velocity is assumed to vary), the model I consider here consists of two half-spaces with acoustic velocities c_0 and c_1 and an interface located at $z=a$ as shown in Figure B-1. If I put the source and receiver on the surface, $z=0$, the pressure wave

$$D(t)=R\delta(t-2a/c_0) \quad \text{(B-1)}$$

will be recorded, where the reflection coefficient $R=(c_1-c_0)/(c_1+c_0)$. For this example, $D(t)$ is the only input to the direct ISS inverse and the iterative inversion methods. Because I will assume knowledge of the velocity in the upper half-space, c_0, the location of the reflector at $z=a$ is not an issue. I will focus on only determining the change of velocity across the reflector at $z=a$. The operators L_0 and L in the reference and actual acoustic media are

$$L_0=\frac{d^2}{dz^2}+\frac{\omega^2}{c_0^2} \quad \text{and} \quad L=\frac{d^2}{dz^2}+\frac{\omega^2}{c^2(z)}, \quad \text{(B-2)}$$

and I characterize the velocity perturbation as

$$\alpha(z)\equiv 1-\frac{c_0^2}{c^2(z)}. \quad \text{(B-3)}$$

The perturbation V (Weglein et al., 2003) can be expressed as

$$V(z)=L_0-L=\frac{\omega^2}{c_0^2}-\frac{\omega^2}{c^2(z)}=k_0^2\alpha(z), \quad \text{(B-4)}$$

where ω is the angular frequency and $k_0=\omega/c_0$. The functions c_0 and $c(z)$ are the reference and local acoustic velocity, respectively. Therefore, the inverse series of V (equation 16) becomes

$$\alpha(z)=\alpha_1(z)+\alpha_2(z)+\alpha_3(z)+\cdots. \quad \text{(B-5)}$$

That is,

$$V_1=k_0^2\alpha_1, \quad V_2=k_0^2\alpha_2, \quad \cdots. \quad \text{(B-6)}$$

From the ISS (equations 21–23), Shaw and Weglein (2004) isolate the leading order imaging subseries and the direct nonlinear inversion subseries.

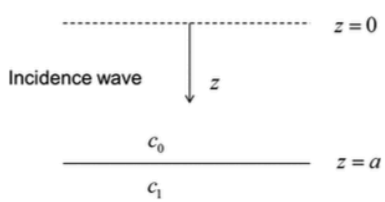

Figure B-1. A 1D acoustic model with velocities c_0 over c_1.

In this section, I will focus on studying the convergence properties of the ISS inversion subseries. The inversion-only terms isolated from the ISS (Zhang, 2006; Li, 2011) are

$$\alpha(z) = \alpha_1(z) - \frac{1}{2}\alpha_1^2(z) + \frac{3}{16}\alpha_1^3(z) + \cdots . \quad \text{(B-7)}$$

For a 1D normal incidence case, the linear equation 21 solves for α_1 in terms of the single trace data $D(t)$ (Shaw and Weglein, 2004) as

$$\alpha_1(z) = 4\int_{-\infty}^{z} D(z')dz', \quad \text{(B-8)}$$

where $z' = c_0 t/2$. For a single reflector, inserting data D (equation B-1) gives

$$\alpha_1 = 4RH(z - a), \quad \text{(B-9)}$$

where R is the reflection coefficient $R = (c_1 - c_0)/(c_1 + c_0)$ and H is the Heaviside function. When $z > a$, substituting α_1 into equation B-7, the ISS direct nonlinear inversion subseries in terms of R can be written as (where α is the magnitude of $\alpha(z)$ for $z > a$)

$$\alpha = 4R - 8R^2 + 12R^3 + \cdots = 4R\sum_{n=0}^{\infty}(n+1)(-R)^n. \quad \text{(B-10)}$$

After solving for α, the inverted velocity $c(z)$ can be obtained through $c_1 = c_0(1-\alpha)^{-1/2}$ (equation B-4).

Considering the convergence property of the series for α or the inversion subseries, I can calculate the ratio test

$$\left|\frac{\alpha_{n+1}}{\alpha_n}\right| = \left|\frac{(n+2)(-R)^{n+1}}{(n+1)(-R)^n}\right| = \left|\frac{n+2}{n+1}R\right|. \quad \text{(B-11)}$$

If $\lim_{n\to\infty}|(\alpha_{n+1}/\alpha_n)| < 1$, this subseries converges absolutely. That is,

$$|R| < \lim_{n\to\infty}\frac{n+1}{n+2} = 1. \quad \text{(B-12)}$$

Therefore, the ISS direct nonlinear inversion subseries converges when the reflection coefficient $|R|$ is less than one, which is always true. Hence, for this example, the ISS inversion subseries will converge under any velocity contrasts between the two media.

For the iterative linear inversion, I use the first linear estimate of $\alpha = \alpha_1^1$ to compute the first estimate of $c_1 = c_1^1$. Then, I choose the first estimate of $c_1 = c_0(1-\alpha_1^1)^{-1/2} \equiv c_1^1$ as the new reference velocity, $c_0^1 = c_0(1-\alpha_1^1)^{-1/2}$, where $\alpha_1^1 = 4R_1$ and $R_1 = (c_1 - c_0)/(c_1 + c_0)$. Repeating the linear process with a new reflection coefficient R_2 (again exploiting the analytic inverse generously provided by ISS to benefit the iterative linear inverse approach) gives

$$R_2 = \frac{c_1 - c_0^1}{c_1 + c_0^1}, \; \alpha_1^2 = 4R_2 \text{ and } c_1^2 = c_0^1(1-\alpha_1^2)^{-1/2} = c_0^2, \quad \text{(B-13)}$$

$$\vdots$$

$$R_{n+1} = \frac{c_1 - c_0^n}{c_1 + c_0^n}, \; \alpha_1^{n+1} = 4R_{n+1} \text{ and}$$
$$c_1^n = c_0^{n-1}(1-\alpha_1^n)^{-1/2} = c_0^n, \quad \text{(B-14)}$$

where $\alpha_1^n = n$th estimate of α_1 and $c_1^n = n$th estimate of c_1. The questions are (1) under what conditions does c_1^n approach c_1, and (2) when it converges, what is its rate of convergence?

From the above analysis, I can see that the ISS method for α always converges and the resulting α can be used to find c_1. For the iterative linear inverse, there are values of α_1, such that you cannot compute a real c_1^1. When $\alpha_1^1 > 1$ and $4R > 1$, $R > 1/4$ and you cannot compute an updated reference velocity and the method simply shuts down and fails. The ISS never computes a new reference and does not suffer that problem, with the series for α always converging and then outputting c_1, the correct unknown velocity below the reflector.

The convergence of the ISS direct inversion and iterative inversion

In this section, I will examine and compare the convergence property of the ISS inversion (equation B-10) and the iterative linear inversion for different velocity contrasts in the 1D acoustic case. In the 1D normal incident acoustic model (Figure B-1), only one parameter (velocity) varies and a plane wave propagates into the medium. There is only a single reflector, and I assume the velocity is known above the reflector and unknown below the reflector. I will compare the convergence of the perturbation α and the inversion results by using the ISS direct nonlinear method and the iterative linear method.

With the reference velocity $c_0 = 1500$m/s, two analytic examples with different velocity contrasts for $c_1 = 2000$ and 3000m/s are examined. Figure B-2 shows the estimated α by the ISS method (green line) for $c_1 = 2000$m/s. The red line represents the actual α that is calculated from the model. The horizontal axis represents the order of the ISS inversion subseries. The vertical axis shows the value of α. The updated estimation of α using the iterative inversion method (blue line) is shown in Figure B-3. The horizontal axis represents the iteration numbers in the iterative inversion method. From Figures B-2 and B-3, I can see that at the small velocity contrast, the estimated α by ISS method becomes the actual α after about five orders of calculation and the updated estimation of α by the iterative inversion method goes to zero as expected because after several iterations, the updated model is close to and approaching to the actual model. Figure B-4 represents the velocity estimation. The green and blue lines represent the estimated velocity by using the ISS inversion method and the iterative inversion method, respectively. We can see that at the small velocity contrast, both methods converge and produce correct velocity after five orders of iterations and the ISS inversion method converges faster than the iterative inversion method.

Figure B-5 shows the estimated α by the ISS method (green line) for $c_1 = 3000\text{m/s}$. When the velocity contrast is larger, i.e., $R > 0.25$, the iterative inversion method cannot be computable, but the ISS inversion method always converges (see the green line in Figure B-5) after the summation of more orders in computing α.

As we know, the reflection coefficient R is almost always less than 0.2 in practice, so that the ISS method and the iterative method converge, but the ISS method converges faster than the iterative method. Moreover, for more complicated circumstances (e.g., the elastic nonnormal incidence case), the difference between the ISS method and the iterative method is much greater, not just on the algorithms, but also on data requirements and on how the band-limited noisy nature of the seismic data impacts the inverse operators in the iterative method but not in the ISS method.

The rate of convergence of the ISS inversion subseries

The rate of convergence of the estimated α for the ISS inversion subseries (equation B-10) is analytically examined and studied. Because α is always convergent

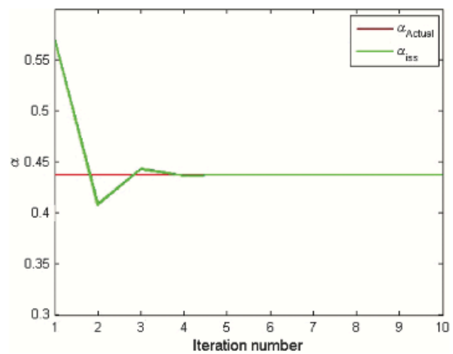

Figure B-2. The estimated α at $R = 0.1429$: The horizontal axis is the order of the ISS subseries and the vertical axis shows the value of α. The red line shows the actual value of $\alpha = 0.4375$. The green line shows the estimation of α using the ISS inversion method order by order.

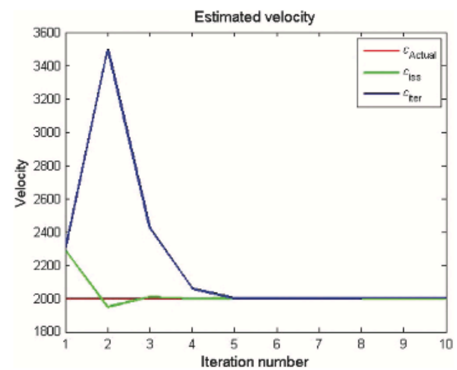

Figure B-4. The estimated velocity by using the ISS inversion method (green line) and the iterative inversion method (blue line).

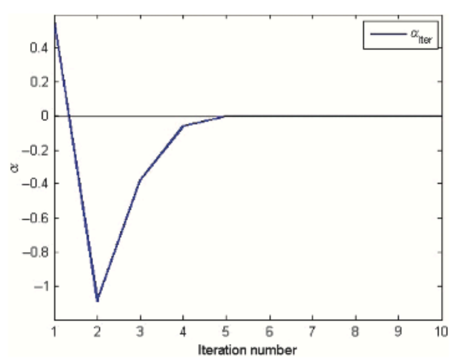

Figure B-3. The updated α at $R = 0.1429$: The horizontal axis is the iteration numbers, and the vertical axis shows the updated value of α. The blue line represents the updated estimation of α using the iterative inversion method.

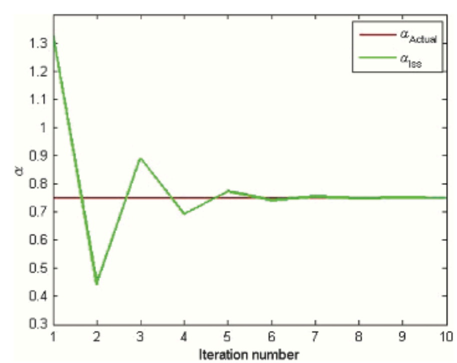

Figure B-5. The estimated α at $R = 0.3333$: The horizontal axis is the order of the ISS subseries, and the vertical axis represents the value of α. The red line shows the actual value of $\alpha = 0.7500$. The green line shows the estimation of α using the ISS inversion method order by order.

when $R<1$, the summation of this subseries (Zhang, 2006) is

$$\alpha = 4R\sum_{n=0}^{\infty}(n+1)(-R)^n = 4R\frac{1}{(1+R)^2}. \tag{B-15}$$

If the error between the estimated and the actual α is monotonically decreasing, it means that the subseries is a term-by-term added-value improvement toward determining the actual medium properties. If this error is increasing before decreasing, it means that the estimate of α becomes worse before it gets better. The error for the first order and the error for the second order have the relation

$$|\alpha-\alpha_1-\alpha_2| > |\alpha-\alpha_1|, \tag{B-16}$$

i.e.,

$$\left|4R\frac{3R^2+2R^3}{(1+R)^2}\right| > \left|4R\frac{-R^2-2R}{(1+R)^2}\right|. \tag{B-17}$$

After simplification, it gives

$$R^2+R-1>0. \tag{B-18}$$

I can solve it and obtain the reflection coefficient $R<[(-1-\sqrt{5})/2]=-1.618$ or $R>[(-1+\sqrt{5})/2]=0.618$. Therefore, when $R>0.618$, the error increases first. Similarly, if the error for the third order is greater than that for the second order, I get $R>0.667$. If the error for the fourth order is greater than that for the third order, I obtain $R>0.721$. In summary, when $R>0.618$, the error increases and the estimated α gets worse before getting better. The sum of terms in the direct inverse ISS solution (for very large contrasts) requires certain partial sums to be temporarily worse in order for the entire series to produce the correct velocity. The dashed green line in Figure B-6 shows that when the reflection coefficient R is equal to 0.618, the error for the first order is equal to the error for the second order.

As the analytic calculation, when the reflection coefficient R is smaller than 0.618, this inversion subseries gives a monotonically term-by-term added-value improvement toward determining c_1. When the reflection coefficient is larger than 0.618, the ISS inversion series still converges, but the estimation of α will become worse before it gets better. Each term in the series works toward the final goal. Sometimes when more terms in the series are included, the estimation looks temporarily worse, but once it starts to improve the estimation at a specific order, the approximations never become worse again, and every single term after that order will produce an improved estimation. The locally worse partial sum behavior is, in fact, purposeful and essential for convergence to and for computing the exact velocity. The direct inverse solution fulfills its commitment to always predict c_1 and not necessarily to having order-by-order improvement. The ISS direct inversion always converges in contrast to the iterative linear inverse method. This property has also been indicated by Carvalho (1992) in the free-surface-multiple elimination subseries; e.g., what appears to make a second-order free-surface multiple larger with a first-order free-surface algorithm is actually helpful and necessary for preparing the second-order multiple to be removed by the higher order terms.

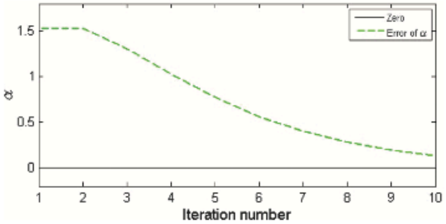

Figure B-6. The error (dashed green line) of estimated α at $R=0.6180$ and $\alpha=0.9443$.

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Arthur Weglein holds the Hugh Roy and Lillie Cranz Cullen Distinguished University Chair in Physics, is Professor in the Department of Physics and the Department of Earth and Atmospheric Sciences, and is Director of the Mission-Oriented Seismic Research Program at the University of Houston. He received his Ph.D. from the City University of New York (CUNY/CCNY), was a research scientist at Cities Service Company, SOHIO Petroleum Company, ARCO Exploration and Production Technology, and Schlumberger Cambridge Research, and was a visiting professor at the Universidade Federal da Bahia. He served as SEG Distinguished Lecturer in 2003, he received the Townsend Harris Medal from CUNY/CCNY in 2008, the Reginald Fessenden Medal from SEG in 2010, and in 2016, he received the highest SEG honor and recognition, the Maurice Ewing Medal.

Direct and indirect inversion and a new and comprehensive perspective on the role of primaries and multiples in seismic data processing for structure determination and amplitude analysis

Arthur B. Weglein*

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ABSTRACT

The removal and use of multiples have a single shared goal and objective: the imaging and inversion of primaries. There are two kinds of primaries: recorded primaries and unrecorded primaries. For imaging recorded primaries using an industry standard practice smooth velocity model, recorded multiples must be removed, to avoid false and misleading images due to the multiples. Similarly, to find an approximate image of an unrecorded primary, that is a subevent of a recorded multiple, unrecorded multiples that are subevents of the recorded multiple must be removed, for exactly the same problem and reason that recorded multiples are needed to be eliminated. Direct inverse methods are employed to derive this new comprehensive perspective on primaries and multiples. Direct inverse methods not only assure that the problem of interest is solved, but equally important, that the problem of interest is the relevant problem that we (the petroleum industry) need to be interested in.

versus use multiples". The premise behind that "versus" phrasing speaks to a competing and adversarial relationship.

A contribution in this paper is placing these two activities and interests within a single comprehensive framework and platform. That in turn reveals and demonstrates their complementary rather than adversarial nature and relationship.

They are in fact after the same single exact goal, that is, to image primaries: both recorded primaries and unrecorded primaries. There are circumstances where a recorded multiple can be used to find an approximate image of an unrecorded subevent primary of the recorded multiple.

All direct methods for imaging and inversion require only primaries as input. To image recorded primaries requires that recorded multiples must first be removed. To try to use a recorded multiple to find an approximate image of an unrecorded primary subevent of the recorded multiple requires that unrecorded multiple subevents of the recorded multiple be removed. All multiples, recorded multiples and unrecorded multiples need to be removed. Not removing those recorded and unrecorded multiples will produce imaging artifacts and false and misleading

A wedge resolution comparison between RTM and the first migration method that is equally effective at all frequencies at the target: tests and analysis with both conventional and broadband data

Yanglei Zou, Qiang Fu, and Arthur B. Weglein, M-OSRP/Physics Dept./University of Houston

SUMMARY

Acquiring lower-frequency seismic data is an industry-wide interest. There are industry reports that (1) when comparing the new and more expensively acquired broad-band lower-frequency data with conventional recorded data, taken over a same region, these two data sets have the expected difference in frequency spectrum and appearance, but (2) they often provide less than the hoped for difference in structural resolution improvement or added benefit for amplitude analysis at the target and reservoir. In Weglein et al. (2016) and Q. Fu et al. (2017), they demonstrate that all current migration and migration-inversion methods make high-resolution asymptotic assumptions. Consequently, in the process of migration, they lose or discount the information in the newly acquired lowest-frequency components in the broadband data. The new Stolt extended Claerbout III migration for heterogeneous media (Weglein et al. 2016) addresses this problem as the first migration method that is equally effective at all frequencies at the target and reservoir. That allows the broadband lower frequency data to provide full benefit for improving structural resolution and amplitude analysis. Q. Fu et al. (2017) provide the first quantification of the difference and impact on resolution for RTM (CII) and Stolt extended CIII. In this paper, we continue to study and quantify these differences in the migration resolution using a wedge model and define the added resolution value provided by the new Stolt extended CIII migration for heterogeneous medium. The side lobes of the images of upper and lower reflectors produce an interference that determines resolution. The migration method with a greater reduction of side lobes will be the migration with a greater ability to resolve two reflectors with a same bandwidth in the data, conventional or band limited.

INTRODUCTION

Migration methods that use wave theory for seismic imaging have two components: (1) a wave-propagation concept and (2) an imaging condition. Today all migration methods make a high-frequency approximation in (1) or (2) or both (1) and (2). Our new migration method, Stolt extended CIII for heterogeneous media is the first migration method that makes no high-frequency approximation in both components (1) and (2), for a heterogeneous medium, and is equally effective at all frequencies at the target and/or the reservoir. Weglein (2016) provides a detailed development of this new migration method.

For the imaging principle component, a good start is Jon Claerbout's 1971 landmark contribution (Claerbout, 1971) where three imaging principles are described. The first is the exploding-reflector model for stacked or zero-offset data, which we call Claerbout imaging principle I (CI). The second is time-space coincidence of upgoing and downgoing waves, which we call Claerbout imaging principle II (CII). Waves propagate down from the source, are incident on the reflector, and the reflector generates a reflected upgoing wave. According to CII, the reflector exists at the location in space where the wave that is downward propagating from the source and the upwave from the reflector are at the same time and space. All RTM methods are based on RTM (CII) imaging principle and we after refer to RTM in this paper as RTM (CII). The third is Claerbout imaging principle III (CIII), which starts with surface source and receiver data and predicts what a source and receiver would record inside the earth. CIII then arranges the predicted source and receiver to be coincident and asks for $t = 0$. If the predicted coincident source and receiver experiment at depth is proximal to a reflector one gets a non-zero result at time equals zero. Stolt and his colleagues provided several major extensions of CIII and we refer to that category of imaging principles/methods as Stolt extended CIII.

RTM (CII) and Stolt extended CIII are of central industry interest today, since we currently process pre-stacked data. RTM (CII) and Stolt extended CIII will produce different results for a separated source and receiver located in a homogeneous half space above a single horizontal reflector. That difference forms a central and key message of this paper.

CII can be expressed in the form

$$I(\vec{x}) = \sum_{\vec{x}_s}\sum_{\omega} S'(\vec{x}_s,\vec{x},\omega)R(\vec{x}_s,\vec{x},\omega), \tag{1}$$

where R is the reflection data (for a shot record), run backwards, and S' is the complex conjugate of the source wavefield.

A realization of CIII is Stolt FK migration (Stolt, 1978)

$$\begin{aligned} M^{stolt}(x,z) = {} & \frac{1}{(2\pi)^3}\iiint d\omega dx_g dx_s dk_{sx} \\ & \times \exp(-i(k_{sz}z + k_{sx}(x-x_s))) \\ & \times \int dk_{gx}\exp(-i(k_{gz}z + k_{gx}(x-x_s))) \\ & \times \int dt \exp(i\omega t)D(x_g,x_s,t). \end{aligned} \tag{2}$$

The weighted sum of recorded data, summed over receivers, basically predicts the receiver experiment at depth, for a source on the surface. The sum over sources predicts the source in the subsurface. Then the predicted source and receiver experiment is output for a coincident source and receiver, and at time equals zero; it defines a Stolt extended CIII image. Each step (integral) in this Stolt extended CIII has a specific physically interpretable purpose towards the Stolt extended CIII image.

RTM IS A HIGH-FREQUENCY APPROXIMATION

Today all migration methods assume a high-frequency approximation in a wave-propagation concept or an imaging condition or both. How does one know if a migration method has made a high-frequency approximation? If you have a ray-based travel time picture of candidate images in the migration process at any step or stage in the migration method, then the migration method has made an asymptotic high-frequency assumption/approximation. As we will see for RTM (CII), for one source and one receiver, the image is an ellipse. If you have a travel-time ellipse of candidate images, that is an absolute indicator that the migration method has made a high-frequency approximation.

In Figure 2 and 3, we compare the results of RTM (CII) and Stolt extended CIII for one source and one receiver, RTM (CII) provides an ellipse while Stolt extended CIII does not. Stolt extended CIII provides a local image. For RTM (CII), in this simplest case, where the data is perfect and the medium is homogeneous, the contribution from one source and one receiver, you obtain a set of candidates. Stolt extended CIII will never provide candidates. Stolt extended CIII will bring you to a point in the earth where you have a coincident source and receiver experiment. At time equals zero, if there is a non zero result, you are at a reflector. There is structure there, not a possible or candidate structure. The result from RTM (CII) is a set of candidates of possible structure. That is intrinsic to CII, hence intrinsic to all current RTM. Hence, if you are imaging with RTM or any extension of RTM, it is worth noting that you have made a high-frequency approximation in your migration methods.

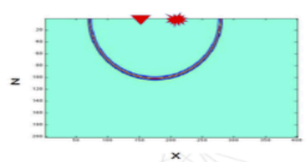

Figure 1: 2D RTM (CII) result for one source and one receiver. High-frequency assumption

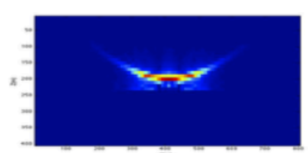

Figure 2: 2D CIII Stolt migration result for one source and one receiver. No high-frequency assumption

All RTM (CII) imaging, i.e., all RTM methods in use today, incorporate high-frequency approximations/assumptions in the imaging principle itself, regardless of how they are implemented. For a heterogeneous medium and assuming one-way propagation (at a point, or overall downgoing between source and reflector and then upgoing from reflector to receiver), a high-frequency approximation has been made, even if you are adopting a CIII imaging principle.

Equation 3 is the new Stolt extended CIII migration method for heterogeneous media of Weglein et al. (2016).

$$P = \int_{S_s}\left[\frac{\partial G_0^{DN}}{\partial z_s}\int_{S_g}\left\{\frac{\partial G_0^{DN}}{\partial z_g}P + \frac{\partial P}{\partial z_g}G_0^{DN}\right\}dS_g + G_0^{DN}\frac{\partial}{\partial z_s}\int_{S_g}\left\{\frac{\partial G_0^{DN}}{\partial z_g}P + \frac{\partial P}{\partial z_g}G_0^{DN}\right\}dS_g\right]dS_s \quad (3)$$

Equation 3 was Stolt extended CIII imaging for a heterogeneous medium, and doesn't assume one-way propagation at either a point, or, separately, overall between source and reflector, and, reflector to receiver. G_0^{DN} is the Green's function for the heterogeneous medium that vanishes along with its normal derivative at the lower surface of the migration volume (Weglein et al., 2011b). Equation 3 is the first migration method that makes no high-frequency approximation in both wave-propagation concept and in the imaging condition for heterogeneous media, i.e., it is equally effective at all frequencies at the target and at the reservoir. For details please see Weglein et al. (2011a,b) and F. Liu and Weglein (2014).

QUANTIFY THE DIFFERENCE AND IMPACT ON RESOLUTION

To quantify the impact and to examine how different migration methods treat different bandwidths in the data, we examine the relative reduction of side-lobe amplitudes for each migration method using conventional and band-limited data. Side lobes in the data are an expression of the band-limited source. For events in the data, the more we extend the low-frequency content of the spectrum, (1) the smaller the amplitude of side lobes and (2) the closer the side lobes move towards the center of the event.

Fu et al. (2017) produced the first direct comparison of differences in structural resolution produced by RTM (CII) and Stolt extended CIII using data with and without low frequencies and the same homogeneous velocity model. There are two factors that contribute to these differences: (1) the imaging condition itself and (2) the way the imaging condition is implemented. In RTM (CII) both the imaging condition and how the imaging condition is implemented are each separately making high-frequency approximations. In the new imaging method (Stolt extended CIII for heterogeneous media) from M-OSRP both the imaging condition and method of implementation are equally effective at all frequencies at the target and reservoir. There are side lobes in the structural image due to the missing low frequencies. With the new imaging method see equation 3 and including low frequencies in the input data the side lobes are reduced 57% (from 0.33 to 0.14) whereas the conventional leading edge RTM only reduced the side lobes by 21% (from 0.78 to 0.62). The new imaging method equation 3 is able to benefit from broadband data for structural resolution improvement to a much greater extent than the current best industry standard.

In this paper we continue to study the resolution differences of RTM (CII) and Stolt extended CIII. We produce the first wedge-model test for the comparison of structural resolution differences with data with and without low frequencies, using the same homogeneous velocity model, comparing RTM (CII) and Stolt extended CIII. With Stolt extended CIII and including low frequencies in the input data the side lobes are reduced 87% whereas RTM (CII) only reduced the side lobes by 50%. More low frequency was included in these tests than in the earlier Q. Fu et al (2017) tests. This result is consistent with the result in Fu et al. (2017). Stolt extended CIII is able to benefit from broadband data for structural resolution improvement to a much greater extent than the current best industry standard. The wedge model test in this paper further demonstrates that the Stolt extended CIII result has better resolution than the RTM (CII) result due to the smaller side lobes in the image. For Stolt extended CIII broadband data, two reflectors can be identified when the distance between 2 reflectors is greater than 25m, while for RTM (CII) broadband data the distance between 2 reflectors must be greater than 50m. For Stolt extended CIII conventional data, two reflectors can be identified when the distance between 2 reflectors is greater than 50m. while for RTM (CII) conventional data, the distance between 2 reflectors must be greater than 75m.

NUMERICAL TEST ON A WEDGE MODEL

The tests and comparisons in this paper had a broad band data that had a high frequency cut-off but the spectrum was full on the low end. That gave a limit or end-member for the most improvement in resolution for a layer that the new migration equation 3 could produce with broadband data. This analysis and conclusion does not depend on having data down to zero frequency. We generate the two events separately and then combine them together to generate the two-event synthetic data. For each event, a two half-space model is used, the velocity of upper half-space is 1500m/s and the lower one is 2000m/s. For the first events the interface between the two half-space is 1500m. For the second event, the location is varying from 1512.5m to 1275m to mimic the wedge model. The purpose of this procedure is to correctly locate both of the two events in the image space using a homogeneous velocity model.

The two wavelets used in the tests are both band-limited spikes. The frequency range of the first one (broadband) is 0Hz-50Hz and of the other one (conventional) is 20Hz-50Hz. Figure 5 (upper left and upper right) shows the frequency spectra of the two wavelets and figure 5 (lower left and lower right) shows the time domain waveforms.

Fig 6 shows the RTM (CII) and Stolt extended CIII images for one reflector at 1500m with the two different wavelets. The upper left is the Stolt extended CIII image with broadband data, the lower left is the Stolt extended CIII image with conventional data, the upper right is the RTM (CII) image with broadband data, and the lower right is the RTM (CII) image with conventional data. For Stolt extended CIII the side lobes are reduced more than 87%, whereas for RTM (CII) the side lobes

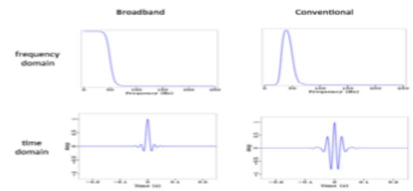

Figure 3: The upper left and upper right show the frequency spectra of the two wavelets; the lower left and lower right show the time-domain waveforms.

reduced only about 50%. This result is consistent with that in Q. Fu et al. (2017).

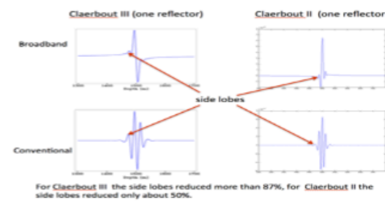

Figure 4: The upper left is the CIII image with broadband data, the lower left is the CIII image with conventional data, the upper right is the RTM (CII) image with broadband data, and the lower right is the RTM (CII) image with conventional data. For CIII the side lobes are reduced more than 87%, whereas for RTM (CII) the side lobes reduced only about 50%. This result is consistent with that in Q. Fu et al. (2017).

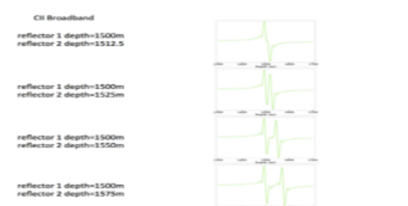

Figure 5: The Stolt extended CIII wedge model image for broadband data with first reflector at 1500m and second reflector at 1512.5m, 1525m, 1550m, 1575m respectively.

Figure 7-10 show the RTM (CII) and Stolt extended CIII image for a wedge model with the broadband data and conventional data. Figure 7 shows the Stolt extended CIII wedge model image for broadband data with the first reflector at 1500m and second reflector at 1512.5m, 1525m, 1550m, 1575m respec-

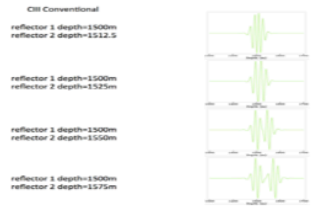

Figure 6: The Stolt extended CIII wedge model image for conventional data with first reflector at 1500m and second reflector at 1512.5m, 1525m, 1550m, 1575m respectively..

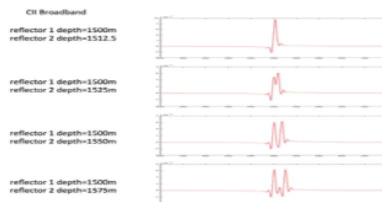

Figure 7: The RTM (CII) wedge model image for broadband data with first reflector at 1500m and second reflector at 1512.5m, 1525m, 1550m, 1575m respectively.

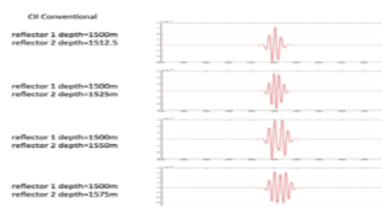

Figure 8: The RTM (CII) wedge model image for conventional data with first reflector at 1500m and second reflector at 1512.5m, 1525m, 1550m, 1575m respectively.

tively. Figure 8 shows Stolt extended CIII wedge model image for conventional data with first reflector at 1500m and second reflector at 1512.5m, 1525m, 1550m, 1575m respectively. Figure 9 shows RTM (CII) wedge model image for broadband data with first reflector at 1500m and second reflector at 1512.5m, 1525m, 1550m, 1575m respectively.Figure 10 shows The RTM (CII) wedge model image for conventional data with first reflector at 1500m and second reflector at 1512.5m, 1525m, 1550m, 1575m respectively.

From the figures we can conclude that two reflectors are separated when the distance between 2 reflectors is greater than 25m for Stolt extended CIII Broadband data, 50m for Stolt extended CIII conventional data, 50m for RTM (CII) Broadband data and 75m for RTM (CII) conventional data.

CONCLUSION

In this paper we produced the first wedge-model test for the comparison of structural resolution differences with data with and without low frequencies, comparing the current leading edge RTM (CII) and the Stolt extended CIII imaging principle. RTM (CII) has a high-frequency assumption in its imaging principle. The Stolt extended CIII imaging principle is not a high-frequency imaging principle. There are side lobes in the structural image due to the missing low frequencies. For a single reflector, including low frequencies in the input data, the side lobes are reduced 87% in Stolt extended CIII whereas the side lobes are only reduced 50% in RTM (CII), which is consistent with the result in Q. Fu et al. (2017). The new imaging method is able to benefit from broadband data for structural resolution improvement to a much greater extent than the current best industry standard migration. The wedge model test in this paper further demonstrates that the Stolt extended CIII result has better resolution than the RTM (CII) result due to the smaller side lobes in the image from each reflector. For Stolt extended CIII with broadband data, two reflectors can be identified when the distance between 2 reflectors is greater than 25m, while for RTM (CII) with broadband data the distance between 2 reflectors must be greater than 50m. For Stolt extended CIII with conventional data, two reflectors can be identified when the distance between 2 reflectors is greater than 50m. While for RTM (CII) with conventional data, the distance between 2 reflectors must be greater than 75m. In this paper we examine the resolution difference for a wedge model. All current migration method (including RTM) assume a one-way propagation model at every point in the subsurface for a smooth velocity model. That one-way propagation model is a high-frequency approximation. The new Stolt extended CIII for heterogeneous media assumes a two-way propagation model at every point in a smoothly varying medium. The next planned tests will include implementation differences (i.e. the wave propagation component of migration) for a smooth velocity model. The differences in resolution derived from the new migration method, Stolt extended CIII for heterogeneous media, that makes no high-frequency approximation in both (A) the wave propagation concept (B) the imaging principle will be greater when both the imaging principle and the wave propagation model are included than we report here for only the imaging principle differences.

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EDITED REFERENCES

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- Link to 2018 M-OSRP Annual Meeting
  - This link below provides a menu for all the video presentation from the 2018 M-OSRP Annual Technical Review- we point out, and possibly of particular interest for this SEG/KOC Workshop, **are the advances by Dr. Jing Wu in on-shore ground roll and reflection data prediction without damaging either, and for on-shore de-ghosting.**
  - M-OSRP Annual Technical Review Presentations: Videos with Synced Slides and Meeting Agenda