Why exactly do multiples need to be removed in direct seismic processing methods? And what about indirect methods?

Arthur B. Weglein[1], M-OSRP/Physics Dept./University of Houston[1]

INTRODUCTION

Multiple removal has been a long-term objective in seismic exploration. Recent methods that use multiples for different processing goals and objectives can be worthwhile. However, their use can also be a source of confusion as to whether the removal is no longer essential, a priority, or even necessary since some may now view multiples as "rehabilitated" and sit along side primaries as entirely useful events. One purpose of this paper is to disabuse us of that seriously flawed and erroneous thinking and to understand that the use of multiples and the removal of multiples have the same exact goal and objective: the imaging of primaries. We explain exactly why multiples need to be removed in all direct and indirect seismic processing methods — and therefore why multiple removal remains a key and central issue and challenge in seismic exploration.

MULTIPLE REMOVAL AND DIRECT AND INDIRECT SEISMIC PROCESSING METHODS

In what follows, we will explain why all direct and indirect seismic processing methods remove multiples at some step or stage within the method. Among references that define (and distinguish between) direct and indirect seismic processing methods are: Weglein (2013, 2017, 2018).

DIRECT SEISMIC METHODS

For the purposes of this paper, it is useful to separate direct methods that require or do not require a velocity model.

DIRECT SEISMIC METHODS WITH A VELOCITY MODEL — MIGRATION, HOMOGENEOUS, CONTINUOUS AND DISCONTINUOUS VELOCITY MODELS

Methods that employ the wave equation for migration have two ingredients: (1) a wave propagation concept and (2) an imaging condition. In Claerbout (1971), he described three imaging conditions for seismic migration. He combined these imaging conditions with one-way wave propagation concepts to determine structure at depth. The three imaging conditions are: (1) the exploding reflector model, (2) the space and time coincidence of an upgoing wave from the reflector and a downgoing wave from the source, and (3) the predicted coincident source and receiver experiment at depth, at time equals zero. We will refer to these original imaging conditions as Claerbout I, II and III. Claerbout I (the exploding reflector model) only relates to stacked or zero offset data. Claerbout II and Claerbout III are valid for prestack data. The third imaging condition, CIII, stood alone in terms of clarity and definitiveness and in its potential to be extended for complex structure

and associated amplitude analysis. For example, Stolt and Weglein (1985) and Stolt and Weglein (2012) extended the original CIII for more physically complete and accommodating structural models, and in addition provides a detailed angle dependent amplitude analysis at the target, for both specular and non-specular reflection. We label the latter extension Stolt Claerbout III migration (or SCIII). Only CIII or SCIII could be extended to accommodate imaging beneath a discontinuous medium, the latter required to analyze (for the first time) and to unambiguously define the role of primaries and multiples in migration. The predicted coincident source and receiver experiment at depth consists of all the events that experiment would record, if you actually had a source and receiver at that subsurface location.

Weglein et al. (2016) provided that extension of SCIII to allow a discontinuous medium above a target and to image above and below each reflector, without any artifacts or issues such as "rabbit ears". That new migration algorithm is represented in equation (3) below.

STOLT CLAERBOUT III MIGRATION IN HOMOGENEOUS, SMOOTHLY VARYING OR DISCONTINUOUS MEDIA

For one-way wave propagation in a homogeneous or smoothly varying 2D medium the predicted source and receiver experiment at depth, $D(x_g, z_g, x_s, z_s, \omega)$ is

$$
\begin{aligned}
&D(x_g, z_g, x_s, z_s, \omega)(\text{at depth}) \\
&= \int_{S_s} \frac{\partial G_0^{-D}(x_s, z_s, x_s', z_s', \omega)}{\partial z_s'} \\
&\times \int_{S_g} \frac{\partial G_0^{-D}(x_g, z_g, x_g', z_g', \omega)}{\partial z_g'} \\
&\times D(x_g', z_g', x_s', z_s', \omega) \, dS_g \, dS_s,
\end{aligned}
\tag{1}
$$

where (x_g, z_g) and (x_s, z_s) are the coordinates of the predicted receiver and source at depth, and D in the integrand is the data, D(on the measurement surface), G_0^{-D} is the anticausal Green's function with Dirichlet boundary condition on the measurement surface, s connotes shot, and g, receiver, respectively. For one way upgoing wave propagation, and this choice of Green's function, the lower surface doesn't contribute to the field inside the finite volume. For two-way propagation, in, e.g., a discontinuous medium above the image point (i.e., above the target reflector), we begin with the recorded data $D(x_g', z_g', x_s', z_s', \omega)$ on horizontal measurement surfaces, with $z_g' = $ constant and $z_s' = $ constant. The predicted experiment for the receiver at x_g, z_g, at depth, and the source at x_s', z_s' (on the measurement surface) is

$$
\int \left\{ \frac{\partial G_0^{DN}}{\partial z_g'}(x_g, z_g, x_g', z_g', \omega) D(x_g', z_g', x_s', z_s', \omega) \right.
$$

$$-G_0^{DN}(x_g,z_g,x_g',z_g',\omega)\,\frac{\partial D}{\partial z_g'}(x_g',z_g',x_s',z_s',\omega)\Bigg\}\underbrace{dx_g'}_{dS_g}$$

$$=D(x_g,z_g,x_s',z_s',\omega) \qquad (2)$$

A second application of Green's theorem inputs (2) and then predicts the experiment for both the receiver at x_g,z_g and the source at x_s,z_s, at depth, using equation (3) below

$$\int\Bigg\{D(x_g,z_g,x_s',z_s',\omega)\,\frac{\partial G_0^{DN}}{\partial z_s'}(x_s,z_s,x_s',z_s',\omega)$$

$$-G_0^{DN}(x_s,z_s,x_s',z_s',\omega)\,\frac{\partial D}{\partial z_s'}(x_g,z_g,x_s',z_s',\omega)\Bigg\}\underbrace{dx_s'}_{dS_s}$$

$$=D(x_g,z_g,x_s,z_s,\omega) \qquad (3)$$

Equation (3) is the prediction required for Stolt Claerbout III migration for heterogeneous (and discontinuous) media. G_0^{DN} is the Green's function for wave propagation in the finite volume that vanishes along with its normal derivative on the lower surface of the finite volume. $dS_g = dx_g'$ and $dS_s = dx_s'$ in a 2D prediction. An integral of equation (3) over ω produces the predicted experiment at t=0 and SCIII migration.

SUMMARY FOR STOLT CIII MIGRATION IN A LAYERED MEDIUM WITH DATA CONSISTING OF PRIMARIES AND MULTIPLES

The analytic analysis of equation (3) for a layered medium is found in Weglein (2016), (following Liu and Weglein, 2014) and demonstrates, for the first time, how the recorded events contribute to the experiment and then to the image for source and receiver experiments predicted above and below each reflector. At each depth, z, below the measurement surface, the predicted coincident source and receiver experiment cares about (depends on) all the actual recorded primary and multiple events on the measurement surface. However, when the imaging condition $t = 0^+$ is applied to the coincident source and receiver experiment at depth, z, only the recorded primaries on the measurement surface contribute to the migration result at any z below the measurement surface. The conclusion: multiples do not contribute to the image at any depth, when using an accurate discontinuous velocity model above the reflector to be imaged. That is, if we migrated data consisting of primaries and multiples with an accurate discontinuous velocity model, and used Stolt CIII migration for heterogeneous media, equation (3) at t=0, then multiples in the recorded data on the measurement surface will not contribute to the image above or below a reflector. For that (correct discontinuous migration velocity) situation, multiples would not cause false images, and would not be harmful, or helpful. Please see figures 1-3 that illustrate these steps and conclusions for recorded data consisting of primaries and free surface multiples, and figures 4-7 for recorded data consisting of primaries and internal multiples, using equation (3) a Stolt Claerbout III migration for heterogeneous media. (Weglein et al., 2016)

However, if the migration (in a discontinuous medium) uses a smooth velocity for the data consisting of primaries and mul-
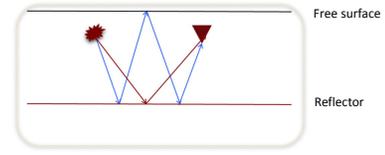
Red event: primary
Blue event: free-surface multiple

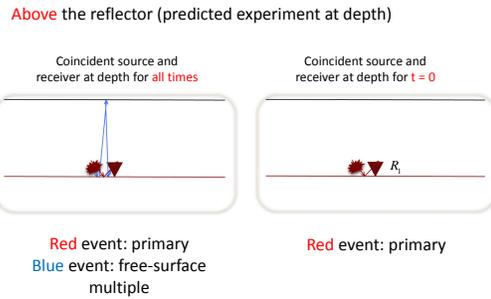Figure 1: *a primary and a free surface multiple (recorded data)*

Red event: primary
Blue event: free-surface multiple

Red event: primary

Figure 2: *the predicted experiment (and the t=0 image) at depth (above the reflector) from a recorded data consisting of a primary and a free-surface multiple*
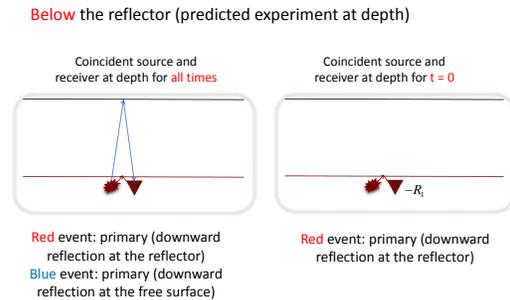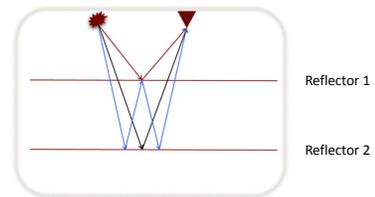

Red event: primary (downward reflection at the reflector)
Blue event: primary (downward reflection at the free surface)

Red event: primary (downward reflection at the reflector)

Figure 3: *the predicted experiment (and t=0 image) at depth beneath the reflector, from a recorded data consisting of a primary and free surface multiple*

Red event: primary from the first reflector
Black event: primary from the second reflector
Blue event: internal multiple

Figure 4: *the recorded data consisting of two primaries and an internal multiple*
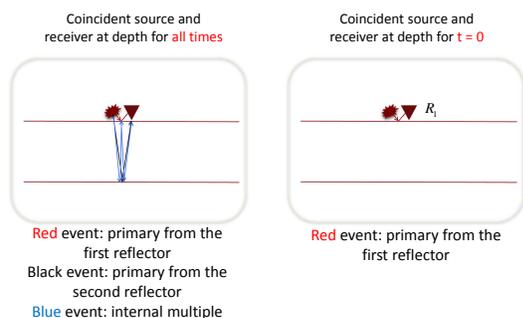
Figure 5: *the predicted experiment (and image) above the first reflector, for a recorded data consisting of two primaries and an internal multiple*
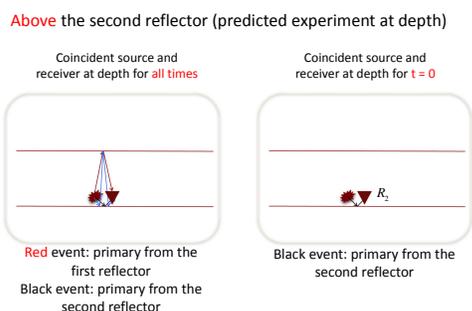

Figure 6: *The predicted experiment (and image at t=0) above the second reflector for a recorded data consisting of two primaries and an internal multiple*
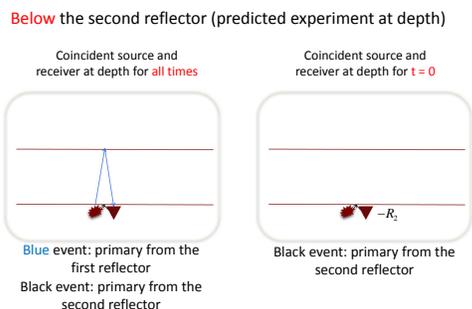

Figure 7: *the predicted experiment at depth (and image at t=0) beneath the second reflector for a recorded data consisting of two primaries and an internal multiple*

tiples, the "predicted" source and receiver experiment at depth will not be the actual source and receiver at depth. That difference and error results in every multiple causing a false image. Hence for a smooth velocity model, multiples must be removed. Since the industry leading edge migration velocity methods can at best find an improved smooth velocity model — and there are currently no candidates to produce an accurate discontinuous velocity model, recorded multiples must be removed now and for the foreseeable future. See e.g. the 2021 SEG/DGS Workshop on Velocity Model Building (Saad et al., 2021) and the final/wrap-up presentation by Weglein (Weglein, 2021)

DIRECT METHODS WITH A VELOCITY MODEL: REMOVING AND USING MULTIPLES

Multiples can at times be useful, see e.g., Lu et al. (2011); Whitmore et al. (2011a,b); Ma and Zou (2015) but the migration of a multiple has no meaning (Weglein, 2019b). Within the sphere of direct seismic methods with a velocity model, certain multiples can be useful to seek an approximate image of an unrecorded primary. The removal of recorded multiples is necessary to image recorded primaries, and the removal of unrecorded multiples is required to find an approximate image of an unrecorded primary. All multiples must be removed to image primaries, recorded and unrecorded primaries. Weglein (2018, 2017, 2019a)

The above methodology (of using multiples) assumes that a recorded free surface multiple consists of two subevents, one that is recorded, and that the second subevent is a primary that is unrecorded. The idea is to extract and predict, from the recorded multiple and its recorded subevent, the approximate image of the unrecorded primary. If all the subevents of the multiple are recorded, the multiple has no use. This use of multiples is itself a testament to the fact that a complete set of recorded primaries is sufficient for imaging the subsurface.

We often hear that multiples can be useful to enhance illumination. To paraphrase Jon Claerbout "Waves, and the reflected seismic wavefield, are ubiquitous, and have no illumination issues. However, seismic processing methods that are asymptotic high frequency approximations, ray-like in nature, (e.g., Kirchhoff and RTM migration) can "squeeze" the wave into ray paths, that leave gaps and produce illumination issues and challenges." In contrast, Stolt CIII migration (Weglein et al., 2016), equation (1) and equation (3), are the only migration methods that make no high frequency approximation in either the imaging condition or the propagation model.

The fact that our most capable migration velocity models (today and for the foreseeable future) are smooth and continuous, remains the key and central reason that all multiples must be removed for imaging and inversion when using any method that requires a velocity model.

DIRECT METHODS WITHOUT A VELOCITY MODEL

The only direct method that can input primaries and multiples, and output all processing objectives without knowing, estimating or determining subsurface information (including velocity) is the isolated task subseries of the inverse scattering series. There are distinct subseries that directly remove free surface and internal multiples (e.g. Weglein et al., 2003; Zou et al., 2019). If multiples were needed to perform tasks such as depth imaging, Q compensation and parameter estimation the ISS would not have subseries whose entire purpose is to remove them. For direct seismic methods that do not require a velocity model multiples must be removed, with primaries (only) as input to the imaging and inversion subseries. See, e.g., Weglein et al. (2012); Zhang and Weglein (2009a,b); Liang et al. (2013); Zou and Weglein (2018).

INDIRECT SEISMIC METHODS, E.G., CIG FLATNESS AND FWI

There are different types of indirect inverse methods. Among them are:
(1) seeking to satisfy a property that a inverse solution would possess;
(2) solving a forward problem in an inverse sense, and
(3) model matching.
The CIG flatness criteria is in the first category, while solving an elastic inverse in terms of PP data and FWI are in the second and third category, respectively. Why each of these is "indirect" is fully detailed in Weglein (2013, 2018, 2017, 2020).

Indirect methods like CIG flatness represent a necessary but not sufficient imaging condition that a correct migration velocity model has been found. The CIG flatness criteria assumes that the data consists of primaries and that multiples have been removed.

References for CIG are Anderson et al. (2012); Baumstein et al. (2009); Ben-Hadj-ali et al. (2008, 2009); Biondi and Sava (1999); Biondi and Symes (2004); Brandsberg-Dahl et al. (1999); Chavent and Jacewitz (1995); Fitchner (2011); Guasch et al. (2012); Kapoor et al. (2012); Rickett and Sava (2002); Sava et al. (2005); Sava and Fomel (2003); Sirgue et al. (2009, 2010, 2012); Symes and Carazzone (1991); Tarantola (1987); Zhang and Biondi (2013). Many wrong velocity models can and will also satisfy a flat common-image-gather criterion, especially under complex imaging circumstances.

Another type of indirect method, FWI, is a model matching methodology that can input any data set, consisting of primaries, free surface multiples and internal multiples. Among FWI references are Brossier et al. (2009); Crase et al. (1990); Gauthier et al. (1986); Nolan and Symes (1997); Pratt (1999); Pratt and Shipp (1999); Sirgue et al. (2010); Symes (2008); Tarantola (1984, 1986); Valenciano et al. (2006); Vigh and Starr (2008); Zhou et al. (2012). In practice, primaries are considered not enough, and primaries and all multiples too much to match. Internal multiples are first removed and then pri-

maries and free surface multiples are matched. As was documented in a recent SEG/DGS Workshop on Velocity Model Building Saad et al. (2021) and the final/wrap-up presentation by Weglein (2021), FWI has been useful in providing an improved smooth velocity for migration. As we pointed out earlier in this paper, with a smooth migration velocity model, all multiples must be removed.

Therefore either initially or ultimately all multiples must be removed in all indirect seismic methods.

CONCLUSIONS

All current migration velocity analysis methods can (at best) produce a smooth continuous migration velocity model. For direct seismic methods, for example, migration with a smooth velocity model, all multiples will cause false images and artifacts — and need to be removed.

To use a recorded multiple, we assume it contains two subevents, one recorded and the other not recorded. Let's further assume that the unrecorded subevent is an unrecorded primary. When using a recorded multiple, and the recorded subevent of the multiple, to seek an estimate of the image of an unrecorded primary subevent of the multiple — to satisfy the latter assumption, unrecorded subevents of the recorded multiple, that are (not unrecorded primaries but rather) unrecorded multiples, must be removed. Furthermore the original recorded multiple must be removed to image recorded primaries. Hence, recorded and unrecorded multiples must be removed to image recorded and unrecorded primaries.

The inverse scattering series, is the only direct inversion method for a multi-dimensional earth that doesn't require any subsurface information (including velocity) to be known, estimated or determined. It contains distinct isolated task subseries that remove free surface and internal multiples. Only primaries are called for in task specific subseries for structure determination, parameter estimation and Q compensation without knowing, estimating or determining Q.

For indirect methods, based on a criteria that only relate to primaries, e.g., CIG flatness, multiples must first be removed. FWI is model matching of primaries and multiples and currently is able to output a smooth velocity for migration. Multiples must be removed when using a smooth velocity for migration. For the smooth migration velocity output of FWI to be useful, for imaging and inversion, multiples must first be removed.

Hence, all direct and indirect seismic processing methods require all multiples to be removed, either initially or eventually.

ACKNOWLEDGEMENTS

REFERENCES

Anderson, J. E., L. Tan, and D. Wang, 2012, Time-reversal checkpointing methods for RTM and FWI: Geophysics, **77**, S93–S103.

Baumstein, A., J. E. Anderson, D. L. Hinkley, and J. R. Krebs, 2009, Scaling of the objective function gradient for full-wavefield inversion: 79th Annual International Meeting, SEG, Expanded Abstracts, 2243–2247.

Ben-Hadj-ali, H., S. Operto, and J. Vireux, 2009, Efficient 3D frequency-domain full-waveform inversion (FWI) with phase encoding: 71st Annual International Conference and Exhibition, EAGE, Extended Abstracts, P004.

Ben-Hadj-ali, H., S. Operto, and J. Virieux, 2008, Velocity model building by 3D frequency-domain, full-waveform inversion of wide-aperture seismic data: Geophysics, **73**, VE101–VE117.

Biondi, B., and P. Sava, 1999, Wave-equation migration velocity analysis: 69th Annual International Meeting, SEG, Expanded Abstracts, 1723–1726.

Biondi, B., and W. Symes, 2004, Angle-domain common-image gathers for migration velocity analysis by wavefield-continuation imaging: Geophysics, **69**, 1283–1298.

Brandsberg-Dahl, S., M. de Hoop, and B. Ursin, 1999, Velocity analysis in the common scattering-angle/azimuth domain: 69th Annual International Meeting, SEG, Expanded Abstracts, 1715–1718.

Brossier, R., S. Operto, and J. Virieux, 2009, Robust elastic frequency-domain full-waveform inversion using the L_1 norm: Geophysical Research Letters, **36**, L20310.

Chavent, G., and C. Jacewitz, 1995, Determination of background velocities by multiple migration fitting: Geophysics, **60**, 476–490.

Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: Geophysics, **36**, 467–481.

Crase, E., A. Pica, M. Noble, J. McDonald, and A. Tarantola, 1990, Robust elastic nonlinear waveform inversion: Application to real data: Geophysics, **55**, 527–538.

Fitchner, A., 2011, Full seismic waveform modeling and inversion: Springer-Verlag.

Gauthier, O., J. Virieux, and A. Tarantola, 1986, Two dimensional nonlinear inversion of seismic waveforms: Geophysics, **51**, 1387–1403.

Guasch, L., M. Warner, T. Nangoo, J. Morgan, A. Umpleby, I. Stekl, and N. Shah, 2012, Elastic 3D full-waveform inversion: 82nd Annual International Meeting, SEG, Expanded Abstracts, 1–5.

Kapoor, S., D. Vigh, H. Li, and D. Derharoutian, 2012, Full-waveform inversion for detailed velocity model building: 74th Annual Conference and Exhibition, EAGE, Extended Abstracts, W011.

Liang, H., C. Ma, and A. B. Weglein, 2013, General theory for accommodating primaries and multiples in internal multiple algorithm: Analysis and numerical tests: 83rd Annual International Meeting, SEG, Expanded Abstracts, 4178–4183.

Liu, F., and A. B. Weglein, 2014, The first wave equation migration RTM with data consisting of primaries and internal multiples: theory and 1D examples: Journal of Seismic Exploration, **23**, 357–366.

Lu, S., N. D. Whitmore, A. A. Valenciano, and N. Chemingui, 2011, Imaging of primaries and multiples with 3D SEAM synthetic: 81st Annual International Meeting, SEG, Expanded Abstracts, 3217–3221.

Ma, C., and Y. Zou, 2015, A clear example using multiples to enhance and improve imaging: Comparison of two imaging conditions relevant to this analysis: The Leading Edge, **34**, 814–816.

Nolan, C., and W. Symes, 1997, Global solution of a linearized inverse problem for the wave equation: Communications on Partial Differential Equations, **22**, 127–149.

Pratt, R. G., 1999, Seismic waveform inversion in the frequency domain, Part 1: Theory and verification in a physical scale model: Geophysics, **64**, 888–901.

Pratt, R. G., and R. M. Shipp, 1999, Seismic waveform inversion in the frequency domain, Part 2: Fault delineation in sediments using crosshole data: Geophysics, **64**, 902–914.

Rickett, J., and P. Sava, 2002, Offset and angle-domain common image-point gathers for shot-profile migration: Geophysics, **67**, 883–889.

Saad, A., F. ten Kroode, M. Jahdhami, A. Al Shuhail, A. Mansour, D. Vigh, E. Verschuur, G. Schuster, J. Etgen, J. E. Vargas, K. Belaid, K. Delaijan, P. Guillaume, R. Al Kalbani, and T. Burley, 2021, SEG | DGS workshop: Challenges & new advances in velocity model building: Presented at the Virtual Workshop, Society of Exploration Geophysicists. (Conference web page at `https://seg.org/Events/Velocity-Model-Building`).

Sava, P., B. Biondi, and J. Etgen, 2005, Wave-equation migration velocity analysis by focusing diffractions and reflections: Geophysics, **70**, U19–U27.

Sava, P., and S. Fomel, 2003, Angle-domain common-image gathers by wavefield continuation methods: Geophysics, **68**, 1065–1074.

Sirgue, L., O. I. Barkved, J. Dellinger, J. Etgen, U. Albertin, and J. H. Kommedal, 2010, Full-waveform inversion: The next leap forward in imaging at Valhall: First Break, **28**, 65–70.

Sirgue, L., O. I. Barkved, J. P. Van Gestel, O. J. Askim, and J. H. Kommedal, 2009, 3D waveform inversion on Valhall wide-azimuth OBC: 71st Annual International Conference and Exhibition, EAGE, Extended Abstracts, U038.

Sirgue, L., B. Denel, and F. Gao, 2012, Challenges and value of applying FWI to depth imaging projects: `http://earthdoc.eage.org/`. (Event name: 74th EAGE Conference and Exhibition - Workshops, Session: From Kinematic to Waveform Inversion - Where Are we and where Do we Want to Go? A Tribute to Patrick Lailly).

Stolt, R. H., and A. B. Weglein, 1985, Migration and inversion of seismic data: Geophysics, **50**, 2458–2472.

———, 2012, Seismic imaging and inversion: Application of linear inverse theory: Cambridge University Press.

Symes, W. W., 2008, Migration velocity analysis and waveform inversion: Geophysical Prospecting, **56**, 765–790.

Symes, W. W., and J. J. Carazzone, 1991, Velocity inversion by differential semblance optimization: Geophysics, **56**, 654–663.

Tarantola, A., 1984, Inversion of seismic reflection data in the

acoustic approximation: Geophysics, **49**, 1259–1266.

———, 1986, A strategy for nonlinear elastic inversion of seismic reflection data: Geophysics, **51**, 1893–1903.

———, 1987, Inverse problem theory: Method for data fitting and model parameter estimation: Elsevier.

Valenciano, A., B. Biondi, and A. Guitton, 2006, Target-oriented wave-equation inversion: Geophysics, **71**, A35–A38.

Vigh, D., and E. W. Starr, 2008, 3D prestack plane-wave, full-waveform inversion: Geophysics, **73**, VE135–VE144.

Weglein, A., 2021, Wrap-up. Presentation given at the SEG | DGS Workshop: Challenges & New Advances in Velocity Model Building, Virtual Workshop, available at `https://drive.google.com/drive/folders/1mwcO9feU41Bk_mPVkk7DHWA9ufqgveGj?usp=sharing`.

Weglein, A., J. Mayhan, Y. Zou, Q. Fu, F. Liu, J. Wu, C. Ma, X. Lin, and R. Stolt, 2016, The first migration method that is equally effective for all acquired frequencies for imaging and inverting at the target and reservoir: 86th Annual International Meeting, SEG, Expanded Abstracts, 4266–4272.

Weglein, A. B., 2013, A timely and necessary antidote to indirect methods and so-called P-wave FWI: The Leading Edge, **32**, 1192–1204.

———, 2016, Multiples: Signal or noise?: Geophysics, **81**, V283–V302.

———, 2017, A direct inverse method for subsurface properties: The conceptual and practical benefit and added value in comparison with all current indirect methods, for example, amplitude-variation-with-offset and full-waveform inversion: Interpretation, **5**, SL89–SL107.

———, 2018, Direct and indirect inversion and a new and comprehensive perspective on the role of primaries and multiples in seismic data processing for structure determination and amplitude analysis: CT&F — Ciencia, Tecnologia y Futuro, **8**, 5–21.

———, 2019a, A new perspective on removing and using multiples — they have the same exact goal — imaging primaries — recent advances in multiple removal. Presentation given at the SEG | KOC Workshop: Seismic Multiples - The Challenges and the Way Forward, Kuwait City, Kuwait, available at `http://mosrp.uh.edu/news/extended-version-weglein-key-note-2019-seg-koc-workshop`.

———, 2019b, A new perspective on removing and using multiples | they have the same exact goal | imaging primaries, recorded and unrecorded primaries | recent advances in multiple removal. Invited key-note address for the SEG/KOC Workshop: Seismic Multiples, the Challenges and the Way Forward in Kuwait December 3-5, available at `https://youtu.be/sD89_418h1A`.

———, 2020, YouTube video with interview of Arthur B. Weglein for the Bahia, Brazil student chapter of the EAGE. Available at `https://www.youtube.com/watch?v=iir4cuk50Cw&feature=youtu.be`.

Weglein, A. B., F. V. Araújo, P. M. Carvalho, R. H. Stolt, K. H. Matson, R. T. Coates, D. Corrigan, D. J. Foster, S. A. Shaw, and H. Zhang, 2003, Inverse scattering series and seismic exploration: Inverse Problems, **19**, R27–R83.

Weglein, A. B., F. Liu, X. Li, P. Terenghi, E. Kragh, J. D.

Mayhan, Z. Wang, J. Mispel, L. Amundsen, H. Liang, L. Tang, and S.-Y. Hsu, 2012, Inverse scattering series direct depth imaging without the velocity model: First field data examples: Journal of Seismic Exploration, **21**, 1–28.

Whitmore, N. D., A. Valenciano, S. Lu, and N. Chemingui, 2011a, Imaging of Primaries and Multiples with Image Space Surface Related Multiple Elimination: Presented at the 73rd EAGE Conference & Exhibition, Extended Abstracts.

Whitmore, N. D., A. A. Valenciano, S. Lu, and N. Chemingui, 2011b, Deep water prestack imaging with primaries and multiples: Presented at the Twelfth International Congress of the Brazilian Geophysical Society, Sociedade Brasileira de Geofísica.

Zhang, H., and A. B. Weglein, 2009a, Direct nonlinear inversion of 1D acoustic media using inverse scattering subseries: Geophysics, **74**, WCD29–WCD39.

———, 2009b, Direct nonlinear inversion of multiparameter 1D elastic media using the inverse scattering series: Geophysics, **74**, WCD15–WCD27.

Zhang, Y., and B. Biondi, 2013, Moveout-based wave-equation migration velocity analysis: Geophysics, **78**, U31–U39.

Zhou, H., L. Amundsen, and G. Zhang, 2012, Fundamental issues in full-waveform inversion: Presented at the 82nd Annual International Meeting, SEG, Expanded Abstracts.

Zou, Y., C. Ma, and A. B. Weglein, 2019, A new multidimensional method that eliminates internal multiples that interfere with primaries, without damaging the primary, without knowledge of subsurface properties, for offshore and onshore conventional and unconventional plays: 89th Annual International Meeting, SEG, Expanded Abstracts, 4525–4529.

Zou, Y., and A. Weglein, 2018, ISS *Q* compensation without knowing, estimating or determining *Q* and without using or needing low and zero frequency data: Journal of Seismic Exploration, **27**, 593–608.