A new method for deghosting data collected on a depth-variable acquisition surface by combining Green's theorem wave separation followed by a Stolt extended Claerbout III wave prediction for one-way propagating waves

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SUMMARY

The Green's theorem wave-separation methods (e.g., separating the reference wave and the reflection data, and deghosting) have an important property of naturally accommodating an arbitrary measurement surface. However, for non-horizontal acquisition, those methods cannot locate the output point on the surface where the input data is acquired. For example, in order to effectively predict the receiver-side deghosted data, the current algorithm originating from Green's theorem wave separation has to output the deghosted result at a depth that is shallower than the actual acquisition location. If the measurement surface is horizontal, the Green's theorem deghosting algorithm can be expressed in the wavenumber domain, and is equivalent to the widely used PVz method, and it can locate the output result on the measurement surface. However, the PVz method is not applicable with a non-horizontal acquisition surface. To overcome this drawback, this paper develops a two-step method that can achieve receiver-side deghosting right on the measurement surface and without assuming the measurement surface to be horizontal. Therefore, it can deghost the actual acquired data. The new method derives from both Green's theorem based wave separation and one-way wave prediction. The shape of the measurement surface is assumed known. This idea is also viable for source-side deghosting. We develop the method and illustrate with synthetic examples, considering a towed streamer in the water column. It is worth noting that onshore and ocean-bottom acquisition can often have a significantly variable topography and can also benefit from the new method in this paper.

INTRODUCTION

Deghosting has obtained a great deal of industry attention because of the increasing interest of marine broadband seismic technology (Amundsen et al., 2013; Amundsen and Zhou, 2013). The destructive interference between primaries and ghosts can lead to notches that can negatively affect the spectral bandwidth of recorded data, damage the temporal resolution of data and therefore compromise data's interpretability (van Borselen et al., 2013). Additionally, the low-frequency information of seismic data can be significant in seismic imaging and inversion and can be discounted by ghost notches (Farouki et al., 2010). In marine exploration, both the traditional shallow-water acquisition and the ocean-bottom acquisition can suffer ghost notch issues. A promising deghosting result can increase the bandwidth of data including the low frequency and enhance the resolution of subsequent imaging results (Wang and Peng, 2012). Aside from the traditional interest, deghosting is a necessary pre-requisite for the inverse-scattering-multiple-removal methods (Araújo, 1994; Weglein et al., 1997, 2003) and the for recent advanced Stolt extended Claerbout III imaging method (Weglein et al., 2011a,b; Weglein, 2016).

Green's theorem derived methods employ a model of the world that consists of a reference medium and sources (Weglein et al., 2003). The freedom of choosing a convenient reference medium means Green's theorem offers a flexible framework for deriving various useful algorithms. Basically, there are two applications: (1) wavefield separation (e.g., separating the reference wave and the reflection data, and deghosting) as a part of seismic pre-processing (Weglein et al., 2002), and (2) wavefield prediction to predict the source and receiver experiment at depth in the subsurface earth before applying the imaging condition, e.g., the Stolt extended Claerbout III imaging condition (Weglein et al., 2011a,b).

The distinct advantages of applying the wave separation method based on Green's theorem have been described by Weglein et al. (2002), Zhang (2007) and Mayhan and Weglein (2013) for marine cases. Wu and Weglein (2014, 2015a,b, 2016a) extended this method to on-shore application for the removal of ground roll and ghosts. In addition, the multi-component data acquired at the ocean bottom was deghosted by adopting this method (Wu and Weglein, 2016b). One unique advantage of this method is that it can accommodate an arbitrary acquisition surface, e.g., horizontal, slanted, or undulating. Preliminary synthetic tests show its capability to deghost marine data from a depth-variable cable (Zhang and Weglein, 2016; Lin and Weglein, 2016). However, it can only output deghosted data at a depth above the cable, but not on the receiver cable. The practical acquisition is always performed with a finite spatial sampling interval; therefore, a finite sum is utilized to numerically approximate the Green's theorem based spatial integral formula. As the output point comes close to the cable, the Green's function in the integrand becomes narrower, and shrinks below what a finite-sampled sum for the integral can adequately approximate (Weglein et al., 2013). Consequently, with the application of this method, you achieve deghosting a new data that is at a new (shallower) depth; you cannot deghost the actual measured data on the cable itself. Furthermore, for those acquisitions (onshore or ocean bottom) at a boundary (air/earth boundary or water/earth boundary), a deghosting result right on the measurement surface is necessary.

If the measurement surface is horizontal, Green's theorem spatial-integral algorithm can be implemented in the wavenumber domain, which is equivalent to the PVz method that is widely used today in industry. In so doing, there is no problem of deghosting measured data. However, that procedure is not available with a non-horizontal measurement surface; e.g., a feathered cable in marine, or an experiment either at the earth's surface or at the ocean bottom with complicated topography.

In order to achieve an effective deghosting result on the actual measurement surface to deghost the acquired data, while accommodating the cable with an arbitrary shape, we propose a two-step strategy. The first step employs the current Green's theorem based deghosting algorithm and outputs the result at a depth which is shallower than the actual cable. The second step employs the Green's theorem formulation of Stolt extended Claerbout III for one-way wave prediction; by doing so, the upgoing wave provided by the first step can be relocated from a shallower depth to the cable. We test the method with three datasets, one is acquired from a horizontal cable, and the other two are acquired from a non-horizontal cable. All examples show: (1) the effectiveness and the accuracy of this new deghosting method to accommodate an arbitrary cable and to separate the data right on the cable, and (2) its capability to deal with a rough sea situation and without requiring any information about the sea surface.

THEORY OF THE TWO-STEP DEGHOSTING METHOD

An experiment with receivers in the water column (Figure 1) is employed to describe how to deghost data right on the receiver cable that has an arbitrary shape. We first study the current Green's theorem deghosting method that is the first step of the new method. The second step is a Green's theorem based one-way wave prediction from the output of step one, above the cable to a point on the cable.

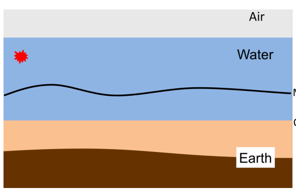

Figure 1: Schematic of a marine acquisition. Red star for an airgun, F.S. for the free surface, or air/water boundary; M.S. for measurement surface, or cable; O.B. for ocean bottom.

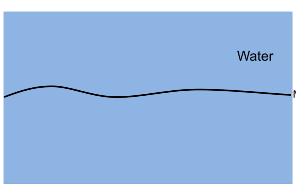

Figure 2: Homogeneous whole-space acoustic reference.

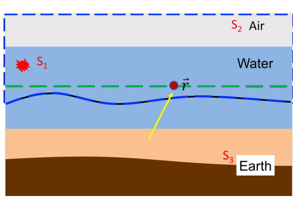

Figure 3: Deghosting at point $\vec{r}$ that is in the volume enclosed by boundaries highlighted in blue. E.S. for evaluation (output) surface.

Step 1: Predicting deghosted data at a depth above cable
We choose the reference medium to be a homogeneous water whole space (Figure 2), whose properties are the same as the medium (Figure 1) along the measurement surface. The differences between the actual and the reference medium are described as sources. As can be seen from Figure 3, S_1 is the energy source; S_2 is the air perturbation; and S_3 is the earth perturbation. We pick a point at $\vec{r}$ that is below S_1 and S_2 and above the cable. With a causal Green's function of the homogenous whole-space reference medium, the contribution from S_1 and S_2 are both downgoing at $\vec{r}$, including the direct wave and its ghost, and the receiver-side ghosts; whereas the contribution from S_3 is the only upgoing portion. Therefore, separating the contribution due to S_1 and S_2 from that due to S_3 is actually separating the total wavefield at $\vec{r}$ into upgoing and downgoing. Following Morse and Feshbach (1953), Weglein and Secrest (1990), and Weglein et al. (2002), selecting a closed semi-infinite surface (highlighted by the dashed blue line in Figure 3) that is bounded below by the measurement surface, making the Green's theorem based surface integral along the closed surface, and evaluating the result at $\vec{r}$ (it is inside the volume), provides the contribution by the source outside the volume (it is S_3) to the total wavefield at $\vec{r}$. For the location of $\vec{r}$ we have specified, the integral (Equation 1) outputs the entire upgoing portion of the total wavefield there. The receiver-side deghosting formula is (Zhang, 2007)

$$P^{up}(\vec{r},\omega) = \int_{m.s.} [P(\vec{r}',\omega)\nabla' G_0^+(\vec{r}',\vec{r},\omega) - G_0^+(\vec{r}',\vec{r},\omega)\nabla' P(\vec{r}',\omega)] \cdot \hat{n} d\vec{r}', \tag{1}$$

where ω is the temporal frequency; P is the measured total pressure data, and $\nabla' P$ is its gradient; $\hat{n}$ is a outward normal unit vector along the measurement surface; G_0^+ is the causal Green's function in the reference medium composed by a homogeneous whole-space of water; and the output result P^{up} is the predicted deghosted data at $\vec{r}$. The integral contribution is only along the measurement surface, because the contribution from other three infinite boundaries approaches zero by invoking the Sommerfeld radiation condition (Sommerfeld, 1949). Similarly, we can output the deghosted result at a series of points, along a fixed depth, called evaluation surface (E.S.) that is indicated by a green dashed line in Figure 3. That is the first step, using the acquired data at measurement surface to predict a receiver-side deghosted wave at a shallower depth in comparison with the cable.

It is worth mentioning two points in terms of Equation 1. First, it effectively accommodates the cable with an arbitrary shape. Second, it is performed without any prior information about the "sources"; i.e., neither the source wavelet, nor the passive sources are required. Specifically, the properties not assumed known include density, velocity, and the shape of the sources. Consequently, as well as accommodating a non-horizontal cable, it accommodates a sea surface with an arbitrary shape and without needing its shape and reflection there.

Step 2: Predicting deghosted data on the cable
To predict the deghosted/upgoing wave right on the cable, the Green's theorem based one-way wave prediction method is applied (Weglein et al., 2011a,b; Weglein, 2016). The basic concept can be described with a simple example, as plotted in Figure 4. There is a finite volume V with top boundary a and lower boundary b, where both pressure P and its normal derivative P_n are measured. Assuming a known property inside V, which

is used to define the reference medium, then Green's theorem integral along a and b can predict the wavefield at any point inside V; e.g., at point $\vec{r}$. If we further assume the medium inside V is homogeneous and P inside is one-way and moving up, and an anti-causal Green's function G_0^- is used, then the wave prediction can be achieved with the integral on the top boundary a only (refer to Weglein (2016) for detail).

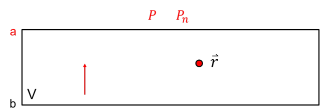

Figure 4: A cartoon of wavefield prediction inside volume V. a and b for top and lower boundaries, respectively.

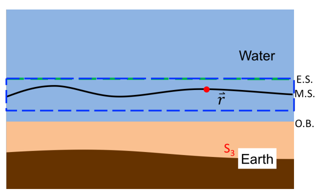

Figure 5: Predicting deghosted data at $\vec{r}$ on the cable.

In a marine experiment, by using the upgoing wave at E.S. as input which is the output result of step 1, the upgoing wave at the cable can be predicted. As is shown in Figure 5, a finite volume enclosed by dashed blue line is selected, whose top boundary is E.S.. The cable is inside the volume. We assume the wavefield has been deghosted and it is going up inside the volume. By selecting a water whole space as the reference medium, S_3 (the earth perturbation) is the only source that contributes to the wavefield inside the volume. Similar to Figure 4, applying the integral (Equation 2) in terms of the G_0^- along E.S., and outputting the result inside the volume, the upgoing wave (e.g., at point $\vec{r}$ on the cable) can be predicted. The algorithm is

$$P^{up}(\vec{r},\omega) = -\int_{e.s.} [P^{up}(\vec{r}',\omega)\partial_{z'}G_0^-(\vec{r}',\vec{r},\omega) - G_0^-(\vec{r}',\vec{r},\omega)\partial_{z'}P^{up}(\vec{r}',\omega)]dx', \quad (2)$$

where $P^{up}(\vec{r}')$ and $P^{up}(\vec{r})$ are deghosted data at the evaluation surface and at the cable, respectively. This step deghosts data on the exact acquisition surface. There are several options to handle $\partial_{z'}P^{up}$. One is making $G_0^-(\vec{r}',\vec{r},\omega) = 0$ when $\vec{r}'$ is on the E.S. to get rid of need of $\partial_{z'}P^{up}$.

NUMERICAL TESTS

Three tests are conducted to examine the effectiveness of this two-step deghosting method. The first one is with a flat cable, while the last two are with the undulating cables. The first two are using flat sea surfaces, while the last one is for an undulating sea surface.

Example 1: Flat sea surface and flat cable
Figure 6 shows that the cable is at a constant depth of 40 m, where data is generated. Though the PVz method can deghost acquired data when the cable is horizontal, this new two-step method can achieve that as well.

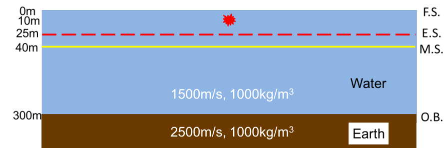

Figure 6: A model with a flat free surface and flat cable at 40 m. Red star for source at 10 m; E.S. is at 25 m.

Figure 7(a) is the total pressure data. There are three events in the shot gather: the direct wave, the primary and the receiver-side ghost of the primary. For all these events, only primary is upgoing, which is expected to be separated from others. Please notice that each event comes along with their source-side ghosts. We don't go into the removal of source-side ghosts (the same for the following two examples), but simply treat them as part those three events.

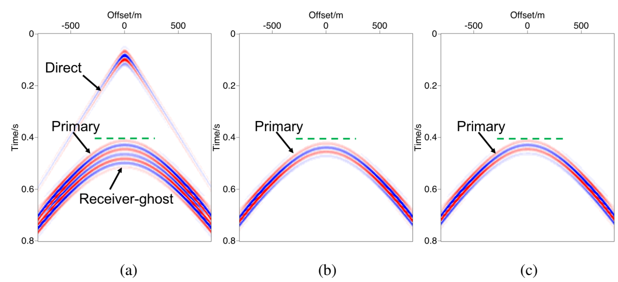

Figure 7: Deghosting results with a flat free surface and a flat cable. (a) the total pressure; (b) deghosted data at E.S.; (c) deghosted data at cable. Green lines locate the arrival time of data's primary.

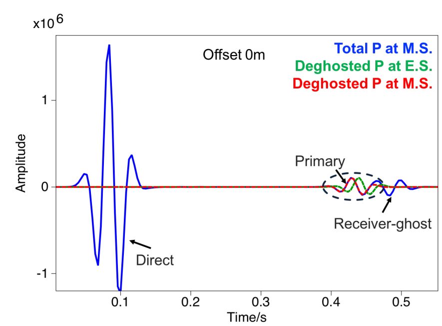

Figure 8: Trace comparison at offset 0 m of Figure 7. Blue for total wave; green for deghosted result from step 1; and red for deghosted result from step 2.

The first step predicts the deghosted data at a shallower depth (25 m), as shown in Figure 7(b), where only primary exists, the direct wave and the receiver ghost are extinguished. In the second step, we downward continue the intermediate result from step 1 to cable's depth of 40 m (in Figure 7(c)). The green dashed lines in the three figures locate the arrival time of the primary in the acquired data at offset 0m. It tells us that the result from step 1 is arriving later, while that from step 2 is with the same time as the acquired primary. The single trace plot (in Figure 8) at offset 0 m illustrates the detail. In the black circle, the predicted primary from step 2 (marked with red) matches well with the primary (marked with blue) of data, whereas the result from step 1 (in green) arrives later.

Example 2: Flat sea surface and undulating cable

We generate data (Figure 10(a)) on a cable with a sine shape

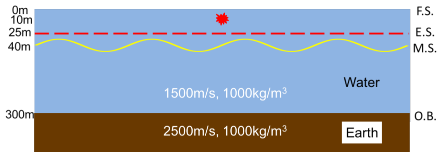

Figure 9: A model with a flat free surface and an undulating cable centered at 40 m.

(centered at 40 m, and with an amplitude of 10 m). With a non-horizontal cable, the events in the data are not symmetric along the two sides of the source. Similar to Example 1, we first predict the upgoing wave at a depth of 25 m (Figure 10(b)). The intermediate output shot gather is symmetric for a horizontal reflector and horizontal output depth. Substituting it into step 2 predicts the deghosted data at the cable (Figure 10(c)). The deghosted portion of acquired data is finally achieved with satisfactory result. Instead of predicting a new deghosted dataset consisting of an up wave, which is what the current Green's theorem method (Equation 1) can deliver, this new two-step method predicts the deghosted data at the same location as the recorded data. Noting that the PVz method is not applicable to this data.

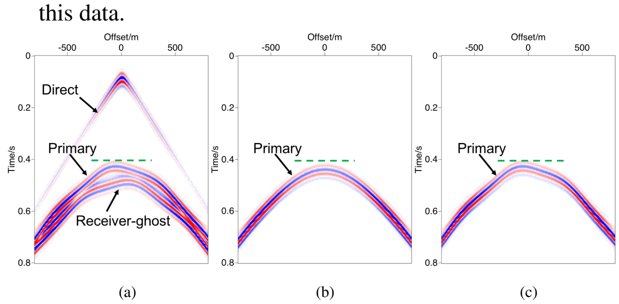

Figure 10: Deghosting with a flat free surface and an undulating cable. (a) the total pressure; (b) deghosted data at E.S.; (c) deghosted data at cable.

Example 3: Undulating sea surface and undulating cable

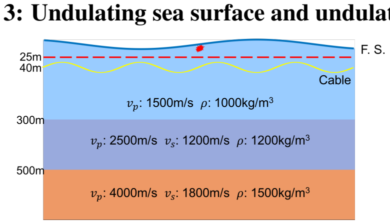

Figure 11: A model with an undulating free surface centered at 0 m and an undulating cable centered at 40 m.

In this example, we consider a rough sea case and a non-horizontal cable (Figure 11). The earth below the ocean bottom is elastic. We generate data (Figure 12(a)) with all the information about the sea surface, the cable and the medium properties. However, only the cable's shape and the water property are needed for the deghosting calculation. The method doesn't need to know the shape of the ocean top, the ocean bottom, and the properties of the elastic earth. We predict the upgoing wave at a depth of 25 m in step 1 (Figure 12(b)). Substituting the upwave at 25 m into Green's theorem one-way wave-prediction algorithm, the deghosted portion of the actual acquired data (Figure 12(c)) can be achieved. A trace plot at offset 0 m (Figure 13) further shows detail. This example illustrates that the method is independent of earth model type.

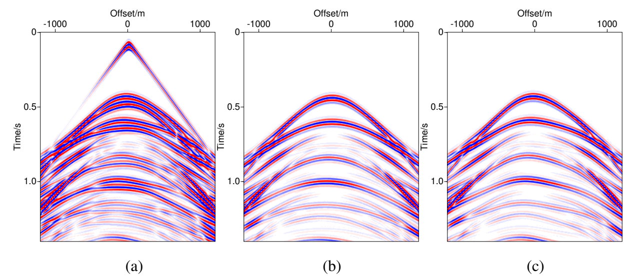

Figure 12: Deghosting result with an undulating sea surface and an undulating cable. (a) the total pressure; (b) deghosted data at E.S.; (c) deghosted data at cable.

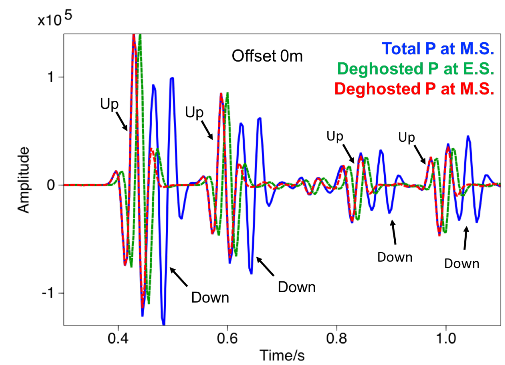

Figure 13: Trace comparison at offset 0 m of Figure 12.

CONCLUSION

The ability to effectively remove ghosts has a positive impact on subsequent processing and interpretation, that can support effective drilling decisions. We provide a two-step strategy that combines the usages of Green's theorem based wave separation and wave prediction algorithms. The new method can successfully deghost actual acquired data at the acquisition depth, without assuming a horizontal measurement surface. As a useful tool, this study extends the capability of current Green's theorem wave separation method towards seismic onshore and ocean-bottom exploration with complicated topographies.

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EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2017 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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